PLANETARY WAVES AND THEIR INTERACTION WITH SMALLER SCALES

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1. INTRODUCTION

Historically, dynamical meteorologists have grown comfortable with two classes of idealized dynamical systems. On the one hand, our understanding of baroclinic instability has evolved by studying the linear and nonlinear development of eddies on the scale of the deformation radius in simple larger-scale environments. On the other hand, there has been a continuing interest in planetary wave theory, aimed at understanding the climatological stationary waves, and a growing interest in the distinctive low-frequency variability observed on these larger scales. The realization that planetary scales have a potentially intricate dynamics of their own has encouraged the use of models in which instability on smaller scales is absent, so that this planetary dynamics can be isolated.

Two examples of papers that have heightened interest in the dynamics of planetary waves in isolation from deformation-scale eddies are Charney and DeVore (1979) on the possibility of multiple equilibria in nonlinear models of topographically forced waves, and Simmons, Wallace, and Branstator (1983) on the barotropic instability of the wintertime planetary wave field. These two papers also serve as examples of two rather distinct streams in research on planetary-scale dynamics -- one focusing on the details of the linear dynamics around some mean flow, and the other taking a nonlinear systems perspective as its starting point.

We are much less comfortable with the complexity of models in which synoptic and planetary scales are both fully active. Our theoretical tools are typically less powerful in this more complex setting. For example, there have been attempts to generalize wave-mean flow interaction theory to the case of zonally asymmetric basic states, but the results are less useful than in the symmetric case. In particular, there typically is no simple relation between "non-conservative" effects on the eddies and their feedback onto the mean flow, as there is for waves on a zonally symmetric flow (Plumb 1986). One might hope that WKB theories dependent on scale separation between the waves and the mean flow would allow one to use some of the insights gained from the study of evolution on zonally symmetric basic states, and such theories are useful on occasion, but one rarely has much scale separation to work with.

The problem of the mutual interaction of planetary and deformation-scale eddies is being attacked from a variety of directions: linear instability and error growth are being studied in ever more complex flows; barotropic models are being used for studying wave breaking scenarios in different larger-scale environments; the simplest two-layer quasi-geostrophic model has a rich dynamics when planetary and synoptic waves are both present, of which we have had a few glimpses; and idealized GCMs are a powerful, as yet nearly untapped source of information about the interaction between storm tracks, low frequency variability, and stationary waves. While there is important ongoing work in all of these areas, much effort will be required to determine the relationships between different classes of models. A critical review at this time seems too daunting a task.

Instead, in the following I try to discuss some confusing aspects of planetary wave dynamics, most of which involve the interaction with smaller scales -- starting with the perspective provided by the classical ideas of geostrophic turbulence. My continuing confusion will be apparent in places.

2. SCALE SEPARATION AND GEOSTROPHIC TURBULENCE

As one moves to larger scales in midlatitudes, the flow becomes more wave-like and, relatively speaking, linear. The simplest explanation for this tendency is based on the Rossby wave dispersion relation, and the comparison between typical eddy velocities u' on a given horizontal scale k^{-1} and the intrinsic Rossby wave phase speed, $c - U = - \beta/k^2$. Streamlines will overturn in the horizontal plane, in a frame of reference moving with a steady wave, if $u' > U - c$. Therefore, one might hope to use the scale

$$k_\beta = (\beta/u')^{1/2} \tag{1}$$

at which this ratio is unity as the dividing line between planetary waves and strongly nonlinear eddies.

As described by Rhines (1975), k_β is also the scale at which a barotropic inverse energy cascade will be halted by the β-effect, for it is at this scale that the nonlinear interactions weaken. The homogeneous turbulence picture of this cascade is familiar. Most importantly, the spectrum peaks at the scale at which the cascade stops, and not where it is being "stirred" by baroclinic activity near the deformation scale λ. The cascade stops when the energy-containing eddies become marginally linear; waves with $k < k_\beta$ are excited inefficiently and selectively.

