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Balancing Dynamic Strength of Spur Gears Operated at Extended Center Distance

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ABSTRACT

This paper presents an analytical study on using hob offset to balance the dynamic tooth strength of spur gears operated at a center distance greater than the standard value. This is an extension of a static study by Mabie and others. The study was limited to the offset values that assure the pinion and gear teeth will neither be undercut nor become pointed. The analysis presented in this paper was performed using DANST-PC, a new version of the NASA gear dynamics code.

The operating speed of the transmission influences the amount of hob offset required to equalize the dynamic stresses in the pinion and gear. The optimum hob offset for the pinion was found to vary within a small range as the speed changes. The optimum value is generally greater than the optimum value found by static procedures. For gears that must operate over a wide range of speeds, an average offset value may be used.

NOMENCLATURE

C	center distance, mm (in)
c	tooth clearance, mm (in)
e	hob offset, mm (in)
k	hob addendum parameter
m	module, mm
N	number of teeth
P_d	diametral pitch, in^{-1}
R_o	outside radius, mm (in)
R	Pitch radius, mm (in)
ϕ	pressure angle, degrees

Subscripts

1	driving gear (pinion)
2	driven gear (gear)

INTRODUCTION

Gears may be designed to operate at a non-standard center distance to introduce backlash, to accommodate space constraints (Townsend, 1991), and to adjust for anticipated deflections under load as well as geometry changes due to thermal effects. In some gear designs it is necessary to connect shafts at a predetermined center distance and to produce a desired velocity ratio. However, when standard gears cannot be used, the design is much more complicated. In these cases, the cutting tool can be offset from the standard setting to produce nonstandard gears which will mesh at the desired center distance.

In designs with large module (small pitch) teeth, the number of teeth is small on both the pinion and gear. If the pinion teeth are enlarged to provide proper strength then the pinion teeth may reach below the base circle of the gear and cause undercut. In such cases, the center distance must be increased. This design is called the extended center distance system. For gears cut by hobs, the adjustment of the cutting tool is called hob cutter offset.

In previous work, Walsh and Mabie (1971), Mabie et al. (1983), and Mabie et al. (1990), procedures and design charts were developed to determine the hob offset for the pinion and gear so that the stress in the pinion and gear teeth were approximately equal. They developed charts relating hob offset at various velocity ratios and changes in center

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distance. However, their analyses dealt with the static tooth strength. They used the Lewis formula (Shigley and Mitchell, 1983) to calculate the tooth root stress and based calculations assuming the maximum static load was applied at the tip of the tooth.

Under dynamic conditions, especially at high speed, the tooth load can be significantly greater than the static load, and the maximum dynamic tooth load may occur at a location other than the tooth tip (Lin et al., 1989). Furthermore, non-standard gears have transmission errors different from gears with standard tooth proportions. The dynamic response of a gear system, excited by the transmission error, can be considerably different for nonstandard gears. Dynamic effects should be considered for the design of high speed gears.

THEORY AND ANALYSIS

Extended Center Distance Gears

The analysis and formulation for nonstandard gears operating at an extended center distance can be found from the previous work of Mabie and Reinholtz (1987). The relationship between the amount of hob offset and gear parameters was found to be:

$$e_1 + e_2 = \frac{(N_1 + N_2)(inv\phi' - inv\phi)}{2P_d \tan\phi} \tag{1}$$

The above equation can be expressed in metric units with m as module,

$$e_1 + e_2 = \frac{m(N_1 + N_2)(inv\phi' - inv\phi)}{2\tan\phi} \tag{2}$$

where e_1 is the hob offset for the pinion and e_2 is the offset for the gear, ϕ is the standard pressure angle and ϕ' is the operating pressure angle. The involute function $inv\,\phi$ is defined as $(\tan\phi - \phi)$.

When a pair of gears (the pinion and the gear) are moved apart a distance ΔC from the standard center distance C, the extended center distance is

$$C' = C + \Delta C = R_1' + R_2' \tag{3}$$

where R_1' and R_2' are the operating pitch radii of the pinion and the gear, respectively. From basic gear geometry, the center distance extension is

$$\Delta C = C\left(\frac{\cos\phi}{\cos\phi'} - 1\right) \tag{4}$$

and the operating pitch radii are

$$\mathrm{R}_1' = C'\left(\frac{N_1}{N_1 + N_2}\right) \tag{5}$$

$$\mathrm{R}_2' = C'\left(\frac{N_2}{N_1 + N_2}\right) \tag{6}$$

From Fig. 1, the outside radii of the gear pair, expressed in U.S. customary units, are

$$R_{o1} = C' - R_2 - e_2 + \frac{k}{P_d} \tag{7}$$

$$R_{o2} = C' - R_1 - e_1 + \frac{k}{P_d} \tag{8}$$

where k is the addendum coefficient value and is equal to 1.00 for a standard full-depth tooth. The above equations can be expressed in metric units as

