CDF Note 8151

Measurement of the Top Quark Mass using the Matrix Element Analysis Technique in the Lepton + Jets Channel with In-Situ $W \to jj$ Calibration

The CDF Collaboration
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We present a measurement of the mass of the top quark from $p\bar{p}$ collisions at 1.96 TeV from CDF Run II at the Fermilab Tevatron. We use events from $p\bar{p}$ to $t\bar{t}$ in the lepton+jets channel and require at least one jet to be identified as coming from a b-quark. The jet energy scale (JES) systematic uncertainty is convoluted with the statistical error using an in-situ measurement of the hadronic W boson mass. A likelihood is calculated for each event using a leading-order $t\bar{t}$ and W+jets differential cross-section and parameterized parton showering. The final measured top quark mass and JES systematic is extracted from a joint likelihood of the product of the individual event likelihoods. From the 118 events observed with 680 pb^{-1} of data we extract a top quark mass of 174.09 $\pm$ 2.54 (stat+JES) $\pm$ 1.35 (syst) $\mathrm{GeV/c}^2$.

Preliminary Results for Winter 2006 Conferences

INTRODUCTION

The top quark mass is a fundamental parameter in the Standard Model. Along with the mass of the W boson, the top quark mass provides the best indication for the value of the Higgs boson mass. This note describes a preliminary measurement of the top quark mass in $\bar{p}p$ collisions at $\sqrt{s} = 1.96$ TeV with the CDF detector at the Fermilab Tevatron using 680 pb^{-1} of data collected from March-2002 to September-2005. The CDF detector is described in detail in [1].

At the Tevatron, top quarks are mainly produced in pairs, either $q\bar{q}$ or gg fusion. Due to its short lifetime, the top quark is detected via its decay products. It decays via the weak interaction to a b quark and a W boson. This is followed by the W decaying into two quarks or a lepton and a neutrino. The final state of the $t\bar{t}$ system has different topological classifications that depend on the decay of the W. The results presented here use the lepton+jets channel, and corresponds to one W decaying leptonically (into a electron or muon), while the other W decays hadronically. The branching ratio of this channel is 30%.

This analysis uses a matrix-element analysis technique in order to maximize the amount of information extracted from the limited amount of $t\bar{t}$ events [2]. This technique uses all measured quantities to characterize each event. All the different jet-parton assignments, as well as all the possible momenta of the neutrino, are taken into account. Therefore, each event enters in this analysis with a different weight do that better measured events contribute more to the final result. Moreover, the jet energy scale (JES) systematic uncertainty is also measured in these events constraining the mass of the W to 80.4 GeV/c^2. The final top quark mass is extracted from a 2-dimensional fit between the top quark mass and the JES.

The web page for this analysis is `http://www-cdf.fnal.gov/physics/new/top/2006/mass/ljets_meat/`

DATA SAMPLE & EVENT SELECTION

Events from the lepton+jets decay channel are selected requiring a single, high-transverse energy, well-isolated lepton, large missing transverse energy from the neutrino, and exactly four central jets with high transverse energy (two from the b quarks and two from the hadronic W). Of these jets, we require at least one to be identified as originating from a b quark by using a secondary vertex tag. The secondary vertex tag identifies tracks associated with the jet originating from a vertex displaced from the primary vertex. We further require the axis between missing transverse energy and leading jet, $\Delta\phi$, not be collinear for the lowest values of missing transverse energy passing our selection. This requirement removes a large fraction of the non-W background while retaining most of the signal. Table I shows the selection criteria used in this analysis.

We use `HERWIG` MC events to determine the mean acceptance and parameterize it with an analytical function [4]. Figure 1 shows the mean acceptance for $t\bar{t}$ events obtained from `HERWIG` MC samples.

