AMP 160

THE ELECTRON PLASMA-EXPERIMENT THEORY AND APPLICATIONS

by

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ABSTRACT

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The basic concepts involved in the production and control of pure electron and electron rich plasmas are reviewed, with particular reference to the different geometries in which equilibrium may be achieved. An inductive method of establishing arbitrary equilibrium profiles is described. Recent theoretical work on the stability of some particularly simple equilibria is then discussed. Instabilities considered are: 1) The so-called diocotron (or slipping stream) instability, this instability (whose importance in magnetron work is well understood) is related to the Kelvin-Helmholtz instability in fluid mechanics; 2) A new mode involving spontaneous coherent radiation into space as a result of bunching. This effect can be described as the "flexible antenna instability". Both these instabilities are considered for low density electron beams (that is, $\omega_p \ll \omega_c$). Two experiments are discussed. The object of the first is to produce a stable electron plasma in a simple geometry. The second is designed to test the practicability of the inductive charging scheme. A brief review of selected applications of the electron plasma is also given; these include space radiation shielding and certain high voltage laboratory devices.

Author

INTRODUCTION

Under suitable conditions, it appears that the crossed-field electron beam can be operated stably; this opens up the possibility of interesting new applications.[1,2] Among others we may mention the generation and maintenance of very high potentials (up to 10^9 volts).

To fix our ideas, consider the configuration illustrated in Fig. 1. A long evacuated cylinder contains a magnetic field parallel to its axis. If we fill the cylinder with electrons, a depression of the electric potential near the axis will result. In the absence of the magnetic field the electrons would, of course, travel straight to the wall of the vessel. The effect of the magnetic field is, however, to cause the electrons to drift in a direction which, on the average, is azimuthal. The important point is that this motion does not carry the electrons to the wall, so that there exists the possibility of establishing a dynamic equilibrium. An analogous geometry is shown in Fig. 2. In this case the magnetic field is due to a current in a long straight wire, and is therefore azimuthal. The wire is charged, and gives rise to an electric field in the outward radial direction; the electric field lines end on electrons in the beam. The electrons then drift parallel to the wire. In both these geometries the end problem can be removed by bending the straight cylinder or wire into a torus or circle. In the first case we arrive at a geometry closely resembling the stellarator;[3] the purpose of our device and the physical conditions under which it is designed to operate are, however, very different. In particular, the strong radial electric field and the absence of any appreciable number of ions are striking features of the present device. In the second case we arrive at a geometry for which it is difficult in the laboratory to provide a vacuum envelope. One possibility is, however, suggested by the Levitron,[4] a similarly shaped device in which the ring is dropped. While falling, the experiment is performed in a few hundred microseconds. A more interesting application of this device is to design a space ship with this geometry, so that the vacuum envelope is the universe.[1] With this geometry the space ship might be raised to a very high potential relative to the surrounding region. In this manner, the occupants of the vehicle would be protected from solar cosmic rays at a relatively modest weight cost.

A common feature of all the devices described is the existence of a large electrostatic potential across the system which may be as much as 10^9 volts; nevertheless, there is only a single material electrode, namely the anode. The role of the cathode is taken by the electron beam which is distributed in the neighborhood of the anode. Clearly this "cathode" is not subject to the ordinary kind of electron emission. Furthermore, there are, in high voltage devices like the Van deGraaff machine, various breakdown mechanisms involving repeated multiplication of collisions at alternate electrodes.[5] Such effects are also absent in our case. The possibility exists therefore of establishing and maintaining extremely high potentials in all the

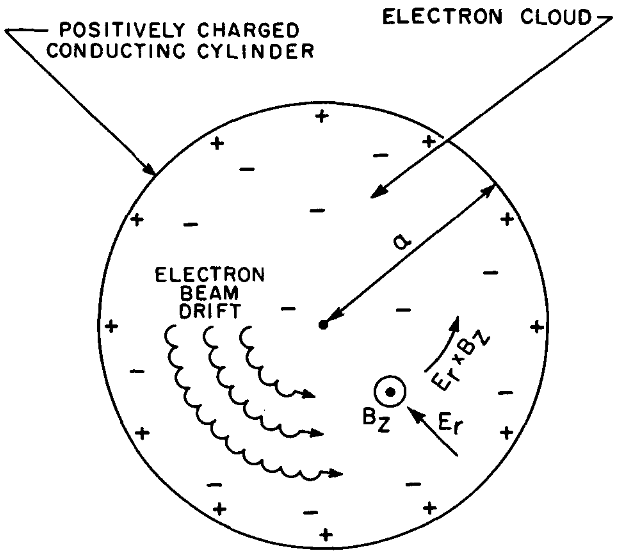

Fig. 1 This illustrates a possible geometry in which an electron cloud could be contained in the laboratory. The magnetic field is parallel to the axis of the cylinder and the electric field is radially in from the walls. The electron orbits are then as shown. The electrons nearest the surface travel fastest. The geometry can be made endless by bending the long cylinder into a curved torus.

