Fitting CP Data for the Cosine Term

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In the Standard Model the weak decay amplitude - both tree and penguin - for $B^0 \to \psi K_S$ is real. This is easy to see by noting that the diagrams for these (in the case of the penguin, the dominant diagram) contain no vertices connecting a first generation quark to a third generation quark, the only vertices with phases (in the Wolfenstein parameterization). As a consequence there should be no direct CP violation, no difference between the partial rates for mesons beginning as B^0 and those beginning as $\overline{B}^0$. However, this prediction could fail if there is new physics contributing to the decay. While the mixing amplitude, which is due to a loop diagram, may be a more propitious place for finding a breakdown of the Standard Model, it is proper to analyze the data without assuming the result we seek to challenge.

Of course, this has been done in the Osaka presentation on $\sin 2\beta$. Here we only want to layout a thorough approach to the analysis and give a suggested means of displaying the result.

When there is direct CP violation as well as indirect CP violation (due to mixing), there are three sorts of time dependence in in the decay rates. In addition to the exponential times a constant and times $\sin \Delta m t$ there is an additional term with $\cos \Delta m t$.

To see how this comes about, consider the time development of states that at $t = 0$ are pure B^0 or $\overline{B}^0$ (dropping the exponential decay):

$$\begin{aligned} |B^0(t)\rangle &= \cos \tfrac{1}{2}\Delta m t|B^0\rangle + i\frac{q}{p}\sin \tfrac{1}{2}\Delta m t|\overline{B}^0\rangle \\ |B^0(t)\rangle &= i\frac{p}{q}\sin \tfrac{1}{2}\Delta m t|B^0\rangle + \cos \tfrac{1}{2}\Delta m t|\overline{B}^0\rangle \end{aligned}$$

As usual we write

$$\begin{aligned} \langle f|\mathcal{H}|B^0\rangle &= A \\ \langle f|\mathcal{H}|\overline{B}^0\rangle &= \overline{A} \end{aligned}$$

and

$$\lambda = \frac{q\overline{A}}{pA}$$

In the familiar case with just indirect CP violation we have

$$\left|\frac{q}{p}\right| = 1; \qquad \left|\frac{\overline{A}}{A}\right| = 1$$

The latter follows from the assumption that there is a single weak phase. Going from A to $\overline{A}$ gives a factor of η, the CP eigenvalue of the final state, and reverses the phases of the weak amplitudes. In the absence of such phases, A and $\overline{A}$ differ only by η (up to a phases convention on B^0 and $\overline{B}^0$, which we shall not belabor).

In general, however, we see that the time-dependent decay amplitudes are

$$\begin{aligned} \mathcal{M}(B^0(t) \to f) &= \cos\tfrac{1}{2}\Delta mtA + i\frac{q}{p}\sin\tfrac{1}{2}\Delta mt\overline{A} \\ &= A[\cos\tfrac{1}{2}\Delta mt + i\lambda\sin\tfrac{1}{2}\Delta mt] \\ \mathcal{M}(B^0(t) \to f) &= \cos\tfrac{1}{2}\Delta mt\overline{A} + i\frac{p}{q}\sin\tfrac{1}{2}\Delta mtA \\ &= \overline{A}[\cos\tfrac{1}{2}\Delta mt + i\frac{1}{\lambda}\sin\tfrac{1}{2}\Delta mt] \end{aligned}$$

For the decay rates we find

$$\begin{aligned} \Gamma(B^0 \to f) &\propto |A|^2\left\{\tfrac{1}{2}(1+\cos\Delta mt) + \tfrac{1}{2}(1-\cos\Delta mt)|\lambda|^2 - \mathrm{Im}\lambda\sin\Delta mt\right\}e^{-\Gamma|t|} \\ &\propto |A|^2\left\{\tfrac{1}{2}(1+|\lambda|^2) + \tfrac{1}{2}(1-|\lambda|^2)\cos\Delta mt - \mathrm{Im}\lambda\sin\Delta mt\right\}e^{-\Gamma|t|} \\ \Gamma(\overline{B}^0 \to f) &\propto |\overline{A}|^2\left\{\tfrac{1}{2}(1+\cos\Delta mt) + \tfrac{1}{2}(1-\cos\Delta mt)\frac{1}{|\lambda|^2} - \mathrm{Im}\frac{1}{\lambda}\sin\Delta mt\right\}e^{-\Gamma|t|} \end{aligned}$$

$$
\begin{aligned}
&\propto \quad |A|^2 \left\{ \tfrac{1}{2}(1+\cos\Delta mt) - \tfrac{1}{2}(1-\cos\Delta mt)|\lambda|^2 + \mathrm{Im}\lambda \sin\Delta mt \right\} e^{-\Gamma|t|} \\
&\propto \quad |A|^2 \left\{ \tfrac{1}{2}(1+|\lambda|^2) - \tfrac{1}{2}(1-|\lambda|^2)\cos\Delta mt + \mathrm{Im}\lambda \sin\Delta mt \right\} e^{-\Gamma|t|}
\end{aligned}
$$

where we have used

$$|\lambda| = \left| \frac{q\overline{A}}{pA} \right|$$

which follows from the usual assumption that $|q/p| = 1$.