TABLE I: Summary of Basic Event Selection

lepton	E_T >20 GeV (electron), p_T >20 GeV/c (μ)
jets	Raw $E_T > 15$ GeV, $\eta < 2.0$
Missing E_T	Missing $E_T > 20$ GeV
b-tag	≥ 1 coming from a secondary vertex
QCD veto	$\Delta\phi < 0.5$ or $\Delta\phi > 2.5$ (Missing $E_T < 30$ GeV)

METHOD

We create a likelihood for each event by combining a signal probability with a background probability. This method has been developed by the D0 collaboration during Run I [2]. This likelihood is minimized for three variables: the top quark mass, JES, and the fraction of events consistent with our signal hypothesis.

$$\begin{aligned}\mathcal{L}(\vec{x}; M_{top}, JES, C_s) &= e^{-NC_s\langle Acc_{t\bar{t}}(M_{top},JES)\rangle} e^{-N(1-C_s)\langle Acc_{W+\mathrm{jets}}(JES)\rangle} \\ &\prod_{i=1}^{N} [C_s P_{t\bar{t}}(\vec{x}; M_{top}, JES) + (1-C_s) P_{W+\mathrm{jets}}(\vec{x}; JES)] \end{aligned} \quad (1)$$

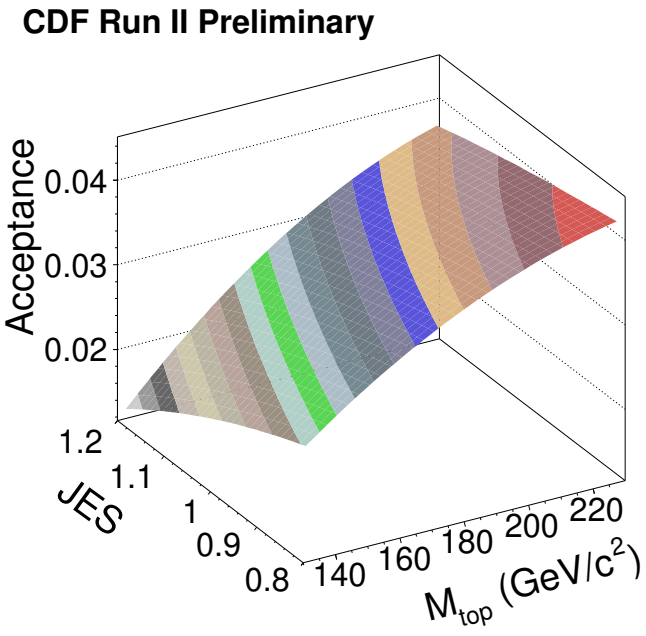

FIG. 1: Mean acceptance of $t\bar{t}$ events for as function of the input top quark mass and JES.

where $\vec{x}$ are the measured quantities. The extracted top quark mass is obtained from a 2-dimensional fit in top quark mass and JES where C_s is minimized via MINUIT. The signal probability measures how well an event describes the $t\bar{t}$ to lepton+jets hypothesis, and the background probability indicates how well an event describes the largest contributing background hypothesis, leptonically decaying W + extra jets. The background probability uses the different matrix-elements of W+4 jets of the VECBOS Monte Carlo routine. The signal probability only uses the leading order matrix-element of the $q\bar{q} \to t\bar{t}$ process. These probabilities are constructed by integrating over the appropriate parton-level differential cross-section, and parton distribution functions and include the detector resolution effects.

$$P(\vec{x}) = \frac{1}{\sigma} \int d\sigma(\vec{y})\ f(\tilde{q}_1) f(\tilde{q}_2)\ W(\vec{x}, \vec{y})\ d\tilde{q}_1 d\tilde{q}_2 \tag{2}$$

The angles of the measured jets and leptons, as well as the momentum of the leptons are considered to be well known. The jet energy resolution is parametrized from the Monte Carlo. We model the resolution effects by a transfer function - the mapping between jet energies and parton energies ($W_{jet}(\vec{x}, \vec{y})$). Signal and background probabilities are evaluated for all the possible permutations among jets and partons. Moreover, the signal probability also considers all the kinematically possible longitudinal momenta of the unmeasured neutrino and the different values of the p_T of the $t\bar{t}$ system.

We are sensitive to the JES through the hadronically decaying W boson. We constrain the invariant mass of 2 untagged jets to be the value of the world average of the W mass, 80.4 GeV/c^2. We define JES to be the multiplicative scale factor applied to the energies of the two untagged jets inside the transfer function ($W_{jet}(\vec{x}, \vec{y})$).

$$E_{jet} = E_{jet}^{MC} / JES \tag{3}$$

We assume the JES determined for the W-jets also applies to b jets and assign an additional systematic uncertainty for difference between this JES and the b jets energy scale.