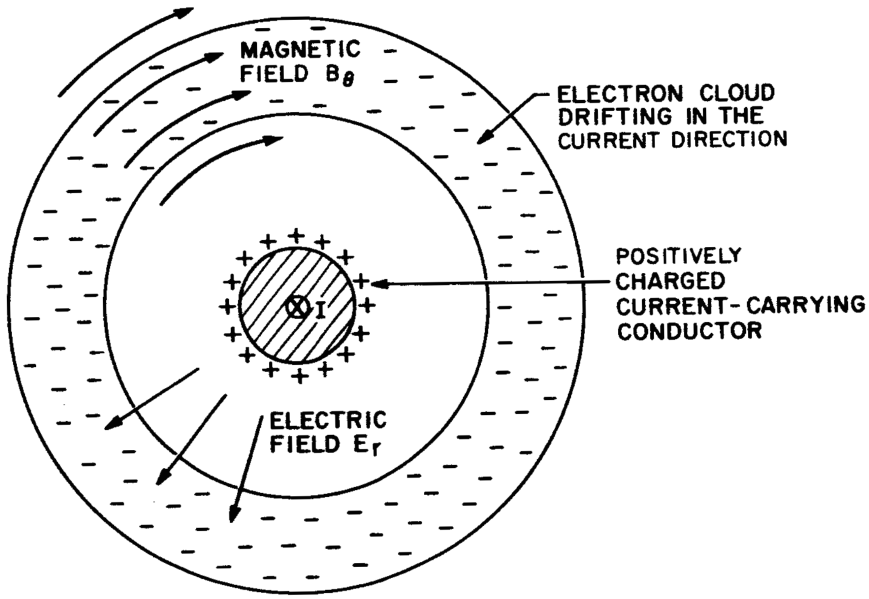

Fig. 2 This illustrates a possible geometry in which an electron cloud could be contained in space. The electric field is radially outwards from the wire, the magnetic field is, as shown, due to a current in the wire. The electron orbits are then parallel to the wire; the electrons nearest the surface travel fastest. The geometry can be made endless by bending the wire into a circular loop.

devices so far described. At the same time, the nature of the topology is such that only a single terminal is available. Any connection between the positive and negative voltages in our systems would necessarily involve a physical obstruction to the electron motion.

In this paper we shall analyze the type of device described from three points of view: establishment, stability, and application.

ESTABLISHMENT

We propose to establish the electrons in their equilibrium orbits by the following scheme:

During the transient process of establishing the magnetic field in the devices illustrated in Figs. 1 or 2, the magnetic field lines can be considered as being "born" near the field coils and subsequently moving to a position further away from the coils. We can keep track of the different field lines as they move by ascribing to each a velocity v which satisfies

$$\vec{E} + \vec{v} \times \vec{B} = 0 \quad (1)$$

where the "E" is the sum of the inward radial E (for Fig. 1) and the azimuthal E induced by the changing B. This yields a motion in the form of a spiral of decreasing radius. Now if the magnetic field changes with a characteristic frequency far less then the electron cyclotron frequency (an easy condition to meet), (1) can be regarded as an adiabatic equation of motion for the guiding center of an electron. Thus we see that an electron injected near the wall onto a field line during the build-up of the magnetic field will subsequently be carried away from the wall towards the center of the device. This process has been aptly described as a "beltless Van deGraaff," the magnetic field playing the role of a (non-ablating) belt. It differs from the betatron acceleration process in that the field is required to change more slowly than the cyclotron period; the result of this is that the electrons increase their potential energy, but not their kinetic energy. Clearly the "belt" in this process is only moving forward just so long as the magnetic field in increasing. Thus a necessary condition for the process to be workable is that losses which cause the electrons to slip back across the field lines to regions of lower potential must have characteristic times longer then the magnetic field build-up time. After the magnetic field has reached its peak value, the electron distribution, and hence the potential across the system would, in the absence of losses, remain fixed as long as the magnetic field is maintained.

