ANALYSIS OF VARIABILITY IN THE DESIGN OF WOOD PRODUCTS UNDER IMPRECISION

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ABSTRACT

The design and analysis of many products is performed with imprecisely known parameters, relationships, and environmental conditions. This is especially true for wood products which exhibit greater variability than most materials. Fuzzy set theory applied to the design and analysis of wood products is regarded as a promising approach for modeling the geometric and mechanical property variability. A fuzzy mathematical model is used to analyze the design of a wood beam structure. The analysis is compared with a Monte-Carlo simulation and a root sum of squares analysis approach. The fuzzy set design approach compares favorably with these approaches and has several distinct advantages. It can model user preference as well as imprecision, it is computationally quick, and it better reduces the design space.

Keywords: Engineering design; fuzzy analysis; fuzzy constraints; decision support tool.

INTRODUCTION

Structural design with wood presents complex problems because wood is a natural material. Since the tree is subject to numerous, constantly changing environmental conditions, the properties tend to vary considerably [20]. In structural design a factor of safety is used to compensate for all these uncertainties and imprecisely known quantities. The factor of safety increases the design requirement for the allowable stress of a material. The resulting structure can support greater loads than the actual loads it will be required to support in service [6]. Clearly, an alternative approach of modeling this imprecision is by using imprecise design parameters in the calculations.

Structural design begins with a design specification, followed by analysis and then redesign as necessary. This adheres to the standard iterative design paradigm. A single, crisp design specification is analyzed and altered until a satisfactory design results. Ward, *et al.*, [19] have studied Toyota and determined they use a set-based approach where sets of design possibilities are simultaneously specified and analyzed. The set-based approach more efficiently explores the feasible design space. This paper presents a fuzzy constraint processing approach for the design and analysis of wood structures. The results are compared to two common methods for analysis under imprecision.

RELATED WORK

Most fuzzy design systems are for selection among alternatives. Many of these systems are for conducting design evaluation [3; 10; 13]. They can be categorized as decision support systems tailored specifically for design decisions. Giachetti, *et al.*, [7] describe a hierarchical model of fuzzy constraint networks for attaining *precision convergence* in the design process. This is the systematic reduction of design imprecision. Zimmermann and Sebastian [25] propose an Intelligent Design Support System. This system would combine many technologies for solving design problems. A compositional hierarchy is used to store knowledge. The problem is modeled as a multi-attribute decision making model or as a linear programming model. Heuristics, neural nets, or genetic algorithms can be used to find a solution to the problem. A set of tools, embodied in a software system KONWERK, are suggested to support this solution method.

Approaches applying algebraic constraints on domains of real numbers have been proposed for mechanical design [18]. In the larger context of concurrent engineering there is a need for logical constraints, linguistic constraints, and database or relational constraints [14]. Conditional constraints, conjunction, and disjunction are important operators in a concurrent engineering system. Young, *et al*., [23] have developed a fuzzy constraint processing system, called FuzCon, that supports these functions.

Another approach to modeling imprecision in design is the method of imprecision (MoI) developed by [22]. Rather than direct calculations the MoI technique allows a designer to estimate the strength of the coupling between design parameters and performance parameters over a wide range of values. The design parameters are ranked according to the strength of their coupling by a metric. Those with the strongest coupling become candidates for modification in an attempt to reduce the imprecision of the output. In this sense, it is related to Taguchi's method [15] and to utility theory [16].

Interval approaches are more numerous. Boettner and Ward [1] built the Mechanical Design Compiler which uses a labeled interval calculus to determine which components in a design catalog are feasible with respect to algebraic constraints of real variables. Kim *et al*. [9] created a logic based constraint processing system which incorporated interval arithmetic into the logic structure. Other approaches to representing imprecision in design include using utility theory, implicit representations using optimization methods, matrix methods such as Quality Function Deployment, probability methods, and necessity methods. These methods have all had limited success in solving design problems with imprecision. Otto and Antonsson [17] provide an extensive review of these approaches and the reader is directed to their paper for a more detailed discussion.