There are, of course, two free parameters, aside from the uninteresting A. We can view these as $\mathrm{Re}\lambda$, $\mathrm{Im}\lambda$, or $|\lambda|^2$ and $\mathrm{Im}\lambda$, or simply as the coefficients A and B in

$$(1 \mp A\sin\Delta mt \pm B\cos\Delta mt)$$

with

$$A = \frac{2\mathrm{Im}\lambda}{1+|\lambda|^2}; \qquad B = \frac{1-|\lambda|^2}{1+|\lambda|^2}$$

Naturally, it is straightforward to fit for A and B in the context of tFit or CPExtract.

In practice we do not have perfect tagging. If we assume that the wrong tag fractions are the same for B^0 and $\overline{B}^0$, then the observed distributions will instead be

$$(1 \mp DA\sin\Delta mt \pm DB\cos\Delta mt)$$

Allowing another free parameter in the CP sector will increase the error on A. In the simple approximation in which we ignore resolution effects we can find the expectation for the covariance matrix by computing

$$
\begin{aligned}
V^{-1}_{AA} &= N\int_{-\infty}^{\infty} dt \left[\frac{1}{f_{B^0}} \left(\frac{\partial f_{B^0}}{\partial A} \right)^2 + \frac{1}{f_{\overline{B}^0}} \left(\frac{\partial f_{\overline{B}^0}}{\partial A} \right)^2 \right] \\
V^{-1}_{AB} &= N\int_{-\infty}^{\infty} dt \left[\frac{1}{f_{B^0}} \frac{\partial f_{B^0}}{\partial A}\frac{\partial f_{B^0}}{\partial B} + \frac{1}{f_{\overline{B}^0}} \frac{\partial f_{\overline{B}^0}}{\partial A}\frac{\partial f_{\overline{B}^0}}{\partial B} \right] \\
V^{-1}_{BB} &= N\int_{-\infty}^{\infty} dt \left[\frac{1}{f_{B^0}} \left(\frac{\partial f_{B^0}}{\partial B} \right)^2 + \frac{1}{f_{\overline{B}^0}} \left(\frac{\partial f_{\overline{B}^0}}{\partial B} \right)^2 \right]
\end{aligned}
$$

with

$$f_{B^0} = \frac{\Gamma}{4} e^{-\Gamma|t|}(1 - A\sin\Delta mt + B\cos\Delta mt)$$

$$f_{\overline{B}^0} = \frac{\Gamma}{4} e^{-\Gamma|t|}(1 + A\sin\Delta mt - B\cos\Delta mt)$$

The normalization is such that

$$\int_{-\infty}^{\infty} dt(f_{B^0} + f_{\overline{B}^0}) = 1$$

as required by the log-likelihood method.

If we make the approximation that we can expand in DA and DB, we find, to lowest order

$$\begin{aligned}
V^{-1}_{AA} &= ND^2 \frac{2x^2}{1+4x^2} \approx \frac{ND^2}{3} \\
V^{-1}_{AB} &= 0 \\
V^{-1}_{BB} &= ND^2 \frac{1+2x^2}{1+4x^2} \approx \frac{2ND^2}{3}
\end{aligned}$$

The covariance matrix is

$$\begin{aligned}
V_{AA} &= \frac{1+4x^2}{2x^2 ND^2} \approx \frac{3}{ND^2} \\
V_{AB} &= 0 \\
V_{BB} &= \frac{1+4x^2}{1+2x^2 ND^2} \approx \frac{3}{2ND^2}
\end{aligned}$$

so

$$\begin{aligned}
\sigma_A &= \sqrt{\frac{3}{ND^2}} \\
\sigma_B &= \sqrt{\frac{3}{2ND^2}}
\end{aligned}$$

Perhaps the most straightforward way to present results is to show contours of χ^2 in the complex λ plane. The standard model result lies on the curve $|\lambda| = 1$, at $\lambda = e^{2i\beta}$. With high statistics, the contours will be ellipses, but with the current statistics, more complicated curves are likely.

The Osaka data sample showed some asymmetry in the number of B^0 and $\overline{B}^0$ events, 70 vs. 50, but nothing beyond a possible statistical fluctuation. What we see above is that a real direct CP violation must be seen not just in the total numbers of events, but in the time distribution as well. In particular, it is not permissible to fit the data assuming the presence of both sin and cos terms and leaving the relative normalization of the B^0 and $\overline{B}^0$ events free as well. This introduces one more free parameter than is actually present. It is important to remember that just as for A, in extracting B, a correction needs to be made for the dilution, which increases the size of the extracted parameter.