Figure 3 is a schematic diagram of an apparatus designed to test the charging principle (called "inductive charging") described above. It consists of two concentric cyclinders, of which the inner one is lightly silvered on its outside and the outer one is lightly silvered on its inside. A coil wound around

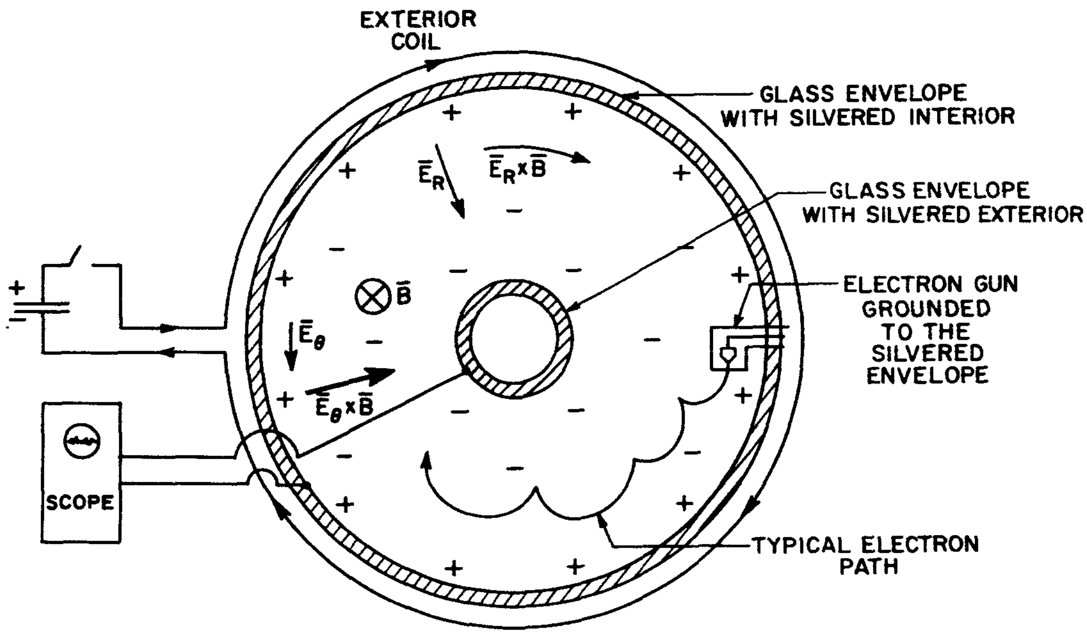

Fig. 3 This schematic diagram shows the apparatus used to test the inductive charging scheme described in the text. Electrons emitted at low energy by the gun were transported by compression of the magnetic field to the inner cylinder; the resulting potential between the inner and outer cylinders far exceeded the injection energy. The E_θ and the $E_\theta \times B$ shown are present only when the magnetic field in increasing with time. The E_R and $E_R \times B$ are always present.

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the outside provides an axial magnetic field which reaches 3 kGauss in a time of approximately 200μ secs. Figure 4 is an oscillogram obtained with this apparatus. As indicated, an electron gun (gun voltage ~ 200 volts) was fired for a short period (about 10μ secs) when the magnetic field strength was rising through 300 Gauss. As the magnetic field increased, the injected electrons were carried inwards towards the inner cyclinder thereby increasing the potential measured between the inner and outer cylinder. The majority of the field lines carrying electrons were compressed into a radius smaller than the radius of the inner cylinder. As a result, most of the electrons were collected by the inner cylinder with the result that the potential stayed up even after the magnetic field relaxed. The potential obtained in this experiment was 3 kV. The background pressure in the apparatus was about 10^{-7} mm Hg. This preliminary experiment can be regarded as supporting quite strongly the theoretical picture of the charging mechanism. Further details of the experimental set-up and results are given by Janes.[6]

STABILITY

We have described above a number of possible equilibrium states for the crossed-field electron beam, together with a method of establishing them. We turn now to a most important consideration, namely the likely stability of the various equilibria. Clearly, in the absence of stability, the prospects for useful applications of the high voltages obtainable are limited. Experience in the general field of plasma physics prompts us to treat with considerable caution the likelihood of attaining a stable confinement of the electron beam. In all the devices described, a great deal of energy is stored in the electrostatic field and this energy is readily available to drive any one of a number of possible mechanisms of instability. On the other hand, the mere availability of an energy reservoir does not by itself imply that stable confinement is impossible; we propose in this section to review the state of the stability problem as it applies to electron beams of the relevant kind, both from the experimental and the theoretical point of view.

We commence by observing that in crossed-field electron beams for which the electric field is entirely produced by the beam itself (i.e., where we do not apply an electric field by external means) it is essential that

$$\omega_p^{\ 2}/\omega_c^{\ 2} \ll 1 \tag{2}$$

This result follows from the observation that it is the condition that the centrifugal term omitted in (1) should, in fact, be negligible. Now the importance of the inequality (2) for our stability theory is that we are at once in a regime different from that to which the magnetron theory has been applied. Virtually all the linear theory of magnetron stability[7,8] applies to the condition $\omega_p^2/\omega_c^2 = 1$. Experimental and theoretical evidence on the existence of growing waves, especially at short wavelengths, is therefore to be treated with caution as regards its applicability to the circumstances of present interest.

