Stochastic Mean-field methods

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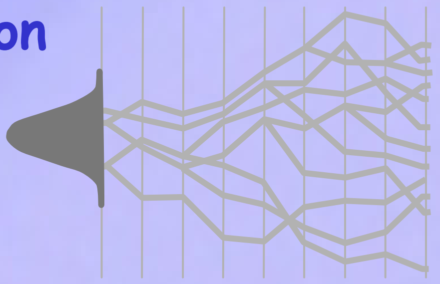
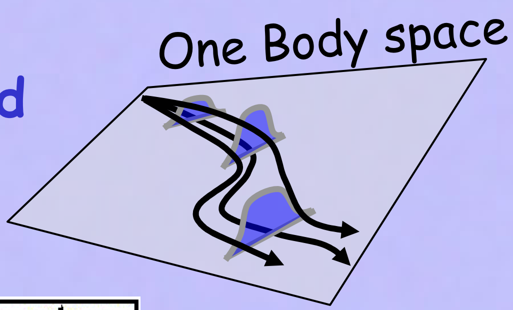
Part 1: Exact evolution of quantum systems

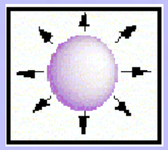
- -Introduction to stochastic Schroedinger Equation
- -illustration : system-environment
- -application to self-interacting system



Part 2: Approximate evolution of quantum systems

- -Dissipation and fluctuations beyond mean-field
- -Quantum jump approach to the many-body problem



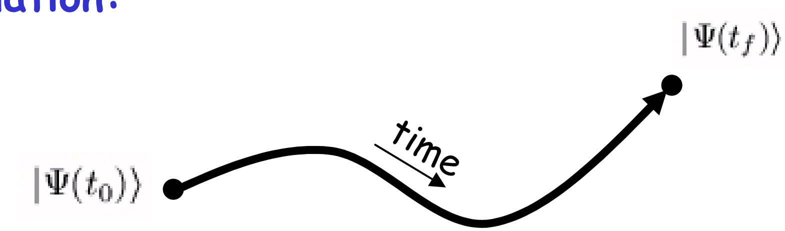


What is a Stochastic Schroedinger equation?

Standard Schroedinger equation:

$$d|\Psi\rangle = \frac{dt}{i\hbar} H |\Psi\rangle$$

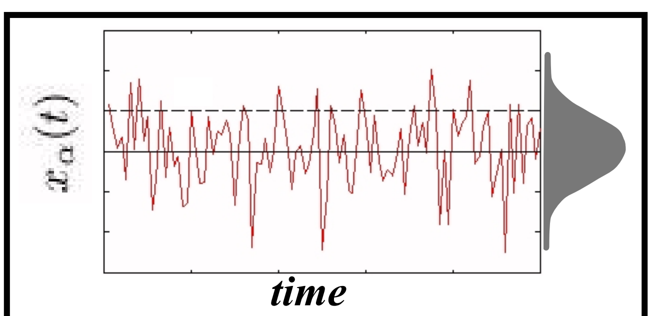
➡ **Deterministic evolution**

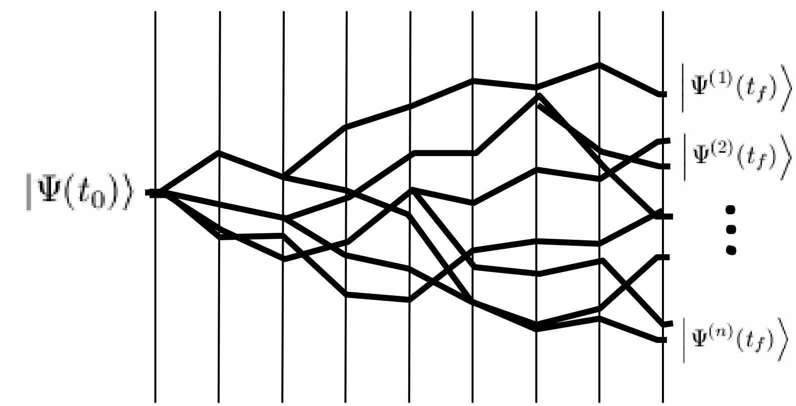


Stochastic Schroedinger equation (SSE):

$$d|\Psi\rangle = \left\{ \frac{dt}{i\hbar} H + dB_{sto} \right\} |\Psi\rangle$$

Stochastic operator : $dB_{sto} = \sum_{\alpha} x_{\alpha}(t) O_{\alpha}$

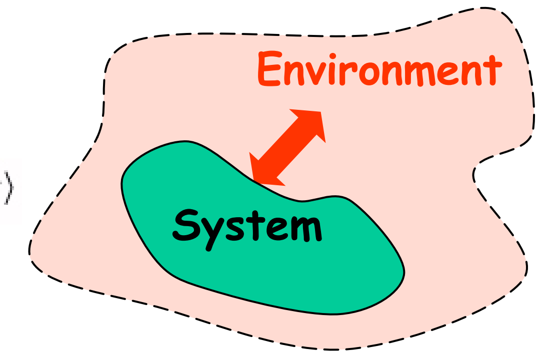




Exact dynamics of a systems coupled to an environment with SSE

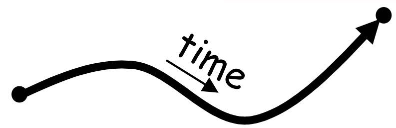
Hamiltonian

$$H = H_S + H_E + \sum_\alpha B_\alpha(S) \otimes C_\alpha(E)$$



Exact dynamics

At t=0 $\quad |\Psi(t_0)\rangle = |\Phi(t_0)\rangle \otimes |\chi(t_0)\rangle$



$$|\Psi(t)\rangle \neq |\Phi\rangle \otimes |\chi\rangle$$

A stochastic version

$$\begin{cases} d\,|\Phi\rangle = \left\{ \dfrac{dt}{i\hbar} H_S + \sum_\alpha d\xi_\alpha(t)\, B_\alpha \right\} |\Phi\rangle \\ d\,|\chi\rangle = \left\{ \dfrac{dt}{i\hbar} H_E + \sum_\alpha d\xi_\alpha(t)\, C_\alpha \right\} |\chi\rangle \end{cases}$$