It is of central importance to the Earth's atmosphere that the inverse energy cascade is, in fact, quite modest, and does not extend over a significant range of scales: the difference between k_β and λ^{-1} is subtle. If we want to understand why the cascade is weak, we have to understand the eddy energy level in the atmosphere. Recent work of V. Larichev and the author (Larichev and Held 1995, Held and Larichev 1996, HL hereafter) suggests that one should expect $u' \sim U_T(k_\beta\lambda)^{-1}$ if the cascade is stopped by the β-effect rather than the size of the domain, where U_T is the magnitude of the thermal wind through the troposphere, so that

$$k_\beta\lambda \sim (\beta\lambda^2)/U_T. \tag{2}$$

Since the RHS of this expression is of order unity in our atmosphere ($U_T = \beta\lambda^2$ is essentially the critical shear for instability in Phillips' two-layer model), this scaling is consistent with the fact that $k_\beta\lambda \sim 1$. Whether or not this scaling is completely accurate, it is clear that a very substantial cascade requires $u' >> U_T$, which is not observed.

The estimate (2) of HL derives from a study of homogeneous turbulence, but similar results are found for baroclinically unstable jets. In particular, in the two-layer unstable jet model of Whitaker and Barcilon (1995) the wavenumber of the energy containing eddies is also found to be approximately proportional to β. Whether the dynamics underlying the choice of this scale is the same in homogeneous and jet-like flows is still unclear. In particular, Whitaker and Barcilon argue that the linear growth rates for unstable normal modes continue to play an important role in this scale-selection, even when the flow is strongly nonlinear, while these linear growth rates play no role in the discussion of HL.

The wavelength of a stationary external Rossby wave is $k_s = (\beta/U_b)^{1/2}$, where U_b is the mean zonal wind weighted in the vertical by the amplitude of the external mode geopotential (e.g., Held et al. 1985). But in the atmosphere U_b is comparable to U_T. If we take $u' \sim U_T \sim U_b$, then, $k_\beta \sim k_s$. From the perspective of this crude scaling, we might expect the inverse cascade to carry energy from eastward propagating into quasi-stationary disturbances, perhaps, but not much further. However, this same scaling suggests that there is nothing fundamental that requires the energy containing eddies to be quasi-stationary. More generally, by accepting (2) we have

$$(\beta/k_\beta^2)/U_T \sim U_T/(\beta\lambda^2), \tag{3}$$

implying that the energy containing scales would propagate westward if the thermal wind were strong enough.

The cascade appears to be particularly weak in Southern summer. The eddy kinetic energy is often sharply peaked at zonal wavenumbers 6-7, and can be organized into coherently propagating wavepackets (Lee and Held 1993). Occasionally, energy makes its way to wavenumber 5 which can then dominate the circulation, evolving more slowly and propagating eastward at ~5 m/s (Salby 1982, Hamilton 1983). The flow appears to be more wave-like and "quasi-linear" when this occurs.

The larger energy levels in Northern winter imply a somewhat stronger inverse cascade; even in the absence of external sources of planetary waves, these energy levels should result in disturbances of larger scale that are quasi-stationary or even westward propagating. Of course, orography and quasi-stationary heating are a major source of planetary waves in Northern winter, primarily at scales near k_s within the troposphere. In addition, the weak low-level static stability in the eddy generation regions of the oceanic storm tracks favors the growth of smaller scale eddies, as does latent heat release. The result is a sense of scale separation, by a factor of two perhaps, between planetary and synoptic scales. But there is no clear energy gap between the scale of the quasi-stationary forced waves and the scales reached by the inverse energy cascade, and this is a major source of complexity.

It should also be emphasized that one can get a sense of scale separation between energy-containing "long waves" and weather-producing "short waves" even in an atmosphere with a single peak in its energy spectrum. The baroclinic eddies that produce the greatest vertical motion and divergence, and the most weather, will naturally tend to be of somewhat smaller scale than the eddies with the most energy, and standard quasi-geostrophic scaling tells us that this distinction can be substantial, since

$$w' \sim (H/L)R_o u' \sim k^2 \qquad (4)$$

for a fixed strength of the horizontal flow and vertical scale (here R_o is the Rossby number). The spectrum of the variance of vertical velocity differs by a factor of the *fourth* power of the wavenumber from that of the horizontal velocity.