CONSTRAINT SATISFACTION

The representation of a constraint satisfaction problem, defined as a constraint network problem, can be defined as follows (this is adapted from [2]):

Fuzzy Constraint Network Problem: A fuzzy constraint network problem consists of a set of *n* variables, $\tilde{X} = \{\tilde{X}_1, \tilde{X}_2, ..., \tilde{X}_n\}$, and a set of *m* constraints, $C = \{C_1, C_2, \ldots, C_m\}$. A fuzzy variable $\tilde{X}_i$ has its domain, Ω_i, which defines the set of values that the variable can have. A constraint C_i is a *k-ary relation* on $\tilde{X}_i' = \{\tilde{X}_{i1}, \tilde{X}_{i2}, \ldots, \tilde{X}_{ik}\} \subseteq \tilde{X}$ and $k \leq n$, i.e., $C_i(\tilde{X}_{i1}, \tilde{X}_{i2}, \ldots, \tilde{X}_{ik})$, and is a subset of the Cartesian product $\Omega_{i1} \times \Omega_{i2} \times \cdots \times \Omega_{ik}$.

In this formulation each constraint is satisfied to a degree, $\mu_{Ci} \in [0,1]$. This is the membership value of the constraint. A solution of the network are defined as an assignment of values to all the variables such that the constraints are satisfied. The constraints are satisfied when $\mu_{Ci} \geq \alpha_S$ where α_S is the *system truth threshold*. It is a level of satisfaction a solution must fulfill within the entire network to be accepted by the designer. This value is set *a priori* by the user [23].

The constraint processing system used to implement the beam design and analysis described in this paper is called FuzCon. In Young, *et al*., [23] the system and its operator set are described along with a brief review of prior fuzzy constraint processing work. FuzCon is the latest generation in a series of constraint processing systems which include; SPARK [24], SATURN [5], and, JUPITER [11]. The approach taken in these systems is to view each constraint as a logic sentence with an associated truth value. (See [8] for an elaboration of the equivalency of a logic-based system and the constraint satisfaction problem defined above.) The constraint processing system's objective is to make all the constraints "*satisfied*". It performs this by interacting with the user. A user supplies values for a variable and the system propagates those values through the system inferring unknown values where possible and calculating the state of the constraints where possible. Consequently, this system supports an intelligent user in finding a feasible solution. It does not perform automatic constraint satisfaction. In the context of engineering design, constraints represent the requirements an artifact must satisfy. The designer then interacts with the system, testing different design alternatives in a solution space bounded by the constraints.

DESIGN INITIALIZATION

The functional requirements for a deck are to support a sufficient load and to cover a sufficient area within given cost requirements. Beam design requires consideration of many factors, such as the type of construction, materials, loads, and environmental conditions. However, in many cases, this task can be reduced to the selection of a particular beam shape and dimensions such that the realized stresses do not exceed the allowable stresses [6]. Figure 1 shows a simply supported beam. In this simplified analysis we will only consider the bending stress and the shear stress. Other considerations, such as buckling and stress concentrations will not be included in this analysis. The design parameters and constraints are shown in Figures 2 and 3 respectively.

Overall, wood exhibits greater imprecision in mechanical properties than most building materials. It is for this reason the mechanical properties of the beams can be modeled as fuzzy numbers. Histograms for 16 species of pine were compiled for the modulus of elasticity and the yield stress. The yield stress of pine is between 41.4 MPa and 62.1 Mpa. The ultimate stress for pine is between 55.2 Mpa and 96.5 Mpa [20]. Dubois and Prade [4] outline a procedure for obtaining a possibility function from a histogram. This data can be roughly represented as triangular fuzzy numbers for the purpose of computation in the fuzzy constraint processing system. Yield stress is represented as ⟨41.4, 51.7, 62.1⟩ MPa and the modulus of elasticity is represented as ⟨8.3, 10.4, 13.7⟩ GPa.

Beam design begins with known loads, desired area, and beam dimensions. The input values are uniform load q, ranges from ⟨2803, 3153, 3504⟩ kg/cm, and desired length l, is about 2.75 m ⟨2.5, 2.75, 3⟩ m. Standard beam dimensions are height $\tilde{h}$ between 3.8 cm and 7.6 cm and base $\tilde{b}$ between 8.9 cm and 15.2 cm. Design requirements are the maximum deflection f be less than 1.25 cm and the section modulus S be greater than the ratio of moment to allowable stress. Given this information, the rest of the information will be propagated in the constraint network.

