The SOPHI Project:
Phasing Lightweight Space Optics Using An Exact Solution

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ABSTRACT

An ultra lightweight space optics system will require active optical control to achieve high optical quality.

The goal of active control is to phase-up the lightweight mirror. Phasing is achieved by controlling the local piston on actuated monolithic or segmented primaries, secondaries or higher corrective mirrors. The problem is that a surface actuated at discrete points cannot arbitrarily take on the shape required to achieve the highest optical quality.

We have found an exact solution[1] to this phasing problem that optimizes the optical quality for any hardware implementation. We have implemented this solution in the SOPHI hardware design.

We discuss how various wave front sensor methods[2, 3] can be used with this exact solution.

Actuated Surface Control Problem and Exact Solution

In order to satisfy the launch package size and weight requirements, the primary mirror of a large space telescope must be segmented or flexible in order to be compacted and then re-deployable at altitude. High optical performance will require that the primary be actively controlled and/or adaptive optics be employed somewhere in the telescope.

Active control of a lightweight mirror requires a method to sense the state of the mirror which typically involves a beacon (reference laser or guide star) and a wavefront sensor. The goal of the active control is to phase-up the lightweight mirror. Phasing is achieved by controlling the local piston on the active actuated surfaces. The lightweight mirror surface is phased-up when the surface reproduces the near-field reflected beacon phase front at the level of precision this phase front is measured by the wavefront sensor.

The control problem arises because a surface actuated at discrete points cannot arbitrarily take on the shape required to reproduce the conjugate beacon phase front as measured by the wavefront sensor. For a simple example of the problem, consider 4 neighboring actuators, A_1 to A_4, arranged in a square as illustrated in Figure 1 on the next page. Each actuator controls the local piston of the mirror surface. If we move all of the actuators in the mirror the same amount the optical quality does not change, only the focal plane is translated. Hence, the global piston of the mirror is an irrelevant degree of freedom with respect to mirror surface control and we are free to fix one piston in the mirror and set all other pistons relative to this one. For simplicity we can choose this to be actuator A_1. Alternatively, we can think of A_1 as fixed from phasing up a neighboring square of actuators. In either case, we next adjust A_2 to phase up the mirror in the region around A_2 with the mirror at A_1. Now A_1 and A_2 are fixed. Continuing around the square, we next adjust A_3 to phase up the mirror in the region around A_3 with A_2. A_3 is also now fixed. A_4 is then adjusted to phase up the region around A_4 with A_3. At this point all 4 actuators are fixed. However, we have no ability left to adjust A_1 relative to A_4. The region around A_4 remains unphased relative to A_1. In general, the two regions will be out of phase relative to each other. This is the fundamental actuated mirror surface control problem.

The phasing information used to adjust A_2, A_3, and A_4 comes from the wavefront sensor measurements of the beacon wavefront. Suppose, for example, that the wavefront sensor measures

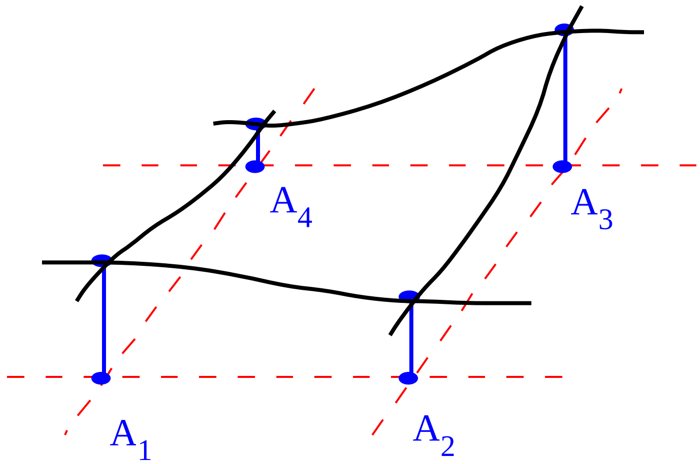

Figure 1 Four neighboring actuators forming a square in an actuated surface. There are more measured beacon wavefront input data for any square than piston degrees of freedom.