The absence of clear scale separation suggests that we still have to contend with turbulent cascades when thinking about the interaction between these planetary waves and deformation-scale eddies, even when stationary forcing is providing a large direct source of planetary waves. In particular, the turbulence perspective reminds us not to be surprised to find deformation-scale eddies reinforcing the kinetic energy of structures with larger scale. Blocking anticyclones are an important example. A tension exists in the literature on blocking between the role played by the large scale flow itself, as it falls into a coherent blocked state with intrinsically small time tendency, and the role played by eddy driving. Given our dissipative and turbulent atmosphere, it seems a safe bet to assume that both are required for a long-lived pattern: a large-scale coherent structure *and* positive reinforcement from smaller scales. If the large-scale pattern has a strong tendency to evolve, there is little hope of creating a long-lived anomaly, whatever the interaction with small scales, while a coherent large-scale structure that is not actively maintained will quickly dissipate. In the idealized barotropic models in which eddies are successful in maintaining a blocked flow, the flow itself is generally close to a stationary state (Shutts 1983, Anderson 1995).

Jupiter's Great Red Spot (GRS) is a useful analogy. There is clearly an underlying coherent structure, but it is actively maintained by an inverse energy cascade as well. The manner in which the anticyclonic vorticity is resupplied is distinct in Jovian and terrestrial cases, however: the GRS swallows smaller anticyclonic vortices that are trapped in the same latitude belt; in terrestrial blocks the PV is instead resupplied by baroclinic waves that sweep in lower latitude air as they approach the block.

The distinction between free and forced structures is of central importance to planetary wave-synoptic eddy interaction. If one forces some zonally asymmetric structure on the atmosphere, the storm tracks will be deformed, and the resulting PV fluxes may or may not feed back positively on the forced eddy. But from the perspective of the inverse energy cascade, we *expect* to see *free* large-scale structures generated and maintained by smaller eddies.

In addition to the inverse kinetic energy cascade, there is a less familiar direct cascade of available potential energy. In baroclinic geostrophic turbulence models with a substantial inverse cascade, there is a sharp distinction on large scales between the barotropic energy, which cascades upscale, and the baroclinic energy, which cascades downscale (Rhines 1977, Salmon 1980, Hoyer and Sadourny 1982). On scales much larger than the deformation radius, the baroclinic energy is essentially the available potential energy, or temperature variance. To the extent that the inverse cascade proceeds for a large range of scales, the flow becomes more and more barotropic, and the baroclinic potential vorticity (or the temperature at the surface and tropopause) are *passively* advected by this barotropic flow. The advection is passive because these fields are not inducing the bulk of the flow that is advecting them. Just as for any passive tracer, the temperature variance and available potential energy cascade to smaller scales. This cascade changes its character at the deformation radius, where the flow is no longer predominately barotropic, and the advection is no longer passive.

Much of the atmospheric flow on planetary scales is *equivalent* barotropic and not fully barotropic. Yet some of this flavor carries over to the atmosphere. The planetary scales, whether excited externally or by an inverse energy cascade, deform the surface and tropopause temperature fields. The smaller scale, weather producing eddies see this deformed baroclinicity and, in their distinctive way, generate a cascade of temperature to even smaller scales as they simultaneously remove some of the baroclinic energy and transfer it to the barotropic component of the flow, re-energizing the inverse cascade. We speak of short waves as steered by and extracting available potential energy from the planetary scales, but perhaps it is useful to think of this interaction as part of a more continuous downscale cascade of *baroclinic* variance. On the smaller scales on which the surface and tropopause disengage, frontal instabilities can be thought of as another step in the cascade of surface temperature variance to small scales.

3. EXTERNAL ROSSBY WAVES

Given that planetary-scale motions are somewhat Rossby wave-like, we are obligated to take a close look at the relevant dispersion relation. Most planetary waves of interest are stationary or quasi-stationary, and are strongly affected by horizontal and vertical shears in the mean zonal flow. At first glance, it is not self-evident which dispersion relation to focus on. To help decide, it is useful to think about the linear stationary response to a localized source (thermal or orographic) in the midlatitude westerlies.

