



# memorandum

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## SUBJECT: Superconducting Cavity Design for SNS

This memo reports a method for optimizing elliptical cavity shapes for potential use in a superconducting part of the SNS linac. The work was done at the request of Tom Wangler, who has been laying out a two-cavity-type superconducting linac.

### **Requirements**

It is not the intention of the memo to justify all the parameter choices. However, I will, where appropriate, mention what kind of tradeoffs are involved.

In the present superconducting concept for SNS, the linac uses room-temperature structures up to a beam energy of 194 MeV. This energy corresponds to the end of a coupled-cavity linac rf module with an exit beam energy near ~200 MeV, below which elliptical niobium cavities are not as attractive because they are so short compared to their diameter. From previous design studies, we have already decided on two superconducting structure types. The first section uses a cavity with geometrical  $\beta = 0.61$  to span the particle velocity range from  $\beta = 0.560$  to 0.688 (194 MeV to 355 MeV). The other section uses a cavity with geometrical  $\beta = 0.76$  for the range from  $\beta = 0.688$  to 0.875 (355 MeV to 1 GeV).

Each cavity structure consists of eight cells operating in the  $\pi$  mode. One would like a large number of cells per cavity to reduce the number of rf drives and the number of control systems. However, for a given cell-to-cell coupling, increasing the number of cells per cavity degrades the field stability. If we can achieve ~2% cell-to-cell coupling in an 8-cell cavity, the field errors will be tolerable. An 8-cell cavity has six identical internal cells plus the two end cells. The inside wall of an end cell is the same as that of an internal cell. The outside walls will differ in shape for proper tuning. In addition, the bore tube on the end of the cavity is larger than the internal bore radius to help achieve sufficient coupling to the rf power supply. Previous work by Frank may limit the performance of a superconducting cavity. In regions of high magnetic field, the surface may become normal conducting causing a quench, and in regions of high electric field, electron emission may occur. A discussion of the details of these effects is beyond the scope of this memo. For guidance in the design process, we apply the experience to date in achieving high fields. Henri Safa suggested a procedure that we have adopted for deciding whether a cavity will be limited by maximum E or B fields. First, define the maximum magnetic and electric fields ( $B_{MA}$  and  $E_{MA}$ ) that have been achieved experimentally. (Ideally, these fields would correspond to the same cavity materials and fabrication techniques planned for the project of interest. Thus, the definitions are based upon subjective judgement to some extent.) Then, for a particular cavity design, compute the ratio  $B_{Max}/E_{Max}$  where  $B_{Max}$  is the peak magnetic field and  $E_{Max}$  is the peak electric field in the cavity at a given excitation level. If  $B_{Max}/E_{Max} > B_{MA}/E_{MA}$ , then the cavity is more likely to be limited by quenching and if  $B_{Max}/E_{Max} < B_{MA}/E_{MA}$  the cavity is more likely to be limited by quenching and if  $B_{Max}/E_{Max} < B_{MA}/E_{MA}$  the cavity is more likely to be limited by field emission. After discussions with Safa, Tom Wangler has chosen the following ratio for the present design effort:

$$\frac{B_{_{MA}}}{E_{_{MA}}} = \frac{120\,\text{mT}}{70\,\text{MV}/\text{m}} \approx 1.71 \frac{\text{mT}}{(\text{MV}/\text{m})} \,. \label{eq:massed_matrix}$$

There are some mechanical constraints. For rigidity, we would like a wall angle in the range 6 to 9 degrees. Dale Schrage suggests that no radius of curvature in the cavity be smaller than about twice the material thickness for ease of forming the cavity. Niobium thickness values of 3 mm and 5 mm are being considered. However, some manufacturing techniques (e.g., spinning) would likely not have this requirement on the radii of curvature.