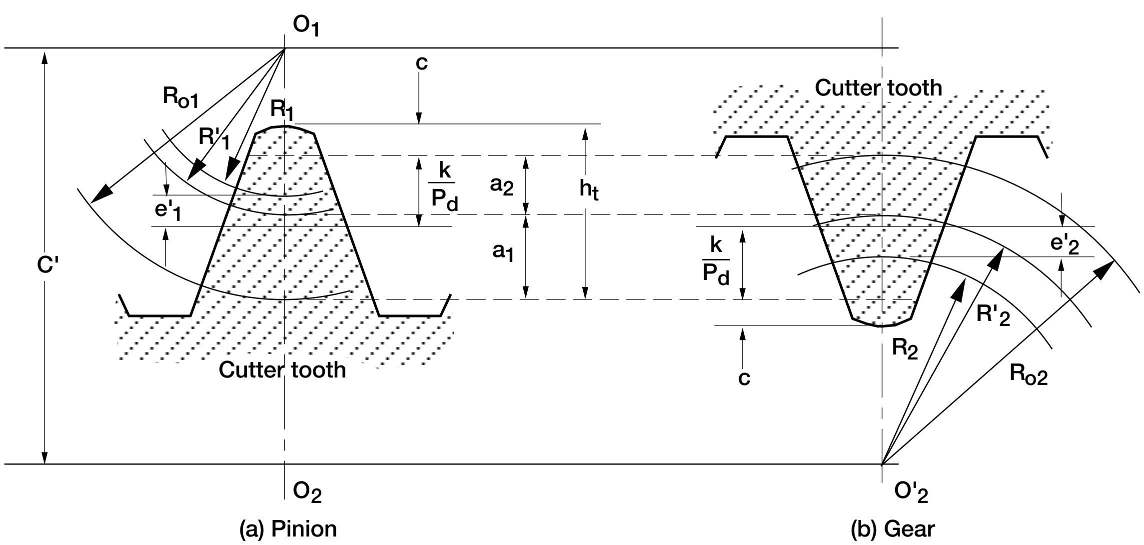

Figure 1.—Gear pair cut by offset hobs to mesh at an extended center distance.

$$R_{o1} = C' - R_2 - e_2 + m \tag{9}$$

$$R_{o2} = C' - R_1 - e_1 + m \tag{10}$$

Note that the sum of the hob offsets, e_1 and e_2, does not necessarily equal the increase in the center distance (C) over the standard value. There is no easy way of determining the hob offsets independently of each other. The values are usually selected by assuming one of them and calculating the other from either Eqs.1 or 2 depending on the units used (U.S. or metric).

Gear Dynamic Model

The computer program DANST-PC was used for the dynamic analysis (Oswald et al., 1993). The DANST model employs four torsional degrees of freedom to represent a typical gear transmission. The model includes driving pinion and driven gear, connecting shafts, motor, and load. DANST predicts the dynamic response of a transmission for several parameters including dynamic load and tooth bending stress. In two validation studies, the predictions of DANST compared very well with experimental measurements (Oswald et al., 1991 and Oswald et al., 1996).

A more detailed discussion of the dynamic model and tooth root stress calculation can be found in the literature (Cornell, 1981, and Lin et al., 1993).

RESULTS AND DISCUSSION

The following section describes a parameter study in which the dynamic analysis was applied to a sample gear set to compare results with the static results obtained by Mabie, et al. (1990). An example problem from their paper was used for the investigation. Parameters for the sample gears used are given in Table 1. The gear set has a standard center distance of 5.00 inches (127 mm) but the operating center distance was extended by 0.25 inches (6.35 mm). In the study, the hob offset on the pinion was treated as a variable and the corresponding offset on the gear was calculated from Eq. (1).

The sample gears for this study have a perfect involute tooth profiles with no modifications. From previous work (Lin, et al., 1993 and 1989) we know that modifying the tooth profile can significantly reduce gear dynamics. However, in this study, no modifications were considered.