A localized source excites disturbances of all scales, and the largest of these propagate through the tropospheric westerlies into the stratosphere. As one moves farther from the source, this vertically propagating component becomes less important than the part of the spectrum trapped in the *tropospheric planetary waveguide*. As the disturbance propagates downstream, the vertical structure within the troposphere quickly settles into the dominant vertical mode within this waveguide, the external Rossby wave. External Rossby wavetrains then move downstream as they are refracted by the spherical geometry and horizontal shears.

Explicit in this description is the claim that it is useful to think about modal structures in the vertical (the external Rossby wave, at least), but not in the horizontal. This is partly a question of time scale -- the wave feels the vertical waveguide structure much sooner than it feels the global constraints on the horizontal structure. But more importantly, many disturbances never feel these global constraints at all; external mode wavetrains are often refracted into the tropics, where they break and can be efficiently absorbed. While there may be cases where quasi-stationary waves are effectively trapped in the large PV gradient along a strong jet, it seems to be a useful approximation, for quasi-stationary waves, to assume absorption due to wave-breaking in the tropics. Results consistent with this picture are obtained when idealized GCMs with a zonally symmetric climate are perturbed by a localized mountain providing a relatively simple localized source of stationary waves. As described in Cook and Held (1992), the dominant ray paths change as the amplitude of the mountain increases (the "mountain" in this work has the scale of the Tibetan plateau), but in all cases the wavetrains propagate rather sharply into the tropics and do not emerge. More recent higher resolution calculations confirm this picture.

Rapidly westward propagating waves, such as the "5-day wave", the gravest symmetric wavenumber 1 Rossby mode, hardly feel the horizontal and vertical shears and do not break in the tropics, and so are free to settle into global modal structures. These are clearly waves with "$k < k_\beta$". There exist several good summaries of the modal structure of the westward propagating variance (Madden 1979, Salby 1984, Venne 1989). As the westward phase speed of a disturbance decreases, the modal picture based on a zonally symmetric basic state becomes less useful; the "16-day wave", the second gravest symmetric wavenumber 1 mode, may be marginal in this sense, as argued by Branstator and Held (1995).

Are there any "internal" Rossby waves? There are vertically propagating waves, of course, but for realistic parameter settings there are no other neutral modes trapped in the tropospheric waveguide (Held et al. 1985). This has numerous consequences, the first of which is that quasi-stationary waves, removed from their source, should have a single vertical structure, that of the external mode. Therefore, we expect an equivalent barotropic structure with no nodes in the vertical, as long as the wave remains in the westerlies. The external mode's geopotential increases with height in the troposphere, and has maximum amplitude near the tropopause, resulting in cold lows and warm highs. A good rule of thumb exists as a consequence: warm lows are locally forced; warm highs are not.

One can understand this vertical structure by considering the dominant balance in the temperature equation for a wave on scales larger than the deformation radius:

$$(U - c)\, \partial_x \partial_z \psi - \partial_x \psi \partial_z U \sim 0, \qquad (4)$$

which implies that $\psi \sim U - c$. In the frame of reference of the wave, the zonal advection of the perturbation temperature $\partial_z \psi$ is primarily balanced by the meridional advection of the mean temperature (with gradient $\partial_z U$) by the perturbation meridional flow $\partial_x \psi$: the high must be warm for the eastward advection of the warm air to balance the cooling in the northerlies to the east of the high. The effects of vertical motion are relegated to a secondary role on these scales.

The external mode dispersion relation in vertical shear, and the accompanying modal structures, have not received as much attention as they deserve, given their central importance to planetary wave structure in the atmosphere. Due to the equivalent barotropic structure, we have been drawn towards thinking in terms of the simplest barotropic models, and have learned a great deal. But frankly, I remain confused as to how we have gotten away with it!

For the scales of most interest, those that produce stationary or low frequency disturbances, external Rossby waves do not so much resemble simple barotropic Rossby waves as they do baroclinically unstable waves. The implications of this fact are, I think, most easily understood by thinking in terms of the pseudomomentum of the wave.