### **Cavity Design Features**

We use the program ELLFISH in the Poisson Superfish code distribution to design the cavity shape and compute surface fields and rf power losses. Figure 1 shows the basic outline for half of an internal cell in a multicell elliptical cavity. The term "elliptical cavity" has become the common jargon used to refer to these cavities. As shown in the figure, a cross section of the right half of a cavity, the actual shape consists of a circular arc of radius  $R_c$  at the outer diameter, a sloping straight section at angle  $\alpha_w$  from the vertical, and a segment of an ellipse of height 2b and width 2a near the bore. The lower left corner is the center of the cell. The left edge is the cavity midplane and the bottom edge is the beam axis. With appropriate boundary conditions, the half cavity is sufficient to model the interior cavities of a superconducting multicell cavity. The boundary conditions are Neumann at the left edge and Dirichlet at the right edge.



Figure 1. The right half of an elliptical cavity. The left edge is a symmetry plane with a

side. The bore tube of radius  $R_{b,R}$  extends a distance  $\delta T_R$  beyond the nominal right edge of the cavity. If necessary, the geometry can include a second bore tube extension of length  $\delta T_2$  and radius  $R_{b,2}$ . The cavity length is  $L = \beta \lambda/2$ , so the full length of the problem geometry is  $L + \delta T_R + \delta T_2$ . The code uses a Dirichlet boundary condition on the left and right edges of the problem geometry. On the right side, the bore tube is long enough so the fields fall to negligible values at the edge of the geometry. Neither Dirichlet nor Neumann boundaries would be appropriate at this edge.



Figure 2. Full-cell elliptical cavity. The shape of the cavity wall on left and right can be different. The ELLFISH program can tune full cavities by adjusting the right-side radius or the right-side wall angle.

The only common parameters on left and right sides are the cavity diameter D and the half length L/2. As shown in Figure 2, all other parameters, including the ellipse aspect ratios, can have different values on the left and right sides. Even if  $(a/b)_L = (a/b)_R$ , the actual size of the two ellipse segments may be different depending on choices for the other geometrical parameters. The wall angle is  $\alpha_L$  on the left and  $\alpha_R$  on the right; the outer corner radius is  $R_L$  on the left and  $R_R$  on the

Program ELLFISH can tune the full-cell cavity by adjusting either the outer diameter D, the rightside corner radius  $R_R$ , or the right-side wall angle  $\alpha_R$ . The usual procedure is to allow ELLFISH to tune the interior half cell by adjusting the cavity diameter for chosen settings of the other geometrical parameters. For the end cell, the designer fixes the geometry of the left side at the values found for interior cells and tunes the cell by adjusting the shape of the right-side cavity wall. The recommended method is to tune the full cavity by varying the right-side outer corner radius  $R_R$ .

### **Optimization Studies**

For each design particle velocity ( $\beta = 0.61$  and  $\beta = 0.76$ ) we tuned a series of cavities with different choices for some of the geometrical parameters. The particle velocity fixes the length since L =  $\beta\lambda/2$ . At 805 MHz, the  $\beta = 0.61$  cavity has a length of 11.3586 cm and the  $\beta = 0.76$  cavity has a length of 14.1517 cm. The cell-to-cell coupling depends mainly on the cavity bore radius, but it also depends on the shape of elliptical segment between cavities. We investigated a few different bore radii and found adequate coupling with an internal bore radius R<sub>b.L</sub> = 5.0 cm.

Optimizing the cavity to balance the peak surface fields involved varying the ellipse aspect ratio for several choices of the cavity corner radius  $R_c$  and the wall angle  $\alpha_w$ . Figure 3 shows plots of the peak field ratio  $B_{Max}/E_{Max}$  versus the ellipse aspect ratio for three choices of  $R_c$ . The low- $\beta$ cavity has  $\alpha_w = 7$  degrees and the high- $\beta$  cavity has  $\alpha_w = 9$  degrees. The circled points are candidate cavities having the desired ratio  $B_{Max}/E_{Max} \approx 1.71$  mT/(MV/m). At  $\beta = 0.61$  nearly all of the points lie above this value. Cavities with aspect ratios smaller than ~0.35 become difficult to fabricate so we look to larger values of a/b. There is little room to increase  $R_c$  much beyond 3.2 cm, so we really have only one choice (a/b = 1.5) for the low- $\beta$  cavity.