with $\quad \overline{d\xi_\alpha d\xi_\beta} = \dfrac{dt}{i\hbar}\delta_{\alpha\beta}$

$$|\Psi^{(n)}(t)\rangle = |\Phi^{(n)}\rangle \otimes |\chi^{(n)}\rangle$$

$|\Psi(t_0)\rangle$

Average evolution

$$\overline{d\{|\Phi\rangle \otimes |\chi\rangle\}} = \overline{|d\Phi\rangle \otimes |\chi\rangle} + \overline{|\Phi\rangle \otimes |d\chi\rangle} + \overline{|d\Phi\rangle \otimes |d\chi\rangle}$$

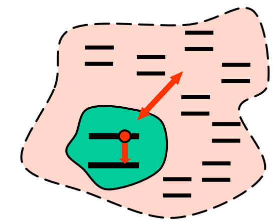
$$\frac{dt}{i\hbar} H_S \quad + \quad \frac{dt}{i\hbar} H_E \quad + \quad \frac{dt}{i\hbar} \sum_\alpha B_\alpha \otimes C_\alpha$$

> The dynamics of the system+environment can be simulated exactly with quantum jumps (or SSE) between "simple" state.

➡ Average density $\quad D = \overline{|\Psi_1\rangle\langle\Psi_2|}$

An simple illustration: spin systems

Lacroix, Phys. Rev. A72, 013805 (2005).

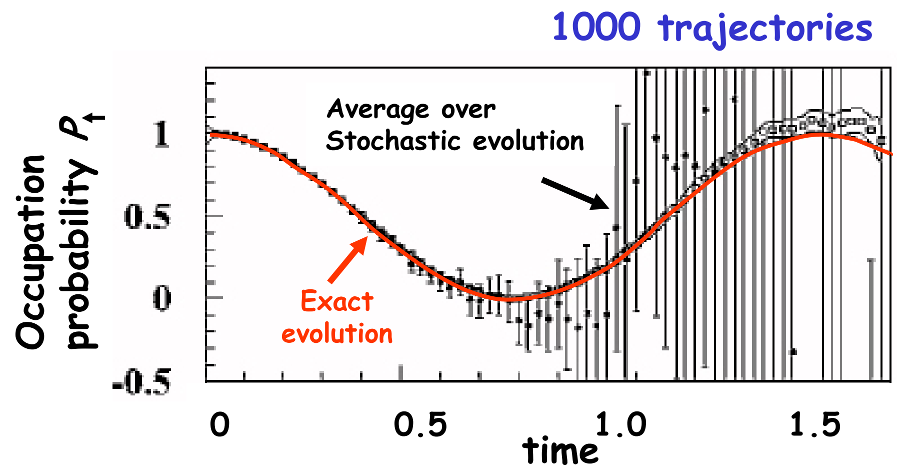


A two-level system interacting with a bath of spin systems

system

$$H = 2\sum_{\alpha} C_{\alpha}(\sigma_{+}\sigma_{-}^{(\alpha)} + \sigma_{-}\sigma_{+}^{(\alpha)})$$

environment



Direct application of SSE:

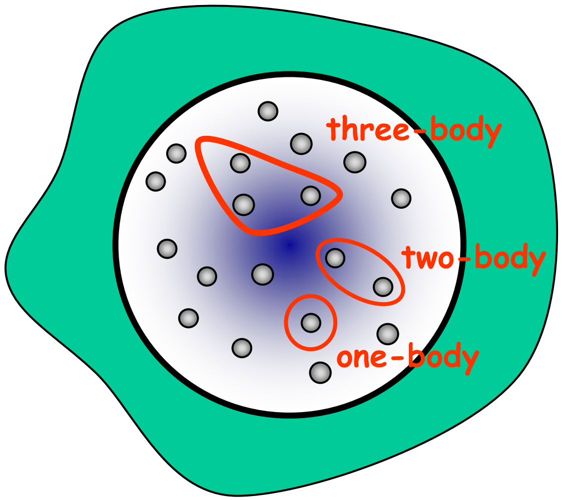
H ➡ "Noise"

Introduction of mean-field:

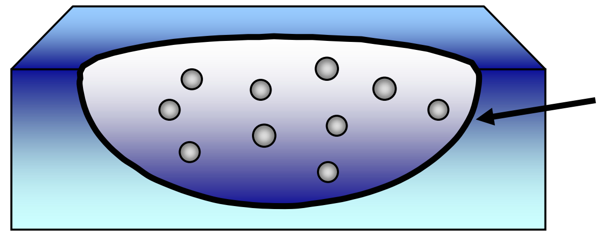
H ➡ mean-field + "Noise"

Stochastic equation are not unique. One can take advantage of this flexibility (mean-field)

Simulation of self-interacting system with 'simple state': the nuclei case



Mean-field:
(DFT)