the local phase gradients (tip and tilt) of the beacon. In this case A_2 is adjusted to match the phase tilt between A_1 and A_2 as measured by the wavefront sensor. A_3 is adjusted to match the measured phase tilt between A_2 and A_3, and A_4 is adjusted to match the measured phase tilt between A_3 and A_4. However, the tilt between A_1 and A_4 will not match the measured phase tilt, and we have no freedom to adjust any of the 4 actuators without ruining the match of the tilts already set. In order for the tilt between A_1 and A_4 by accident to match the measured beacon tilt, the measured tilt vectors would have to form a vector field. This will not happen because it is impossible for the wavefront sensor to measure the phase gradient at a single point or at the actuator grid frequency with infinite precision. In general, the measured gradients are averaged over a finite region (the size of the sensor subaperture) and will contain noise.

This fundamental control problem exists whether the mirror is composed of small segments, large segments, or a continuous actuated monolithic surface. In fact, the control problem is essentially identical for small segmented, large segmented, or continuous surfaces. A wavefront sensor with finite-sized subapertures defines a natural segmentation of the beacon wavefront. Besides the wavefront sensor, the discretization is defined by the mirror actuator grid. Thus, any actuated optical surface can be thought of as segmented with virtual segments. Similarly, large segments with many actuators can be thought of as composed of smaller virtual segments.

As can be seen in the example above, the system of equations for the actuator pistons is overdetermined in terms of the wavefront sensor input. Since the equations are overdetermined, the phase reconstruction must be optimized. Iterative algorithms have been employed to optimize the phase reconstruction from the input data. Iterative algorithms are undesirable since they are slow, especially for large mirrors with many actuators, and they necessarily demand low noise, highly stable actuation and sensing to avoid the possibility of secular noise growth across the mirror which is introduced by iteration.

We have identified surface constraints that have allowed us to exactly and analytically solve for the mirror actuator configuration or phase reconstruction that the iterative methods were attempting to converge to. Hence, our solution to actuated surface control eliminates the need to do iterations, and eliminates the need to develop iterative control algorithms. The mathematical details can be found in Reference 1.

Our analytic solution determines a minimal tiled surface for a given set of surface normals. Since the surface is minimal, there exist no 2π ambiguities. Furthermore, we have shown that the number of independent surface constraints exactly reduces input degrees of freedom so that the equations are no longer overdetermined, but are in fact well-determined so that the solution is unique. We have shown that this is true for any surface topology (number of holes). This means that a well-defined phase reconstruction can be generated even when the beacon is highly scintillated or when the optical telescope assembly contains multiple or complicated obscurations.

Advantages of Small Segments

While the control solution can be applied to small or large segments, or to continuous monolithic surfaces, there are advantages to constructing large space optics with relatively small segments.

- Small segments can be made of stiff, thermally stable material without infringing on mass limits or requirements.
- Furthermore, the individual stiffness of small segments suppresses any error from higher spatial frequencies.
- Clusters of small segments can be made self similar thereby making it natural to disassemble and reassemble a very large mirror. The mirror can be launched into space in stages with the reassembly steps nearly identical.
- All of the technology associated with making and controlling small segments is completely scalable. Larger apertures are simply made by adding more segments. The control solution is known for an arbitrarily large number of segments.
- Small segments are individually controlled by just three degrees of freedom: tip, tilt and piston. Due to the stiffness and low inertia of small segments, actuator influence functions are dynamically irrelevant.
- By contrast, large segments require many actuators and the control of large segments is much more complicated (because the dynamical behavior of large segments is much more complicated). Actuator influence functions are dynamically extremely important, and therefore must be well understood and stable.
- Control of large segments using many actuators includes trying to control and maintain the segment shape along with its overall tip, tilt and piston. Small segment control does not require surface shape control - just tip, tilt and piston.
- Thus, the segmented surface dynamics is linear for small segments, and **nonlinear** for large segments or monolithic actuated surfaces.
- When an actuator fails in a large segment, it is difficult to compensate for. There is no way to mask the mirror at that point. The behavior of the segment in the vicinity of the actuator may be difficult to predict and adjustments to the control algorithm are non-trivial at best.
- When an actuator fails on a small segment, the segment can be tilted out of the optical path, effectively masking the mirror at that point. The control algorithm (exact solution is known) is simple to adjust (the genus (number of holes) of the surface has merely increased by 1).
- Metrology concepts are basically the same for both large and small segments; the metrology is just simpler for the small segments, because their dynamical behavior is easier to understand (fewer degrees of freedom per segment and the surface dynamics are linear).