For the high- $\beta$  cavity, Figure 3 shows three candidates with  $B_{Max}/E_{Max} \approx 1.71 \text{ mT/(MV/m)}$ . Obviously, it is possible to generate more cavities with this field ratio for any radius  $R_c$  between 4.5 and 5 cm. In order to choose among these cavities, we list in Table 1 the actual values of the peak fields and the cell-to-cell coupling for the three candidate cavities from Figure 3. The table shows that cavities 1 and 2 have substantially higher coupling than cavity 3 has. In terms of cavity fields, which have been normalized to  $E_0T = 8.5 \text{ MV/m}$ , cavity 2 has a slight advantage over cavity1 and a little larger advantage over cavity 3. Since there is little room to increase  $R_c$  much beyond 5.0 cm, we choose cavity 2 (a/b = 0.9) for the high- $\beta$  cavity.



Figure 3. Ratio of maximum magnetic and electric fields versus ellipse aspect ratio for

Parameter	Cavity 1	Cavity 2	Cavity 3
a/b	0.6	0.9	1.4
R <sub>c</sub> (cm)	5.0	5.0	4.5
H <sub>Max</sub> (kA/m)	32.16	31.95	33.26
B <sub>Max</sub> (mT)	40.41	40.15	41.80
E <sub>Max</sub> (MV/m)	23.45	23.32	24.17
B <sub>Max</sub> /E <sub>Max</sub>	1.72	1.72	1.73
Coupling (%)	2.18	2.09	1.54

Table 1. Candidate cavities for  $\mathbf{b} = 0.76$ .

## **Design Parameters for SNS**

Having chosen the basic shape of the internal cells, we proceeded to design the end cavity for the 8-cell structure. As illustrated by Figure 2, which shows the end cavity on the right-hand side of the structure, we make the end cavity's inner-wall shape identical to the shape of the internal cells. If we did the same for the outer wall, the frequency of the end cavity would be a few MHz lower than the internal cells because of the electric fields penetrating into the bore tube. We raise the frequency of the end cell by reducing the corner radius R Table 2 lists cavity parameters for both design velocities. Refer to Figures 1 and 2 for definitions of symbols listed for the geometric parameters.

After tuning the internal cell and the end cell to 805 MHz, we construct the geometry for half of the 8-cell cavity and run SUPERFISH to compute the frequencies and axial field distributions for all eight modes in the TM<sub>010</sub> pass band. Figure 4 is a plot of field contours computed by SUPERFISH for the 805-MHz mode of each cavityFigure 5 shows the eight axial field distributions in the  $\beta = 0.61$  cavity. The patterns are nearly identical for th $\beta = 0.76$  cavity. The field plots correspond to the right half of the cavity. Each mode in the TM<sub>0</sub> passband is labeled by its phase coordinate on the dispersion curve (seFigure 6), which was fitted by the program DISPER. Frequencies of all the modes appear inTable 3. Although it is common practice to refer to the highest frequency mode as the *f*t mode," this term properly applies only to an infinitely long structure. The curves plotted inFigure 6 correspond to a finite number of cavities (eight) terminated by full cells. The values of the nearest neighbor coupling k and second neighb