Self-consistent
Mean-field

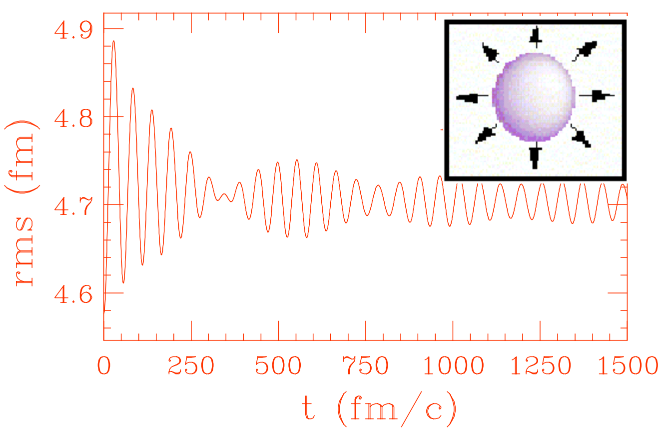
"Simple" Trial state:

$$\Psi(r_1, \cdots, r_{12}, \cdots, r_{123}, \cdots) \longrightarrow |\Phi_{HF}\rangle = \Pi a^+_\alpha |0\rangle$$

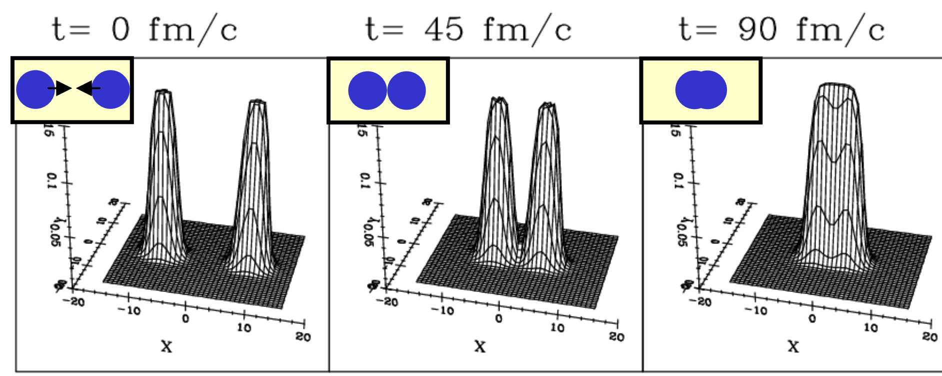
Success of the mean-field:

Dynamics

Vibrations



Collisions of nucleus



3D TDHF-Sly4d (P. Bonche)

Critical aspects
- Static: some important long range correlations are neglected.
- Dynamics: correlations (fluctuations) are underestimated.

Exact Many-Body with SSE on "simple" state: the Functional integral method

General strategy

S. Levit, PRC21 (1980) 1594.

Given a Hamiltonian and an initial State

$$|\Phi(t+\Delta t)\rangle = \exp\left(\frac{\Delta t}{i\hbar} H\right)|\Phi(t)\rangle$$

Write H into a quadratic form

$$H|\Phi\rangle = \left(H_1 - O^2\right)|\Phi\rangle$$

Use the Hubbard Stratonovich transformation

$$\exp\left(-\frac{\Delta t}{i\hbar} O^2\right)|\Phi(t)\rangle = \int d\sigma G(\sigma)\exp(a\sigma O)|\Phi(t)\rangle$$

$$\Rightarrow |\Phi(t+\Delta t)\rangle = \int d\sigma G(\sigma)|\Phi_\sigma(t+\Delta t)\rangle$$

Interpretation of the integral in terms of stochastic Schrödinger equation

$$|\Phi_\sigma(t+\Delta t)\rangle = \exp\left(\frac{\Delta t}{i\hbar} H_1 + a\sigma O\right)|\Phi(t)\rangle$$

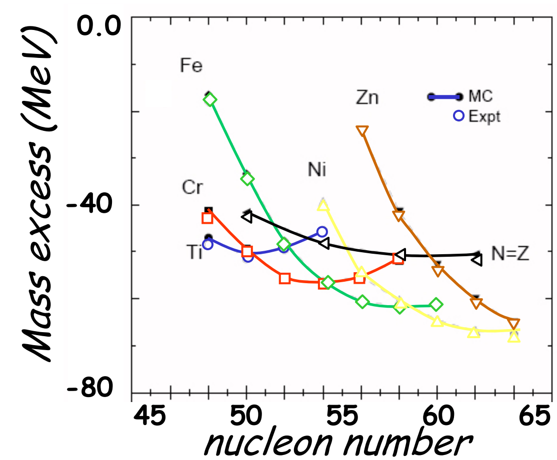
$$\Rightarrow \Delta|\Phi_\sigma\rangle = \left(\frac{\Delta t}{i\hbar} H + a\sigma O\right)|\Phi\rangle$$

The many-body problem

$$H = \sum_{ij} T_{ij} a_i^+ a_j + \frac{1}{4}\sum_{ijkl} V_{ijkl} a_i^+ a_j^+ a_l a_k$$

$a_i^+ a_j \rightarrow O_{ij}$ $\qquad$ $a_i^+ a_j^+ a_l a_k \rightarrow O_{il}\, O_{jk}$

Example of application in nuclear physics:
-Shell Model Monte-Carlo ...



Adapted from:
S.E.Koonin, D.J.Dean, K.Langanke, Ann.Rev.Nucl.Part.Sci. 47, 463 (1997).

Recent progress for dynamics: stochastic mean-field

Functional techniques

$|\Phi\rangle = \Pi_\alpha a^+_\alpha |0\rangle$

$$H = \sum_{ij} T_{ij} a^+_i a_j + \frac{1}{4} \sum_{ijkl} V_{ijkl} a^+_i a^+_j a_l a_k$$

Deterministic part — noise

New approach: mean-field+ noise

$$H |\Phi\rangle = \left(\sum_{ij} T_{ij} a^+_i a_j + \frac{1}{4} \sum_{ijkl} V_{ijkl} a^+_i a^+_j a_l a_k \right) |\Phi\rangle$$

Deterministic mean-field — noise Residual inter.

