

THEORY MANUAL FOR FAROW VERSION 1.1:

# A Numerical Analysis of the Fatigue and Reliability of Wind Turbine Components

Steven R. Winterstein  
Department of Civil Engineering  
Stanford University  
Stanford, California

Paul S. Veers  
Wind Energy Technology Department  
Sandia National Laboratories  
Albuquerque, NM

Prepared by  
Sandia National Laboratories  
Albuquerque, New Mexico 87185 and Livermore, California 94550

Sandia is a multiprogram laboratory operated by Sandia Corporation,  
a Lockheed Martin Company, for the United States Department of Energy  
under contract DE-AC04-94AL85000.

Approved for public release; further dissemination unlimited.

SAND94-2459  
Unlimited Release  
Printed January 2000



Issued by Sandia National Laboratories, operated for the United States Department of Energy by Sandia Corporation.

**NOTICE:** This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government, nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, make any warranty, express or implied, or assume any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represent that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government, any agency thereof, or any of their contractors or subcontractors. The views and opinions expressed herein do not necessarily state or reflect those of the United States Government, any agency thereof, or any of their contractors.

Printed in the United States of America. This report has been reproduced directly from the best available copy.

Available to DOE and DOE contractors from  
Office of Scientific and Technical Information  
P.O. Box 62  
Oak Ridge, TN 37831

Prices available from (703) 605-6000  
Web site: <http://www.ntis.gov/ordering.htm>

Available to the public from  
National Technical Information Service  
U.S. Department of Commerce  
5285 Port Royal Rd  
Springfield, VA 22161

NTIS price codes  
Printed copy: A02  
Microfiche copy: A01



SAND94-2459  
Unlimited Release  
Printed February 2000

# Theory Manual for FAROW Version 1.1: A Numerical Analysis of the Fatigue and Reliability of Wind Turbine Components

Steven R. Winterstein  
Department of Civil Engineering  
Stanford University  
Stanford, CA 94305

Paul S. Veers  
Wind Energy Technology Department  
Sandia National Laboratories  
Albuquerque, NM 87185-0708

## **Abstract**

Because the fatigue lifetime of wind turbine components depends on several factors that are highly variable, a numerical analysis tool called FAROW has been created to cast the problem of component fatigue life in a probabilistic framework. The probabilistic analysis is accomplished using methods of structural reliability (FORM/SORM). While the workings of the FAROW software package are defined in the user's manual, this theory manual outlines the mathematical basis. A deterministic solution for the time to failure is made possible by assuming analytical forms for the basic inputs of wind speed, stress response, and material resistance. Each parameter of the assumed forms for the inputs can be defined to be a random variable. The analytical framework is described and the solution for time to failure is derived.



## Introduction

The FAROW code numerically estimates the probability of premature failure for wind turbine components. It uses parametrically defined load spectrum and fatigue life (S-N) curves to calculate time to fatigue failure. If the calculated time falls short of the desired target lifetime, the failure is deemed premature. By allowing the governing parameters of the loading, environment, and material properties to be described as random variables, the issue of fatigue lifetime is cast in a probabilistic framework. FAROW repeatedly solves for the time to failure using different sets of values of the random variables until it has determined (using an optimization scheme) the most likely set to produce failure at the target lifetime. It then integrates over the combination of values of the random variables that would produce premature failure to calculate the probability of such early failure. FAROW also outputs the most likely set of random variable values to produce failure at the target lifetime, together with the importance factors defining the relative contribution of each variable to the probability of failure. Sensitivity factors are also estimated by repeatedly solving for probability of failure with slightly perturbed inputs. The version 1.1 user's manual [1] describes the inputs and shows how to interpret the outputs.

FAROW estimates the probability of failure using what are usually called Structural Reliability Methods, or FORM/SORM (First Order Reliability Methods/Second Order Reliability Methods). FORM/SORM procedures are efficient means of numerical probability calculations for multiple random variables. These methods have only become popular since the advent of inexpensive computing. They are often confused with other so-called first order methods that may rely on assumptions of normality and/or small numbers of random variables to achieve closed-form solutions. FORM/SORM, on the other hand, can deal with random variables having arbitrary probability distributions and can estimate reliability results for large numbers of random variables. Unlike other methods such as Monte-Carlo simulation, results of FORM/SORM become more accurate in cases when failures become increasingly rare. As a result, FORM/SORM can be considered a useful method to complement Monte-Carlo simulation, which in turn is more efficient in estimating probabilities of frequently occurring events.

Rackwitz and Fiessler [2] originally developed these methods, and Rackwitz [3] authored the module used in FAROW. Additional features that aid modeling of correlated random variables were added by Winterstein et al. [4]. The FORM/SORM methods are briefly outlined here to give the user a conceptual framework to better understand the results. However, the details of how the methods work are left to the references; a good summary of FORM/SORM can be found in Madsen, et al. [5] or Thoft-Christensen [6].

FAROW's FORM/SORM reliability analysis requires an embedded mathematical model of the interaction between the environment, the loading response to the environment, and the cumulative damage process underlying the lifetime calculation. Certain forms for the governing equations have been assumed here so that the problem can be conveniently parameterized. Each governing parameter can be defined as a random variable, which describes its variability and uncertainty. The parameters and the suite of random variable definitions available in the code are fully defined in the version 1.1 users' manual [1]. It

also shows the governing equation derived by integrating over environment, turbine load response, and material fatigue strength. The derivation of this equation is the topic of the Fatigue Life Calculation portion of this report.

