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An Update on the FFAG lattice Without Opposite Bends with Distributed RF

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Berkeley National Laboratory

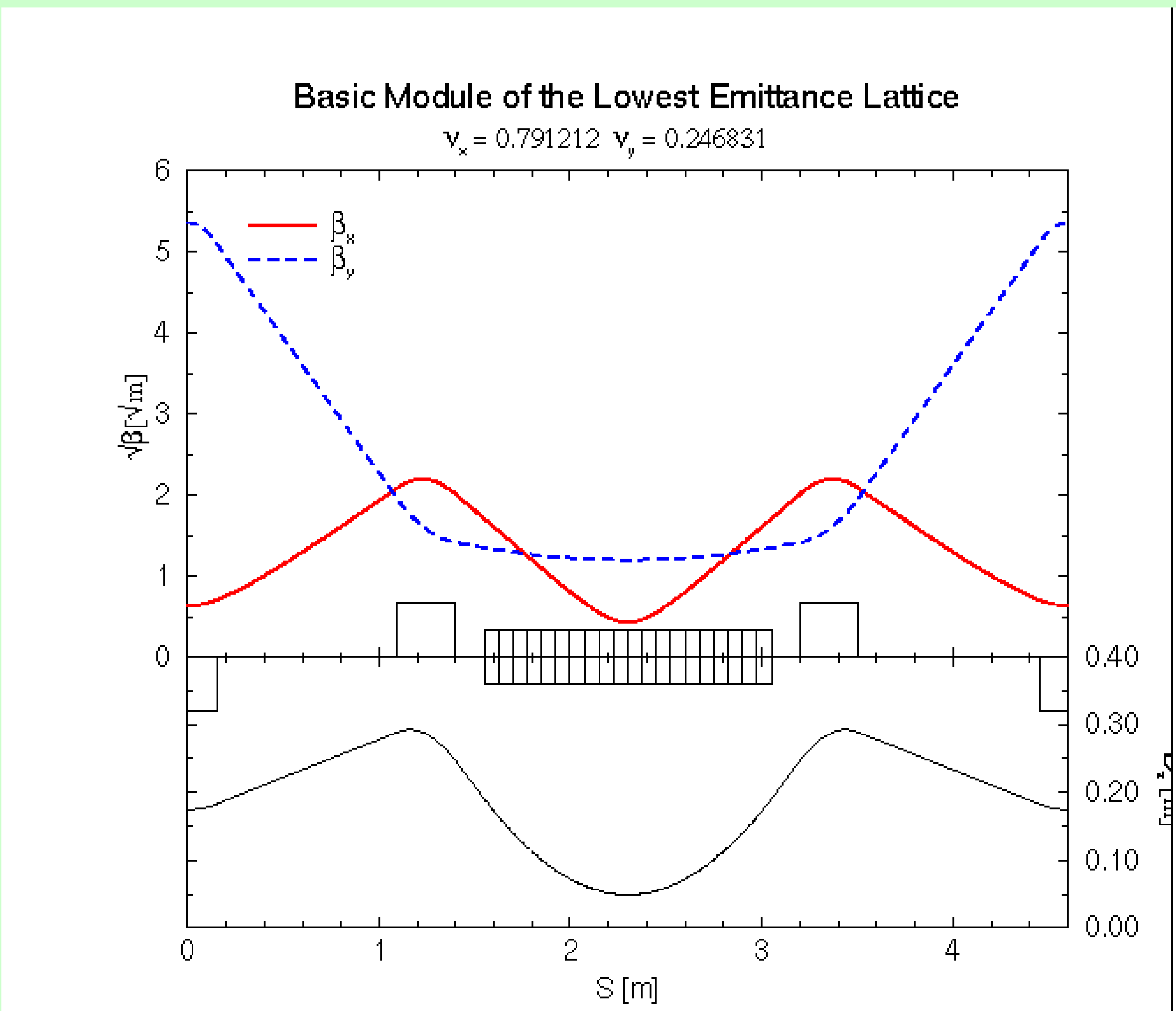
CONTENT:

Update with our lattice design:

- **Checking the tools: New SYNCH, COSY, MAD8, MADX, TEAPOT.**
- **Promising results from COSY Lattice properties – a ring picture.**
- **Dynamical aperture @ central energy**
- **Longitudinal simulation of the acceleration with the latest lattice solutions (Mike Blaskiewicz).**
- **Conclusions**

This is an old slide as a reminder of the the Montauk 99 presentation: a relevance to the minimum emittance lattice and muon acceleration lattice.

- The minimum emittance lattice requires reduction of the function H:
  - The normalized dispersion amplitude corresponds to the $\langle H \rangle^{1/2}$!!!



What are the basic parameters?

- Required Range of Energies (or dp/p)
  - the “central” energy or momentum p_o is in two examples presented later set to 10 GEV. The acceleration would be possible from 10 GeV up to 20 GeV.
  - Aperture limitation is defined by the maximum value of the DISPERSION function:
    - $\Delta x < +/- 30$ mm
    - if the 0.5 < dp/p < 1.5 then:
      - **$D_x < 60$ mm**
- Why is the Minimum Emittance Lattice for the electronic Storage Rings Relevant?
  - The normalized dispersion amplitude Corresponds to the $<H>^{1/2}$!!!

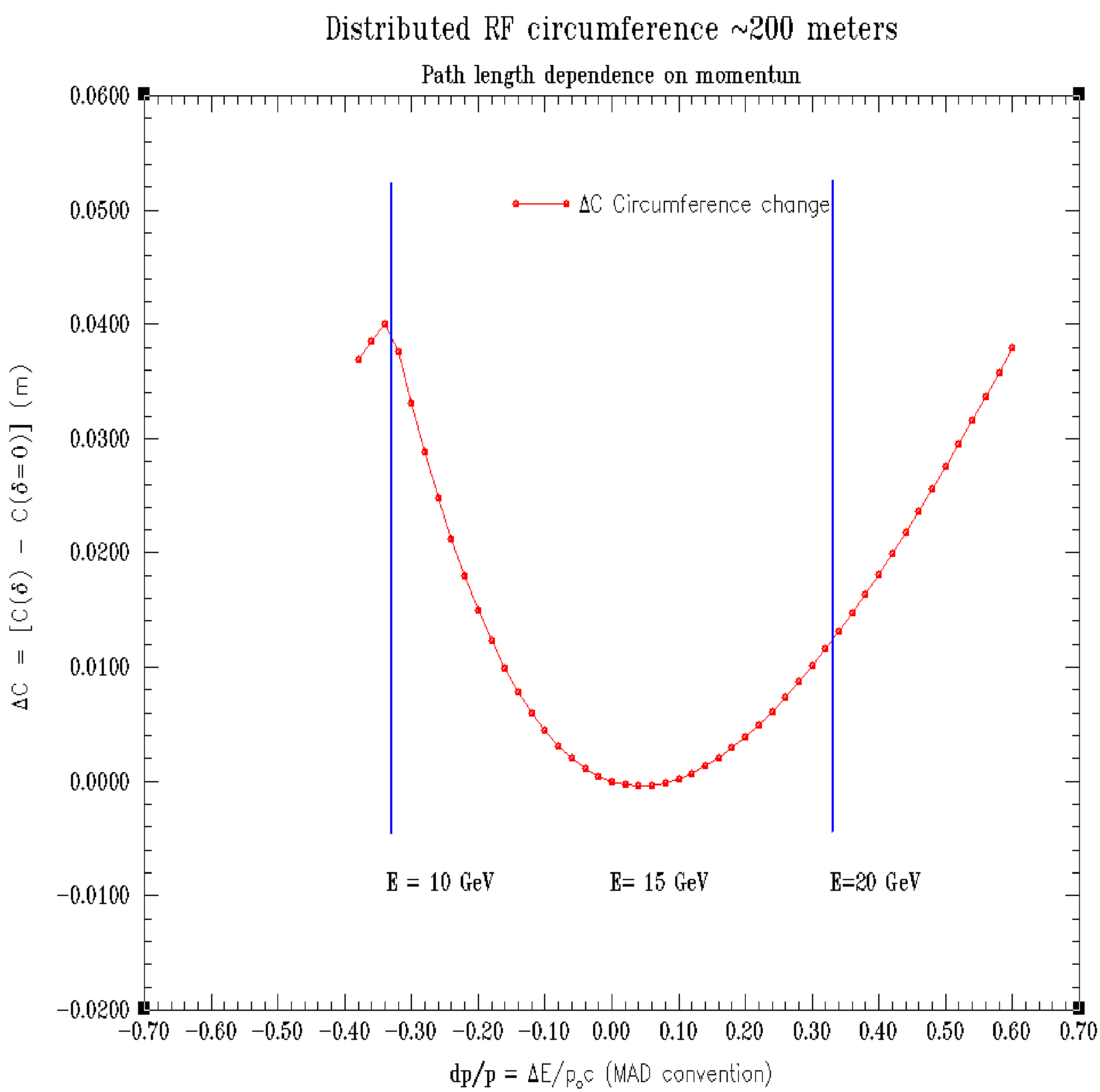
What was our promise given at the last meeting (BNL editors meeting):

- Construct a lattice where the dispersion will oscillate between positive and negative values but not exceeding 6 cm without opposite bending magnets.