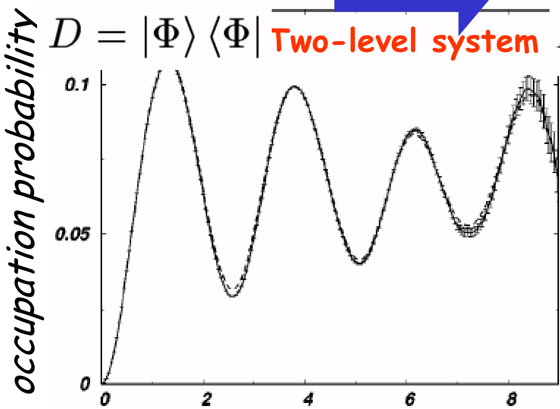
Bosons

Deterministic evolution

Trial state: Bose condensate

Quantum jump between Slater

$D = |\Phi\rangle \langle\Phi|$ — Two-level system — $D(t) = \overline{|\Phi_a\rangle \langle\Phi_b|}$



Carusotto, Y. Castin and J. Dalibard, PRA63 (2001)

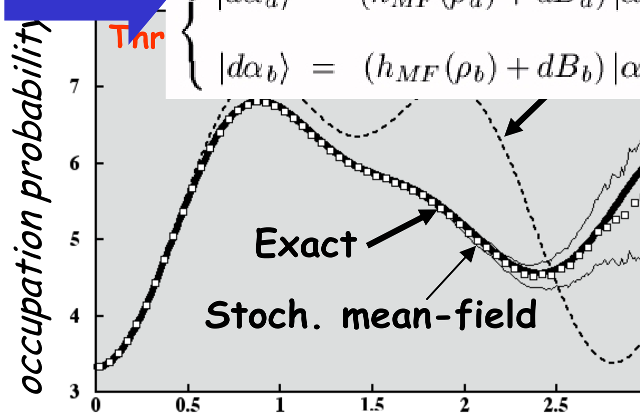
Fermions

Stochastic evolution of single-particle

Trial state: Slater determinant

Thr

$$\begin{cases} |d\alpha_a\rangle = (h_{MF}(\rho_a) + dB_a) |\alpha_a\rangle \\ |d\alpha_b\rangle = (h_{MF}(\rho_b) + dB_b) |\alpha_b\rangle \end{cases}$$



O. Juillet and Ph. Chomaz, PRL 88 (2002)

- The link with observable evolution is not simple *(D. Lacroix , PRC71, 064322 (2005))*
- A systematic method is desirable
- The numerical effort is huge

Mean-field from variational principle

More insight in mean-field dynamics:

Exact state → **Trial states**

$$|\Psi(t)\rangle \rightarrow \begin{cases} |Q(t)\rangle \\ |Q+\delta Q\rangle = e^{\sum_\alpha \delta q_\alpha A_\alpha}|Q\rangle \end{cases}$$

The approximate evolution is obtained by minimizing the action:

$$S = \int_{t_0}^{t_1} ds \langle Q | i\hbar\partial_t - H | Q\rangle$$

Good part: average evolution

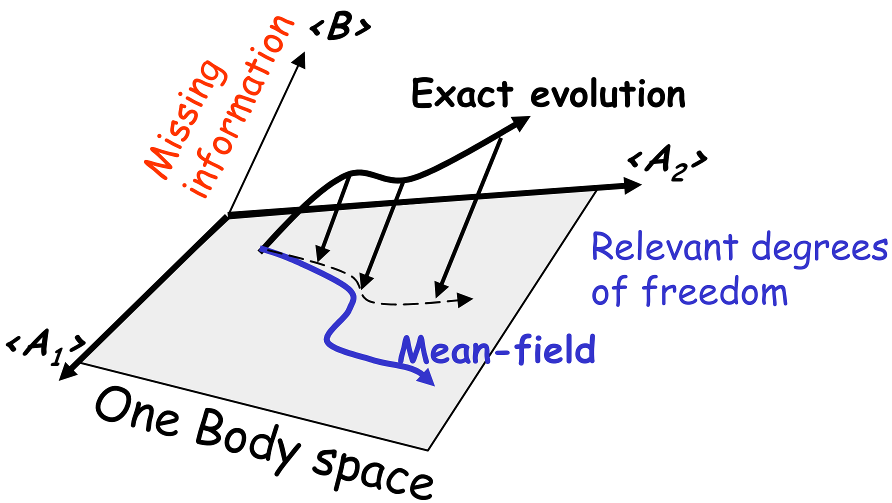
$$i\hbar \frac{d\langle A_\alpha\rangle}{dt} = \langle [A_\alpha, H]\rangle$$ → **exact Ehrenfest evolution**

$$H = \mathcal{P}_1 H + (1-\mathcal{P}_1)H$$

Missing part: correlations

$$|dQ\rangle = \sum_\alpha dq_\alpha A_\alpha |dQ\rangle = \frac{dt}{i\hbar}\mathcal{P}_1(t) H |Q\rangle$$

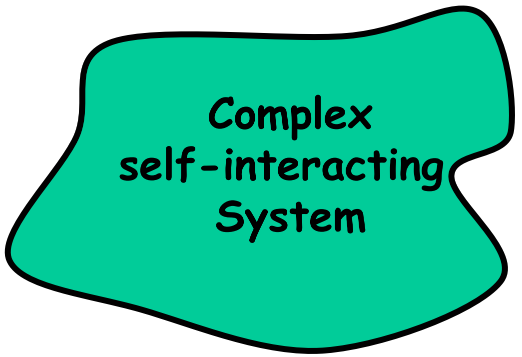
$$\rightarrow i\hbar \frac{d\langle A_\alpha A_\beta\rangle}{dt} \neq \langle [A_\alpha A_\beta, H]\rangle$$