The pseudomomentum M is a quadratic measure of wave amplitude that is conserved in adiabatic, inviscid flow, and that tells one how the wave modifies the zonal mean flow when it propagates from one region to another or is dissipated. The flux of pseudomomentum in the meridional-vertical plane is also known as the Eliassen-Palm flux (Edmon et al. 1980). The pseudomomentum of a barotropic Rossby wave at a given latitude is simply

$$M = - \{\eta'^2\}\ \beta/2 \qquad (5)$$

where η' is the meridional particle displacement (the meridional displacement of a streamline in a steady wave). The brackets are an average over the phase of the wave. More generally, in the case of an unbounded atmosphere,

$$2M = \{\eta'^2\}\ (f^2/N^2)\ \partial_z U \big|_{z=o} - ?\rho\{\eta'^2\}\ \partial_y Q\ dz \qquad (6)$$

where $\rho = e^{-z/H}$. In a flow without horizontal curvature, the potential vorticity gradient is $\partial_y Q = \beta - f\rho^{-1}\partial_z(\rho S)$, where S is the isentropic slope; therefore, if we idealize the tropopause as a discontinuity in isentropic slope, $\Delta_T S$, we can separate off its Eady-like contribution:

$$2M = \{\eta'^2\}\ (f^2/N^2)\ \partial_z U \big|_{z=o} - \{\eta'^2\}\ f\ \Delta_T S\big|_{z=trop} \qquad (7)$$
$$- ?\rho\{\eta'^2\}\partial_y Q\ dz$$

The remaining integral is understood to extend over the smooth part of the PV gradient. In an unstable baroclinic mode, the pseudomomentum must sum to zero; otherwise it would increase exponentially, which is impossible, since it is conserved. The positive contribution from the particle displacements at the surface exactly cancel the negative contribution from the interior (including the tropopause).

The pseudomomentum of a quasi-stationary external Rossby wave is not zero, but it is small. The negative contribution from the interior is dominant, so the mode's dynamics can be thought of as being somewhat more strongly controlled by upper level PV evolution than by the surface, but the compensation is close. Using mean shear and stability appropriate for midlatitude winter, one finds that a wave moving 5m/s westward with respect to the surface wind has roughly 80% of its interior pseudomomentum compensated at the surface. By varying the phase speed of the wave, one find that the phase speed must be westward and of magnitude comparable to the tropospheric thermal wind before the compensation drops below 50%.

When this compensation is strong, it is easy to excite the wave to large amplitude. To see this, define the operator M so that the pseudo-norm $\{\eta, M\eta\}$ is the expression in (6) or (7). The external mode is orthogonal to all other modes using this weight (Held 1985). Therefore, an arbitrary initial condition $\eta_i = A\ \eta_e + \ldots$ can be projected onto the external mode by setting

$$A = \{\eta_e, M\eta_i\}/\{\eta_e, M\eta_e\}. \tag{8}$$

Suppose, for example, that η_i is confined to the upper troposphere. Then there will be no compensation between positive and negative values in the numerator; but there will be in the denominator. Therefore, the amplitude of the particle displacements in the external mode will be larger than those in the initial condition. The energy required for this growth is extracted from the mean available potential energy, just as in normal mode baroclinic instability. Other perspectives on this non-normal growth have been discussed by Farrell (1984).

In thinking about synoptic eddies, we are accustomed to focusing equally on upper tropospheric and surface disturbances. We should do the same when thinking about planetary waves. Focusing only on upper tropospheric PV, or thinking in terms of barotropic dynamics, overemphasizes the difficulty in exciting the external mode, and prevents one from appreciating that much planetary wave development must have a baroclinic character.

The fact that a wave as a whole has pseudomomentum of opposite sign to that of the surface disturbance may have other interesting implications. It is reasonable to assume that smaller-scale eddies try to damp the surface temperature signal in a planetary wave through their low level PV (or heat) flux. Held et al. (1986) show that the net effect of this low-level damping is to increase the amplitude of an external mode wavetrain. If the eddy damping is localized, the argument predicts larger amplitudes downstream of the damping. One can think (counter-?)intuitively of the damping as reducing the positive M near the surface, but the disturbance as a whole has negative M, so this actually increases its amplitude. One can also think of the damping as creating a vertical phase shift that extracts more energy from the basic state. Quantitatively, this does not appear to be a very important effect for external mode wavetrains propagating from the tropics to higher latitudes, perhaps because their group velocity is large enough that they pass through the midlatitude storm track rather quickly. This catalytic effect of low level thermal damping may be of greater importance for localized blocks. To the author's knowledge, the potential importance of downgradient near-surface diffusive heat flux for maintaining blocks in baroclinic mean flows has not been examined.