AMPERES

Knowledge of surface constraints allowed us to find an exact and unique solution to the piston system of equations. The surface constraints have a simple geometrical interpretation[1] which we have exploited to construct an automated control solution generator.

The AMPERES Segment Design Tool is a desktop wave-optics model of active and adaptive segmented telescopes developed for NASA. This design tool contains an automated solution generator. The user need only input the mirror configuration and AMPERES does the rest. In this sense, AMPERES also acts as a controller of control solutions. The performance of a particular segmented mirror and solution is displayed through simulated astronomical and terrestrial images.

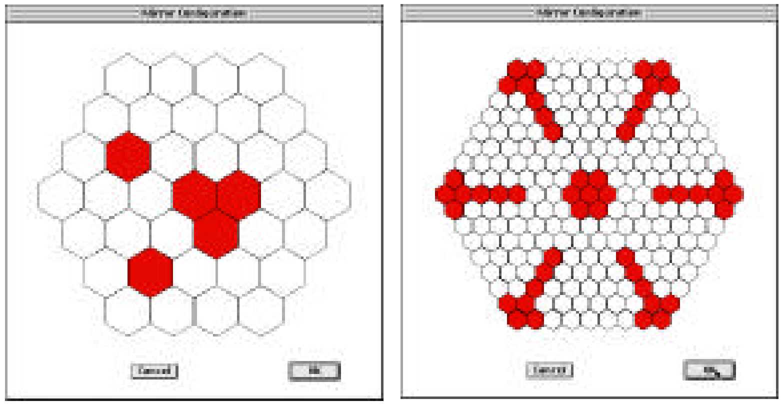

Figure2 Mirrorconfigurationinputwindowwithagenus3,3-ringhexagonalmirrordefined(left)andamirrorwithconvex and concave regions (right). Red (dark) segments are disabled or missing.

AMPERES is capable of simulating images degraded by atmospheric turbulence with and without adaptive correction. In addition, AMPERES can employ a laser guide star model to include anisoplanatic effects that arise from a non ideal beacon.

The Solution Generator is completely menu driven. You select the segment shape (square or hexagonal) and the number of segments in the mirror. AMPERES will display the segmented mirror you just defined. Click on any segment that you want to disable or remove. When you do so, the segment will turn red to indicate that it is no longer part of the mirror. Click on any disabled segment to re-enable it. Figure 2 above displays two examples of input mirror configurations for control solution construction. The surface constraints allow for an exact solution even for mirrors with many holes or mirrors made up of concave regions and odd boundaries.

Figures 3–5 contain a sample gallery of results from AMPERES. The mirror configuration used for the sample images consists of 36 hexagonal segments arranged in a 3 ring annulus In building the segmented telescope, spherical, paraboloidal or flat segments may be used. Noise can also be added to the sensor and actuator measurements.

SOPHI

SOPHI (Segmented Optics Phase Integration) is an active and adaptive segmented optics breadboard technology demonstrator that will soon be under construction. The final goal of the breadboard is the development of a complete 1m telescope with a segmented primary. The segments are relatively small (about 10cm) and segment control will be based on the exact solution. The