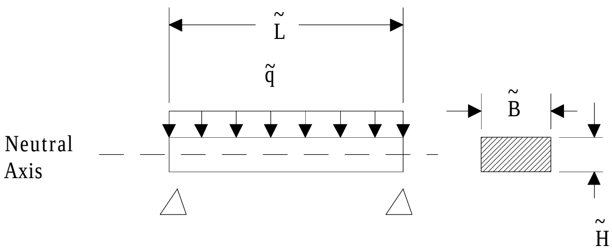

Figure 1. Conceptual Diagram of a Simply Supported Beam

Variables	Description Units
q	Uniform Load (kg/cm)
I	Moment of Inertia (cm^4)
M_max	Maximum bending moment (cm-kg)
V_max	Maximum shear force (kg)
E	Young's modulus of elasticity, ⟨8.3, 10.4, 13.7⟩ GPa
f	Deflection at beam center (cm)
strs_all	Maximum allowable stress ⟨41.4, 51.7, 62.1⟩ MPa
t_max	Tau, shear stress (Pa)
S	Sectional modulus (cm^3)
f_max	Max desired deflection (cm)

Figure 2. Problem Definition Parameters

b	Base of beam (cm)
h	Height of beam (cm)
l	Length of beam (m)

Figure 2. Design Parameters

$f = \dfrac{5ql^4}{384EI}$	deflection at beam center
$max\ \ t = \dfrac{(max\ V)h^2}{8I}$	maximum shear stress at ends
$max\ M = \dfrac{ql^2}{8}$	maximum moment beam center
$max\ V = \dfrac{ql}{2}$	maximum shear force
$S = \dfrac{max\ M}{str\_all}$	section modulus = ratio of *max M* to allowable stress
$I = \dfrac{bh^3}{12}$	inertia of rectangular cross-section
$S = \dfrac{bh^2}{6}$	section modulus definition from beam dimensions
$max\ t \tilde{<} strs\_all$	maximum shear stress less than maximum allowable shear stress
$f \tilde{<} \max\ f$	actual deflection less than desired maximum deflection

Figure 3. Constraints For Simply Supported Beam

SOLUTION ANALYSIS USING FUZZY MEMBERSHIP FUNCTIONS

This section analyzes the solution space of the example problem. The two decision variables are $\tilde{b}$ and $\tilde{h}$. These are plotted to determine the solution space. The fuzzy set design approach used here is compared with two other common methods. The first method is to perform a root sum of squares analysis of the variance. The second method is to perform a Monte-Carlo simulation. The results using these three methods are obtained and compared with each other.

The problem is reduced to the selection of a height and base dimension of a beam by holding all other parameters constant. This is a reasonable assumption, since the only other design parameter is the deck length and the desired length is customer specified. The constraint with the known values instantiated is rewritten as a function of only $\tilde{b}$ and $\tilde{h}$. As previously stated, the deflection constraint is the critical constraint and defines a more restrictive design space. The deflection constraint is,

$$f = \frac{5ql^4}{384EI}$$

Substitute for *I*,

$$I = \frac{bh^3}{12}$$

Substitute values for *q*, *E*, and *l* and rewrite the deflection expression to obtain two expressions in terms of $\tilde{b}$ and $\tilde{h}$,

$$\tilde{h} = \left\langle \frac{7.17}{\sqrt[3]{b}}, \frac{9.20}{\sqrt[3]{b}}, \frac{11.45}{\sqrt[3]{b}} \right\rangle \qquad (1)$$

In Figure 4 an upper limit on feasible base dimensions is drawn. This constraint is based on the widest beam available, 15.25 cm. This constraint limits the feasible design space to beams of at least 11.2 cm in height. This dimension is found as the intersection of the upper dashed (worst case) line with the 15.25 cm base constraint.

Suppose a fuzzy dimension for $\tilde{b}$ is selected as $\langle 9.5, 10.2, 10.8 \rangle$. Then the fuzzy values for $\tilde{h}$ can be determined from the graph. The upper value on $\tilde{h}$ is:

$$h_{upper} = \max \left\{ f(x) \middle| \ x \in \tilde{b} \right\} \qquad (2)$$

h_{upper} = 16.5 cm and the lower value on $\tilde{h}$ is;

$$h_{lower} = \min \left\{ f(x) \middle| x \in \tilde{b} \right\} \qquad (3)$$

h_{lower} = 9.75 cm.