VALIDATION OF THE METHOD

Many tests of the performance of the analysis are done using simulated (Monte Carlo) samples generated with different top quark masses. The extracted top quark mass is found to be unbiased with respect to the top quark mass

for which the input events were generated. Figure 2 shows these results. Similar tests are performed for the JES output. In this case, as shown by Fig. 3, we find a slope different than one. The origin of this bias is understood to come from a non-perfect parametrization of the transfer function. This bias does not affect our measurement of the top quark mass, but we correct for it so that we properly interpret the output of the JES. Figure 4 shows that this average correction can be applied to different η and p_T regions and we can recover the reconstructed W mass as a function of η and p_T.

A linear relationship between the output value C_s (determined in the likelihood minimization) and the signal fraction is found which is parameterized by a mapping function. Fig. 5 shows this mapping function which depends on the underlying assumptions made in constructing our signal and background probability. The evaluation of the errors on the extracted top quark mass is tested by calculating pull distributions. We find that the RMS of the pull distribution to be 1.06 on average. Thus, the statistical plus JES systematic error is inflated by 6% after the top quark mass measurement is made. Figure 7 shows the width of the pull distribution as a function of the input top quark mass.

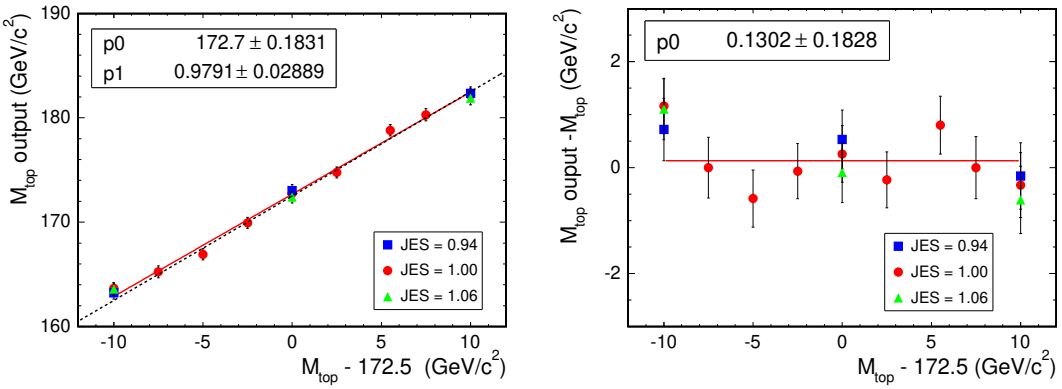

FIG. 2: Output top quark mass as determined by the likelihood as a function of the top quark mass of the input MC events. We see no bias or offset. In the left plot, we expect p0 = 0. In the right plot, we expect p0 = 172.5 and p1 = 1.

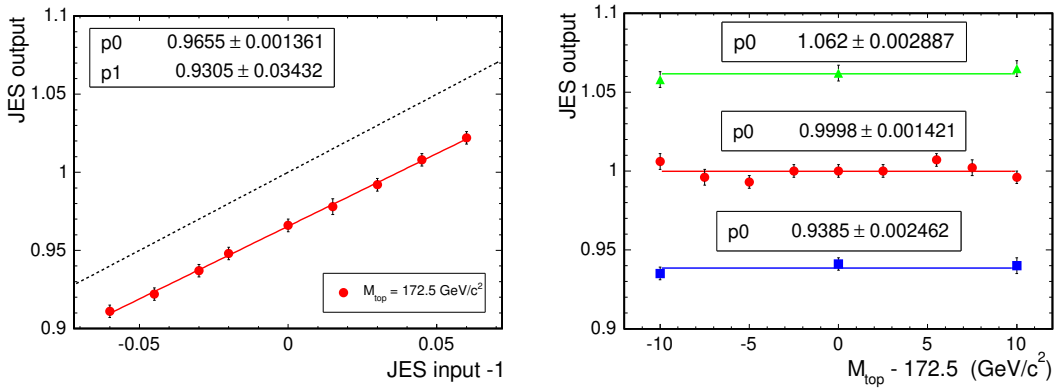

FIG. 3: Left: output JES as determined by the likelihood vs. JES of input MC events before mapping, we expect p0 = 1 and p1 = 1. Right: output JES after mapping as a function of the input top quark mass for different JES input samples of 1.06, 1, and 0.94.