This report starts by describing the difference between FAROW version 1.1 and the later development contained in version 2.0. Structural reliability analysis is then briefly outlined. A short section on some properties of the Weibull distribution is included to help with later derivations. Then the assumptions inherent in version 1.1 are enumerated along with the defining parameters that result from the assumptions. The closed-form solution for the time to failure based on the above assumptions and included in version 1.1 is included last.

## Note on Version 1.1 and Later Developments

The release and user's documentation on FAROW version 1.1 was completed in late 1994. The publication of this theory manual was nearly complete at that time. In fact, it existed as part of the user's manual [1] but was excised at the last minute in an effort to alleviate the theoretical burden on potential readers. Because version 1.1 has seen little industrial application, immediate publication of the theoretical background was not seen as a high priority, and publication of this manual languished. However, recent improvements in FAROW embodied in version 2.0 have led to the need for an updated user's manual and therefore, the publication of the theoretical underpinnings. Before version 2.0 can be fully explained, it is necessary to document the fundamental theory behind the FAROW software in general. This document is intended to fill that need.

One of the greatest drawbacks in implementation of version 1.1 lay in the restrictive definition of the stress response of the turbine. As stated elsewhere in this document, it was assumed that the stress response had a Weibull distribution (often an adequate assumption) that could be described by the same two parameters independent of wind speed (an often inadequate assumption). Research in the interim has focussed on the loads definition issue and has come up with a more comprehensive method for describing loads as a function of both wind speed and turbulence. This leads to a method of describing the uncertainty in the distribution parameters based on analysis of loads data [7, 8, 9, 10]. There has also been substantial progress in the area of reliability-based fatigue design for wind turbines centered in Denmark and published by Ronold, et al. [11, 12].

The yet-to-be-released FAROW version 2.0 is based on substantial research into wind turbine fatigue loads. The loads distribution model in version 2.0 matches the first three statistical moments of the measured rainflow-counted range distribution. Generalized Weibull distributions are used to create analytical load distributions capable of matching all three statistical moments at a particular wind condition. Each of the moments is then described as a function of both wind speed and turbulence level. Finally, correlation between the moments and uncertainty in parameter estimates are determined through preprocessing statistical analysis software. The generalized Weibull distributions are generally better able to match the measured load distributions, and the definition of the moments as functions of wind speed and turbulence permits great flexibility in modeling even quite complex turbine response behavior. Because of the more complicated loading definition and expanded choices of distribution types in version 2.0, the closed-form solution is replaced by a numerical integration scheme.

## Structural Reliability Analysis

In general each problem in structural reliability can be defined in terms of a failure state function,  $g(\mathbf{x})$ , which compares a calculated result with a desired outcome. The failure state function depends on a set of governing parameters, which includes a vector of random variables,  $\mathbf{x}$ . The random variables may represent inherent randomness, parameter uncertainty, or a combination of both. A common application has been to compare a calculated load level with a component strength, such as the load in a beam compared with the beam's load carrying capacity. In equation form,  $g$  is the failure state function (or safety margin function),  $R$  is the resistance or strength, and  $L$  is the load:

$$g = R - L. \quad (1)$$

Both load and strength may be uncertain and thus may contain several random variables (e.g., beam dimensions, yield strength, plasticity model, applied load magnitude and location, stress concentrations, etc.). When  $g$  is negative, failure is predicted.

In the FAROW case, the failure state function compares a calculated time to failure  $T_f$  with a desired, or target, lifetime  $T_T$ :

$$g = T_f - T_T. \quad (2)$$

We need to define *failure* in a reliability sense as the condition that the calculated time to failure is less than the target time to failure, Although this perhaps requires double duty of a single word, failure, the meaning should be clear from context.

The  $n$  random variables form an  $n$ -space of all possible combinations of values. Each point in the space can be deterministically mapped to a resulting value of the failure state function. The task that remains is to estimate the amount of probability that lies in the portion of the space with a negative  $g$  function. It should be clear before starting the reliability analysis that the conditions resulting in failure (negative  $g$  function) are rare, or the reliability calculation would not be of interest. In other words, the analyst should already know that when average values for the random variables are used, an acceptable result is obtained (i.e., failure is avoided). Therefore, the portion of space over which to integrate, which denotes failure, is generally far removed from the mean. This leads to simplifications of the problem, which are exploited by the FORM/SORM procedure.

FORM/SORM may be explained by dividing the process into four steps: formulation, transformation, approximation, and computation.

### Formulation

The problem is formulated by defining a failure state function that delineates between safe and failed states. In this case, Equation 2 provides the formulation, comparing the calculated life with the desired target. The derivation of the expression to calculate  $T_f$  is shown in the section titled Derivation of Time to Failure.