Hamiltonian splitting

$$H = \mathcal{P}_1 H + (1-\mathcal{P}_1)H$$

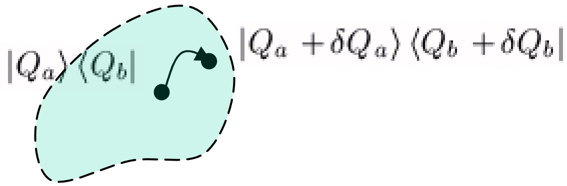
System Environment



The idea is now to treat the missing information as the *Environment* for the Relevant part (*System*)

Existence theorem : Optimal stochastic path from observable evolution

D. Lacroix, Annals of Physics (2006), in press.



Theorem :

One can always find a stochastic process for trial states such that $\overline{\langle A_\alpha \rangle}, \ \overline{\langle A_\alpha A_\beta \rangle}, \cdots \overline{\langle A_{\alpha_1} A_{\alpha_2} \cdots A_{\alpha_k} \rangle}$ *evolves exactly over a short time scale.*

with

$$|Q_a+\delta Q_a\rangle = e^{\sum_\alpha \delta q_\alpha^{[a]} A_\alpha}|Q_a\rangle$$
$$|Q_b+\delta Q_b\rangle = e^{\sum_\alpha \delta q_\alpha^{[b]} A_\alpha}|Q_b\rangle$$

Valid for $D=|Q_a\rangle\langle Q_b|$

or $D=\dfrac{|Q_a\rangle\langle Q_b|}{\langle Q_b|Q_a\rangle}$

In practice

Mean-field level

$$\begin{cases}\delta q_\alpha^{[a]} = \delta q_\alpha^a \\ \delta q_\alpha^{[b]*} = \delta q_\alpha^{b\,*}\end{cases} \quad \Longleftarrow \quad i\hbar\frac{d}{dt}\langle A_\alpha\rangle = \langle [A_\alpha, H]\rangle$$

Mean-field + noise

$$\begin{cases}\delta q_\alpha^{[a]} = \delta q_\alpha^a + \delta\xi_\alpha^{[2]} \\ \delta q_\alpha^{[b]*} = \delta q_\alpha^{b\,*} + \delta\eta_\alpha^{[2]}\end{cases} \quad \Longleftarrow \quad \begin{aligned} i\hbar\frac{d\overline{\langle A_\alpha\rangle}}{dt} &= \overline{\langle [A_\alpha, H]\rangle} \\ i\hbar\frac{d\overline{\langle A_\alpha A_\beta\rangle}}{dt} &= \overline{\langle [A_\alpha A_\beta, H]\rangle}\end{aligned}$$

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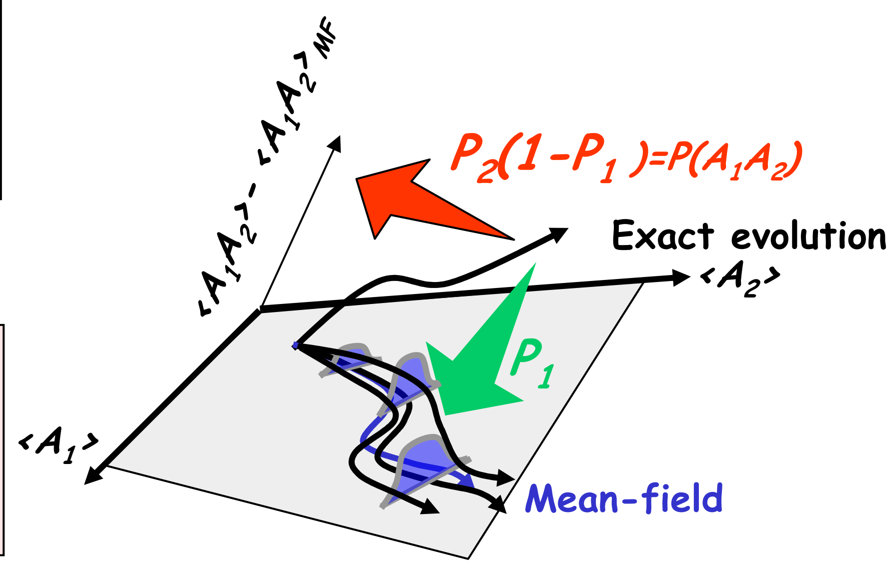
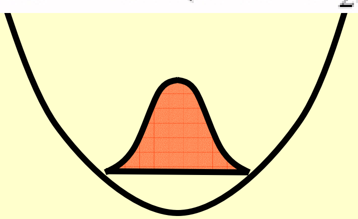


Simple illustration: simulation of the free wave spreading with "quasi-classical states"

$$H_0 = \hbar\omega(a^+ a + \tfrac{1}{2})$$

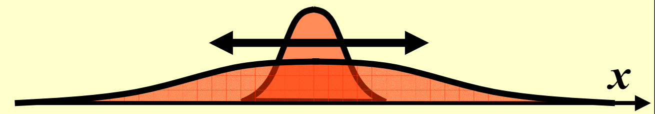
t=0



$\langle x^2 \rangle = \frac{1}{2\eta}$ with $\eta = \frac{m\omega}{\hbar}$

t>0

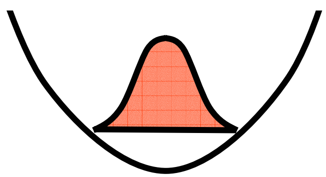
$$H = \frac{p^2}{2m} = -\frac{\hbar\omega}{4}(a^+ - a)^2$$



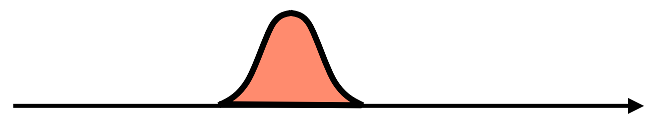
$$\langle x^2 \rangle = \frac{1}{2\eta} + \frac{\hbar^2\eta}{2m^2}t^2$$