Figure 4 shows for a desired base between 9.5 cm and 10.8 cm the corresponding height is between 9.75 cm and 16.5 cm. The dashed lines of the graph define a feasible design space with respect to this constraint.

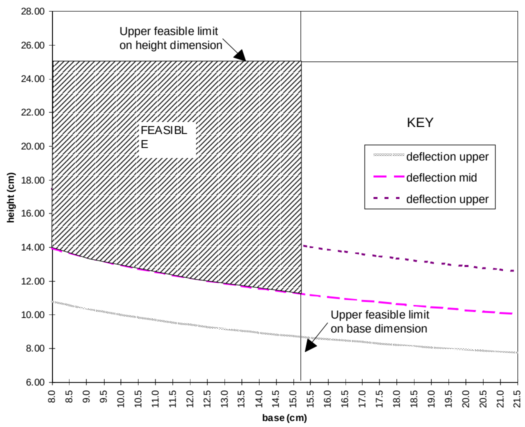

Figure 4. Plot of Section and Deflection Constraint with Limits on Dimensions

Figures 4 provide important information for analyzing the problem. The designer can visually see the trade-offs made by selecting one beam dimension. Since the constraints are "greater than constraints", the designer would be advised to only consider the solution space greater than the solid line of each plot. The upper solution limit shows a "worst case" scenario and designs with values on this line are conservative. Solutions much greater than this line become less desirable since cost constraints (not shown in these graphs) would probably become important. The feasible design space is defined by the solid horizontal and vertical lines, which denote upper limits on beam sizes available in the vendor catalog, and *the Deflection Lower* line. This is denoted in Figure 4 as Area 1.

ALTERNATIVE ANALYSIS APPROACHES

Designing with uncertain parameters and properties has traditionally been handled using statistical approaches. Two commonly used approaches are the root sum of squares analysis method and Monte-Carlo simulation.

Approach 1: Root Sum of Squares

The root sum of squares (RSS) approach analyzes the functional equation governing the output [12]. In this approach the design goal, minimize the variance in the deflection is found from the variance of the input parameters. These parameters are the length, base, and height of the beam and are assumed to be normally distributed. The load q is treated as a known constant, and E is a known constant. If the variance of the length is 1 inch and the variance of height and base are 0.5 inches than the problem can be solved.

$$f = \frac{5ql^4}{384EI} \quad \text{and} \quad I = \frac{bh^3}{12}$$

The RSS method is used to determine the variance of f.

$$\sigma^2(f) = \left(\frac{\partial f}{\partial l}\right)^2 \sigma^2(l) + \left(\frac{\partial f}{\partial b}\right)^2 \sigma^2(b) + \left(\frac{\partial f}{\partial h}\right)^2 \sigma^2(h) \qquad (4)$$

taking the partial derivatives the expression for the variance of f is obtained.

$$\sigma^2(f) = \left(\frac{20ql^3}{32Ebh^3}\right)^2 \sigma^2(l) + \left(\frac{-5ql^4}{32Eb^2h^3}\right)^2 \sigma^2(b) + \left(\frac{-15ql^4}{32Ebh^4}\right)^2 \sigma^2(h) \qquad (5)$$

The problem is set up as a non-linear programming problem with the following values,

q = 3153 kg/cm
E = 10.4 GPa
l = 2.75 m σ^2 (l) = 0.08
b = 10.2 cm, σ^2 (b) = 1
h = 13 cm, σ^2 (h) = 1

The objective is to minimize the variance. The decision variables are the length, width, and base.

The solution of expression (5) for the deflection is;

f = 1.25 cm, σ^2 (f) = 0.057, σ = 0.241

In non-linear problems it is noted that this method is only an approximation. The deflection in this example is highly non-linear. Consequently, this is not an exact solution.

Approach 2: Monte-Carlo Simulation

A Monte-Carlo simulation of 100,000 samples was conducted with the data. As in the RSS approach normal distributions are assumed. The mean deflection was 1.31 cm and the standard deviation was 0.71 cm.