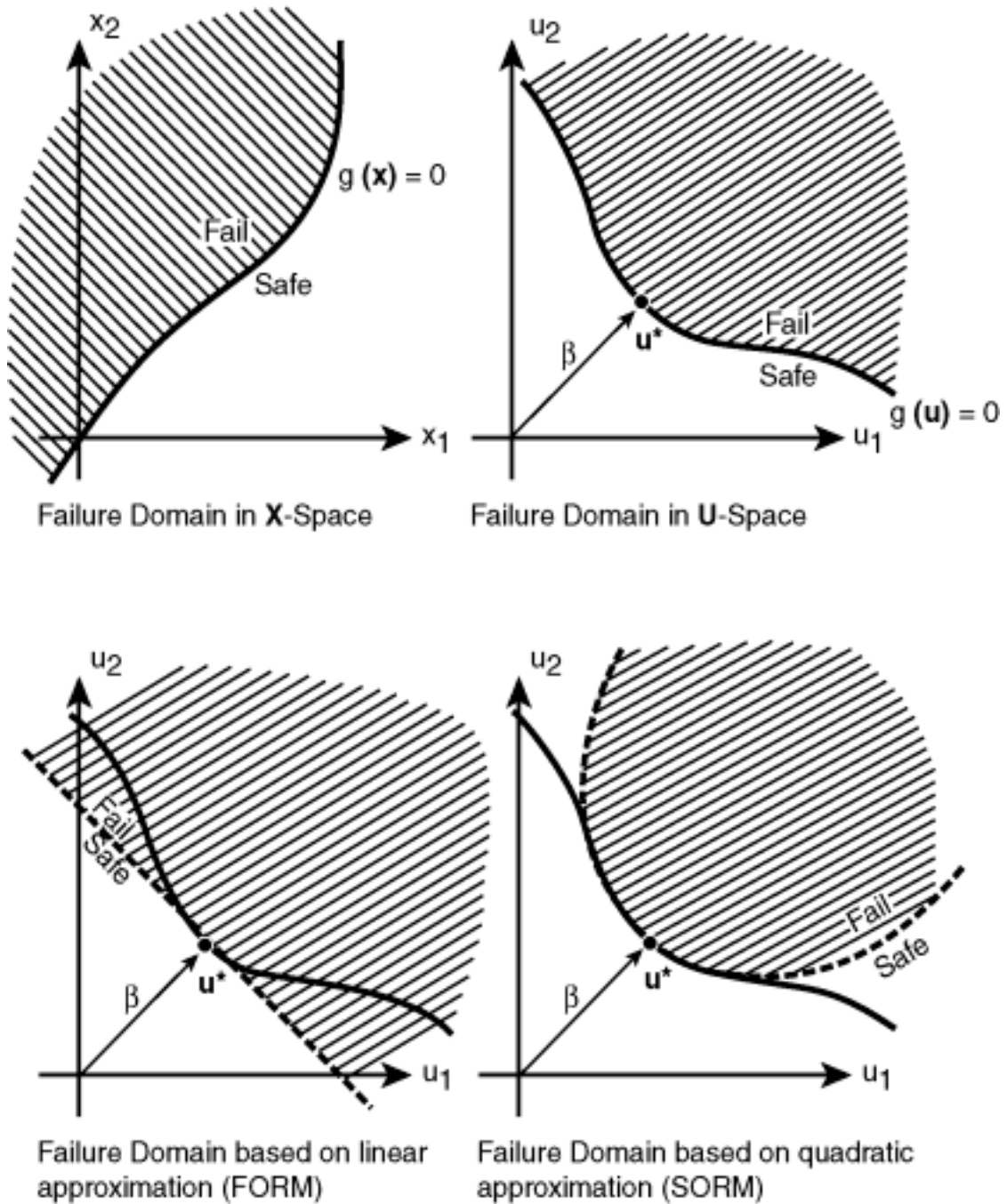


Figure 1: The failure domain in physical space, standard-normal space, and with both first-order and second-order approximations [14, 15].

The formulation can be best illustrated using only two random variables. A two-dimensional plot can then show the  $g$  function. Figure 1 shows the physical space of the random variables,  $\mathbf{x} = (x_1, x_2)$ , along with an example failure state function in the upper left quadrant. One could imagine that  $x_1$  is a strength variable,  $x_2$  is a load variable and the safe region is some arbitrarily complicated relationship between the two (e.g., it could come from a non-linear finite element calculation). Because  $x_1$  and  $x_2$  can have arbitrary



distributions, calculating the probability in the failure region in this physical space could be quite difficult. Therefore, it is expedient to take the next step.

### Transformation

Each random variable can be transformed (mapped) into a standard normal random variable (i.e., with a Gaussian probability distribution that has zero mean and unit variance). The mapping, as illustrated in Figure 2, matches the physical value of each random variable,  $x$ , with a standard normal variable,  $u$ , by matching probability levels of the cumulative distribution function (cdf). For example, the median value of  $x$  is mapped to  $u=0$  (the median value for a standard normal variable), the .84 percentile value of  $x$  is mapped to  $u=1$  (its .84 percentile), and so forth.

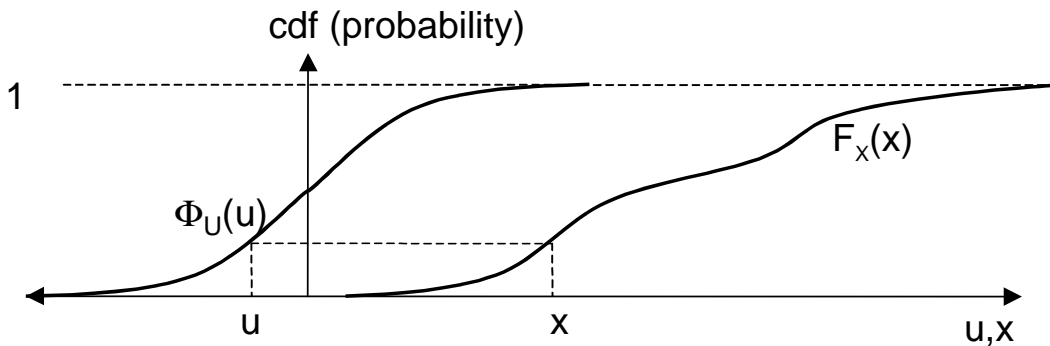


Figure 2 Transformation between a standard normal variate  $U$  and the physical variate  $X$ . ( $\Phi_U(u)$  is the standard normal cdf, and  $F_X(x)$  is the cdf in physical units.)  $x$  and  $u$  have equal probability levels.