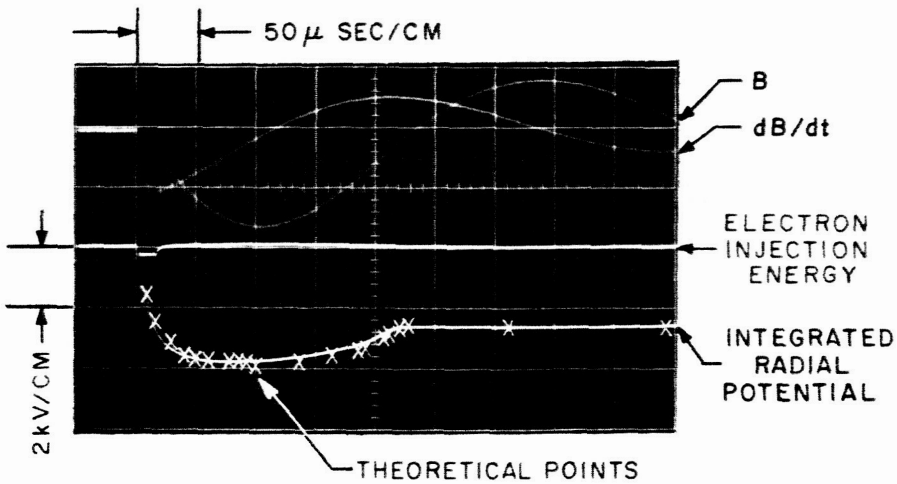

Fig. 4 This oscillogram was obtained with the apparatus shown in Fig. 2. The experiment was initiated after 50μ secs; the two traces in the upper half of the diagram represent B and $\dot{B}$. The electron gun is pulsed about 200 volts negative for about 15μ secs. Subsequently, the electrons emitted during this pulse are transported by the rising field to the inner cylinder giving rise to a total potential of nearly 4 kV. Those electrons actually collected by the inner cylinder cause the potential to stay up even after the magnetic field is relaxed. The crosses represent a semi-empirical calculation assuming ideally that each electron accepted by the system stays with the magnetic field line on which it was born until collected by one or the other cylindrical walls of the vessel.

It is, of course, the case of magnetrons that, for the most part the applied electric fields are greater than the space charge fields. There is, however, a body of experimental evidence available to us which does satisfy the condition (2). We refer to the operation of Ion Pumps or Penning Discharges. Data quoted by Knauer[9] shows that under conditions for which $\omega_p^2/\omega_c^2 \approx 0.07$, stable confinement of electron beams in a geometry resembling that of Fig. 3 is possible. By stable in this context we mean that a leakage current is observed which is consistent with the hypothesis that the loss is entirely due to collisions with the neutral gas atoms present in the system. This hypothesis is additionally supported by further evidence quoted by Knauer. As the background pressure is reduced in the range 10^{-4} mm Hg to 10^{-6} mm Hg, the loss current decreases linearly from about 10^{-6} of the circulating current to about 10^{-8} of the circulating current. This data implies that the electrons travel around the system over a million times in the azimuthal direction before crossing it once in the radial direction and being collected. This corresponds in numbers to confinement times of over 1 millisecond. Since the characteristic times for all the growth mechanisms corresponding to instabilities of one sort or another are much shorter than this time, it may be assumed that, at least for the conditions of Knauer's experiment, stable confinement has been demonstrated. An experiment similar to Knauer's, but using a hot cathode to ensure that conditions were not emission limited, has been reported by Janes[6] with essentially similar results.

From the theoretical point of view it is never possible to prove the stability of a configuration of the type under discussion; it is only possible to consider possible mechanisms of instability and show, one by one, that they do not cause growth in the particular circumstances. We have treated however, the mechanism which, it appears, has the greatest relevance to the conditions of the experiments described. We refer to the so-called "diocotron or slipping stream" instability. We have been able to show[10] that under conditions where the electron beam closes on itself, i.e., is periodic, a sufficiently thick beam is not subject to this instability.