SYSTEMATIC UNCERTAINTIES

Table II lists the systematic uncertainties estimated from various Monte Carlo samples. To first order, we fit the JES systematic in our likelihood, but we also apply a residual systematic to cover any higher order effects such as variations in the expected η or p_T distribution. This higher order effect is estimated by fluctuating the jet energies up and down by one sigma as defined by the CDF Jet Energy and Resolution group [5] . The generator systematic

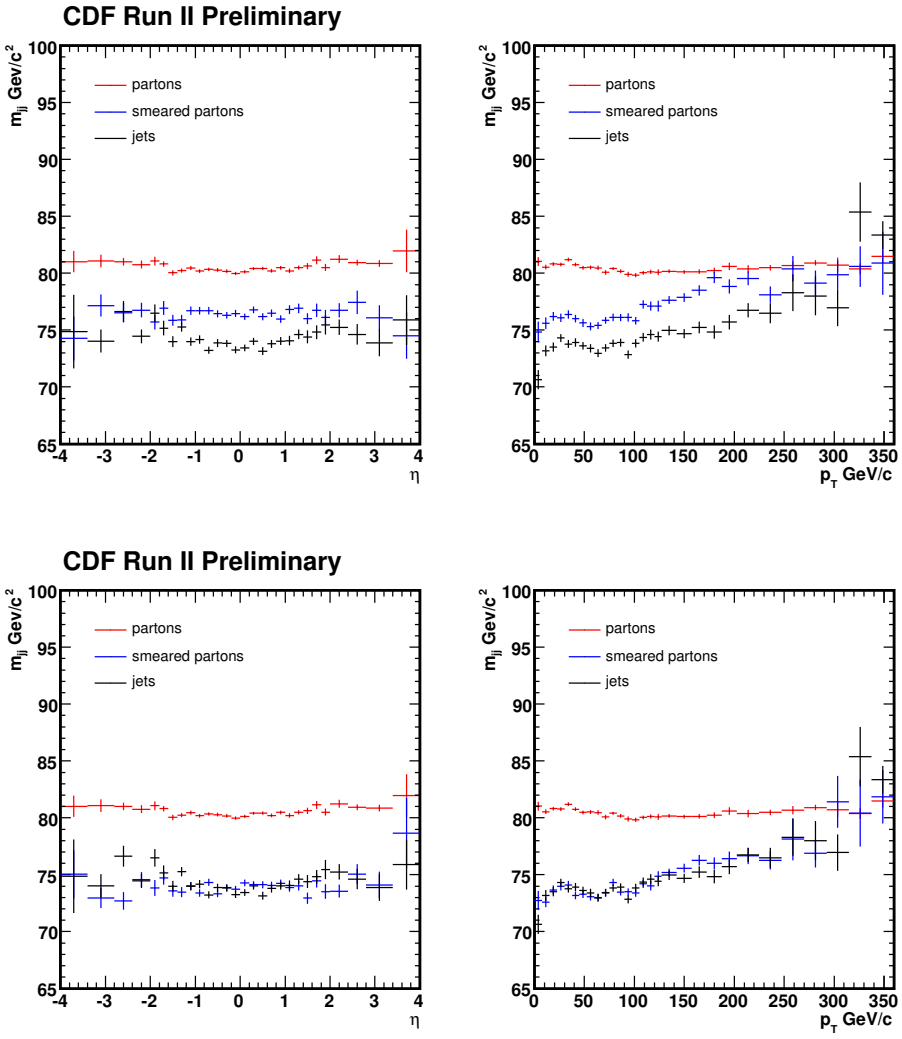

FIG. 4: Comparison of partons from $t\bar{t}$ events (red), jets (black) and partons smeared with transfer functions (blue). We expect the smeared partons to match closely the jets. Plot on left is before application of JES mapping and plot on right is after. This demonstrates the JES mapping corrects problems with the transfer functions and we observe no bias as a function of η or p_T.