Reduction of the information: I want to simulate the expansion with Gaussian wave-function having fixed widths. $\langle x^2 \rangle = cte, \quad \langle p^2 \rangle = cte$

Mean-field evolution:



t>0



Relevant/Missing information:

Relevant degrees of freedom

$\langle x \rangle, \quad \langle p \rangle$

$\langle a^+ \rangle, \quad \langle a \rangle$

Missing information

$\langle x^2 \rangle, \quad \langle p^2 \rangle, \quad \langle xp \rangle$

$\langle {a^+}^2 \rangle, \quad \langle a^2 \rangle, \quad \langle a^+ a \rangle$

Trial states

$$|Q + \delta Q\rangle = e^{\sum_\alpha \delta q_\alpha A_\alpha} |Q\rangle$$

Coherent states

$$|\alpha + d\alpha\rangle = e^{d\alpha a^+} |\alpha\rangle$$

Guess of the SSE from the existence theorem

Densities

$$D = \frac{|\alpha\rangle\langle\beta|}{\langle\beta|\alpha\rangle} \quad \text{with} \quad \begin{array}{l} \langle\beta + \mathrm{d}\beta| = \langle\beta|\mathrm{e}^{\mathrm{d}\beta^* a} \\ |\alpha + \mathrm{d}\alpha\rangle = \mathrm{e}^{\mathrm{d}\alpha a^+}|\alpha\rangle \end{array}$$

Stochastic c-number evolution from Ehrenfest theorem

$$\begin{cases} \mathrm{d}\alpha = \overline{\mathrm{d}\alpha} + \mathrm{d}\xi^{[2]}, \\ \mathrm{d}\beta^* = \overline{\mathrm{d}\beta^*} + \mathrm{d}\eta^{[2]} \end{cases}$$

mean values

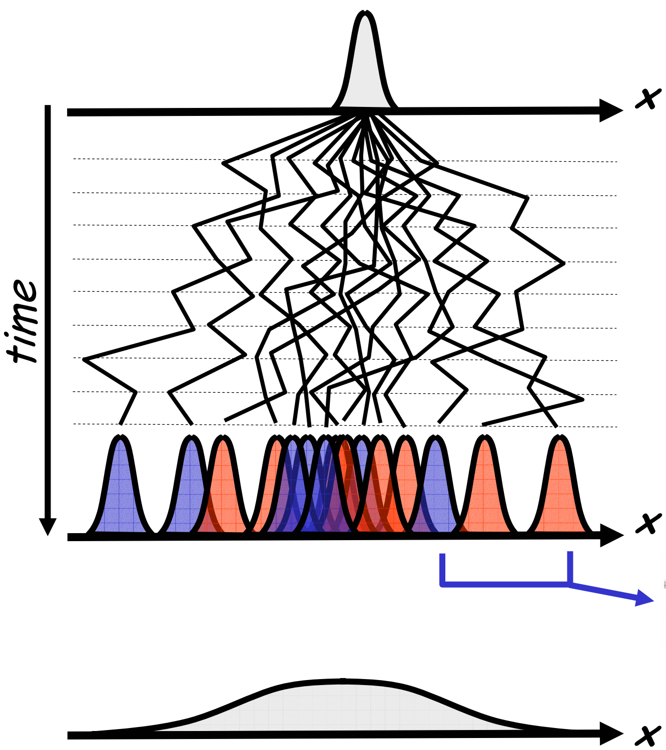
$$\overline{d\langle a\rangle} = \overline{d\alpha}$$

$$\overline{d\langle a^+\rangle} = \overline{d\beta^*}$$

fluctuations

$$\overline{d\langle a^2\rangle} = 2\alpha\overline{d\alpha} + \overline{d\xi^{[2]}d\xi^{[2]}}$$

$$\overline{d\langle a^{+2}\rangle} = 2\beta^*\overline{d\beta^*} + \overline{d\eta^{[2]}d\eta^{[2]}}$$



$$\mathrm{Tr}(Dx^2) = \frac{1}{2\eta} + X^2$$

$$\overline{\mathrm{Tr}(Dx^2)} = \frac{1}{2\eta} + \frac{\hbar^2\eta}{2m^2}t^2$$

Nature of the stochastic mechanics

$$\begin{cases} X = \frac{1}{\sqrt{2\eta}}(\alpha + \beta^*), \\ P = \mathrm{i}\hbar\sqrt{\frac{\eta}{2}}(\beta^* - \alpha) \end{cases} \Rightarrow \begin{cases} \mathrm{d}X = \frac{P}{m}\,\mathrm{d}t + \mathrm{d}\chi_1 \\ \mathrm{d}P = \mathrm{d}\chi_2, \end{cases}$$

with $\overline{\mathrm{d}\chi_1\,\mathrm{d}\chi_2} = \frac{\hbar^2\eta}{2m}\,\mathrm{d}t$

the quantum wave spreading can be simulated by a classical brownian motion in the complex plane

SSE for Many-Body Fermions and bosons

D. Lacroix, Annals of Physics (2006), in press.

Starting point:

$$H = \sum_{ij} \langle i|T|j\rangle a_i^+ a_j + \frac{1}{2}\sum_{ijkl} \langle ij|v_{12}|lk\rangle a_i^+ a_j^+ a_l a_k$$

$$D_{ab} = |\Phi_a\rangle\langle\Phi_b| \quad \text{with} \quad \langle\Phi_b \mid \Phi_a\rangle = 1$$

$$\rho_1 = \sum |\alpha_i\rangle\langle\beta_i|$$

Observables $\langle j|\rho_1|i\rangle = \langle a_i^+ a_j\rangle$

Fluctuations $\langle ij|\rho_{12}|kl\rangle = \langle a_k^+ a_l^+ a_j a_i\rangle$