We illustrate the nature of the diocotron instability here by relating it to the well-known Kelvin-Helmholtz instability of shearing fluid flows. Let us suppose that the motion of an electron beam is described by (1), and furthermore that there are no electric fields or motions parallel to the magnetic field lines. In addition, suppose that the frequencies under consideration are much smaller than the frequency of electromagnetic radiations having wavelengths comparable to the size of the system. Then we may assume

$$\text{curl}\,\underline{E} = 0 \quad \text{or} \quad \underline{E} = -\frac{\partial \psi}{\partial x},\ -\frac{\partial \phi}{\partial y},\ 0 \qquad (3)$$

It follows from the above assumptions that

$$\underline{v} = -\frac{1}{B}\frac{\partial \phi}{\partial y},\ \frac{1}{B}\frac{\partial \phi}{\partial x},\ 0 \quad \text{or} \quad \text{div}\,\underline{v} = 0 \qquad (4)$$

Hence, the motions of the electron beam are "incompressible". Conservation of electrons then requires:

$$\frac{Dn}{Dt} \propto \frac{D(\nabla^2 \phi)}{Dt} = 0 \tag{5}$$

If we compare the electron number density to the vorticity of an incompressible fluid moving in two dimensions, we observe that we have a complete analogy; this analogy includes the boundary condition at a conducting wall where the vanishing of the tangential component of E corresponds through Eq. (1) to the vanishing of the normal component of v.

We conclude on the basis of the above analogy that the Kelvin-Helmholtz instability of shearing fluids can be carried over directly to the present study whenever we restrict our attention to waves having $k_{\parallel} = 0$. It is well known that a surface of slip (corresponding to a very thin electron beam) is unstable to perturbations of all wavelengths. It is also known that a region of distributed shear is unstable to perturbations having a sufficiently long wavelength. This corresponds to an instability to very long wavelength perturbations of electron beams of finite thickness. We now observe that in closed systems of the type discussed, our electron beams are periodic in the flow direction. This means that the only wavelengths permitted are the perimeter of the beam, and submultiples of this length. In particular, there exists a longest permitted wavelength, namely just the perimeter of the system. If now the beam is so thick that it is stable to perturbations of this wavelength, it follows that it will be stable to perturbations of all permitted wavelengths. In the absence of conducting walls, circular beams are stable when their outer to inner radius ratios exceed 2:1. In the presence of conducting walls the requirements are less stringent.

The fluid dynamic analogy to this geometric situation is the flow between two rotating cylinders. The stability of such flows is usually treated by deriving the condition of Rayleigh that interchange type motions should be energetically unfavorable. This condition requires:

$$\frac{d}{dr} (r^2 \Omega)^2 > 0 \tag{6}$$

or, more physically, that the angular momentum distribution should increase outward. The connection between the two fluid dynamic mechanisms discussed above lies in the following observations: although (6) can be derived on purely energetic grounds, the actual accomplishment of the interchange motion implied is possible only when $k_z \neq 0$. This corresponds to the well-known fact that unstable motions between rotating cylinders break up into cells corresponding to the most unstable axial wave number. Energetically unstable situations can be stabilized experimentally be making the system shorter in the axial direction than the fundamental wavelength. Theoretically, instabilities are necessarily present only when $k_z \neq 0$. Now it was pointed out above that the fluid dynamic analogy for our electron beams was only meaningful when $k_z = 0$. We conclude that results of the type (6) cannot be applied directly to the study of the stability of crossed-field electron beams.

A more useful, though somewhat vague conclusion can, however, be drawn from the above discussion. If the electronic system is infinitely long in the direction of the magnetic field, effects associated with the electronic inertia will permit the development of electric fields in the magnetic field direction. We might therefore suspect, though at the moment the conclusion is tentative, that there exists in the electrical case a maximum permissible length of the system in the magnetic field direction. This length, however, is certainly different from that calculated for the case of fluid rotations. This is essentially because the dynamics of the system in the axial direction are governed by different considerations in the two cases. In particular the mobility of the electrons parallel to the field is very high. It may therefore be guessed that the maximum permissible length is much greater for the electronic than for the fluid dynamic case. It is hoped that this guess can in due course be justified analytically.

It is doubtful that more information than has already been discussed can be drawn from the fluid dynamic analogy. We turn next to a consideration of other possible mechanisms of instability which may be important. The first of these would appear to be the fact that when the electron velocities approach the speed of light, the displacement current can no longer be ignored. It will be seen later that it is desirable to contemplate operating useful devices in the range $v/c \approx 1/2$. This is therefore an important consideration.

It is known that the introduction of the displacement current into Maxwell's equations makes possible the phenomenon of radiation. We expect therefore that possible instabilities may arise when radiation is taken into account. A well-known example of such an instability is the classical behavior of the hydrogen atom. In the classical theory, the centrifugal acceleration of an electron in orbit around a nucleus causes it to radiate; the radiated energy is supplied from the potential energy the charge has by virtue of being a certain distance from the nucleus. Thus, as the electron radiates, it would tend to fall into the nucleus. That this effect does not take place is, of course, due to the fact that quantum rather than classical theory should be applied to this particular situation. In our case, classical considerations, however, are appropriate and we must therefore seek mechanisms of the type described. Before discussing such mechanisms in detail, however, it is important to distinguish between the incoherent and the coherent radiation of a group of N electrons moving under an acceleration a. For the incoherent radiation the total radiated power is:

$$P \propto N e^2 a^2 \tag{7}$$

On the other hand, if all N electrons take part in a coherent acceleration a, the radiation is

$$P \propto N^2 e^2 a^2 \tag{8}$$

Since in applications of this concept N is often in excess of 10^{15}, it is easy to see that it is the coherent effects which must be considered. The total power in the incoherent radiation in the devices considered is generally less than 1 watt.