  $\Delta x < D\ dp/p = 0.06 * (+-0.5) = +-0.03$ m

- Make a change in the circumference smaller to reduce the RF phase change.
- Try to combine the linac with a single arc.
- Or make enough room for the cavities within the ring.
- Longitudinal simulation of the multiple turns (10 – 20 turns)

The major result: reduced change of the circumference the 'SYNCH' result (with Ernie's combined function dipole subroutine correction)



TEST data for different tools: Cyclotron

Equation of motion (The first lecture in Accelerator Physics Course 1982 Ernest Courant):

$$\frac{\partial^2 x}{\partial s^2} = -\frac{x}{\rho^2} + \frac{B_y - B_0(s)}{B\rho} \quad \cdots(1.6)$$

$$\frac{\partial^2 y}{\partial s^2} = -\frac{B_x}{B\rho} \quad \cdots(1.7)$$

$$B_y = B_0\left(1 - \frac{n\,x}{\rho} + \cdots\right) = B_0 + G\,x$$

$$B_x = B_0\,\frac{n\,y}{\rho} \quad \cdots(1.8)$$

$$(B_0 + G\,x)(r_0 + x) = p_0\,(1+\delta) \ \ldots(1.9)$$

where δ *is*: $\delta = \dfrac{dp}{p_0}$ *where* $G = -\dfrac{n\,B_0}{\rho}$. *If* $u = \dfrac{x}{r_0}$ *then*:

$n_0 u^2 + (1 - n_0)u + \delta = 0,$ *and the two solutuions of the quadratic equaton are*:

$$u_{1,2} = \lambda \pm \sqrt{\lambda^2 - \frac{\delta}{n_0}}.$$

$n \equiv -\dfrac{\rho}{B_0}\dfrac{dB}{dx}$ *where* $n_0 = -\dfrac{r_0}{B_0}G_0$, *and* $r_0 = \dfrac{B_0\,r_0}{B_0} = \dfrac{p_0}{B_0}.$

The two transverse equations of motion are:

$$\frac{\partial^2 x}{\partial s^2} = -\frac{(1-n)}{\rho^2}\,x$$

$$\frac{\partial^2 y}{\partial s^2} = -\frac{n}{\rho^2}\,y \quad \cdots(1.9) \quad \textit{with a condition} \quad 0 < n < 1.$$

Solutions for the Courant − Snider parameters are:

Tunes: $\nu_x = \sqrt{1-n}$, $\nu_y = \sqrt{n}$, *betatron funcitons*: $\beta_x = \dfrac{\rho}{\sqrt{1-n}}, \beta_y = \dfrac{\rho}{\sqrt{n}},$

Dispersion function: $D_x = \dfrac{\rho}{1-n}$, *Chromaticities*: $\xi_{x(\delta)} = \dfrac{\nu_x - \nu_{x0}}{\delta}$, $\xi_{y(\delta)} = \dfrac{\nu_y - \nu_{y0}}{\delta}$

EXAMPLE of the Cyclotron − weak focusing synchrotron made of five combined function dipoles: $n = 0.5$, $C = 100\ m$, $B\rho = 50.0$, $n_{dipoles} = 5$,

$r_0 = \dfrac{100}{2\pi}$ $B_0 = \dfrac{50}{r_0}$. *Solutions for* $n_0 = 0.5$ *and* $\delta = 0.001$:

$: u_{1,2} = 0.5 \pm 0.4979959839\,2, \quad x_{C0(\delta=0.01)} = 0.0318949065\,2\,m$

TEST data for different tools: Cyclotron

	δ	x (mm)	ΔC (m)	νx	βx	ηx	ξ	γt/νx	Dx/Dp	Dq/Dp	ΔC/2 pi x	νy	βy
SYNCHF	-0.001	-47.55651	-0.1996	0.70816	22.42946	31.67272	-1.05175				0.667991		
	0	0	0	0.70711	22.50791	31.83099	-1.06066		47.7475	-1.06			
	0.001	47.93849	0.2004	0.70604	22.58702	31.99104	-1.06978				0.665324		
	0.049	2974.68381	11.01282	0.64233	27.50639	42.82268	-2.02609				0.589221		
	0.05	3054.26691	11.27017	0.64065	27.64276	43.14831	-2.06935		80.13792	-1.695	0.587278		
	0.051	3134.95965	11.52923	0.63894	27.78116	43.48025	-2.11469				0.585313		
MAD	-0.001	-31.767573	-0.025823726	0.708164	22.429457	31.672708	-1.054916	1.000998			0.129376		
	0	0	0	0.707107	22.507908	31.830993	-1.06066	1	31.831236	-1.061			
	0.001	31.894899	0.200040159	0.706042	22.587016	31.991029	-1.066492	0.999			0.998197		
	0.049	1751.466824	9.963493683	0.642983	27.478961	42.748099	-1.509104	0.948873			0.905378		
	0.05	1792.329883	13.21546832	0.641328	27.613771	43.069209	-1.523726	0.947765	40.9973295	-1.665	1.173504		
	0.051	1833.461483	13.57585077	0.639653	27.750549	43.396393	-1.538702	0.946652			1.178461		
								1					
Mathematica	-0.001	-31.7676	-0.199602	0.708163	22.4295	31.6727	-1.05276	1			1.000001		
	0	0	0	0.707107	22.5079	31.831	-1.06066	1	31.83125	-1.0605			
	0.001	31.8949	0.200402	0.706042	22.587	31.991	-1.06867	1			1.000002		
	0.049	1752.75	11.0128	0.642332	27.5064	42.8227	-1.67705	1			0.999995		
	0.05	1793.7	11.2702	0.640645	27.6428	43.1483	-1.69725	1	41.09	-1.697	1.000004		
	0.051	1834.93	11.5292	0.638938	27.7812	43.4802	-1.71796	1			1		
COSY	-0.01	31.767758		0.708163484	22.45191502								
	0.051	1834.812428		0.638888733	26.46187649								
TEAPOT													

TEST data for different tools: Cyclotron

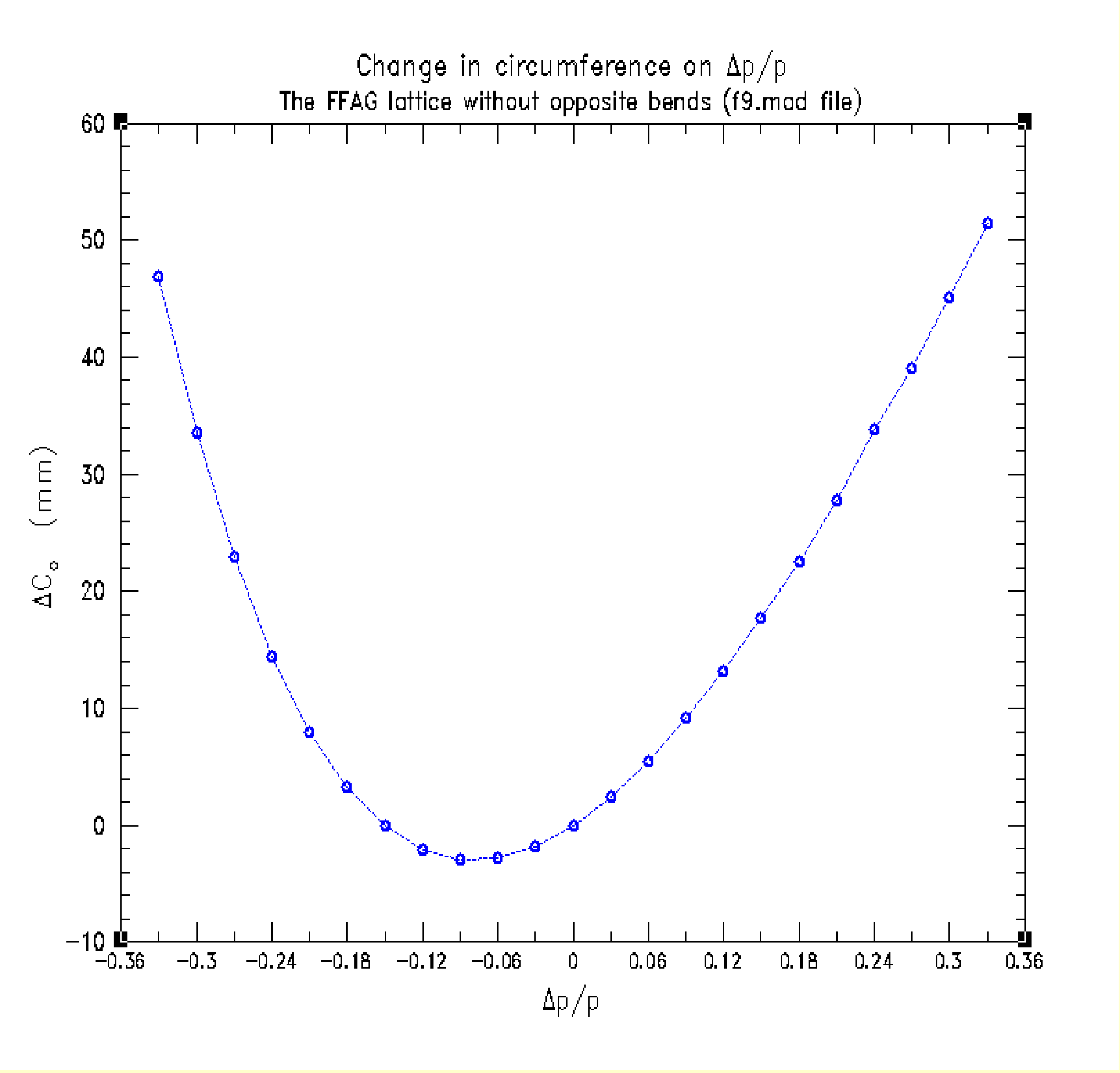
	δ	x (mm)	ΔC (m)	νx	βx	ηx	ξ	γt/νx	Dx/Dp	Dq/Dp	ΔC/2 pi x	νy	βy
COSY	-0.01	31.76776		0.708163	22.45192							0.706048	22.51917
	0.051	1834.812		0.638889	26.46188							0.769257	21.95733
Mathematica	-0.001	-31.7676	-0.1996	0.708163	22.4295	31.6727	-1.05276	1			1.000001		
MAD	-0.001	-31.7676	-0.02582	0.708164	22.42946	31.67271	-1.05492	1.000998			0.129376		
	0.051	1833.461	13.57585	0.639653	27.75055	43.39639	-1.5387	0.946652			1.178461		