Parameter	Symbol	$\beta = 0.61$	$\beta = 0.76$
Nominal cell length (cm)	L	11.3586	14.1517
Cavity diameter (cm)	D	33.1629	33.0032
Internal cell (left side) bore radius (cm)	R <sub>b,L</sub>	5.0	5.0
External (right side) bore radius (cm)	$R_{b,R}$	5.5	5.5
Internal cell (left side) corner radius (cm )	R <sub>L</sub>	3.2	5.0
External (right side) corner radius (cm)	R <sub>R</sub>	3.1382	4.5794
Wall angle (degrees)	$\alpha_{\rm L}$	7.0	9.0
Equator flat length (cm)	$F_{Eq}$	0.0	0.0
Ellipse aspect ratio	a/b	1.50	0.90
Ellipse center to mid cell on left side (cm)	$X_{0,L}$	5.6793	7.0758
Ellipse center to axis on left side (cm)	$Y_{0,L}$	6.0318	6.3022
Horizontal semi-axis on left side (cm)	$a_{L}$	1.5477	1.1720
Vertical semi-axis on left side (cm)	$a_{\rm R}$	1.0318	1.3022
Ellipse center to mid cell on right side (cm)	$X_{0,R}$	5.6793	7.0758
Ellipse center to axis on right side (cm)	$Y_{0,R}$	6.6158	7.3825
Horizontal semi-axis on right side (cm)	$b_L$	1.6736	1.6943
Vertical semi-axis on right side (cm)	b <sub>R</sub>	1.1158	1.8825
External beam tube length (cm)	$\delta T_R$	12.5	12.5
Second beam tube length (cm)	$\delta T_2$	10.0	10.0
Second beam tube radius (cm)	$R_{b,2}$	4.2	4.2
Frequency (MHz)	f <sub>8π/9</sub>	805.000	805.000
Nearest neighbor coupling (%)	k	2.037	2.194
Second neighbor coupling (%)	<b>k</b> <sub>2</sub>	0.205	0.199
Frequency difference to nearest mode (kHz)	f <sub>8π/9</sub> -f <sub>7π/9</sub>	575	642

 Table 2. Cavity Parameters.





Figure 4. SUPERFISH generated cavity shapes for  $\mathbf{b} = 0.61$  (top) and  $\mathbf{b} = 0.76$  (bottom) for half of an 8-cell cavity. The contour lines correspond to constant values of the magnetic field  $H_{f}$ . The electric field is parallel to these contours.

Phase	$\beta = 0.61$	$\beta = 0.76$
π/9	790.070	788.966
2π/9	791.798	790.756
3π/9	794.361	793.442
4 - 10	707 252	706 622

Table 3. Mode frequencies in MHz for the  $TM_{010}$  pass band.





0.790

# **Transit-Time Factors**

The energy gain of a charged particle through a cavity is

$$\delta W = qE_0Abs(T+iS)cos\phi_sL$$
,

where q is the charge, L is the cell length, and  $\phi_s$  is the synchronous phase. The quantity  $E_0$  is the average axial electric field given by

$$\mathbf{E}_0 = \frac{1}{\mathbf{L}} \int_{\mathbf{Z}_s}^{\mathbf{Z}_e} \left| \mathbf{E}_z \right| dz \,,$$

where  $Z_s$  and  $Z_e$  are starting and ending longitudinal coordinates that span the nonzero axial fields of the cavity. Variables T and S are the real and imaginary parts of the transit-time factor, which are defined as follows:

$$T = \frac{1}{E_0 L} \int_{Z_s}^{Z_e} (z) \cos[k(z - Z_c)] dz,$$

 $S = \frac{1}{E_0 L} \int_{Z_c}^{Z_e} (z) \sin[k(z-Z_c)] dz,$ 

and

where  $k = 2\pi/\beta\lambda$  is the wave number, and  $\beta\lambda$  is the distance traveled in one rf period by a particle of velocity  $\beta c$ . The resonant mode has frequency  $f = c/\lambda$ , where  $\lambda$  is the free-space wavelength. For beam dynamics simulations in the superconducting part of a linac, one needs the transit-time factors as a function of particle velocity. The postprocessor program SFO in the Poisson Superfish code distribution computes T and S as well as their derivatives with respect to the wave number k:

$$T' = \frac{1}{E_0 L^2} \int_{Z_s}^{Z_e} (z - Z_c) E_z(z) \sin[k(z - Z_c)] dz,$$
  
$$S' = \frac{1}{E_0 L^2} \int_{Z_s}^{Z_e} (z - Z_c) E_z(z) \cos[k(z - Z_c)] dz,$$