It is fairly clear that the interaction of an electron beam with a coherent radiation field cannot be studied without reference to the properties of the containing vessel as a waveguide. For this reason a great many separate cases present themselves for study of which number we shall consider only one. We consider the geometry illustrated in Fig. 5. It consists of a long conducting cylinder, positively charged, surrounded by a crossed-field electron beam. A magnetic field is imposed on the whole device which is purely axial in direction. The geometry of the beam, i. e., its thickness, is supposed such that the beam is stable to the diocotron effect, and we consider the situation that results when the displacement current is not negligible. Any bunching of the electron beam will clearly result in a radiation field being set up. In the geometry illustrated the condition to be applied to the field at infinity is just the radiation condition that all waves present be outgoing. The object of the analysis is to study the effect of the radiation pressure on the dynamics of the beam. For this reason we have called our study "flexible antenna theory". Two possibilities can be distinguished: the energy supplied to the radiation field can come from the potential energy associated with the formation of the bunch: in this case the effect of radiation is to damp out the oscillation. In the second case, the energy of the radiation field can come from the potential energy of the electron beam, i. e., from the electrostatic field. In this case, the effect of radiation is to cause an instability.

The results of our study[11] are as follows: if the inner edge of the beam is detached from the surface of the cylinder (i. e., $a < b$) a single unstable mode exists for each azimuthal mode number. The real part of the frequency associated with this mode is on the order of ω_p^2/ω_c. The growth rate of the mode is given very roughly by the expression:

$$\frac{\omega_p^2}{\omega_c} \cdot \left(\frac{v}{c}\right)^2 \left(1 - \frac{a}{b}\right)^2 \tag{9}$$

The conclusion to be drawn from this result appears to be chiefly that the properties of the surface of the cylinder viewed as a waveguide are all important. It may be possible, by properly adjusting the effective admittance of the wall to damp out all unstable radiating modes.

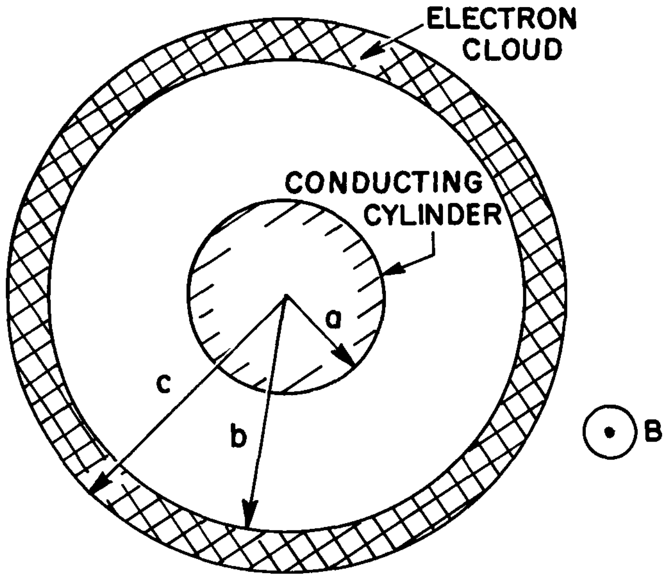

Fig. 5 This illustrates the geometry in which the radiative stability problem was studied. The electrons drift azimuthally around the cylinder and if they bunch, will emit an electromagnetic wave in the outer direction. The energy for this wave comes out of the energy of the electrostatic field, however, the instability is not present when $b = a$, i.e., the inner edge of the beam is in contact with the outer wall of the conducting cylinder.

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We conclude this brief survey of the present situation in low density crossed-field electron beam stability theory by listing without comment additional effects which might well be expected to provide growth mechanisms but which, at this date, remain largely uninvestigated. These are:

1) Magnetron work suggests that when $\omega_p^2/\omega_c^2 = 1$ instability inevitably results. In the limit $\omega_p^2/\omega_c^2 \rightarrow 0$, stability has been shown to be attainable. However, ω_p^2/ω_c^2 must, of course, be finite. Is there a range of small but finite values of ω_p^2/ω_c^2 for which stability can be assured?