The result is the transformation of the problem of calculating the probability of failure into standard-normal space ( $u$ -space) where the calculations become simple. The generality of the procedure should be obvious from the figure – the distribution of  $x$  need not follow any specific analytical distribution type, but could also be empirical. Transforming a single random variable is simple, and transforming a set of statistically independent random variables can be done sequentially and independently. However, the process for transforming correlated variables is somewhat more involved and is not covered here. For more information see Winterstein, et al. [4].

The result of the transformation is illustrated schematically in the upper right quadrant of Figure 1. Notice that the failure state function has been transformed as well. The failure region lies away from the origin in the standard normal space.

### Approximation

The upper right quadrant of Figure 1 indicates a point called the design point, designated as  $\mathbf{u}^*$ . Formally, the design point is defined as the point in the failure domain that is most likely to occur (i.e., has highest value of probability density). In  $u$ -space, the mean of all random variables lies at the origin, and the probability drops off symmetrically in all directions as  $e^{-u^2/2}$ . Therefore, the design point is equivalently defined as the point on the failure surface boundary that lies nearest to the origin. The location of this design

point is found with an optimization routine that uses gradient search methods. This gradient search procedure is the main numerical expense of the FORM calculation. The vector from the origin to the design point is called  $\beta$ . The direction cosines of  $\beta$  indicate the relative importance of each random variable.

The first-order approximation to the failure probability is illustrated in the lower left hand part of Figure 1. A tangent line is fit to the failure boundary at the design point. Because of the  $e^{-u^2/2}$  nature of the drop-off in probability in any direction from the origin, a fit that is good near  $\mathbf{u}^*$  may yield a good reliability approximation even if the fit is poor elsewhere.

A second order approximation (SORM) fits the curvature of the surface at the design point for an improved estimate of the failed region and is illustrated in the lower right quadrant of Figure 1. Calculating the curvature of the  $g = 0$  boundary at the design point requires many more evaluations of  $g$ . Thus, the SORM estimate is much more numerically expensive than the simple FORM result. It is often the case that the two estimates agree within a few percent, indicating a small curvature and an adequate FORM estimate.

### Computation

Because of the symmetry of probability in  $u$ -space, the calculation of the probability lying outside both the first and second order approximations is quite simple. For the first order approximation, the  $n^{th}$  dimensional calculation is the same as for one variable. The probability of failure is  $1-\Phi(|\beta|)$ . ( $\Phi$  is the symbol used for the standard normal distribution cdf, as in Figure 2.) The second order approximation is similarly done, but is based on a corrected length of  $\beta$  based on the local curvatures of the  $g = 0$  boundary. Thus when the FORM and SORM estimates of the probability of failure are compared, one can obtain a sense for how well the failure surface is approximated. A large difference implies a highly curved surface at the design point and inaccuracy in the approximations. However, close agreement implies the failure surface has been well approximated even with the FORM solution.

## The Weibull Distribution

The Weibull probability distribution is a very commonly occurring function in natural processes related to dynamic response of elastic systems. This is especially true of wind turbines, which are aeroelastic systems driven by the random excitation of the atmospheric turbulence. In addition, the Weibull distribution is quite flexible in representing the distribution of many single-sided random variables (i.e., positive valued variables such as speed, amplitude, cycles to failure, etc.) whether or not the source is one particularly expected to be of Weibull form from the physics of the process.

The mathematical form of the Weibull distribution is simplest when viewed from the perspective of the cumulative distribution function (cdf)  $F_X(x)$ . The cdf is the integral of the probability density function (pdf)  $f_X(x)$ . It therefore shows the probability that the random variable is less than  $x$ :

$$F_X(x) = P(X < x) = \int_0^x f_X(x) dx = 1 - \exp\left[-\left(\frac{x}{\beta}\right)^\alpha\right]. \quad (3)$$

The more complicated, but in many ways more useful form is the pdf described in Eq. (4), shown using two common groupings of parameters.

$$f_X(x) = \frac{\alpha x^{\alpha-1}}{\beta^\alpha} \exp\left[-\left(\frac{x}{\beta}\right)^\alpha\right] = \left(\frac{\alpha}{\beta}\right) \left(\frac{x}{\beta}\right)^{\alpha-1} \exp\left[-\left(\frac{x}{\beta}\right)^\alpha\right]. \quad (4)$$

There are two parameters of the distribution that fully describe it and govern its statistical properties. The relative variance of the distribution is controlled by  $\alpha$ , also known as the Weibull shape factor. The other parameter  $\beta$  is related to the mean as shown below. With specific values of the shape factor, the Weibull distribution reduces to special cases. When  $\alpha = 1$  the Weibull becomes the exponential distribution. When  $\alpha = 2$  the special case of the Rayleigh distribution results.