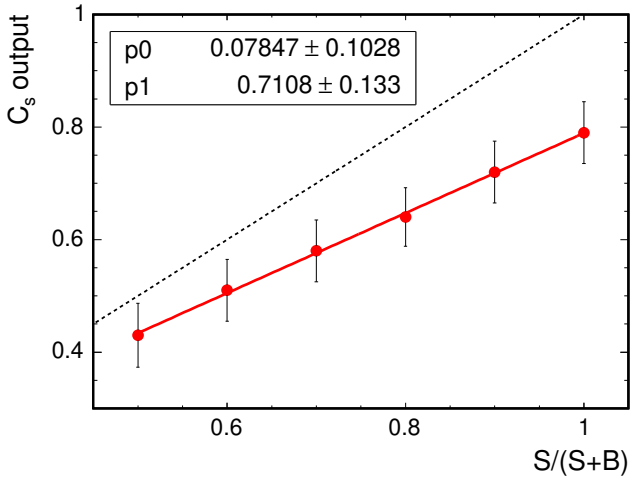

FIG. 5: C_s vs. input signal fraction.

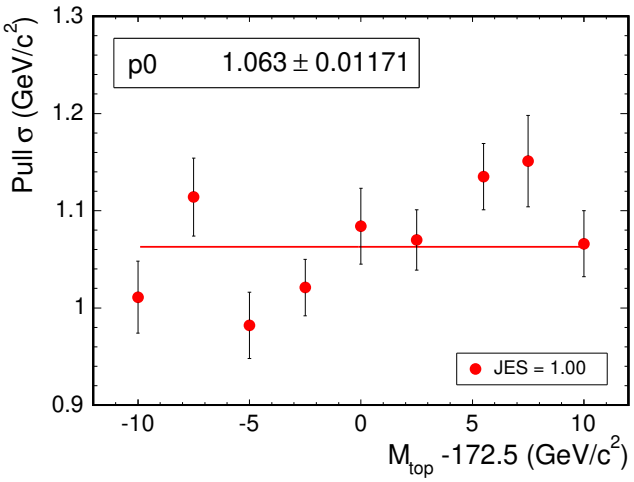

FIG. 6: Pull RMS of distribution of pseudo-experiment results in coverage tests as a function of the top quark mass in input MC events. We inflate our error by 6% to obtain proper coverage. The pseudo-experiments were constructed with 0.83 signal events and 118 total number of events.

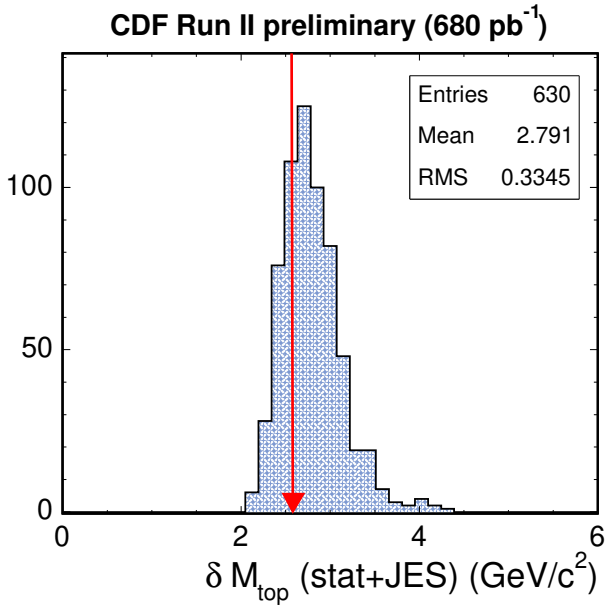

FIG. 7: Statistical plus JES systematic expected error after inflation by 6%. These pseudo-experiments were done with a total of 118 events, a signal fraction of 83% and an input top quark mass of 175 GeV/c^2. The red line indicates our measured error in the data after inflation.