Throughout the preceding discussion, the β-effect has played an important role. This is probably troubling to synopticians who see the β-effect as being at best of marginal importance on synoptic scales. Tropospheric PV-gradients are poorly defined and rarely resemble β. One expects baroclinic waves to mix PV near their midtropospheric steering levels, and they may be fairly successful at homogenizing it is some layers of the troposphere (e.g., Sun and Lindzen 1994). Therefore, it is important to ask if the external mode is sensitive to the tropospheric PV distribution.

Fortunately, it is not, as described by Swanson and Pierrehumbert (1995). If one starts with a profile in which the vertical shear and N^2 are independent of height, and then adjusts N^2 so as to counterbalance β and produce uniform PV, the changes in vertical structure of the mode are very small, as are changes in its wavelength. The compensation between the positive and negative contributions to the pseudomomentum of the wave is also nearly unchanged, even though the negative part is now concentrated at the tropopause.

In this homogenized atmosphere, the external mode feels both the δ-function PV gradients at the surface and the tropopause. Because of the elliptic relation between PV and the flow, the wave feels some average value of these gradients. For a flow with vertical shear only, the vertically averaged quasi-geostrophic PV gradient equals β (if one integrates through the lower boundary, with the standard convention that the vertical shear vanishes beneath the surface), so one should not be totally surprised by the insensitivity to the redistribution of the gradient. The degree of insensitivity is impressive nevertheless. It is worth noting that the argument for the vertical structure of the mode based on (4) does not make any reference to the PV distribution or N^2.

4. RELEVANCE OF THE TIME MEAN FLOW

Given the strong zonal asymmetries in the climatology of the Northern Hemisphere, one is led to consider the dynamics in the neighborhood of, or even linearized about, a zonally asymmetric flow resembling the time-mean. How much can we learn about storm tracks, low-frequency variability, and the response to changes in boundary conditions within this linear framework? To what extent is this placing too much dynamical weight on the time mean flow?

To fix ideas, consider the extratropical response to the tropical heating anomaly associated with an El-Nino event. From observational and modeling studies, the mean response in wintertime has the appearance of an equivalent barotropic wavetrain arcing across the North Pacific and North America (Horel and Wallace 1981). The simplest interpretation of this pattern (Hoskins and

Karoly 1981) a linear stationary external Rossby wave propagating on the zonal mean winds, directly forced by tropical heating, is very appealing but has proven to be insufficient for several reasons.

This inadequacy is most clearly shown by experiments with a variety of models (from barotropic, Simmons et al. 1983, to GCMs, Geisler et al. 1985) which show that the extratropical response is not simply translated zonally when one shifts the longitude of the tropical heating. Rather, there is a preferred region in which the response is large, the PNA (Pacific-North American) region. This result has motivated the study of models linearized about asymmetric basic states. While this may seem like a small modification to the theory, it raises the level of complexity substantially. In particular, nearly neutral normal modes that owe their existence to the zonal asymmetries of the basic state are created with which the stationary forcing can resonate. Despite the added complications, one is tempted to pursue this approach since a linear, or weakly nonlinear, model would allow one to go rather far in analyzing the details of the response.

But the planetary wave pattern itself undergoes dramatic fluctuations within a season which are of more than enough amplitude to call into question the linearization about the time mean. One can raise the same concern about models linearized about zonally symmetric states, but variability in the zonal mean is less dramatic than in the planetary wave pattern. As a result, interest shifts to the structure of the probability distribution of the planetary wave pattern. If this distribution is more or less normally distributed about the mean, there might be some hope of justifying the study of dynamics in the neighborhood of the mean. In the other extreme, the picture in Figure 1 of the response to El-Nino is a possibility (e.g., Molteni et al. 1993). The principle response to the change in external conditions in this schematic is in the occupancy rate of two distinct "regimes". Under these circumstances, it is unlikely that linearizing about the time mean, which is not visited very often, will be of much value. While there is no question that some idealized models will behave in this way when perturbed, the jury is still out with regard to more realistic models and the atmosphere itself. A bimodal distribution is not needed to make this kind of picture relevant. A skewed distribution in which the mode differs significantly from the mean is sufficient, especially if the skewness changes when the system is perturbed.