Figures 2 and 3 illustrate the effect of pinion hob offset (e_1) as well as rotation speed on the dynamic stress of the pinion. The most prominent feature in both figures is the resonant response which occurs at 13500 rpm. The “notches” which appear along the resonant speed contours result from numerical error. Minimum values of dynamic tooth stress appear as a valleys in the three dimensional figure (Fig. 2) and can be located in the contour diagram (Fig. 3). The system resonant speed was minimally affected by the pinion hob offset. In this and the cases to follow, we maintained the center distance at 5.25" (133.35 mm). The hob offset for the pinion was varied as shown in the figure and the offset for the gear was calculated from Eq. (1).

The pinion hob offset was varied between 0.015 and 0.213 inch (0.38 and 5.41 mm). These limits were required to avoid having undercut or pointed teeth in either pinion or gear. The optimal pinion hob offset from the static procedure is 0.159 inch (4.04 mm). This is shown as a dashed line in the figure. At lower speeds (1000 to 5000 rpm) the pinion dynamic stress reaches its minimum when the pinion hob offset is less than the optimal static value. (Two areas marked with the letter “A” in Fig. 3.) However, at higher speeds (6000 to 13000 rpm), the minimum dynamic pinion stress occurs at hob offsets greater than the static optimal value (letter “B” in Fig. 3).

Figures 4 and 5 show the effect of pinion hob offset and rotation speed on dynamic stress of the gear. Unlike Figs. 2 and 3, the minimum

TABLE 1.—Gear Parameters

Gear type	Full depth, involute tooth
Number of teeth (Pinion/Gear)	20, 30
Module M, mm (diametral pitch P, 1/in.)	5.08 (5)
Pressure angle, deg	20
Face width, mm (in.)	6.35 (0.25)
Applied torque, N-m (lb-in.)	54.25 (480)
Static tooth load, N/m (lb/in.)	178 850 (1022)

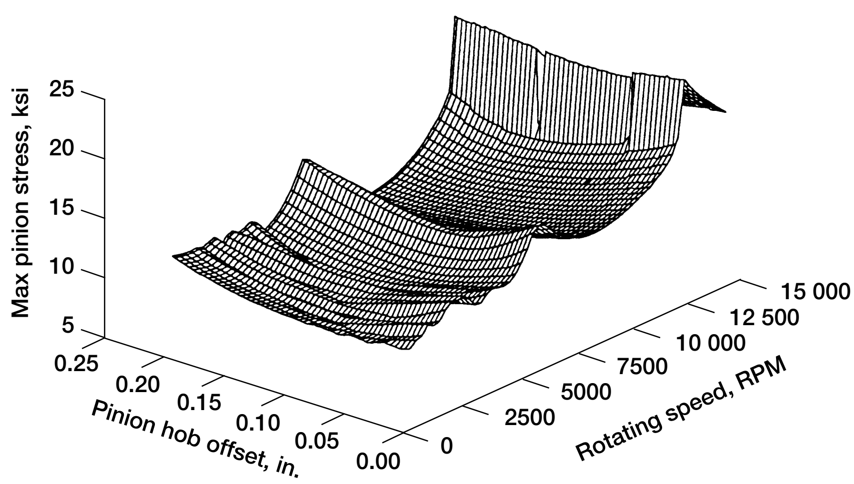

Figure 2.—Effect of pinion hob offset (e_1) and rotating speed on pinion dynamic stress.