It is also useful to note a general result for Weibull distributions. The expected value of a Weibull distributed random variable raised to an arbitrary power is

$$E[X^z] = \beta^z \left(\frac{z}{\alpha}\right)! . \quad (5)$$

Two special cases for the mean ( $z = 1$ ) and mean square ( $z = 2$ ) of the Weibull variable can be noted for later use.

$$z = 1 \quad E[X] = \bar{X} = \beta(1/\alpha)! , \quad (5.1)$$

$$z = 2 \quad E[X^2] = \beta^2(2/\alpha)! . \quad (5.2)$$

Notice that the scale parameter  $\beta$  can thus be substituted for in terms of the shape parameter  $\alpha$  and the mean  $\bar{X}$  :

$$\beta = \frac{\bar{X}}{(1/\alpha)!} . \quad (6)$$

It is often useful to approximate the variance of the Weibull directly from the shape factor. A useful approximation for coefficient of variation (cov), the ratio of the standard deviation divided by the mean, in the range of  $0.5 < \alpha < 2$  is  $1/\alpha$  . Both exact and approximate solutions are given in Eq. (7):

$$\text{cov} = \frac{E[(X - \bar{X})^2]^{1/2}}{E[X]} = \left[ \frac{E[X^2] - \bar{X}^2}{\bar{X}^2} \right]^{1/2} = \left[ \frac{(2/\alpha)!}{(1/\alpha)!^2} - 1 \right]^{1/2} \cong \frac{1}{\alpha} . \quad (7)$$

The approximation is exact when  $\alpha = 0$  and  $\alpha = 1$  and deteriorates in between and as you move above 1.

## Assumptions and Resulting Parameters

The failure state function implicit in FAROW version 1.1 compares the calculated fatigue life with the target life. The fatigue lifetime is governed by five related inputs:

1. The wind environment,
2. the magnitude of the stress response to a given wind environment,
3. the distribution of stress cycle amplitudes defined by the magnitude of the response,
4. the fatigue strength properties of the material at a given stress amplitude, and
5. the cumulative damage rule used to assess the damage due to the full distribution of stress amplitudes.

Each of these inputs has been assumed to follow a form that can be defined by some governing parameters. All or some of the governing parameters may be defined as random variables. The forms and resulting parameters are described below. From the equations that define the inputs, an expression for the time to failure is derived in the following section.

The assumptions and parameters are as follows:

1. The wind speed,  $V$ , is assumed to have a Weibull probability distribution,  $f(V)$ :

$$f_V(v) = \left( \frac{\alpha_V}{\beta_V} \right) \left( \frac{v}{\beta_V} \right)^{\alpha_V - 1} \exp \left[ - \left( \frac{v}{\beta_V} \right)^{\alpha_V} \right]. \quad (8)$$

The average wind speed  $\bar{V}$  is related to the Weibull parameter  $\beta_V$  by  $\bar{V} = \beta_V (1/\alpha_V)!$ .

**Resulting uncertain parameters:**  $\bar{V}, \alpha_V$  = mean wind speed, Weibull shape factor.

2. Stresses are described in terms of the root mean square (RMS) of the instantaneous stress variations about the mean value. The RMS of the instantaneous (global) stress,  $\sigma_g$ , is assumed to be of the form  $\sigma_{char} (V / V_{char})^p$ , i.e., increasing in power law fashion with the wind speed  $V$ . The local stress at the fatigue-sensitive detail is further scaled by a stress concentration factor  $K$ . The resulting RMS at a point,  $\sigma$ , is then

$$\sigma = K \sigma_g \quad ; \quad \sigma_g = \sigma_{char} \left( \frac{V}{V_{char}} \right)^p. \quad (9)$$

**Resulting uncertain parameters:**  $V_{char}, \sigma_{char}, p, K$  = characteristic levels of wind speed and resulting RMS stress, power-law exponent, and stress



concentration factor. Remember that  $\sigma_{char}$  is the RMS of the instantaneous time varying stress, not the root mean square of the stress cycle amplitude distribution.

3. The probability density of stress amplitude  $S$  for a given wind speed  $V$ ,  $f(S|V)$ , is also assumed to have a Weibull distribution.

**Resulting uncertain parameter:**  $\alpha_s$  = Weibull shape parameter of stress distribution  $f(S|V)$ . Typical range: between  $\alpha_s = 1$  (exponential distribution) and  $\alpha_s = 2$  (Rayleigh distribution). Note that the other parameter,  $\beta_s$ , of the Weibull model of  $f(S|V)$  is assigned from random vibration theory, assuming mean-square value  $E[S^2] = 2\sigma^2$ , with  $\sigma$  from Eq. 9.

4. The S-N curve is taken here as a straight line on log-log scale, with an effective intercept  $C_0$  that includes the Goodman correction for mean stress effects:

$$N_f(S) = C \left( \frac{S}{1 - K |S_m| / S_u} \right)^{-b} = C_0 S^{-b} ; C_0 = C (1 - K |S_m| / S_u)^b . \quad (10)$$

**Resulting uncertain parameters:**  $C, b$  = S-N curve parameters;  $S_m, S_u$  = mean stress and ultimate strength levels:

5. The mean damage rate per unit time,  $\bar{D}$ , is estimated from Miner's rule:

$$\bar{D} = \int_{V=0}^{V_c} \int_{S=0}^{\infty} F(V) \frac{f(S|V)f(V)}{N_f(S)} dS dV . \quad (11)$$

The upper cutoff wind speed,  $V_c$ , is the highest wind speed for which the turbine is assumed to operate.  $F(V)$  is the mean rate of stress cycles as a function of mean wind speed  $V$ . The cycle rate has been included as a power series in wind speed defined by constant, linear, and quadratic coefficients,  $f_0, f_1$ , and  $f_2$ :

$$F(V) = f_0 + f_1 \left( \frac{V}{V_{char}} \right) + f_2 \left( \frac{V}{V_{char}} \right)^2 . \quad (12)$$

Equation (12) can also be written in summation form:

$$F(V) = \sum_{k=0}^2 f_k (V/V_{char})^k . \quad (12.1)$$

After the many cycles that contribute to high-cycle fatigue, the actual damage varies negligibly from its average value of  $\bar{D}$  per unit time.