Ehrenfest theorem ➡ BBGKY hierarchy

$$i\hbar\frac{d}{dt}\rho_1 = [h_{MF}, \rho_1],$$

$$i\hbar\frac{d}{dt}\rho_{12} = [h_{MF}(1) + h_{MF}(2), \rho_{12}] + (1-\rho_1)(1-\rho_2)v_{12}\rho_1\rho_2 - \rho_1\rho_2 v_{12}(1-\rho_1)(1-\rho_2)$$

$$v_{12} = \sum_\lambda O_\lambda(1)O_\lambda(2)$$

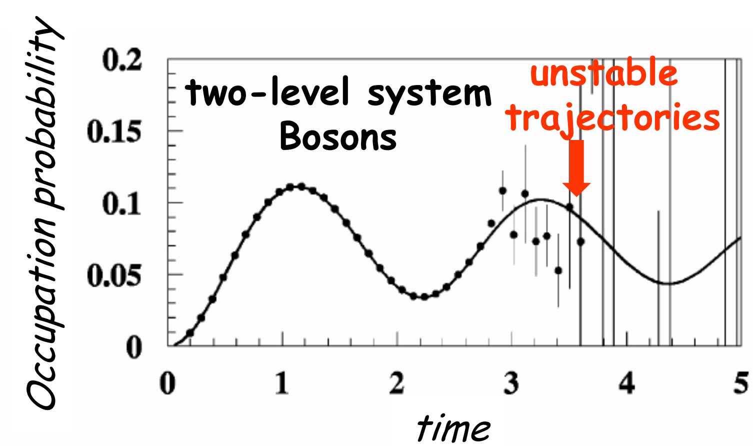
Stochastic one-body evolution

$$d\rho_1 = [h_{MF}, \rho_1] + \sum_\lambda d\xi_\lambda^{[2]}(1-\rho_1)O_\lambda\rho_1 + \sum_\lambda d\eta_\lambda^{[2]}(1-\rho_1)O_\lambda\rho_1$$

with $\overline{d\xi_\lambda^{[2]} d\xi_{\lambda'}^{[2]}} = -\overline{d\eta_\lambda^{[2]} d\eta_{\lambda'}^{[2]}} = \delta_{\lambda\lambda'}\frac{dt}{i\hbar}$

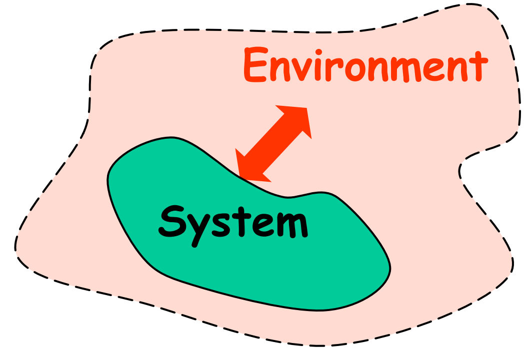
- The method is general.
  the SSE are deduced easily
  ➡ extension to Stochastic TDHFB
  D. Lacroix, arXiv nucl-th 0605033
- The mean-field appears naturally and the interpretation is easier
- the numerical effort can be reduced by reducing the number of observables

but...



Part II
Dissipation in Many-Body Systems with SSE

Quantum jump method -Dissipation



$$H = H_S + H_E + \sum_\alpha B_\alpha(S) \otimes C_\alpha(E)$$

Exact dynamics

with SSE on simple state $|\Psi\rangle = |\Phi\rangle \otimes |\chi\rangle$

$$|\Psi^{(n)}(t)\rangle = |\Phi^{(n)}\rangle \otimes |\chi^{(n)}\rangle$$

Then, the average dyn. identifies with the exact one

1. For total wave $\overline{d|\Psi\rangle} = \left\{ \frac{dt}{i\hbar} H + \mathcal{O}(dt) \right\} |\Psi\rangle$
2. For total density $D = \overline{|\Psi_1\rangle\langle\Psi_2|}$

Application to self-interacting system
Interpretation as a "system+environment"

Approximate Dissipative dynamics

At t=0 $D(t=0) = \rho_S \otimes \rho_E$

- Weak coupling approx.
- Projection technique
- Markovian approx.

Lindblad master equation:

$$i\hbar \frac{d}{dt}\rho_S = [H_S, \rho_S] + \sum_k \gamma_k \left(A_k A_k \rho_S + \rho_S A_k A_k - 2 A_k \rho_S A_k \right)$$

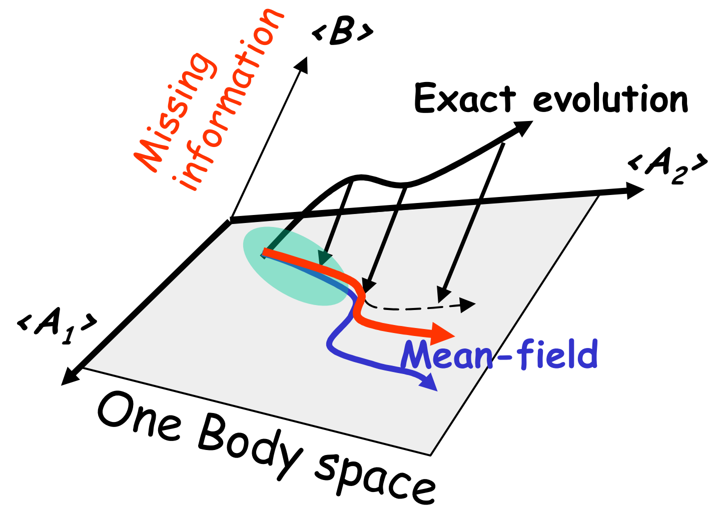
Can be simulated by stochastic eq. on |Φ>,
The Master equation being recovered using :

$$\rho_S = \overline{|\Phi\rangle\langle\Phi|}$$

Gardiner and Zoller, *Quantum noise* (2000)
Breuer and Petruccione, *The Theory of Open Quant. Syst.*