TEST data by different tools: Cyclotron

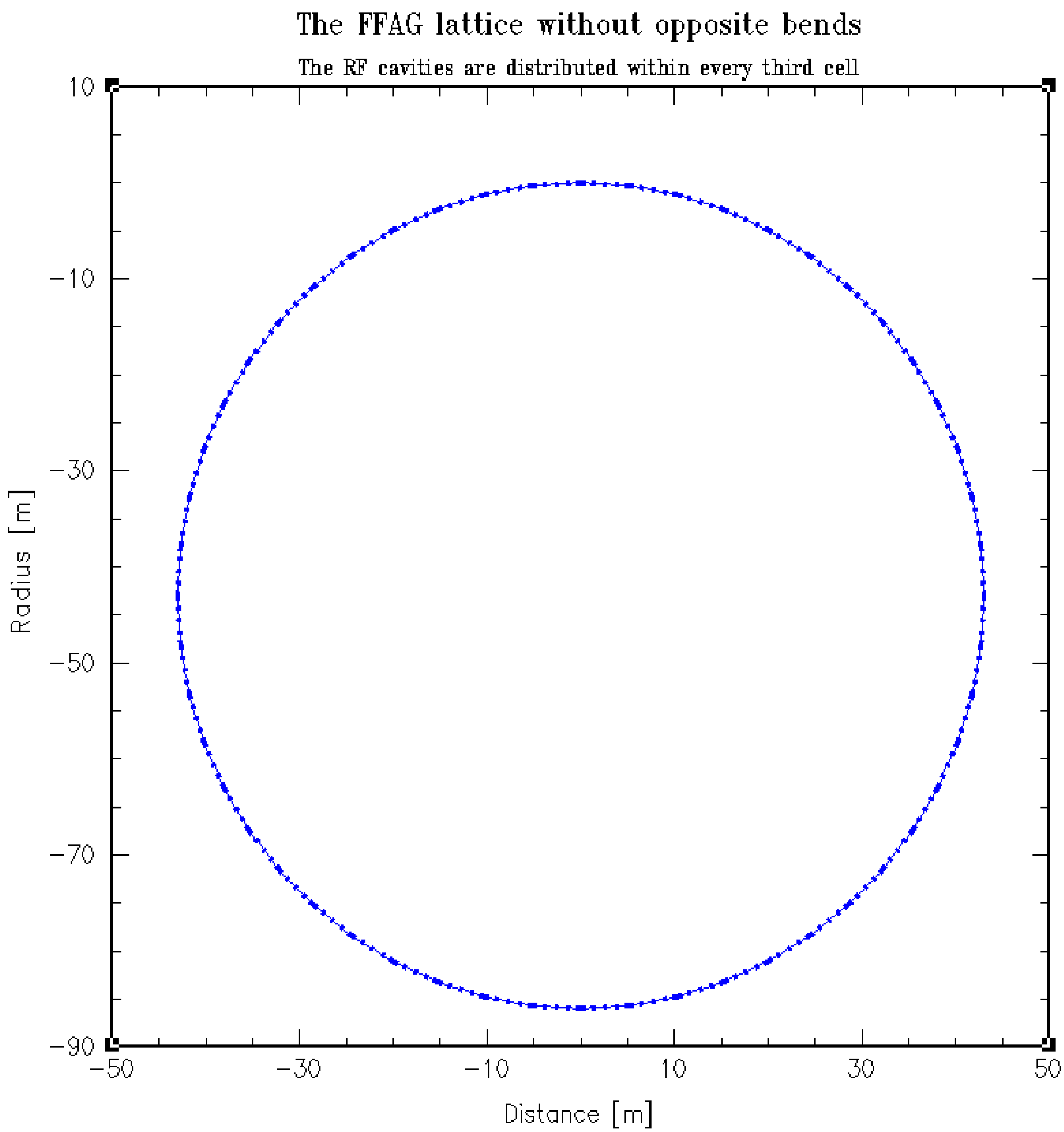
TEST data for different tools: FFAG COSY vs. SYNCH

dpp	dC	xc0 (mm)	xc0max	bx_in	by_in	nux	nuy
-0.3 SYNCH	0.00066	-14.42	-23.54130	0.5830	11.0908	0.65631	0.16568
-0.3 COSY	-0.00188	-14.06520		0.44529 18.78855		0.67946	0.12692
-0.2 SYNCH	0.00027	-9.37	-14.98552	0.5439	16.2845	0.67876	0.13404
-0.2 COSY	-0.0007366	-9.2466796		0.6720	21.1695	0.69281	0.11903
-0.1 SYNCH	0.00007	-4.57	-7.18385	0.5270	20.2421	0.68984	0.12178
-0.1 COSY	-0.000180	-4.5476		0.58455	22.8343	0.69045	0.120172
0.0	0.0	0.0	0.0	0.52712	22.5536	0.69290	0.11925
0.0	0.0	0.0	0.0	0.52712	22.5536	0.69290	0.11925
0.1 SYNCH	0.00003	4.38	6.67579	0.5396	23.7829	0.69060	0.12033
0.1 COSY	-0.0000691	4.39534		0.48912	21.41111	0.69098	0.12166
0.2 SYNCH	0.00013	8.58	12.93920	0.5611	24.5005	0.68485	0.12195
0.2 COSY	-0.00030702	8.6476		0.46367	20.0778	0.68612	0.12454
0.3 SYNCH	0.00028	12.64	18.88140	0.5895	25.0235	0.67696	0.12288
0.3 cosy	-0.000660	12.76687		0.4453	18.7885	0.67946	0.126923
0.4 SYNCH	0.00047	16.55	24.53229	0.6236	25.5028	0.66781	0.12272
0.4 COSY	-0.001088	16.753		0.42471	17.48073	0.67242	0.129552
0.5 SYNCH	0.00068	20.35	29.92676	0.6627	26.0117	0.65796	0.12144
0.5 COSY	-0.00154	20.5693981		0.37913	15.7978	0.66765	0.135832
0.6 SYNCH	0.00092	24.04	35.09355	0.7067	26.5915	0.64776	0.11914
0.6 COSY	0.0019236	24.087769		0.26180	13.2517539	0.67048	
	0.1532735						

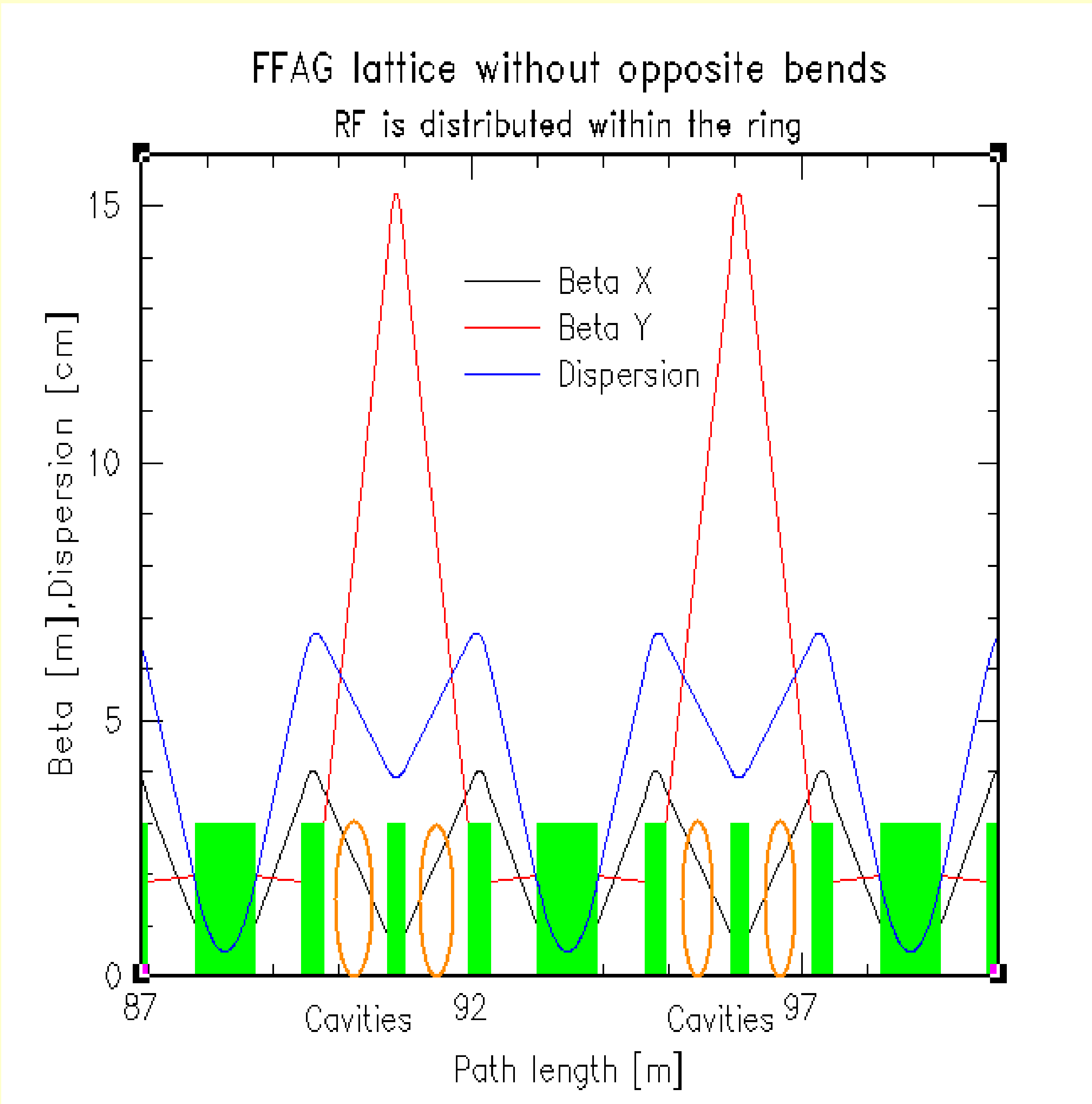
The major result: reduced change of the circumference the 'MAD' file result