Attempts at estimating the probability distribution of the large-scale flow continue to be controversial (Cheng and Wallace 1993, Kimoto and Ghil 1993a,b, Hansen and Sutera 1986, Nitsche et al. 1994). Verifying model results against observations will be difficult if the details of the distributions are important, given the length of the time series needed to estimate these distributions.

In addition to the problems introduced by this potential for non-normal planetary wave statistics, several linear modeling studies of the extratropical response to tropical heating in GCMs and observations suggest that another ingredient must be taken into account: the response of the eddies in the Pacific storm track (Kok and Opsteegh 1985, Held et al. 1989, Hoerling and Ting 1994). Let's take a moment to consider what such a linear diagnosis can and cannot tell us.

Suppose that our model is

$$\partial_t A_i = N_{ijk} A_j A_k + Q_i, \tag{9}$$

so that

$$0 = N_{ijk}[A_j][A_k] + N_{ijk}[A_j'A_k'] + [Q_i], \tag{10}$$

where brackets represent the time mean and primes the deviations from this mean (transient eddies). Denoting the unperturbed climate by the subscript zero, and the perturbation in the climate by an asterisk, we have, in response to a perturbation in [Q],

$$\begin{aligned} 0 = {} & N_{ijk}([A_j]_0[A_k]_* + [A_k]_0[A_j]_* + [A_j]_*[A_k]_*) \\ & + N_{ijk}[A_j'A_k']_* + [Q_i]_* \end{aligned} \tag{11}$$

Suppose that the final two terms, the anomalous transient eddy fluxes and the anomalous forcing, are both thought of as given from models or observations. By disregarding the third term on the first line (stationary nonlinearity) one can solve the resulting linear problem for the perturbed climate. More ambitiously, one can iterate to a steady nonlinear solution so as to take this term into account. In the few partial diagnoses of El-Nino responses that have been attempted along these lines, the direct response to tropical heating and the response to modified transient eddies are generally both important. The anomalies in the upper tropospheric momentum fluxes are the dominant part of the forcing by transients.

There is much that is confusing about this storm track response and its back effect on the mean flow. For example, using an idealized GCM in which the unperturbed climate is zonally symmetric, Ting and Held (1990) find that the storm track eddies play only a minor role in shaping the response. Our suspicion is that the eddy momentum fluxes must be concentrated in the barotropic decay region of a localized storm track, and this barotropic decay region must be strongly modified by the perturbation, for these fluxes to play an important role. Recent work of Branstator (personal communication) appears to be consistent with this picture: in a GCM with realistic boundary conditions, he finds that forcing by anomalous transients is dominant for heating anomalies in the central Pacific, while the direct response to heating is dominant for anomalies in the western Pacific.

If a theory could be found for the storm track eddy fluxes as a function of the time-mean flow, then (11) would provide a deductive explanation for the perturbed climate, given $[Q]_*$. By linearizing this functional dependence, the eddy fluxes would become part of the linear operator, rather than the forcing. Branstator (1995) has examined a particularly simple algorithm for this purpose and tested it against GCM fluxes (although not in the context of an El-Nino simulation). He linearizes the GCM dynamics about a given mean flow, then performs multiple integrations of this linear model for a specified length of time (which can be thought of as a fitting parameter) initializing with noisy initial conditions. The predicted eddy statistics are obtained by averaging over this ensemble. The results look promising.

The technique is closely related to that advocated by Farrell and Ioannou (1994), in which a model linearized about the time mean is driven stochastically. Extra damping is included so as to stabilize the linear model. The stochastic stirring and the damping are both thought of as mimicking the effects of nonlinearities, and the strength of the damping plays a role similar to that played by the length of the linear integration in Branstator's algorithm. As it is a direct attempt at simulating a statistically steady state, this approach may be more amenable to the elaboration of theories for the structure and parameter dependence of the required stirring and damping. But the hope would be that one could extract useful information from such a linear model without any detailed understanding of this nonlinearity.