2) In interesting applications, the ideal long straight cylinder is inevitably bent into a circular shape. This gives rise to non-uniformity in the magnetic field in any section, and these non-uniformities in turn give rise to particle drifts. In the presence of such drifts is equilibrium attainable, and if so, is the resulting stability picture changed from the ideal constant field approximation? This question is related to the use of the rotational transform in the stellarator.

3) When v/c becomes of order unity, two effects arise; one is the displacement current-radiation effect already dealt with. The other is that the electron beam dynamics are themselves modified, especially at high frequencies. Virtually nothing is known regarding effects of this kind.

4) If the surface of the vessel exhibits a small but finite resistance, growing waves may result; it seems appropriate to lump the AC properties of the containing vessel into a single complex admittance, and hence to combine this effect with that of other effects such as radiation. In connection with pure resistance, an instability of importance in particle accelerator design[12] has been effectively stabilized by using a feedback mechanism operating at RF frequencies.

To sum up, the stability picture is still quite cloudy. Experimental evidence, however, is encouraging and rather clear directions for future work on stability related problems can be established. On the whole, it may be said that the situation is not without hope, and it certainly seems a worthwhile exercise to try and estimate the properties of possible electron plasma devices on the assumption that stability will eventually ensue. It should always be kept in mind, however, that the results of such an exercise are of value only in relation to the answer to the stability problem. In the remainder of the paper, we shall assume the answer to this question in the affirmative, and see what can be done.

APPLICATIONS

Only two geometries are known in which it may be possible to attain very large potential differences by the scheme outlined. These are derived respectively from Figs. 1 and 2 by closing the sections shown on themselves in the axial direction. The former yields a device with the general form of a stellarator and is suitable for laboratory development. The second requires in the laboratory a support which must necessarily cross magnetic field lines. It is better adapted to use in space; in the latter configuration it has been proposed as a space radiation shield:[1] the high positive potential effectively repels energetic protons from solar flares, while the magnetic field requirement is relatively modest. Both these devices can be represented in a very rough way by the map, Fig. 6. On this map, the ordinate represents the total potential across the device. In the space ship, this potential exists between the surface of the ship, and the magnetic field line that coincides with the axis of the device. In the laboratory geometry, the same statement is true, though the field line in question now runs along the "minor" axis of the device and closes on itself.

To relate the potential of the device to its other properties, we must fix a size. We have chosen to fix this size at 1 meter for the minor radius, since this represents a dimension appropriate to large manned space ships. It would of course have been possible to do the same calculations for a lab device of 10 cm or 10 meters; the scaling between these sizes stems from Gauss' law and is straightforward.

Once given a potential and a size, we can estimate the electric field at the surface of the device by division. Knowing E, we know the lowest value B can take, namely the value E/c which would cause the electrons to drift at the speed of light. B must, of course, be greater than the value given by this limit. We have chosen for the abscissa of Fig. 6 essentially the ratio of the actual B to the B given by this limit, or put differently, the value of the drift speed at the surface of the device, normalized to the speed of light. Instead of plotting directly in this variable, we have however, chosen to use something uniquely related to it, namely the ratio of the kinetic energy of any particle moving at this speed, normalized to its rest mass. In this way, the region of interest near $v/c = 1$ is made to cover a larger part of the area of the graph. For reference, on the same abscissa, we indicate the corresponding values of v/c.

Having established the axes of the map, we can now plot lines on it corresponding to the various other parameters of the system. Thus, given the potential, the size, and the drift speed, we can calculate the magnetic field, the electric field, the number density and the ratio $\omega_p{}^2/\omega_c{}^2$. All these quantities are shown on Fig. 6. The following points deserve notice.

In the first place, it is clear the enormous potentials appear to be attainable. We recall the instance that the highest potentials available today are supplied by Van de Graaff machines which, in large sizes can make up to a few tens of millions of volts. Our map exhibits clearly the possibility of making potentials up to possibly a billion volts or higher.

In the second place, we observe that for a given magnetic field, a maximum in the potential appears when v/c is about $1/2$. This maximum arises as a consequence of the following considerations. When the electrons are moving very slowly, the total current in the electron beam is much less than the total current in the field coils. When they are moving at the velocity