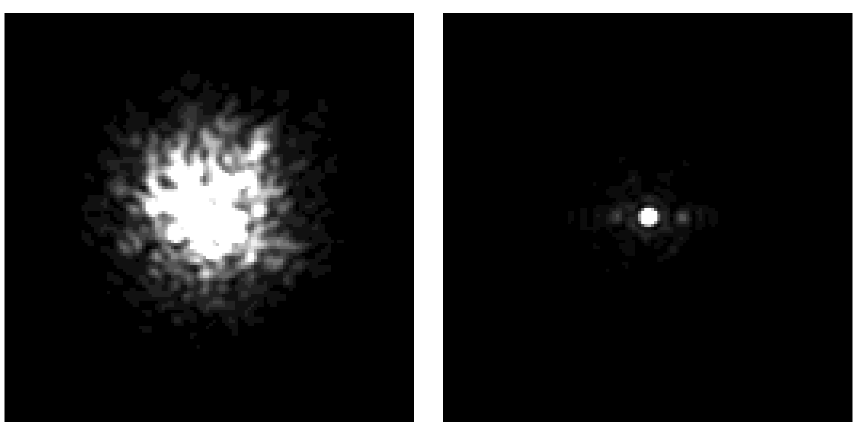

Figure 3 Left: Uncompensated (short exposure) star image produced by AMPERES for a 60 inch ground-based segmented telescope. $r_0 = 15$ cm. Right: Corresponding compensated star image. Strehl is about 0.51.

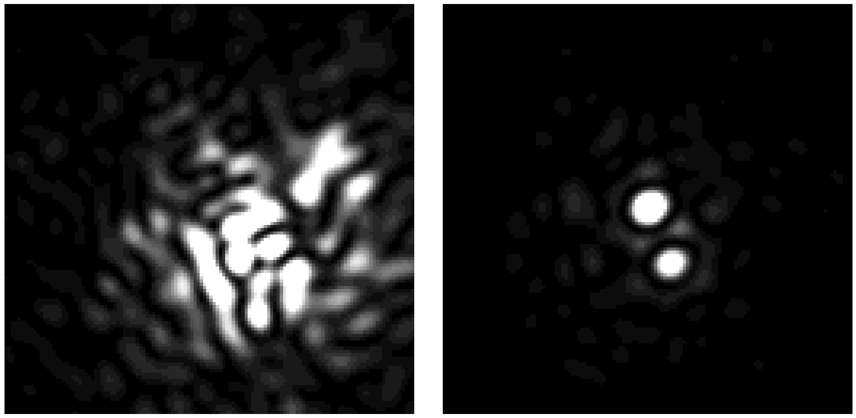

Figure 4 Left: Uncompensated (short exposure) double star image produced by AMPERES. The star separation is 1 arc second and the relative magnitude of the two stars is 1. $r_0 = 1.22$ times the segment size. Right: Corresponding compensated double star image. Strehl is about 0.56.

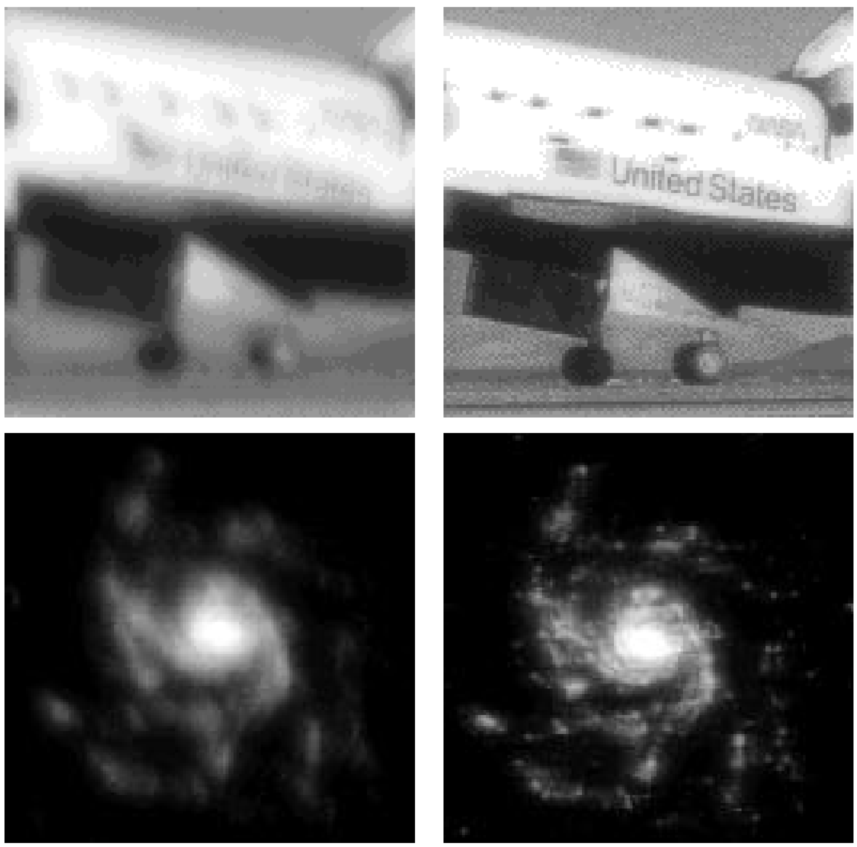

Figure 5 Left: Uncompensated images. Right: Corresponding compensated images. The Strehl is about 0.44.