uncertainty takes into account differences in the fragmentation and showering comparing the different output masses obtained with two different Monte Carlo models `PYTHIA` and `HERWIG`. Possible biases originating from differences in the amount of radiation between the data and the Monte Carlo are estimated using `PYTHIA` samples generated with a different amount of initial and final state radiation. The uncertainty on the parton distribution functions (PDF) is evaluated as the sum in quadrature of the difference between MRST and CTEQ parton distribution functions, MRST with $\Lambda_{QCD} = 228$ MeV and $\Lambda_{QCD} = 300$ MeV, and the variation in the 20 CTEQ eigenvectors. Systematic effects from the dependence of the b-tagging with p_T are evaluated changing the dependence by one sigma. The background composition and modeling systematic error is the sum in quadrature of three effects: the largest variation when fluctuating the contribution of the each individual background by 100%, the change in the signal fraction by plus or minus 10%, and the largest variation due to changing the Q^2 scale in W + jet production. To understand the effects of limited statistics in the background sample, we divide the smallest sample in half (non-W background)

and compare the results using each half separately. This experiment is repeated several times dividing the sample in different ways and we take one-half the RMS of the distribution of the differences as a systematic error.

TABLE II: Summary of Systematic uncertainties

Source of systematic uncertainty	Magnitude (GeV/c^2)
Residual JES	0.42
b-JES	0.60
Generator	0.19
ISR	0.72
FSR	0.76
b-tag E_T dependence	0.31
Background composition	0.21
PDF	0.12
Monte Carlo statistics	0.04
Total	1.35

RESULTS

The output likelihood is a simultaneous fit to three parameters: top quark mass, JES and C_s. In the extraction of the top quark mass we do not use any prior knowledge of the JES or C_s. The data used in this analysis corresponds to the period of March 2002 to September 2005 with a total integrated luminosity of 680 pb^{-1} where 118 events pass our event selection. Figure 8 shows the fit to this data and different ΔlnL contours.

Figure 9 and 10 show the fits to the data for different divisions of the data depending on the lepton flavor and the amount of b-tagging in the sample. Although, the contours in ΔlnL were not adjusted by 6% from the RMS of the pull distributions the reported top quark mass uncertainty is inflated. Only statistical uncertainties are reported on JES and C_s. The uncertainty on JES error is inflated by the RMS of the pull distribution (10%). Table III show the results for these data.

TABLE III: Results using 680 pb^{-1}

Data sample	M_{top} (GeV/c^2)	JES	C_s
All data	174.09 ± 2.54 (stat+JES)	1.02 ± 0.02 (stat)	0.67 ± 0.06 (stat)
1 b-tag	169.52 ± 3.44 (stat+JES)	1.00 ± 0.03 (stat)	0.62 ± 0.07 (stat)
2 b-tags	180.29 ± 3.97 (stat+JES)	1.05 ± 0.03 (stat)	0.78 ± 0.09 (stat)
e+jets	172.55 ± 3.33 (stat+JES)	0.98 ± 0.02 (stat)	0.64 ± 0.07 (stat)
μ+jets	177.11 ± 3.91 (stat+JES)	1.08 ± 0.03 (stat)	0.72 ± 0.08 (stat)

CONCLUSIONS

We presented a measurement of the top quark mass in the lepton+jets channel using 680 pb^{-1} of data collected by the CDF experiment. We used a matrix-element analysis approach requiring at least one or more b jets with an in-situ measurement of the JES uncertainty. Our measurement is

$$M_{top} = 174.09 \pm 2.54(\text{stat} + \text{JES}) \pm 1.35(\text{syst})\text{GeV/c}^2 \quad (4)$$

We plan to update this analysis with 1 fb^{-1} of data.