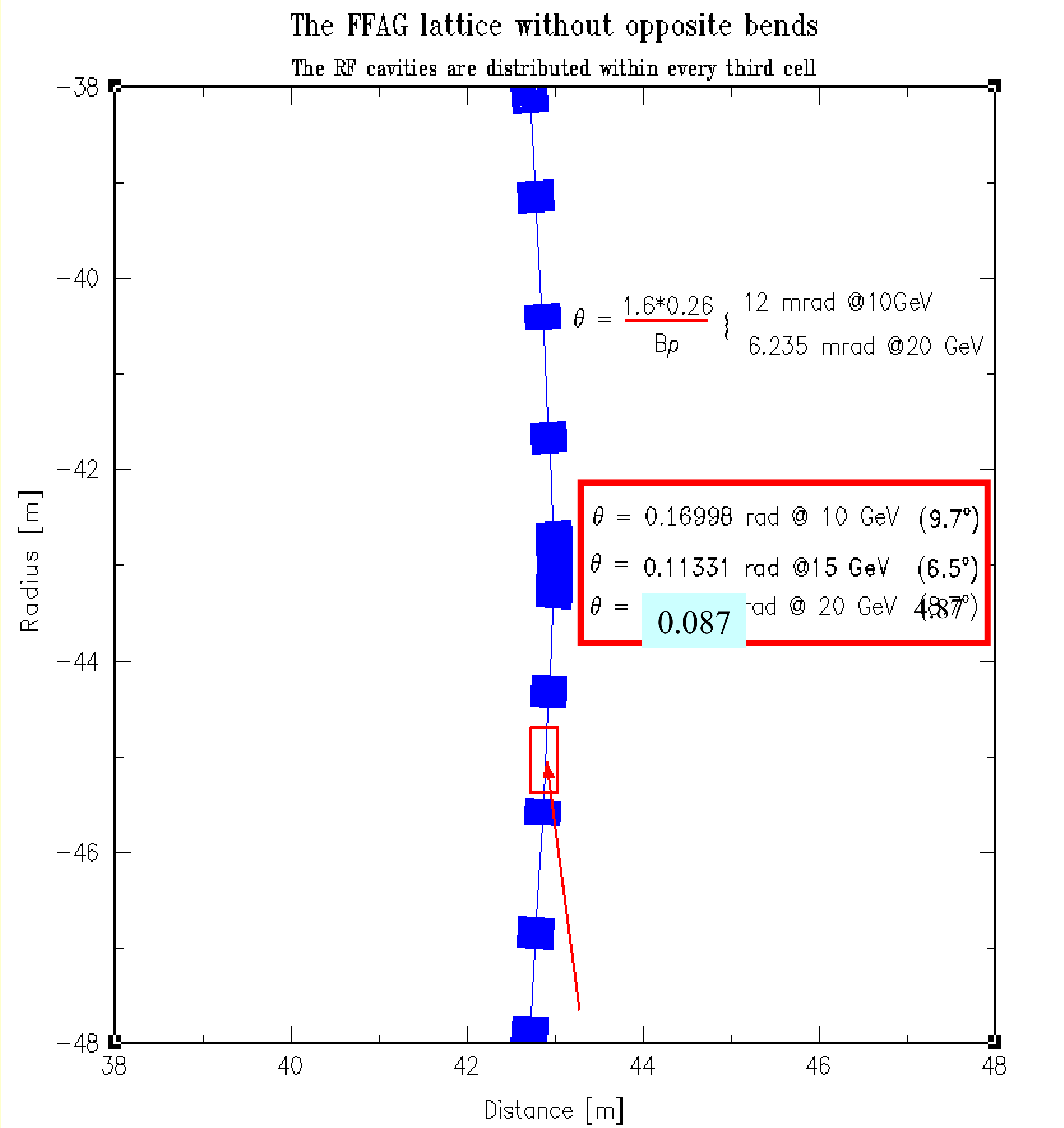
Picture of the 'MAD' ring



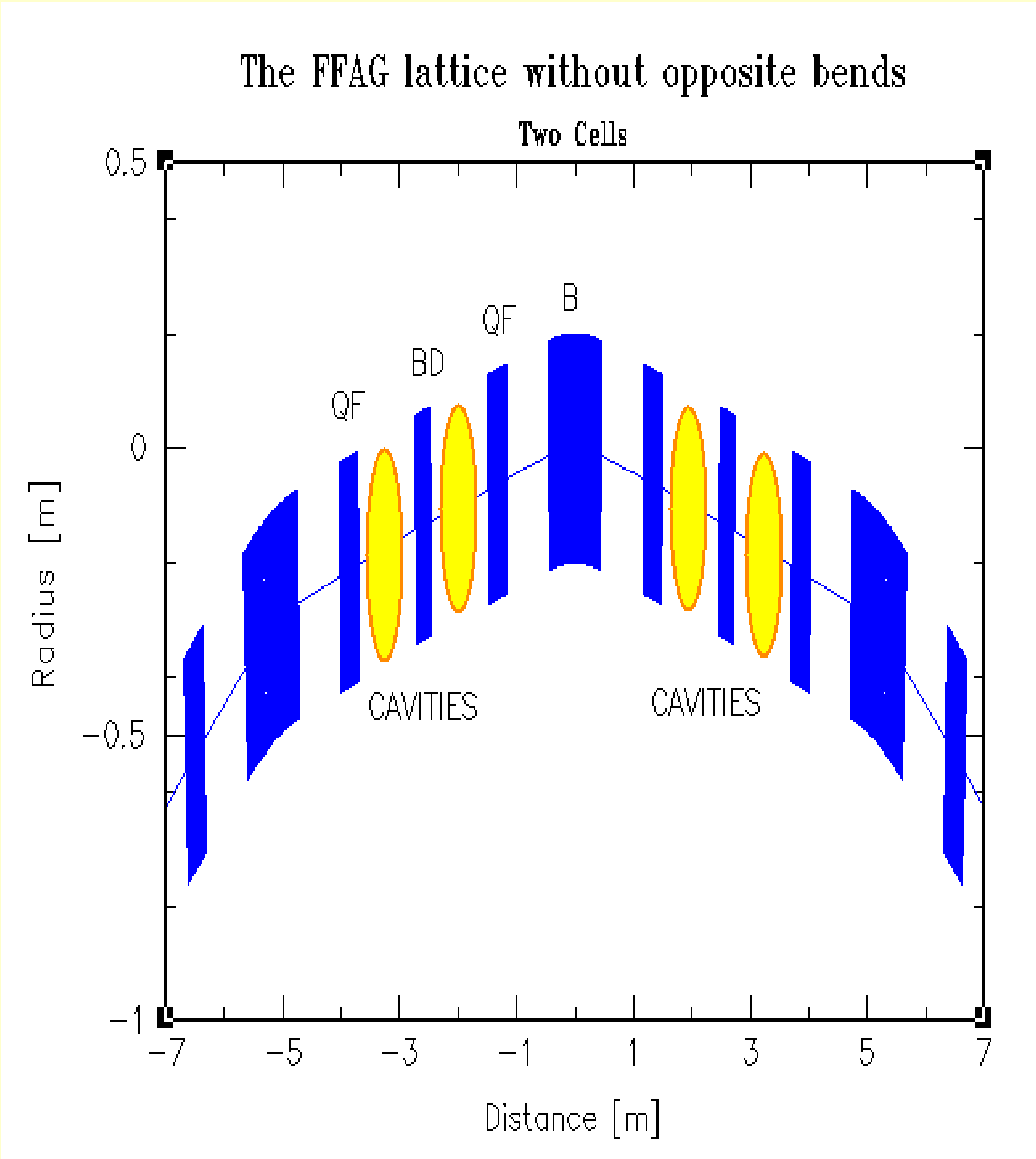
Betatron functions within the two cells



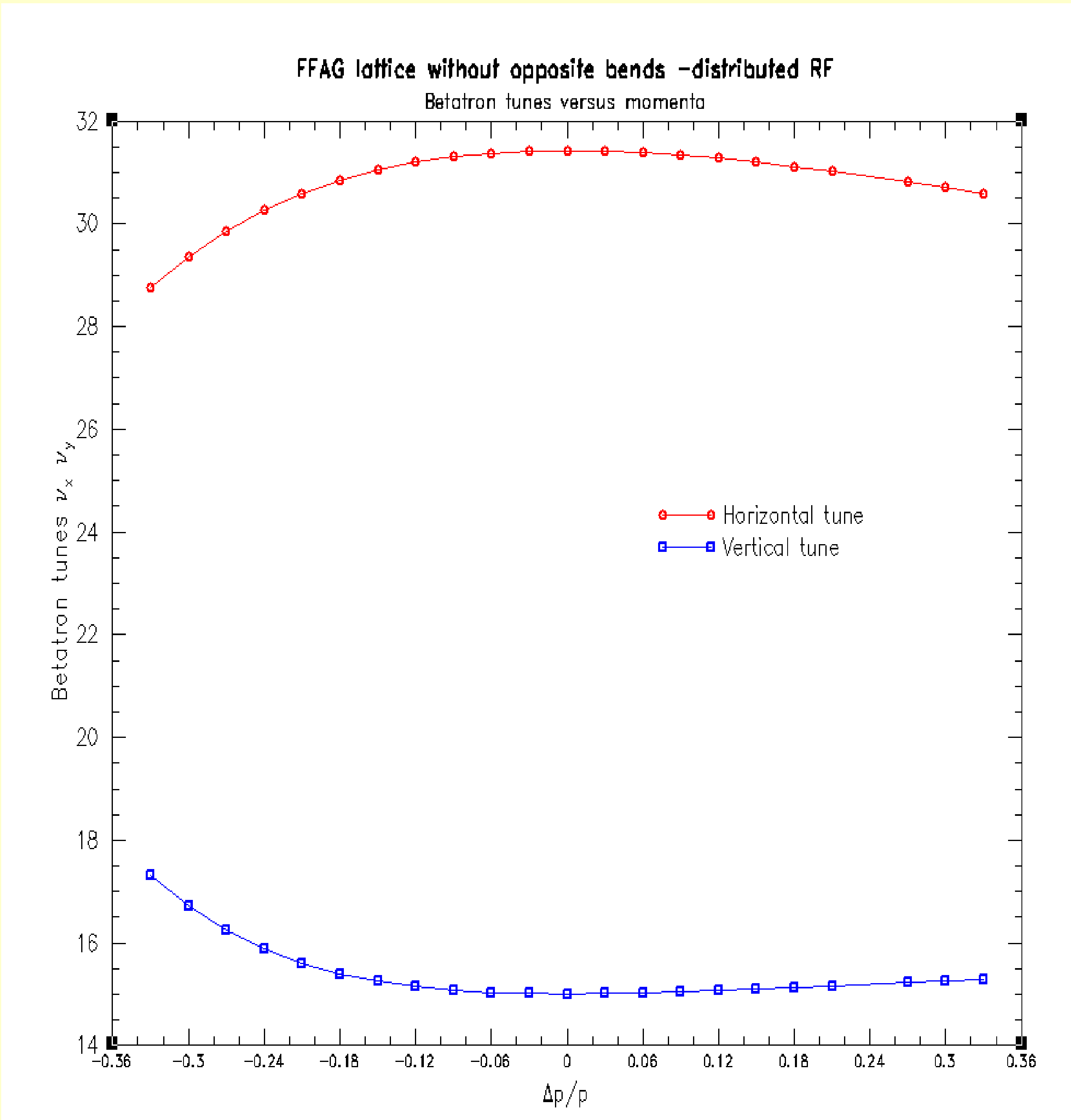
A part of the ring



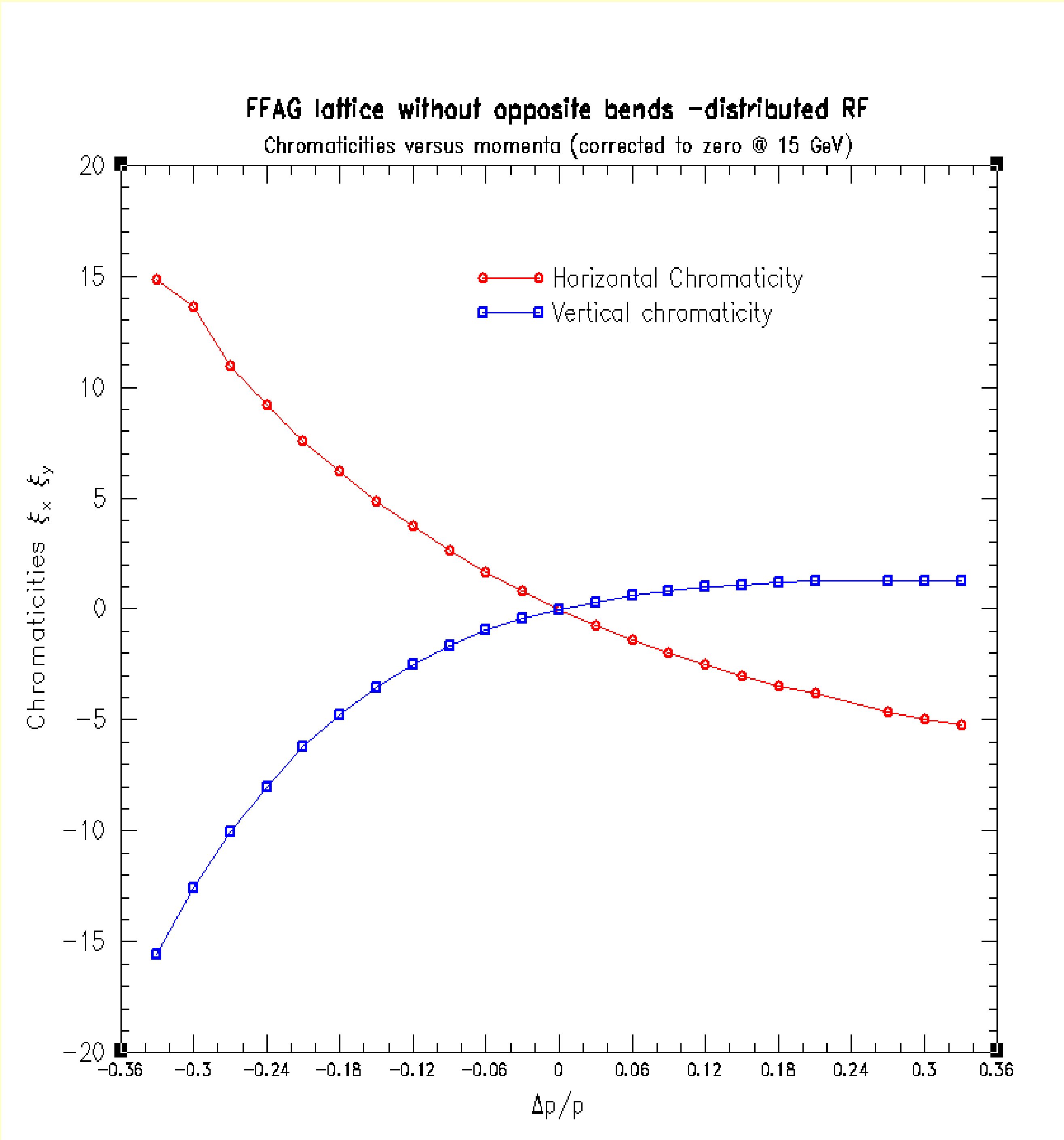
Two CELLS:


The FFAG lattice without opposite bends