But what if the actual dynamics of the planetary scale response is that pictured in Figure 1? Presumably, the anomalous transients would show up as an important forcing term in a diagnosis based on (11), but the implication would be that one could not understand this change in transients in the context of a model linearized about the time mean, so that the stationary wave model would be of little use in predicting the response. For that matter, one could argue that the response of the mean flow is of no great interest under these circumstances. Whether one could ever predict or "understand" the changes in probability distribution of interest in this case without conducting the full model computation, is unclear. On the other hand, to the extent that the program outlined in the previous paragraph is successful -- if a robust algorithm can be developed for predicting transient eddy fluxes given the mean flow, which can then be used in the steady state model (11) to predict the mean response -- one could argue that the complexity of regime diagrams and probability distributions will be of secondary importance for understanding the perturbed climate. It will be interesting, and important, to see which of these perspectives emerges as most useful. Our understanding of the response of planetary waves and storm tracks to global warming, as well as to El-Nino, may be strongly influenced by the outcome.

To further confuse the reader about the physical meaning of a "mean flow" and of the distinction between high and low frequency transients, consider Figure 2. (The following discussion was motivated by discussions with I. Orlanski and R. Panetta.) The upper panel shows a simple vortex street -- a double row of isolated vortices, with the row of cyclones displaced northward from the row of anticyclones. The center panel shows the streamfunction associated with this vorticity distribution, assuming two-dimensional flow. In addition to the localized highs and lows, there is a westerly jet along the center line of the vortex street.

Suppose that this pattern is translating eastward. A time mean flow then exists whose main feature is the westerly jet along the center line. The transient eddies, the deviations from this mean, are displayed in the lower panel (by symmetry, the time mean is equal to the zonal mean). The transient highs and lows are now nearly symmetric about the center line. Furthermore, if this entire pattern undergoes a north-south oscillation on long time scales, there will be a low-frequency flow, an oscillating jet, whose movement will be exactly correlated with the north-south movement of the variance of the high frequency transients.

As observed by Wallace et al. (1988), when one removes the time-mean flow before examining the storm track eddies in the atmosphere, one finds highs and lows that are symmetrically placed along the storm track axis, just as in the lower panel of Figure 2. It is only when one adds in the time mean flow that one sees the synoptically familiar latitudinal displacement of the highs and lows. The transients so defined give the appearance of being more linear and wave-like. This appears to be a legitimate motivation for removing the mean, at least to the extent that linear models are useful in explaining these symmetric transient eddy statistics. But in the case of the vortex street, it is clearly more natural to view the entire pattern as a coherent structure. (While the flow as pictured is not an exact solution of the barotropic vorticity equation, with some effort it could be so modified; the vorticity patches would no longer be circular.)

Lau's (1988) analysis confirms impressions based on familiarity with daily weather that the movement of the variance of the high frequency geopotential height (the storm tracks) is strongly correlated with the variations in the low frequency flow itself. Indeed, this correlation is so strong as to suggest that one is really looking as two parts of the same pattern. Ting and Lau (1993) and Branstator (1995) show that an impressively large fraction of the low frequency variability of the flow in a GCM can be thought of as "forced", in the sense of (11), by vorticity fluxes generated by the high frequencies. But a simpler picture suggested by the vortex street analogy is that this diagnosis is confirming the internal consistency of a coherent structure (speaking loosely) that includes time-varying and steady parts. Low frequency variations in this structure produce coherent low frequency variations in the flow and in the variance of the high frequency eddies.

5. A FINAL REMARK

We have to be alert to the possibility that our analysis tools produce distorted descriptions of the atmosphere that, in isolation, can lead to misinterpretation of the underlying dynamics. Given this potential for distortion, and the complexity of atmospheric flows, a variety of perspectives, tools, and models are clearly required. And we must not be shy about ranging widely into "un-atmospheric" fluid dynamical regimes to test our understanding. Even configurations as extreme as homogeneous geostrophic turbulence have much to tell us about planetary wave-synoptic eddy interactions.

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FIGURE CAPTIONS

Fig. 1. A possible schematic of the probability distribution of the NH winter planetary wave pattern: unperturbed, and as perturbed by El-Nino.

Fig. 2. A storm track as vortex street. Upper panel: vorticity. Middle panel: streamfunction. Lower panel: eddy streamfunction.