COMPARISON OF THREE APPROACHES

The fuzzy analysis approach assumes a triangular membership function for all values. The above problem can be solved using FuzCon. The result using FuzCon for the deflection is, ⟨0.86, 1.25, 1.95⟩. The question arises, how does this result compare to the statistical approaches? The cumulative distribution function is used to determine what percentage of the TFN and the root sum of squares method overlap. Using η(1.25, 0.241) for the root sum of squares normal distribution and the TFN values of , ⟨0.86, 1.25, 1.95⟩ we can compute the overlap as,

$$z_{Lower} = \frac{x_{Lower} - \mu}{\sigma} = \frac{0.86 - 1.25}{0.241} = -1.62$$

$$z_{Upper} = \frac{x_{Upper} - \mu}{\sigma} = \frac{1.95 - 1.25}{0.241} = 2.90$$

This is 94.5% of η(1.25 , 0.241). Thus, the TFN captures 94.5% of the normal distribution obtained by the RSS method. Interestingly, the variance approach is also itself an approximation due to the non-linear functions used. The same method is used to compare the TFN with η(1.31, 0.71) obtained by the Monte-Carlo simulation.

$$z_{Lower} = \frac{0.86 - 1.31}{0.71} = -0.634$$

$$z_{Upper} = \frac{1.95 - 1.31}{0.71} = 0.901$$

This is 56% of η(1.31 , 0.71). Thus, the TFN captures 56% of the normal distribution obtained by the Monte-Carlo simulation. Next, the root sum of squares method and the Monte-Carlo method are both compared. Points from the root sum of squares method are used at two standard deviations.

$$z_{Lower} = \frac{0.768 - 1.31}{0.71} = -0.76$$

$$z_{Upper} = \frac{1.73 - 1.31}{0.71} = 0.59$$

This is 50% of η(1.31 , 0.71). The distributions for all three methods are shown in Figure 5. The data points are for the vertices of the TFN and the points which are two standard deviations from the root sum of squares mean, and the Monte-Carlo determined mean. It is noted that all three methods are approximations. The results indicate that the differences between the root sum of squares method and the Monte-Carlo method are comparable (of the same magnitude) as the differences between the fuzzy set approach and either statistical method.

Additionally, the statistical approaches assign a small probability to deflections which are not physically justified by the input values. The physical limits of deflection are shown by the vertical lines in Figure 5. In the Monte-Carlo simulation this could have been corrected by using Beta distributions instead of normal distributions, but was not done so that the results could be compared.

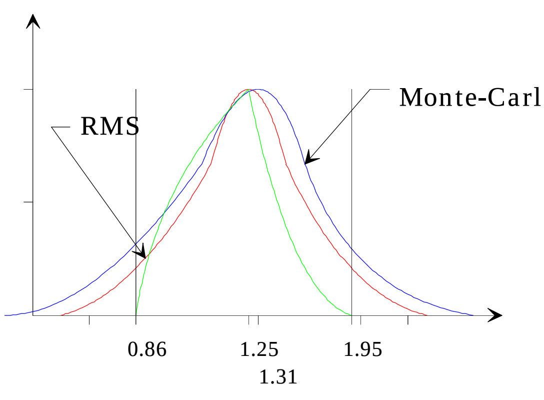

Figure 5. Comparison Between Statistical Approaches and Fuzzy Approach

Some of the differences between these approaches can be attributed to the assumptions used in each approach. The fuzzy approach used triangular membership functions whereas the statistical approaches used a normal distribution.

CONCLUSION

The design approach using fuzzy sets differs from more traditional methods for dealing with imprecision and uncertainty in design. Using fuzzy sets, the imprecision is associated with each variable and not lumped into a factor of safety. Consequently, the fuzzy constraint analysis approach is an integral element of the design process. Additionally, fuzzy sets have richer semantics than using intervals alone. They can represent a combination of preference and possibility in calculations. In this example, imprecisely stated customer preferences were used in conjunction with imprecise engineering data, and precisely known relationships. The fuzzy constraint satisfaction approach requires less computations than the Monte-Carlo simulation, yet it provides similar results to the more common approaches to handling imprecision in design.

The fuzzy set design methodology enables a sensitivity analysis of design parameters not typically performed with the statistical approaches since the time requirements of each simulation discourage designers from exploring the design space. In the conceptual design phase the dimensions and technical parameters are represented by sets of possible values. Since the union of acceptable design values envelop the process variation it indicates all the beams described by these sets will satisfy the constraints. This is a form of robust design since the environmental conditions under which the designed artifact must operate are imprecisely specified. If a designed artifact satisfies these fuzzy conditions then it can be concluded that it will most likely perform under the entire range of environmental conditions. This is an issue which needs further exploration.

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