Two Cells
QF
BD
QF
B
CAVITIES
CAVITIES
Radius [m]
Distance [m]
0.5
0
−0.5
−1
−7
−5
−3
−1
1
3
5
7

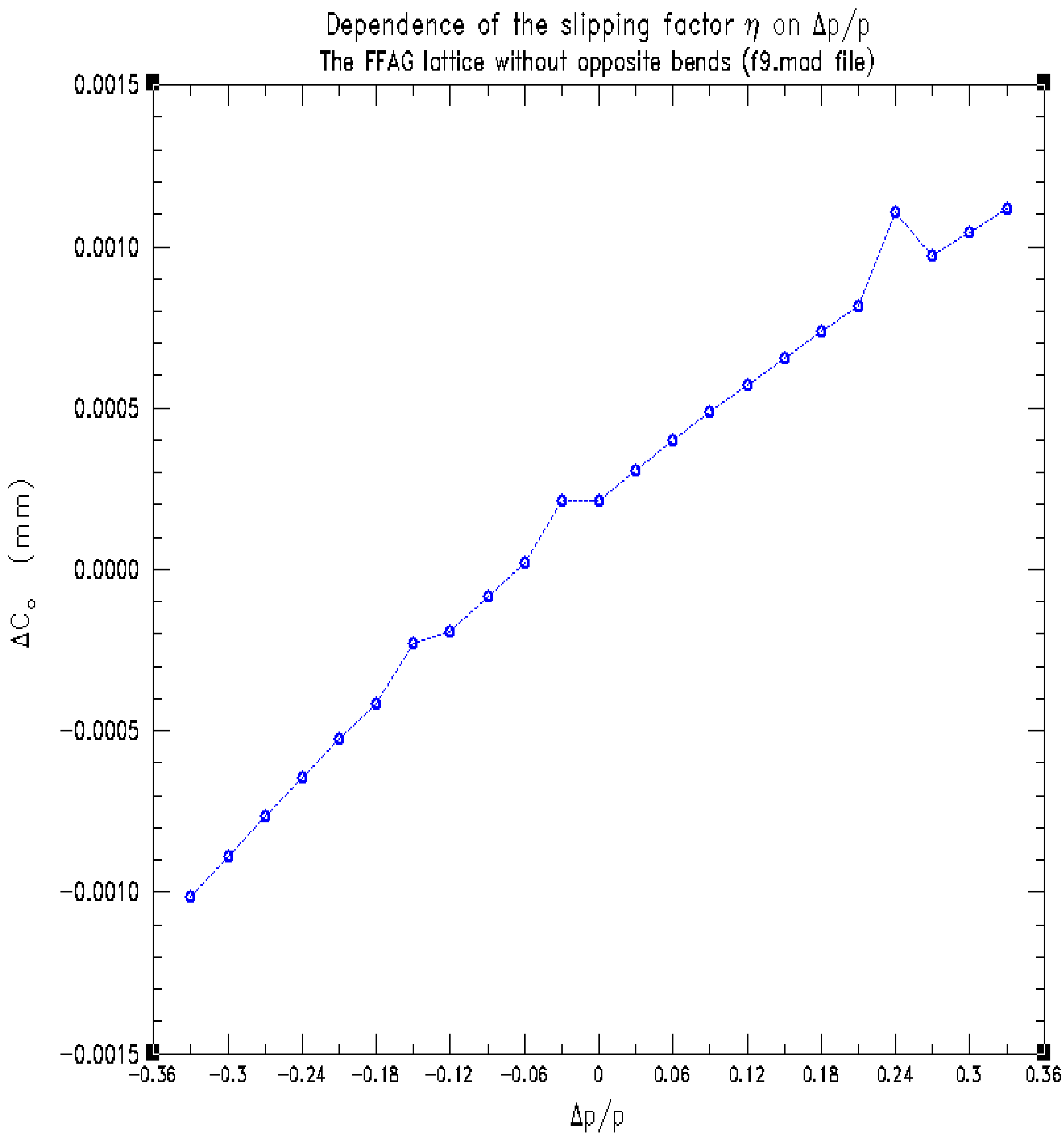
Betatron tunes during acceleration



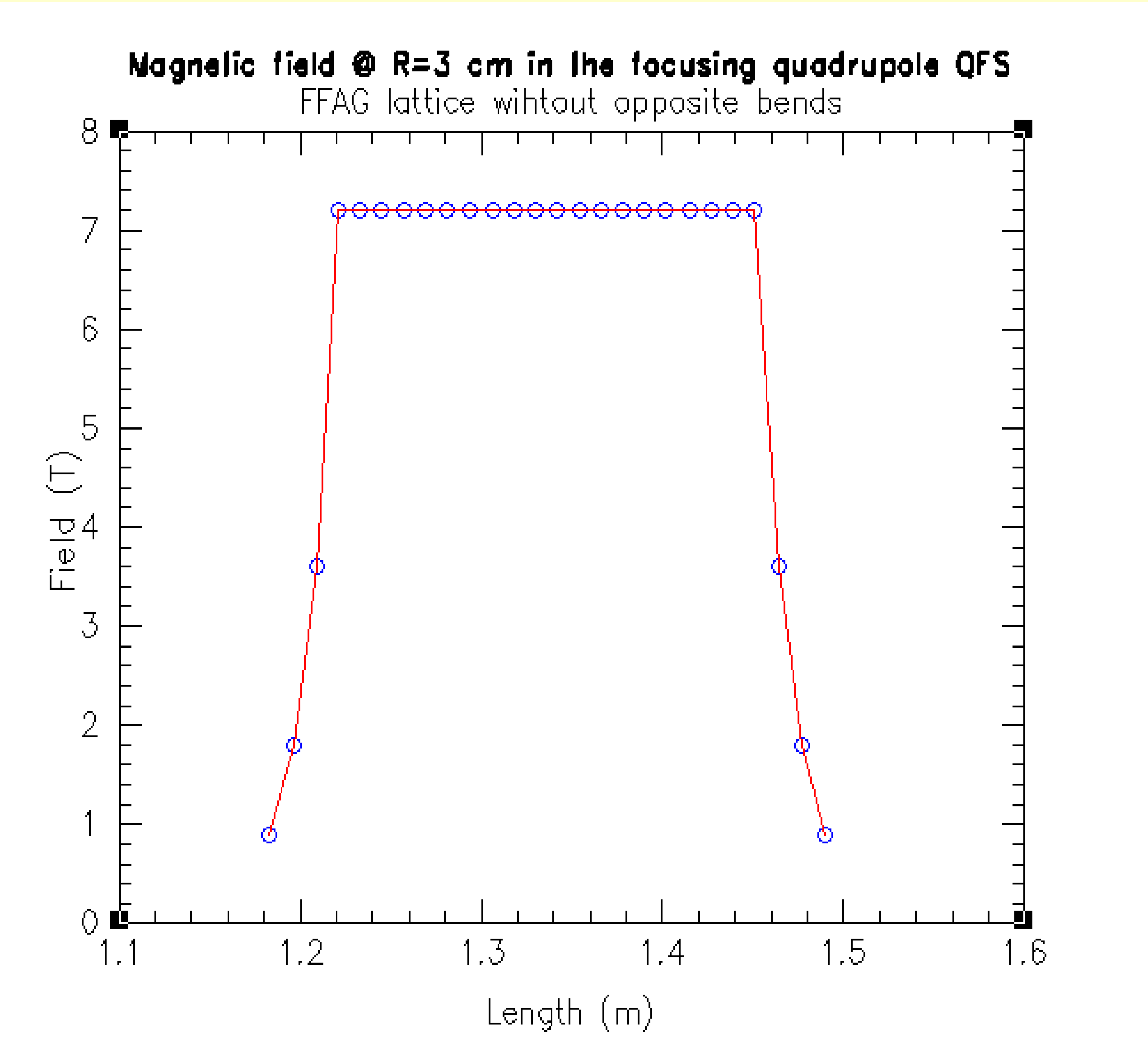
Chromaticities during acceleration – Corrected to zero at the central muon energy of 15 GeV