We introduce two additional factors:  $\Delta$  = the actual level of Miner's damage at which failure occurs, and  $A$  = the fraction of time for which the turbine is available ( $A \leq 1$ ). The failure time is then

$$T_f = \frac{\Delta}{AD} . \quad (13)$$

If Miner's rule is correct we would assign  $\Delta = 1$ . More generally, variability in  $\Delta$  would reflect uncertainty in Miner's rule; e.g., due to load sequence effects.

**Resulting uncertain parameters:**  $V_c, f_0, f_1, f_2, \Delta, A$ .

From the foregoing five assumptions, the fatigue life  $T_f$  is given in terms of a total of  $2+4+1+4+6=17$  uncertain parameters. The target lifetime  $T_t$  is the last parameter in the version 1.1 reliability analysis (making a total of 18) and is treated the same as the other governing parameters even though it may not be used as an uncertain input in most cases.

## Derivation of Time to Failure

The above assumptions are here used to produce a closed-form relation for the time to failure of a component. This is the equation used in FAROW version 1.1 to evaluate the time to failure given realizations of the random variables in the failure state function.

Recall that the general fatigue formulation requires information on three distinct aspects:

1. The loading environment;
2. The gross level of structural response given the load environment; and
3. The local failure criterion given both load environment and gross stress response.

These aspects lead directly to three specific functional inputs: 1) the probability density  $f(V)$  of wind speed  $V$ ; 2) the conditional probability density  $f(S|V)$  of applied stress  $S$  given wind speed  $V$ ; and 3) the mean number of cycles,  $N_f(S)$ , to fatigue failure at stress amplitude level  $S$ .

We first use the assumptions above to develop specific functional forms for these three quantities. We then show how these functions are combined to estimate the mean damage rate under Miner's rule and finally the fatigue life as in Abramowitz and Stegun [13].

### The probability density $f(V)$ of wind speed $V$ .

Our first assumption was that  $V$  follows a Weibull probability distribution, with mean  $\bar{V}$  and shape parameter  $\alpha_v$ . The density  $f(V)$  is then

$$f(V) = \frac{\alpha_v V^{\alpha_v - 1}}{\beta_v^{\alpha_v}} \exp \left[ - \left( \frac{V}{\beta_v} \right)^{\alpha_v} \right]. \quad (14)$$

### The conditional probability density $f(S|V)$ of applied stress amplitudes $S$ given wind speed $V$ .

Here we again adopt a Weibull model (assumption 3, above). In terms of the parameters  $\alpha_s$  and  $\beta_s$ , its form follows Eq. 6:

$$f(S|V) = \frac{\alpha_s S^{\alpha_s - 1}}{(\beta_s(V))^{\alpha_s}} \exp \left[ - \left( \frac{S}{\beta_s(V)} \right)^{\alpha_s} \right]. \quad (15)$$

Note that from random vibration theory, the stress amplitude process may be approximated by a slowly varying envelope,  $e(t) = [s^2(t) + \hat{s}^2(t)]^{1/2}$ , in which  $s(t)$  is the instantaneous stress process (with its mean value removed) and  $\hat{s}(t)$  its Hilbert transform. The envelope process defines the bounds between which the stress is oscillating in the cyclic loading. It follows that

$$E[e^2] = E[s^2] + E[\hat{s}^2] = \sigma^2 + \sigma^2 = 2\sigma(V)^2 \quad (16)$$

in which  $\sigma$  is the RMS stress at wind speed  $V$ , which should therefore be written  $\sigma(V)$ . We assume that the amplitudes of the stress process, and therefore its envelope as well, are Weibull distributed. Then, by applying Eq. (5.2),  $E[e^2]$  can be seen to be

$$E[e^2] = \beta_s^2 (2/\alpha_s)! \quad (17)$$

Equating Equations (16) and (17), and solving for  $\beta_s$  results in a definition of  $\beta_s$  in terms of  $\alpha_s$  and  $\sigma(V)$ :

$$\beta_s = \frac{\sqrt{2}\sigma(V)}{\sqrt{(2/\alpha_s)!}} \quad (18)$$

Therefore, either the RMS of the instantaneous stress process or the scaling parameter of the Weibull distribution of the amplitude process can be used to define the distribution of peaks and ranges.

### The mean number of cycles, $N_f(S)$ , to failure

The form of the S-N curve is exactly as previously stated in Eq. (10).

### Derivation

Equation 11 is the expression into which the various assumed functional forms must be substituted and which must be integrated to come to the solution for the damage rate and hence the time to failure. First, it should be noted that the inner integral is really an expectation operation because it is the integral of the stress pdf over all stress amplitudes.

$$\bar{D} = \int_{V=0}^{V_c} F(V) E \left[ \frac{1}{N_f(S|V)} \right] f(V) dV \quad (19)$$

Equation (10) is substituted for  $N_f$  inside the expectation operator and Eq. (5) is applied to solve for the average value of a Weibull variable raised to the  $b$  power. Equation (18) is also used to cast the solution in terms of the RMS stress level as a function of wind speed,  $\sigma(V)$ .

$$E \left[ \frac{1}{N_f(S|V)} \right] = C_0^{-1} E[S^b|V] = C_0^{-1} \left( \frac{2}{(2/\alpha_s)!} \right)^{b/2} (\sigma(V))^b \left( \frac{b}{\alpha_s} \right)! \quad (20)$$