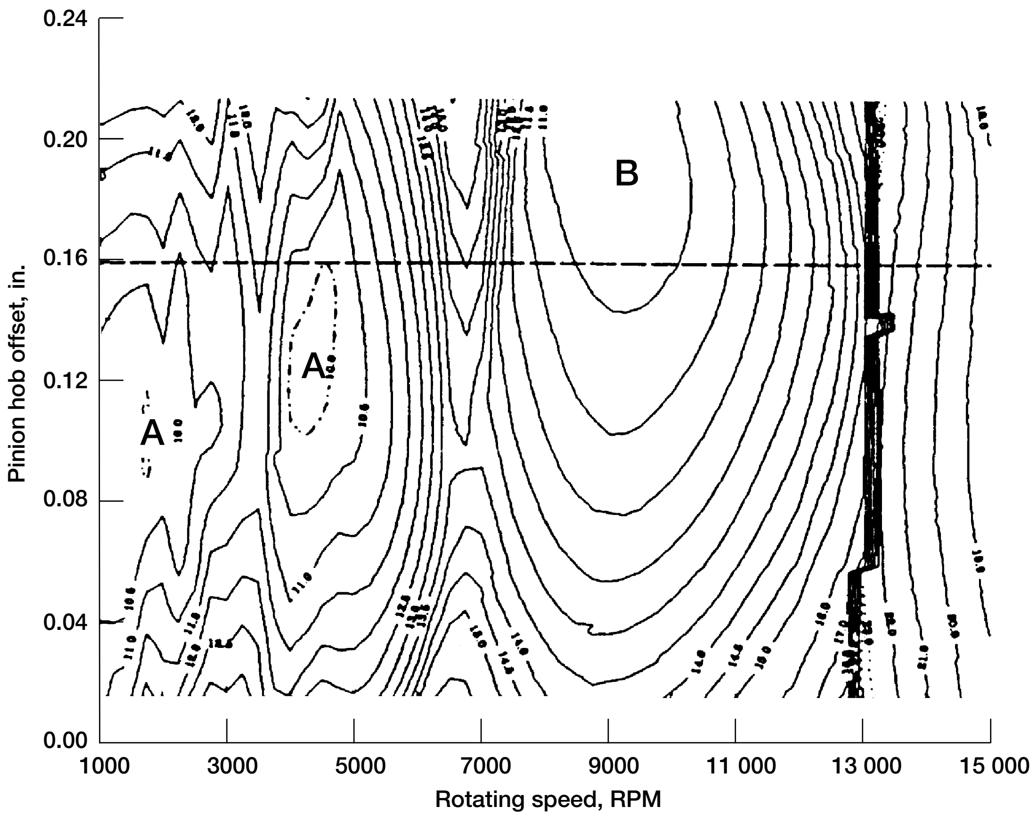

Figure 3.—Contour diagram showing the effect of pinion hob offset and rotating speed on pinion dynamic stress. (The optimal static case hob offset is shown by the dashed line.)

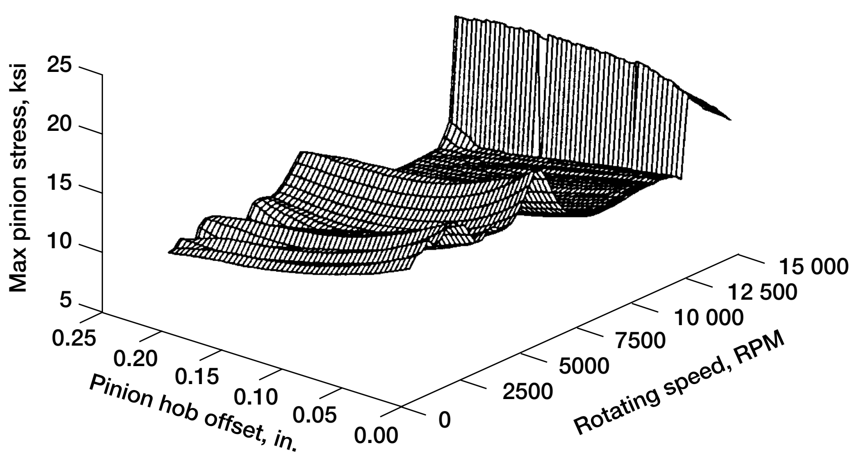

Figure 4.—Effect of pinion hob offset (e_1) and rotating speed on gear dynamic stress. (The hob offset for the gear is determined from eq. 1.)