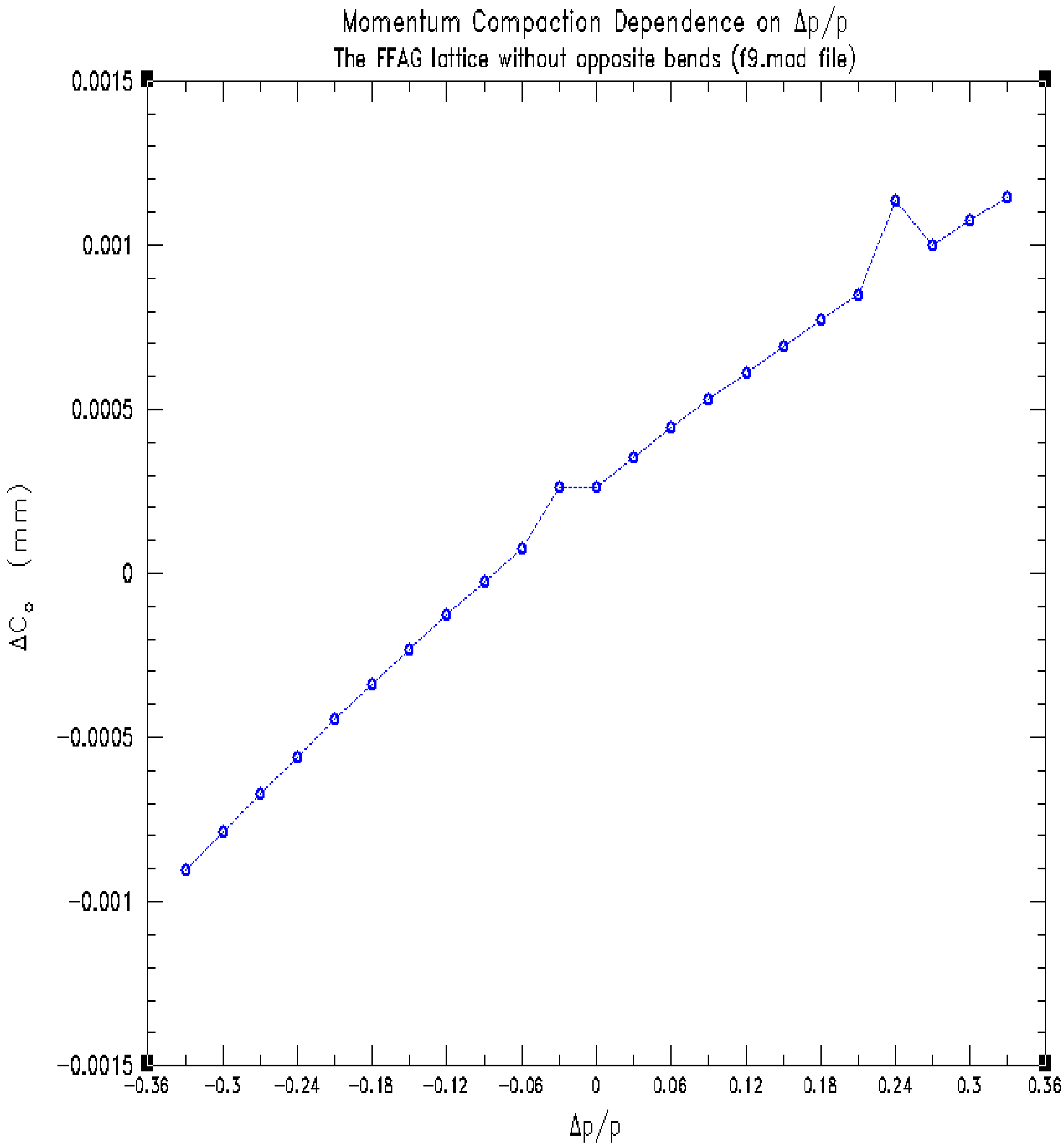
The slipping factor η during acceleration



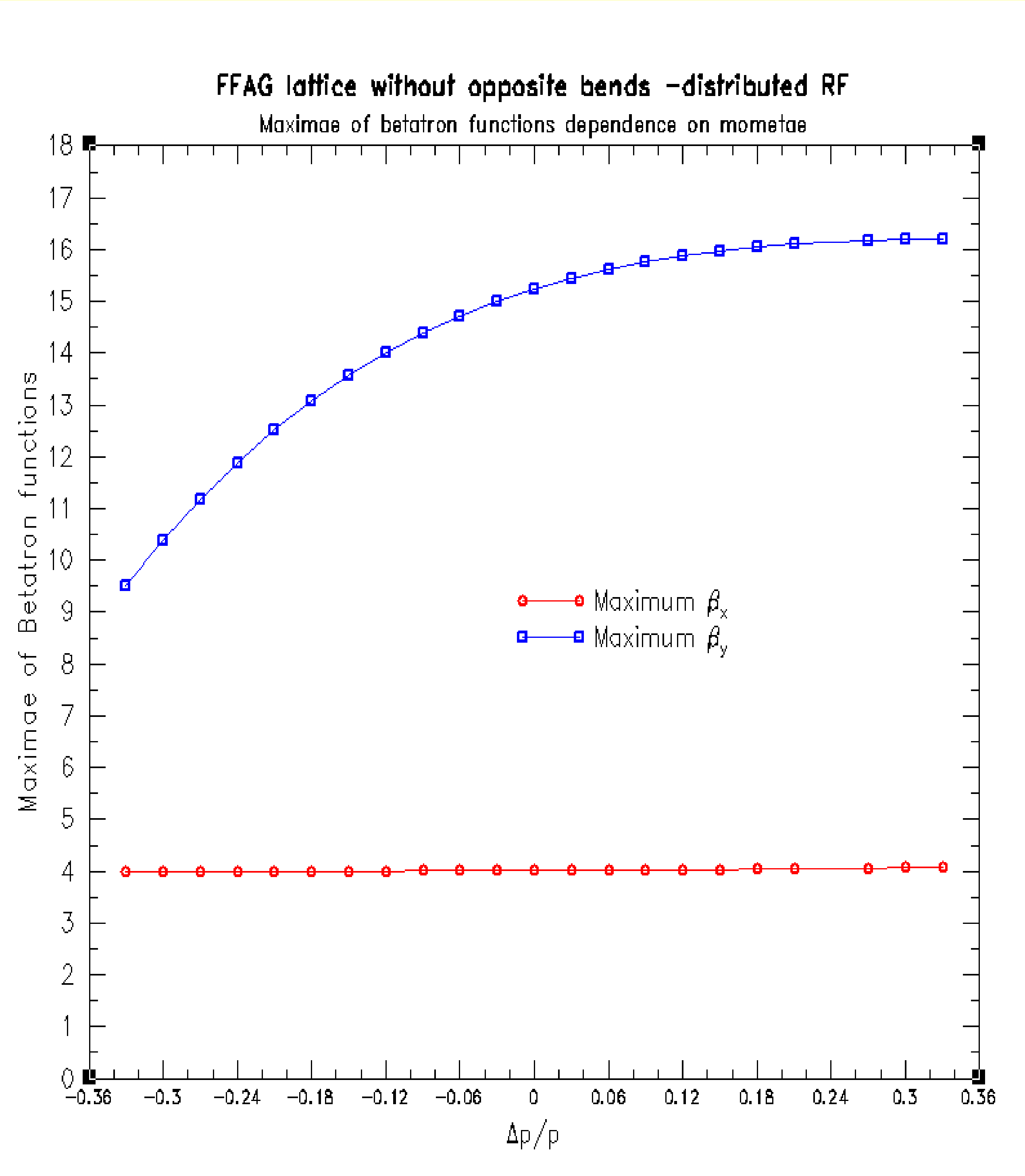
All previous results have a ~1m dipole divided into 100 pieces and quadrupoles divided into 26 and 46 pieces, as well they include the first attempt to include the end of the quadrupole field



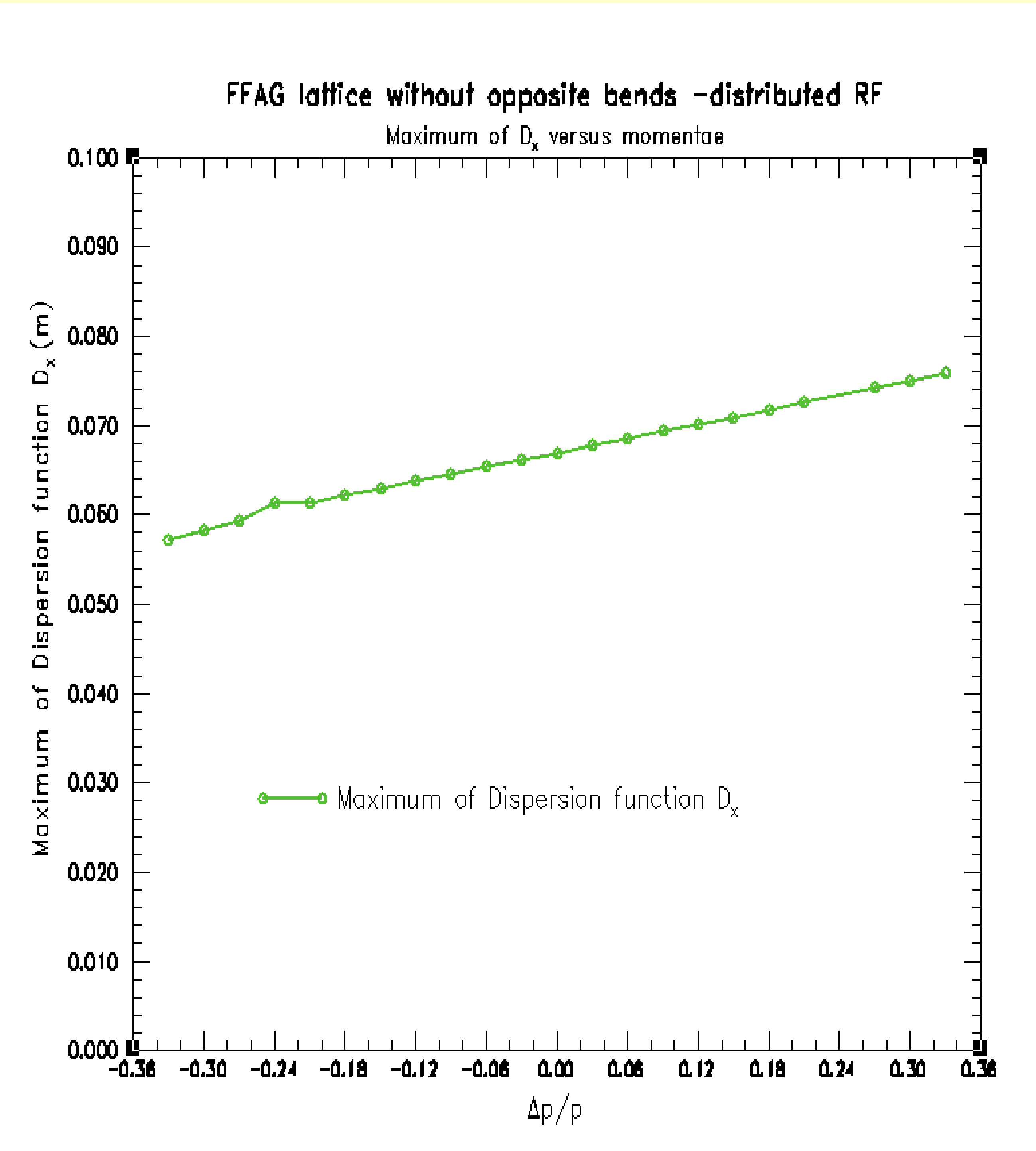
At negative Δp/p lattice is 'imaginary γ_t'