Using the assumed general form of stress increase with wind speed from Eq. (9),  $\sigma(V) = K\sigma_{char}(V/V_{char})^p$ , substitute Eqs. (20) and (8) for  $f(V)$  into Eq. (19).

$$\bar{D} = \frac{1}{C_0} \left( \frac{2}{(2/\alpha_s)!} \right)^{b/2} (K\sigma_{char})^b \left( \frac{b}{\alpha_s} \right)!_{V=0}^{V_c} F(V) \left( \frac{V}{V_{char}} \right)^{bp} \frac{\alpha_V V^{\alpha_V - 1}}{\beta_V^{\alpha_V}} \exp \left[ - \left( \frac{V}{\beta_V} \right)^{\alpha_V} \right] dV \quad (21)$$

#### Case 0: Constant cycle rate and no cut-out wind speed

It simplifies the derivation to consider some special cases first. Let the cyclic frequency be constant,  $F(V)=f_0$ , and remove the cut-out wind speed so the upper limit of integration goes to infinity. Equation (21) then simplifies to

$$\bar{D} = \frac{f_0}{C_0} \left( \frac{2}{(2/\alpha_s)!} \right)^{b/2} (K\sigma_{char})^b \left( \frac{b}{\alpha_s} \right)! \mathbb{E} \left[ \left( \frac{V}{V_{char}} \right)^{bp} \right]. \quad (22)$$

From Eq. (5) for the expected value of a Weibull distributed variable  $V$  raised to an arbitrary power  $bp$ , and keeping Eq. (6) in mind, the result becomes

$$\bar{D} = \frac{f_0}{C} \left( \frac{2}{(2/\alpha_s)!} \right)^{b/2} \left( \frac{K\sigma_{char}}{1 - K|S_m|/S_u} \right)^b \left( \frac{\bar{V}}{V_{char} (1/\alpha_V)!} \right)^{bp} \left( \frac{b}{\alpha_s} \right)! \left( \frac{bp}{\alpha_V} \right)!, \quad (23)$$

which is the source, after substitution into Eq. (13), of the final result cited in Veers, et al. [1] for the time to failure.

#### Case 1: Cycle rate as a function of wind speed

For the case where the cyclic frequency is not fixed, substitute Eq. (12.1) into Eq. (21). Equation (23) then generalizes to produce the following solution including the power series in wind speed that defines a variable frequency.

$$\bar{D} = \frac{1}{C} \left( \frac{2}{(2/\alpha_s)!} \right)^{b/2} \left( \frac{K\sigma_{char}}{1 - K|S_m|/S_u} \right)^b \left( \frac{b}{\alpha_s} \right)! \left[ \sum_{k=0}^{\infty} f_k \left( \frac{\bar{V}}{V_{char} (1/\alpha_V)!} \right)^{bp+k} \left( \frac{bp+k}{\alpha_V} \right)! \right]. \quad (24)$$



## Case 2: Finite cut-out wind speed

To reinstate the finite cut-out wind speed while reverting to a constant cyclic frequency, it is convenient to define a change of variables,  $x = (V / \beta_V)^{\alpha_V}$ ,  $dx = \alpha_V V^{\alpha_V - 1} dV / \beta_V^{\alpha_V}$  and  $V^{bp} = \beta_V^{bp} x^{bp / \alpha_V}$ . Eq. (21) then becomes

$$\bar{D} = \frac{f_0}{C_0} \left( \frac{2}{(2/\alpha_s)!} \right)^{b/2} (K\sigma_{char})^b \left( \frac{\beta_V}{V_{char}} \right)^{bp} \left( \frac{b}{\alpha_s} \right)! \int_{x=0}^{x_c} x^{bp / \alpha_V} \exp(-x) dx. \quad (25)$$

The upper limit of integration is written as

$$x_c = \left( \frac{V_c}{\beta_V} \right)^{\alpha_V} = \left( \frac{V_c (1/\alpha_V)!}{\bar{V}} \right)^{\alpha_V}. \quad (25.1)$$

(The limit of integration was printed in Veers et al. [1], Eq. (2.8), with a typographical error in which  $\bar{V}$  was replaced with  $V_{char}$ . The expression in Eq. (25.1) is the correct one.)

The solution to the integral in Eq. (25) is the incomplete Gamma function:

$\gamma(1 + (bp/\alpha_V), x_c)$  [13].

$$\bar{D} = \frac{f_0}{C} \left( \frac{2}{(2/\alpha_s)!} \right)^{b/2} \left( \frac{K\sigma_{char}}{1 - K|S_m|/S_u} \right)^b \left( \frac{\bar{V}}{V_{char} (1/\alpha_V)!} \right)^{bp} \left( \frac{b}{\alpha_s} \right)! \gamma \left( 1 + \frac{bp}{\alpha_V}, x_c \right). \quad (26)$$

## General Case: Variable cycle rate and finite cut-out wind speed

Putting both generalizations together (for frequency as a function of wind speed and for a finite cut-out wind speed) gives the following result.

$$\bar{D} = \frac{1}{C} \left( \frac{2}{(2/\alpha_s)!} \right)^{b/2} \left( \frac{K\sigma_{char}}{1 - K|S_m|/S_u} \right)^b \left( \frac{b}{\alpha_s} \right)! \left[ \sum_{k=0}^{\infty} f_k \left( \frac{\bar{V}}{V_{char} (1/\alpha_V)!} \right)^{bp+k} \gamma \left( 1 + \frac{bp+k}{\alpha_V}, x_c \right) \right] \quad (27)$$

Again, the solution for time to failure is obtained by substituting the average damage rate into Eq. (13).