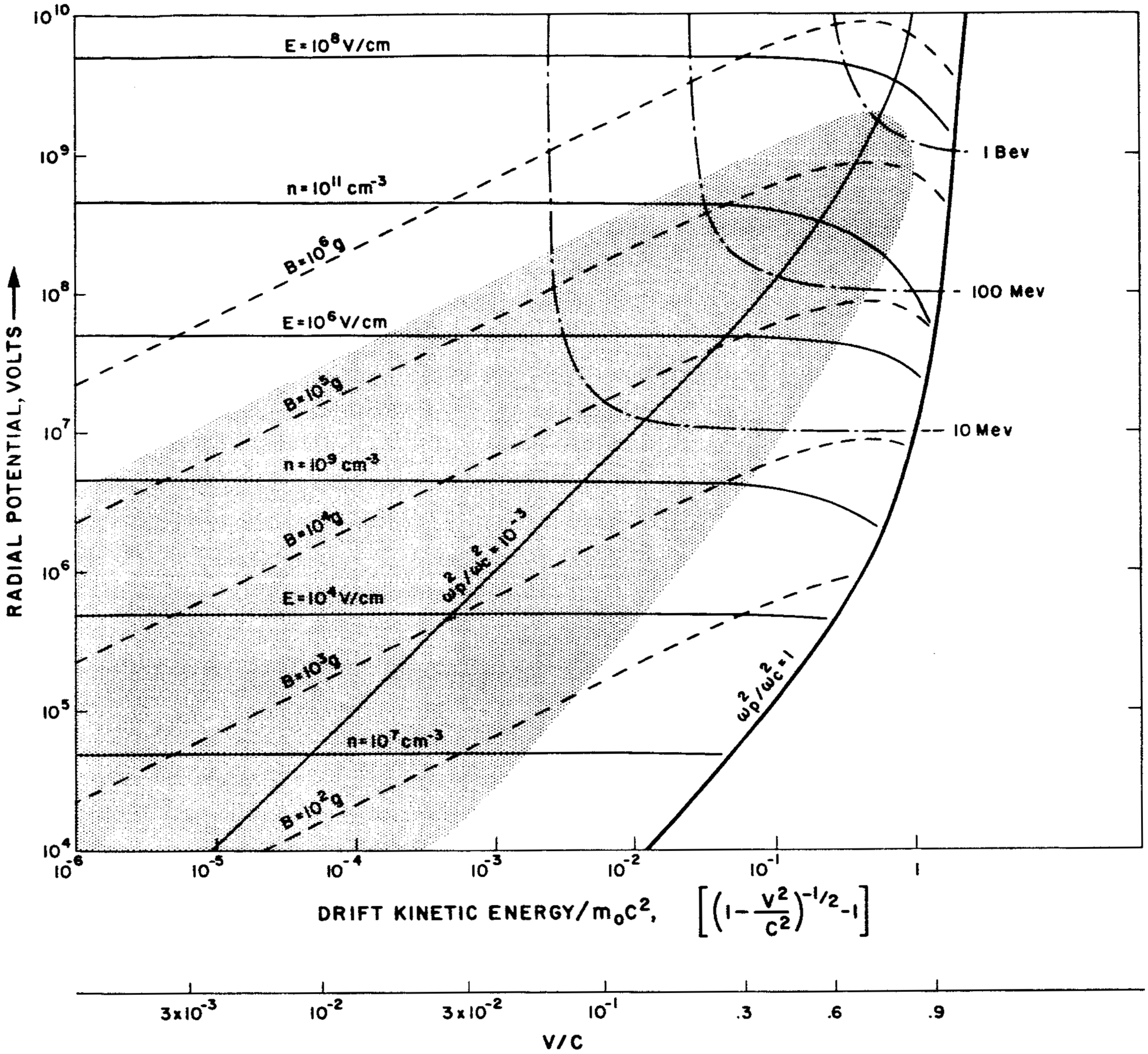

Fig. 6 This map illustrates characteristic parameters of electron plasma devices having a minor radius of 1 meter. The shaded area represents the region presently believed to be accessible. It is bounded on the one hand by practical magnetic field strength limits and on the other by an apparent stability limit when $\omega_p^2/\omega_c^2 \approx .05$. A proton injected into the lab device would acquire the full radial potential in dropping to the central axis unless the magnetic field is too strong, in which case it acquires only the drift energy corresponding to the E/B velocity. These limits are shown on the lines marked 10 MeV, 100 MeV and 1 BeV. Potentials far in excess of 10 million volts (the present limit of Van de Graaff machines) appear accessible.

of light, however, these two currents tend to approach each other, with the result that the magnetic field is reduced below the value it would have in the absence of the beam. In the limit, the magnetic field due to the coils is entirely cancelled and containment is no longer possible. Detailed consideration of this effect yields the maximum attainable potential as:

$$\phi = f\, acB \tag{10}$$

where f is a fraction of the order of 1/3, and a is the size of the device. This maximum occurs when v/c is roughly 1/2.

Finally, if we guess that a field of 100, 000 Gauss is the maximum attainable practical field strength, and that stability can be achieved whenever $(\omega_p/\omega_c)^2$ is less than about .05 (the latter being a distinct speculation), the area between the hatched lines becomes available. The possibilities available within these limits are clearly sufficiently exciting to warrant an intense effort, experimental and theoretical, to see whether the full scale devices implied by these figures can, in fact, be constructed.

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