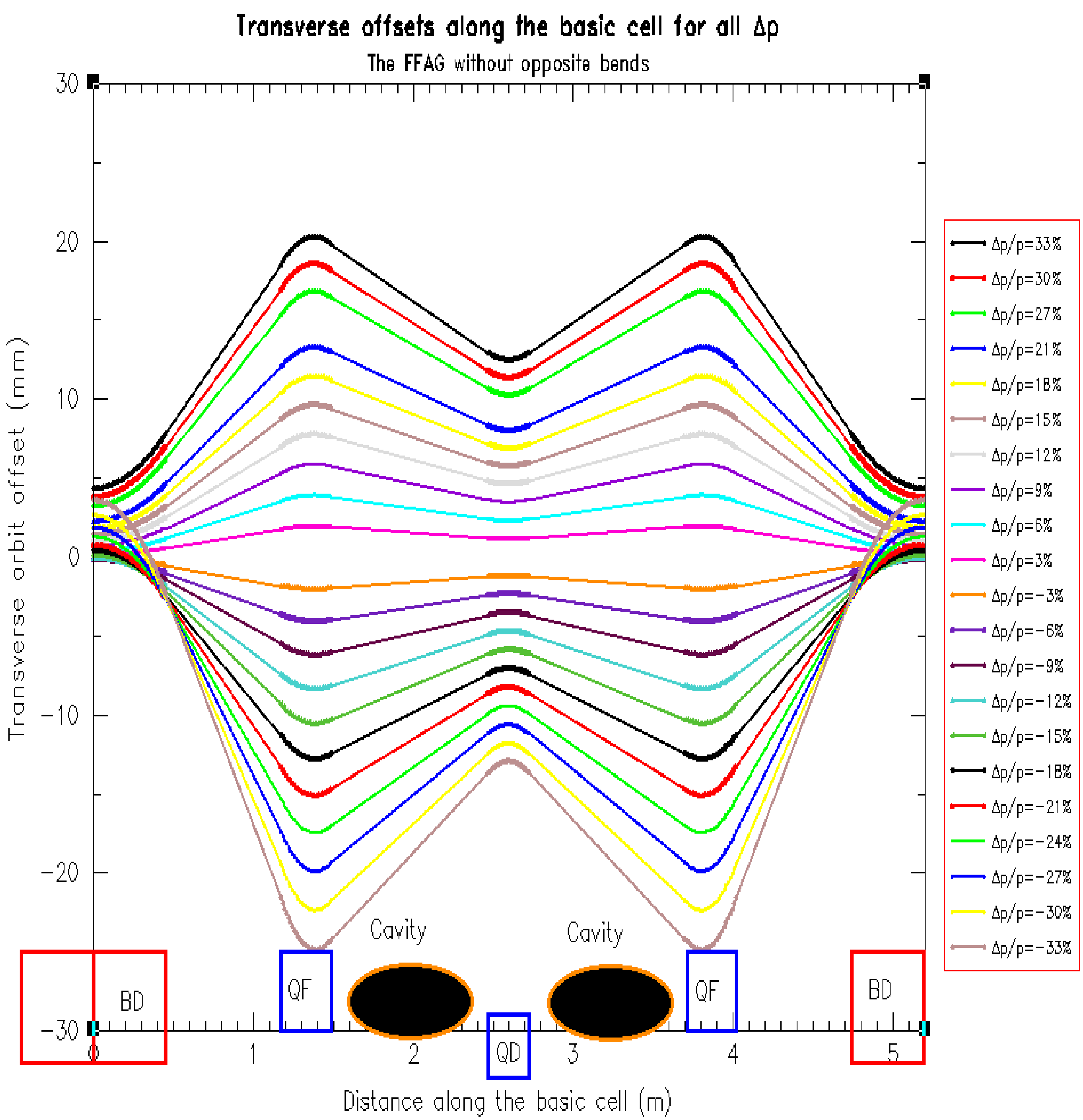
Maximae of the betatron functions during acceleration



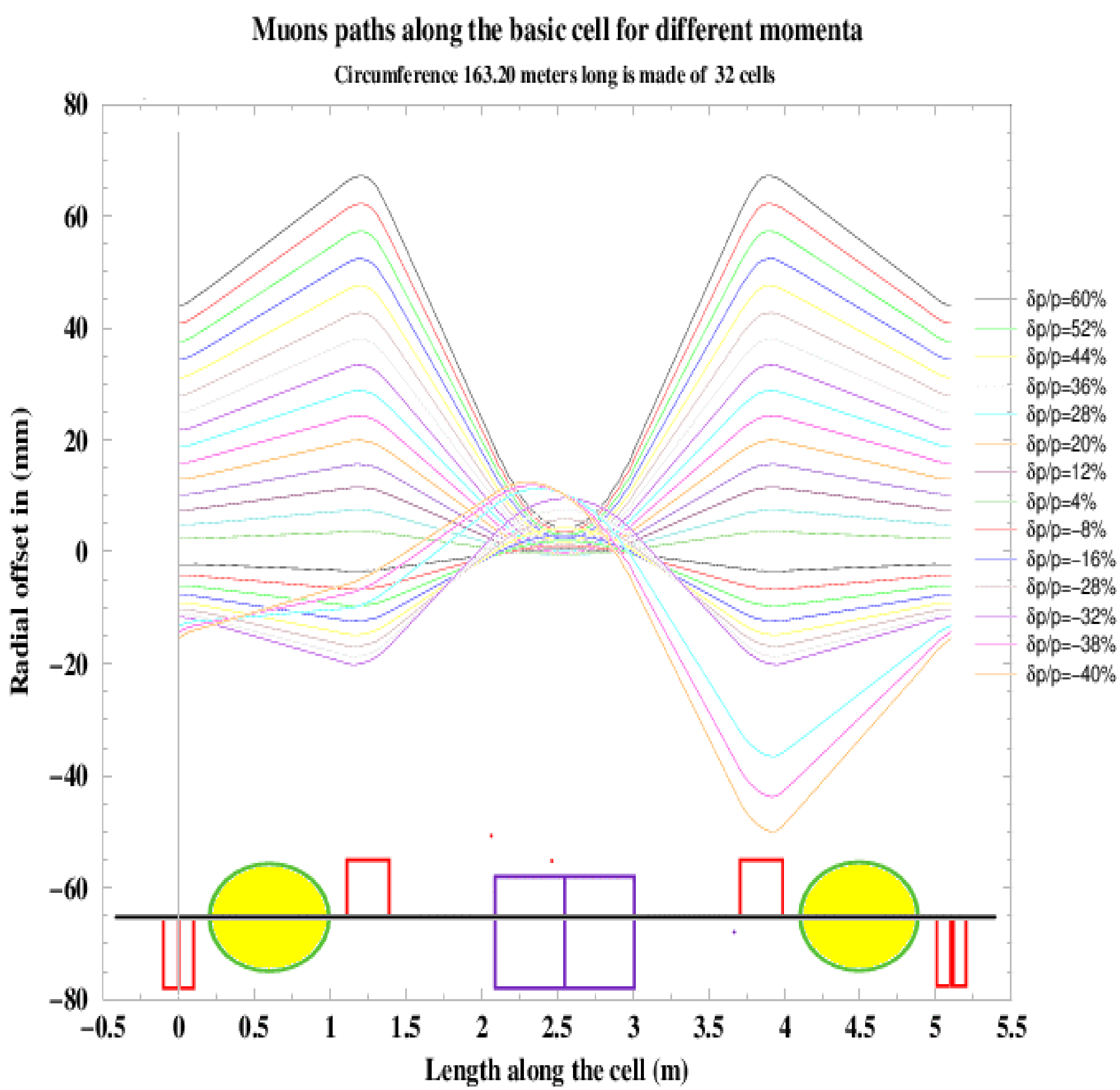
Maximum of the dispersion function during acceleration