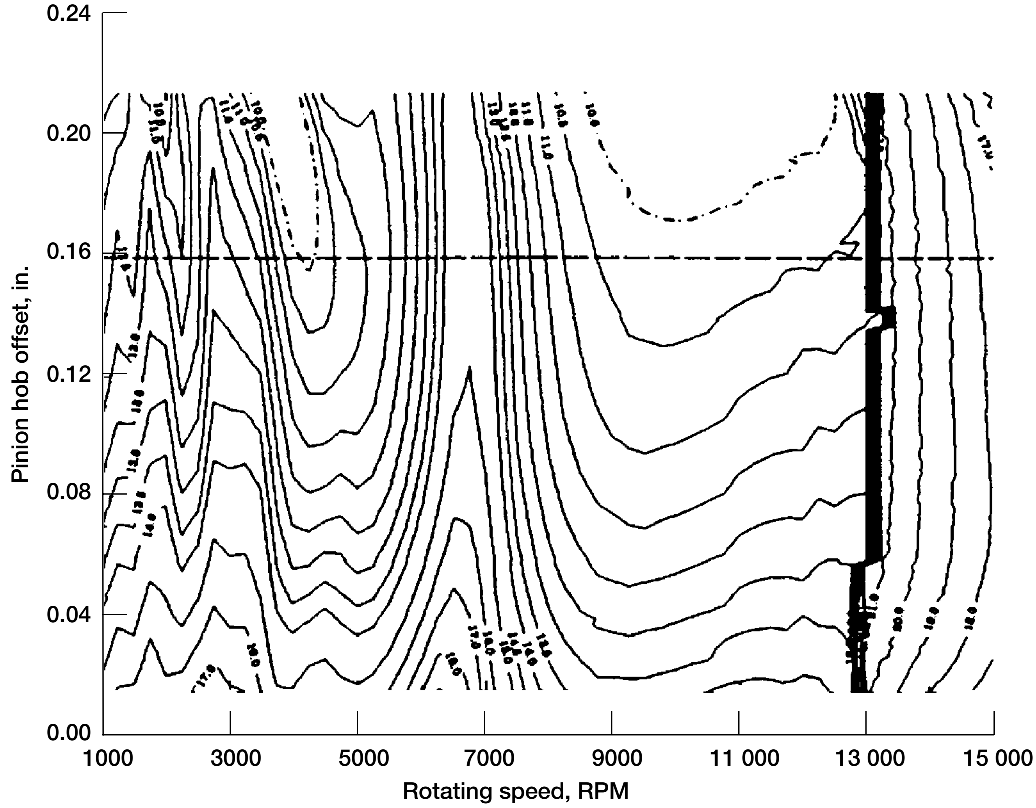

Figure 5.—Contour diagram showing the effect of pinion hob offset and rotating speed on gear dynamic stress. (The hob offset for the gear is determined from eq. 1. The optimal static case hob offset is shown by the dashed line.)

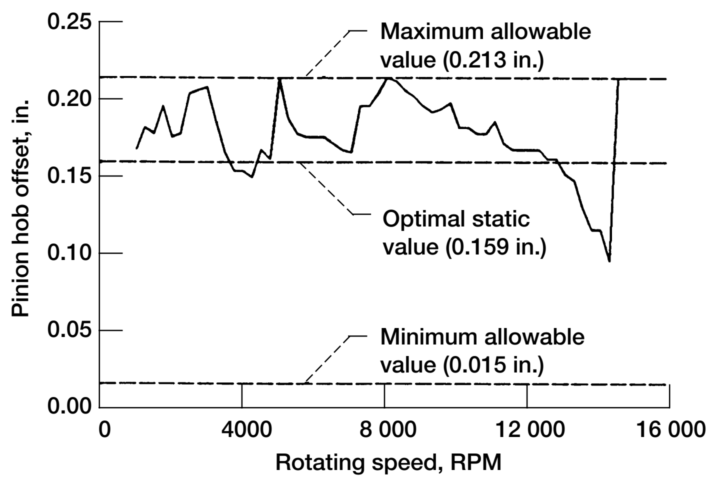

Figure 6.—Determining pinion hob offset to balance dynamic tooth strength of pinion and gear at different speeds.

values in Figs. 4 and 5 occur at an offset greater than the optimal static value for all speeds.

We can optimize the pinion hob offset to balance the dynamic strength of the gear set by minimizing the difference between the pinion and gear dynamic stresses. Figure 6 shows how this optimum pinion hob offset varies with the rotation speed. At most speeds (except near the resonance of 13500 rpm), the best value lies between the optimal static value, 0.159 inch (4.04 mm) and the maximum allowable value, 0.213 inch (5.41 mm). At the resonance speed, the best value was about 60 percent of the optimum static value.

The analysis presented in this paper can be used to determine the hob offset required to balance the dynamic tooth strength of gears operated at extended center distance. The amount of offset depends on the intended operating speed range of the gear pair. For a gear set which will operate over a range of speeds, an averaged hob offset value can be used to balance dynamic strength.

CONCLUSIONS

DANST-PC, a new version of the NASA gear dynamics code, was used to study the dynamic stress of non-standard spur gears cut with offset hobs. The study was applied to gear pairs operated at a center distance greater than the standard value. The operating speed of the transmission was varied over a broad range to evaluate speed effects on the dynamic response. The following conclusions were obtained from the investigation:

1. Over most of the speed range studied, the best value for hob offset is greater than the optimum value obtained from the static procedure. The hob offset tends to make the pinion teeth longer and the gear teeth shorter.

2. For gears operating over a range of speeds, a weighted average hob offset value can be used to balance the dynamic strength of pinion and gear.

3. The best hob offset to minimize dynamic stress in the pinion is different from that required to minimize dynamic stress in the gear.

4. The analysis developed in this study can be used to determine the hob offset value to balance the dynamic tooth strength of the pinion and gear. The balanced design will provide a higher load capacity gear system.

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