β	S	Ś	S″
0.50	0.073793	-0.404288	-0.476691
0.51	0.139670	-0.470249	-0.395158
0.52	0.211400	-0.519741	-0.284747
0.53	0.286026	-0.550394	-0.152143
0.54	0.360670	-0.561082	-0.004699
0.55	0.432692	-0.551799	0.150138
0.56	0.499797	-0.523489	0.305292
0.57	0.560093	-0.477843	0.454391
0.58	0.612124	-0.417100	0.591993
0.59	0.654866	-0.343852	0.713707
0.60	0.687703	-0.260875	0.816229
0.61	0.710389	-0.170984	0.897315
0.62	0.723000	-0.076911	0.955703
0.63	0.725880	0.018778	0.991012
0.64	0.719591	0.113754	1.003615
0.65	0.704856	0.205965	0.994505
0.66	0.682518	0.293655	0.965169
0.67	0.653494	0.375379	0.917461
0.68	0.618739	0.449989	0.853489
0.69	0.579216	0.516626	0.775517
0.70	0.535869	0.574696	0.685876
0.71	0.489606	0.623843	0.586893
0.72	0.441281	0.663925	0.480833
0.73	0.391685	0.694984	0.369857
0.74	0.341539	0.717215	0.255979
0.75	0.291490	0.730944	0.141053
0.76	0.242107	0.736594	0.026751
0.77	0.193887	0.734673	-0.085446
0.78	0.147252	0.725743	-0.194242
0.79	0.102554	0.710405	-0.298532
0.80	0.060082	0.689286	-0.397389

Table 4. Transit-time factors for  $\mathbf{b} = 0.61$  cavity.

β	S	Ś	S″
0.60	-0.025149	-0.271450	-0.558731
0.61	0.014994	-0.343526	-0.541675
0.62	0.062590	-0.409505	-0.499097
0.63	0.116234	-0.466734	-0.433100
0.64	0.174358	-0.513122	-0.346703
0.65	0.235327	-0.547172	-0.243568
0.66	0.297510	-0.567968	-0.127735
0.67	0.359351	-0.575140	-0.003392
0.68	0.419419	-0.568804	0.125325
0.69	0.476437	-0.549496	0.254490
0.70	0.529314	-0.518096	0.380508
0.71	0.577151	-0.475752	0.500199
0.72	0.619246	-0.423803	0.610855
0.73	0.655089	-0.363718	0.710259
0.74	0.684349	-0.297030	0.796692
0.75	0.706864	-0.225284	0.868914
0.76	0.722617	-0.149997	0.926137
0.77	0.731723	-0.072620	0.967983
0.78	0.734405	0.005491	0.994438
0.79	0.730979	0.083089	1.005809
0.80	0.721830	0.159056	1.002668
0.81	0.707398	0.232407	0.985806
0.82	0.688164	0.302294	0.956189
0.83	0.664633	0.368005	0.914911
0.84	0.637320	0.428959	0.863156
0.85	0.606744	0.484703	0.802166
0.86	0.573417	0.534901	0.733207
0.87	0.537835	0.579323	0.657544
0.88	0.500474	0.617841	0.576420
0.89	0.461784	0.650411	0.491037
0.90	0.422189	0.677069	0.402543

Table 5. Transit-time factors for  $\mathbf{b} = 0.76$  cavity.



Figure 7. Whole-cavity transit-time factors for both 8-cell elliptical cavities. The curve on the left corresponds to the  $\mathbf{b} = 0.61$  cavity and the curve on the right corresponds to the  $\mathbf{b} = 0.76$  cavity.

Some beam dynamics codes such as PARMILA step the particles through each cell of the structure, applying impulses at the center of each cell. For input to PARMILA, SFO generates transit-time-factor data in a series of files, one file for each specified particle velocity. Another utility program reads this family of files and creates "SFDATA" tables to include in the PARMILA input file. Though these data are two numerous to list in this memo, they will be available by electronic transfer as described below.

# **Data Files**

The SUPERFISH input and output files for the elliptical cavities are available in shared directories \\pc-billen\Projects\NSNS\SC\beta.61 and \\pc-billen\Projects\NSNS\SC\beta.76. (If you are not a member of the AOT-1 computer domain, log in to machine pc-billen.atdiv.lanl.gov by FTP as user Linac with password aot1ftp.)

## **Electronic Distribution:**

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