A picture tells a story: particle path in the basic cell during acceleration

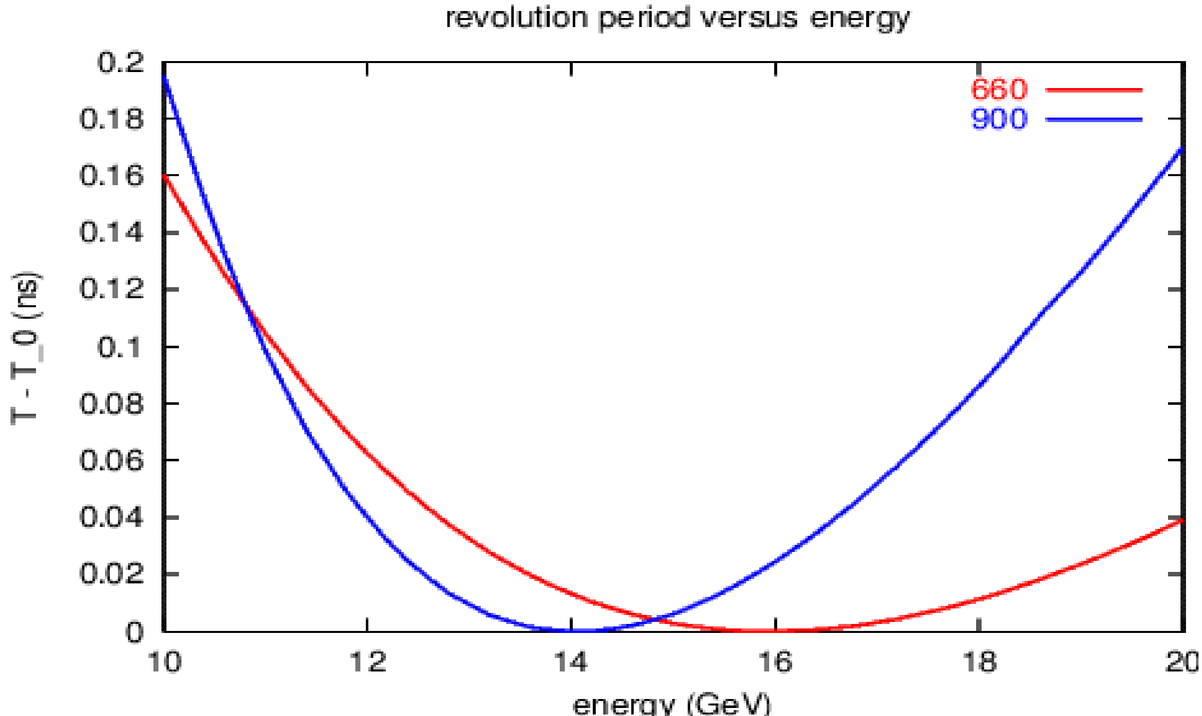


Particle path in one of the recent examples:



Simulation of the acceleration

RF considerations for FFAG rings
M. Blaskiewicz, BNL



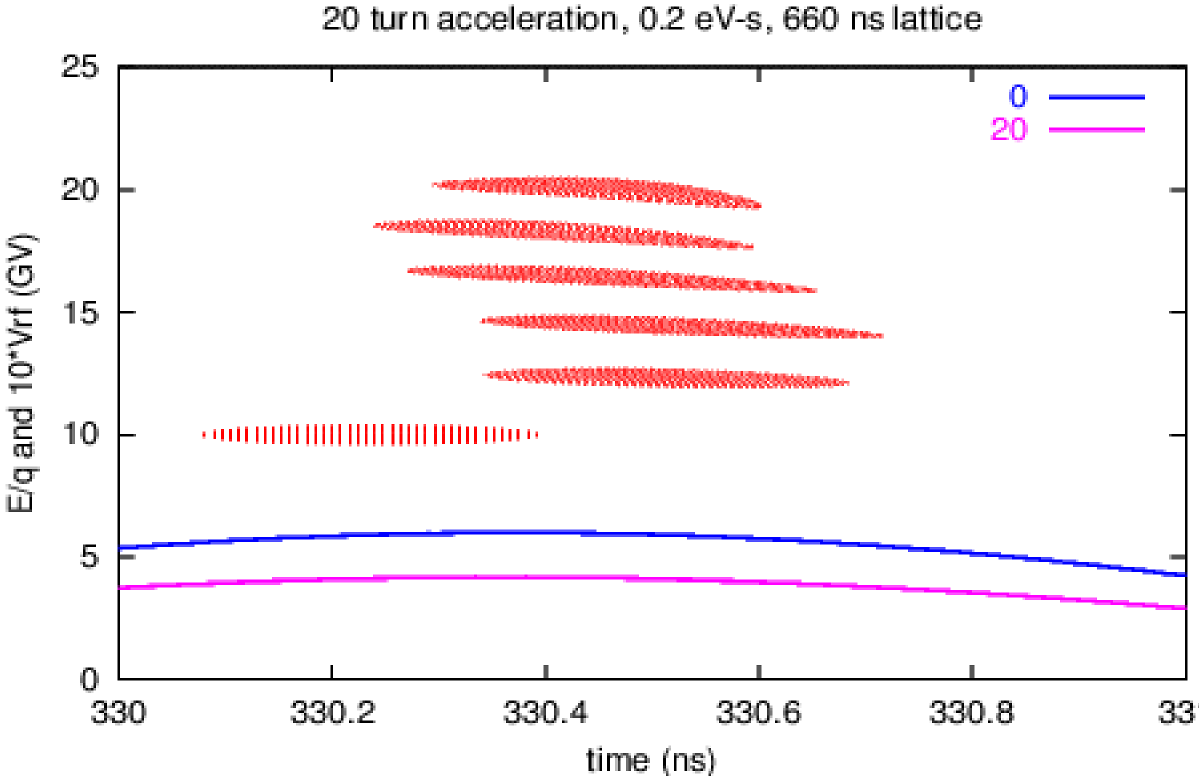
660 ns lattice from D. Trbojevic and 900 ns lattice from E. Courant. Assume negligible energy input to the RF system during acceleration[1] 1-D update equations are

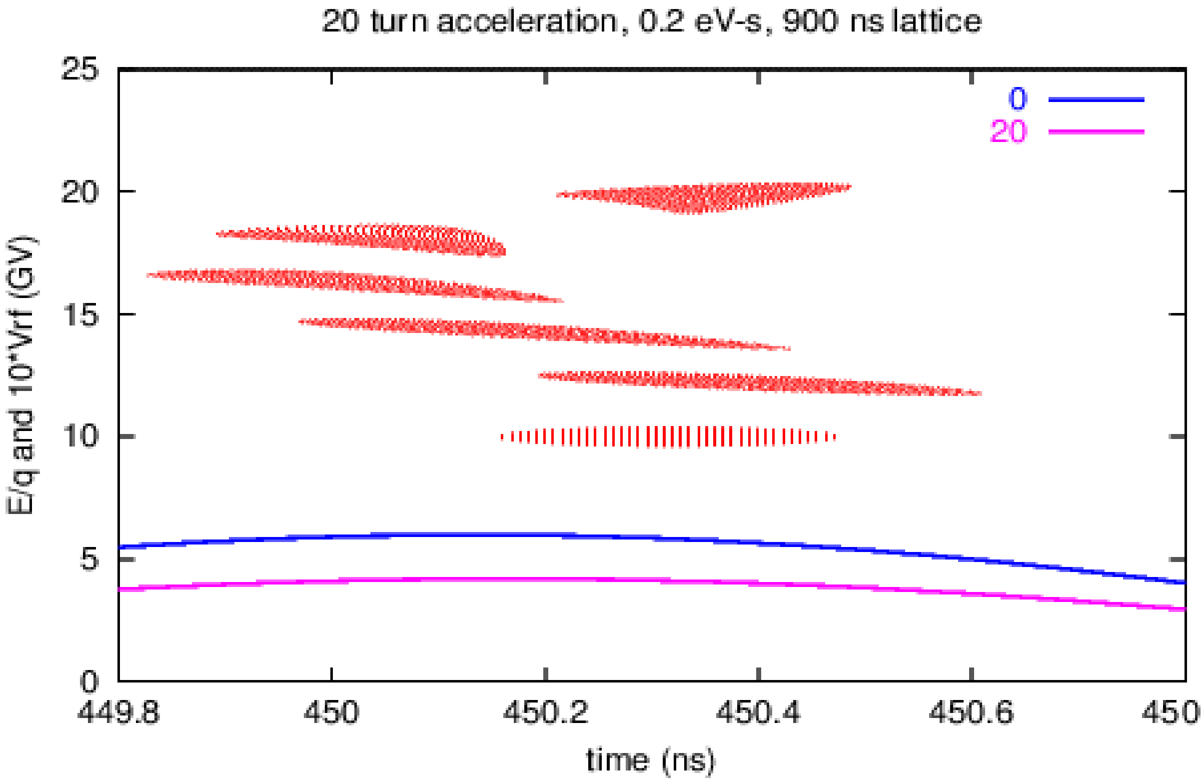
$$\tau_{n+1} = \tau_n + T(E_n) \quad (1)$$

$$\left(\frac{R}{Q}\right) I(t) = \frac{1}{\omega_{rf}} \frac{dV(t)}{dt} + \omega_{rf} \int_0^t dt_1 V(t_1) \quad (2)$$

$$E_{n+1} = E_n + qV(\tau_{n+1}) \quad (3)$$

$I(t)$ smoothed by 0.5 ps. V(t) updated with $\Delta t = 0.15$ ps.


20 turn acceleration, 0.2 eV-s, 660 ns lattice
E/q and 10*Vrf (GV)
time (ns)
0
20
25
20
15
10
5
0
330
330.2
330.4
330.6
330.8


20 turn acceleration, 0.2 eV-s, 900 ns lattice
E/q and 10*Vrf (GV)
time (ns)
0
20
25
20
15
10
5
0
449.8
450
450.2
450.4
450.6

Energetics of the RF system

For 6.25×10^{12} muons the total charge is 1μC.

Assuming a factor of 2 voltage drop the initial stored energy in the RF cavities is

$$U = 10\text{GV} \times 1\mu\text{C} \times \frac{4}{3} = 13\text{kJ}$$

The stored energy is related to the voltage and impedance by

$$U = \frac{V^2}{2\omega_{rf}\left(\frac{R}{Q}\right)}$$

Taking a total voltage of 500 MV and $\omega_{rf} = 2\pi \times 200$MHz one obtains $(R/Q) = 7.6$ kΩ.

The simulations used this impedance and $V = 600$ MV so the voltage dropped to 400 MV at the end of the cycle.

Taking 10 MV per cavity the requisite R/Q per cavity is 126Ω.

The stored energy per cavity is 300 J.

For $E = 10$ MV/m the volume is $0.7m^3$.

With 60 cavities some extra straight sections may be required but, since 10 GeV $\gg$ 106 MeV $= m_\mu c^2$, the straights will have a negligible effect on dT/dE.

References

[1] N. Holtkamp, D. Finley *eds.*, "A feasibility study of a neutrino source based on a muon storage ring", FNAL 2000.

Conclusions:

• **The latest results in the FFAG lattice without opposite bends with distributed RF are very encouraging.**

• **Present codes MAD and SYNCH should be checked by either other codes or by an analytical calculation.**

• **If it is shown that the presented idea is really possible the whole muon acceleration should be redone.**