Equation (27) is the form implemented in FAROW version 1.1.

## References

1. Veers, P. S., S. R. Winterstein, C. H. Lange and T. A. Wilson, "User's Manual for FAROW: Fatigue and Reliability of Wind Turbine Components," SAND94-2460, November 1994.
2. Rackwitz, R., and B. Fiessler, "Structural Reliability Under Combined Load Sequences," *Computers and Structures*, 9, 1978, pp. 489-494.
3. Golweitzer, S., T. Abdo and R. Rackwitz "FORM (First Order Reliability Method) Manual," and S. Golweitzer, F. Guers, and R. Rackwitz, "SORM (Second Order Reliability Method) Manual," RCP GmbH, Munich, 1988.
4. Winterstein, S. R., Rabi S. De, and P. Bjerager, "Correlated Non-Gaussian Models in Offshore Structural Reliability," *Proc., ICOSSAR '89, 5th International Conference on Structural Safety and Reliability, Vol. 1*, 1990, pp. 239-242.
5. Madsen, H. O., S. Krenk, and N. C. Lind, *Methods of Structural Safety*, Prentice Hall, 1986.
6. Thoft-Christensen, P. and M. J. Baker, *Structural Reliability Theory and Its Applications*, Springer-Verlang, New York, 1982.
7. Lange, C. F. and S. R. Winterstein, "Load Models for Fatigue Reliability from Limited Data, *Wind Energy 1995*, ASME, SED-Vol. 16, January 1995.
8. Kashef, T. and S. R. Winterstein, "Relating Turbulence to Wind Turbine Blade Loads: Parametric Study with Multiple Regression Analysis," *AIAA Aerospace Sciences Mtg. / ASME Wind Energy Symp.*, W. Musial ed., SED - Vol. 19, 1998.
9. Veers, P. S. and S. R. Winterstein, "Application of Measured Loads to Wind Turbine Fatigue and Reliability Analysis," *Journal of Solar Energy Engineering, Trans. of the ASME*, Vol. 120, No. 4, November 1998.
10. Winterstein, S. R. and T. Kashef, "Moment-Based Load and Response Models with Wind Engineering Applications," *AIAA Aerospace Sciences Mtg. / ASME Wind Energy Symp.*, T. Ashwill ed., AIAA 99-0068, 1999.
11. Ronold, K. O., J. Wedel-Heinen, C. J. Christensen and E. Jorgensen, "Reliability-Based Calibration of Partial Safety Factors for Design of Wind-Turbine Rotor Blades Against Fatigue," *Proc., 5<sup>th</sup> European Wind Energy Conf., II*, Thessalonika, Greece, 1995, pp. 106-111.
12. Ronold, K. O., J. Wedel-Heinen, and C. J. Christensen, "Reliability-based fatigue design of wind-turbine rotor blades," *Engineering Structures* (21), Elsevier, September 1999, pp. 1101-1114.

13. Abramowitz and Stegun, *Handbook of Mathematical Functions, with Formulas, Graphs and Mathematical Tables*, Dover, New York, 1965.
14. Karamchandani, A., "New Methods in System Reliability," Reliability of Marine Structures Program, Report No. RMS-7, Dept. of Civil Engineering, Stanford University, May, 1990.
15. Veers, P. S., C. H. Lange and S. R. Winterstein, "FAROW: A Tool for Fatigue and Reliability of Wind Turbines," *Proc., Windpower '93*, American Wind Energy Association, San Francisco, California, July 13-16, 1993, pp. 342-349.

## DISTRIBUTION

J. Cadogan  
U.S. Department of Energy  
Office of Geothermal & Wind  
Technology  
EE-12  
1000 Independence Avenue SW  
Washington, DC 20585

S. Calvert  
U.S. Department of Energy  
Office of Geothermal & Wind  
Technology  
EE-12  
1000 Independence Avenue SW  
Washington, DC 20585

P. R. Goldman  
Director  
Office of Geothermal and  
Wind Technology  
EE-12  
U.S. Department of Energy  
1000 Independence Avenue  
Washington, DC 20585

A. Lucero  
Librarian  
National Atomic Museum  
Albuquerque, NM 87185

NWTC Library (5)  
NREL  
1617 Cole Boulevard  
Golden, CO 80401

L. Pratsch  
U.S. Department of Energy  
Office of Geothermal & Wind Technology  
EE-12  
1000 Independence Avenue SW  
Washington, DC 20585

D. Sanchez  
U.S. Dept. of Energy  
Albuquerque Operations Office  
P.O. Box 5400  
Albuquerque, NM 87185

S. R. Winterstein (10)  
Civil Engineering Department  
Stanford University  
Stanford, CA 94305

M.S. 0555	B. Hansche, 9122
M.S. 0557	T. J. Baca, 9125
M.S. 0557	T. G. Carne, 9124
M.S. 0708	H. M. Dodd, 6214 (25)
M.S. 0708	T. D. Ashwill, 6214
M.S. 0708	D. E. Berg, 6214
M.S. 0708	P. L. Jones 6214
M.S. 0708	D. L. Laird, 6214
M.S. 0708	M. A. Rumsey, 6214
M.S. 0708	H. J. Sutherland, 6214
M.S. 0708	P. S. Veers, 6214
M.S. 0708	J. R. Zayas, 6214
M.S. 0847	D. W. Lobitz, 9125
M.S. 0847	D. R. Martinez, 9124
M.S. 0612	Review & Approval Desk, 4912
M.S. 0899	Technical Library, 4916 (2)
M.S. 9018	Central Technical Files, 8940-2