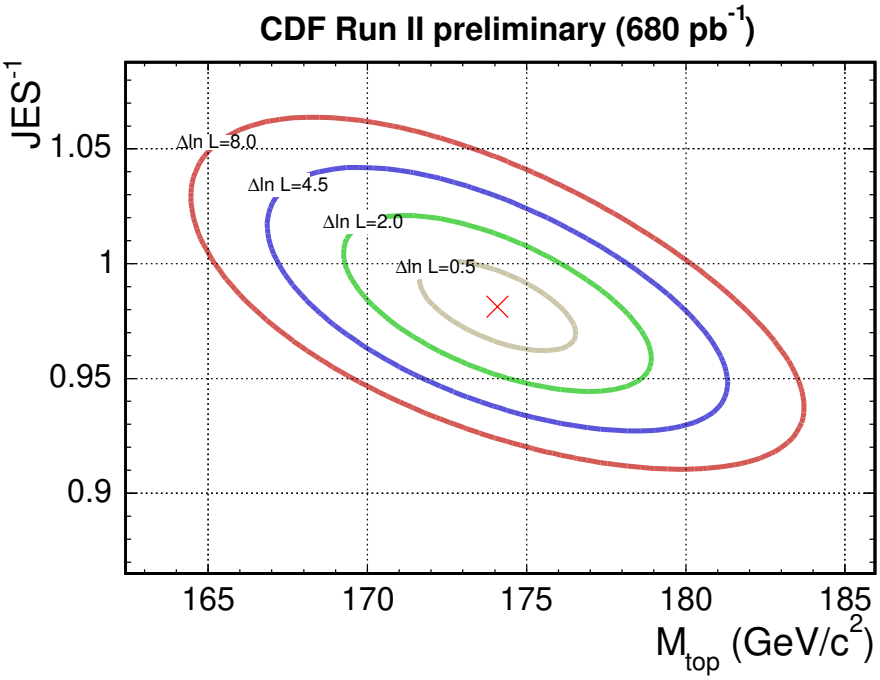

FIG. 8: Top quark mass measurement using 680 pb^{-1}.

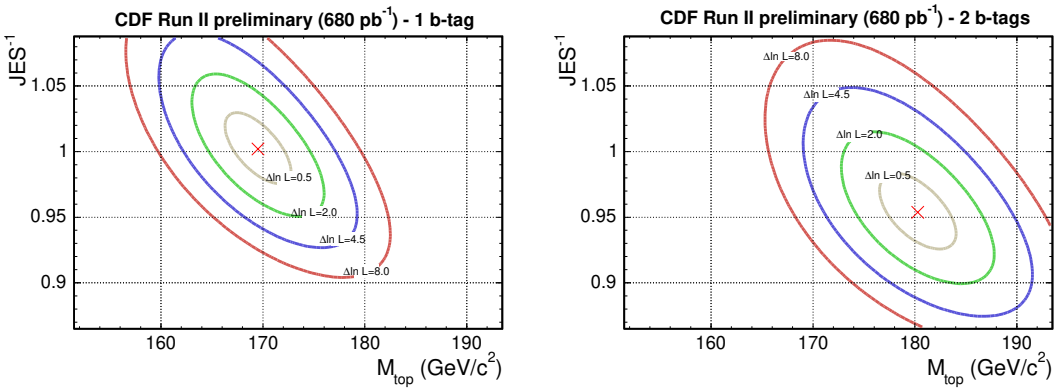

FIG. 9: Top quark mass measurement using 680 pb^{-1} for events with one-tag (left) and 2-tags (right).

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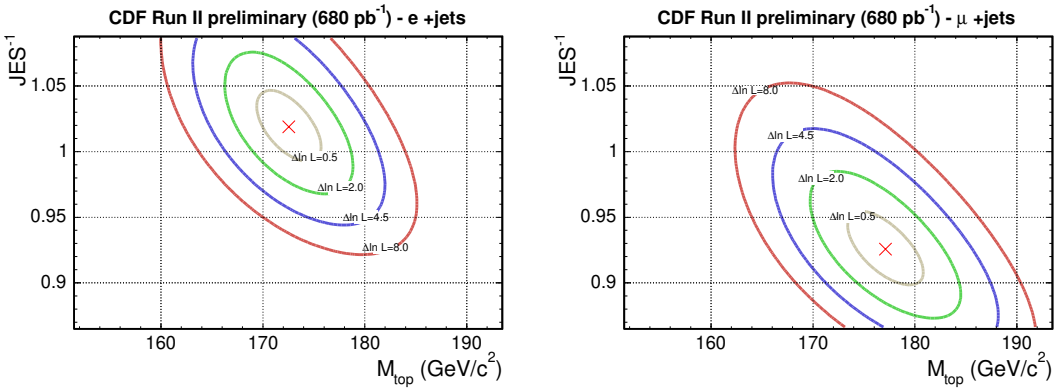

FIG. 10: Top quark mass measurement using 680 pb^{-1} for events with electrons (left) and muons (right).

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