segments as well as the optical structure will be made of C/SiC ceramic. C/SiC is an ideal material for segmented optics because of its high stiffness, high thermal stability, and ease of use in manufacturing optimized structures. Only materials and components that are already space proven will be used, hence the breadboard may be upgraded to a higher qualification level at minimal additional cost so that it may be flown in a later phase on a suitable carrier platform (e.g. SPAS III) for an in-orbit demonstration. In addition, we have developed wavefront sensing and optical metrology concepts that eliminate the need to perform adjacent segment edge midpoint height sensing. Instead we exploit the fact that each segments surface is shaped to bring light to a focus.

A segments piston can then be determined relative to its focus, or equivalently, relative to the radius of curvature of the segment surface. This will be accomplished with traditional wavefront sensing methods used in novel ways. In this approach the relative piston of a segment is measured without reference to neighboring segments, hence 2π ambiguities in the metrology cannot exist.

Discussion

The limit of the ability of an actuated surface to correct optical aberrations is clearly set by the actuator grid spacing. Traditional wavefront sensors with finite subapertures physically subdivide the beacon wavefront and the optimal subaperture size is directly related to the actuator grid spacing. Several questions naturally arise. First of all, how does our solution account for low order aberrations, i.e. aberrations whose spatial extent spans several or many wavefront subapertures? Our solution is an exact solution to the system of piston equations for the actuator grid derived from the classical mechanical requirement of making the mirror surface match the beacon phase front. The piston equations are elliptical (Laplacian type operator), meaning that actual piston value at one point depends on the values at all of the other actuated points. Thus our solution automatically incorporates the correlation between distant actuated points due to low order aberrations. Iterative methods eventually establish the correlations between distant points through the process of iteration. Our solution accomplishes this without iterations.

Another question that naturally arises with finite-sized sensor subapertures involves aliasing. Won't using a wavefront sampling grid that is matched to the actuator grid introduce the possibility of aliasing the higher spatial frequencies aberrations to the wavefront sampling grid? After all, spatial frequencies beyond the actuator grid cannot be corrected for by the actuated surface. We would like to point out that traditional wavefront sensor subapertures work like low-band pass filters. For example, a Hartmann sensor subaperture only measures tip and tilt from the off-axis displacement of the image of the beacon. Higher spatial frequencies are averaged out across the subaperture. Furthermore, the beacon wavefront is physically subdivided, and the actual signals from different subapertures are not mixed. The high spatial frequency components will be spatially localized. With physical wavefront sensor subapertures, they stay localized and are filtered out.

Some wavefront sensing methods, such as phase retrieval, essentially trade physical wavefront sensors for supercomputers. For phase retrieval, the entire beacon wavefront is brought to a focus. Thus, the signal from localized high spatial frequency aberrations do not stay localized and are mixed with everything. Fortunately, the power in high spatial frequency errors is much smaller than lower order errors. However, phase retrieval methods are iterative and non-linear, hence power from high frequency modes can be transferred to lower frequency modes. Iterations also allow for secular error growth from noise. This is a concern for phase retrieval methods since these methods require precise knowledge of the actuator influence functions and therefore require that these influence functions be highly stable. In contrast, our exact solution used with small segments makes the actuator influence functions dynamically irrelevant.

References

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2. S. Enguehard and B. Hatfield, *Adaptive Optics* (Frontiers in Physics Series, Perseus Book Group), in final preparation, under contract to Perseus, 1999.
3. S. Enguehard and B. Hatfield, "Compensated Atmospheric Optics," *Prog. Quant. Electron* **19**, pp 239-301 (1995).