Dissipation in self-interacting systems

Y. Abe et al, Phys. Rep. 275 (1996)
D. Lacroix et al, Progress in Part. and Nucl. Phys. 52 (2004)



Short time evolution

$$i\hbar \frac{\mathrm{d}}{\mathrm{d}t}\rho_1 = [h_{\mathrm{MF}}, \rho_1],$$

$$i\hbar \frac{\mathrm{d}}{\mathrm{d}t}\rho_{12} = [h_{\mathrm{MF}}(1) + h_{\mathrm{MF}}(2), \rho_{12}] + (1-\rho_1)(1-\rho_2)v_{12}\rho_1\rho_2 - \rho_1\rho_2 v_{12}(1-\rho_1)(1-\rho_2)$$

Correlation

$$C_{12} = \rho_{12} - (\rho_1\rho_2)_A$$

Approximate long time evolution+Projection

$$i\hbar\frac{d}{dt}\rho_1 \;=\; [h_{MF}, \rho_1] + Tr_2\,[v_{12}, C_{12}]$$

with

$$C_{12}(t) = -\frac{i}{\hbar}\int_{t_0}^{t} U_{12}(t,s)\, F_{12}(s)\, U_{12}^{\dagger}(t,s)\, ds + \delta C_{12}(t)$$

projected two-body effect

Propagated initial correlation

Dissipation

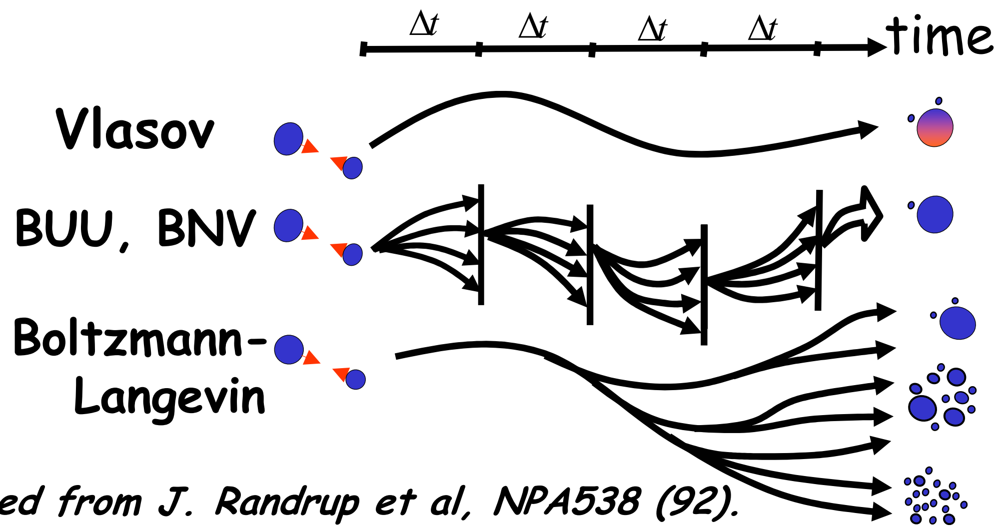
$$i\hbar\frac{d}{dt}\rho \;=\; [h_{MF}, \rho] + K(\rho)$$

Dissipation and fluctuation

$$i\hbar\frac{d}{dt}\rho \;=\; [h_{MF}, \rho] + K(\rho) + \delta K(\rho)$$

Random initial condition

Semiclassical version for approaches in Heavy-Ion collisions



Adapted from J. Randrup et al, NPA538 (92).

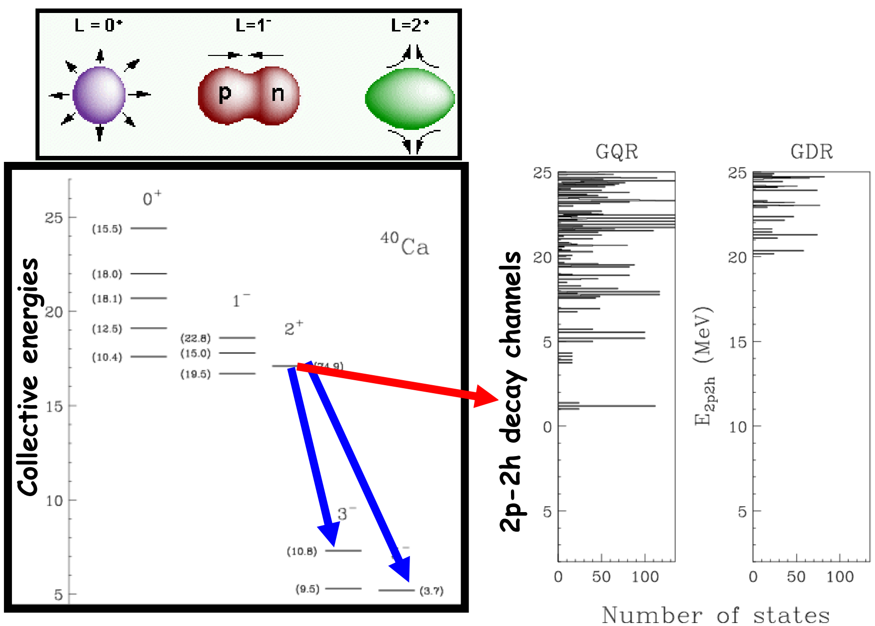
Application in quantum systems

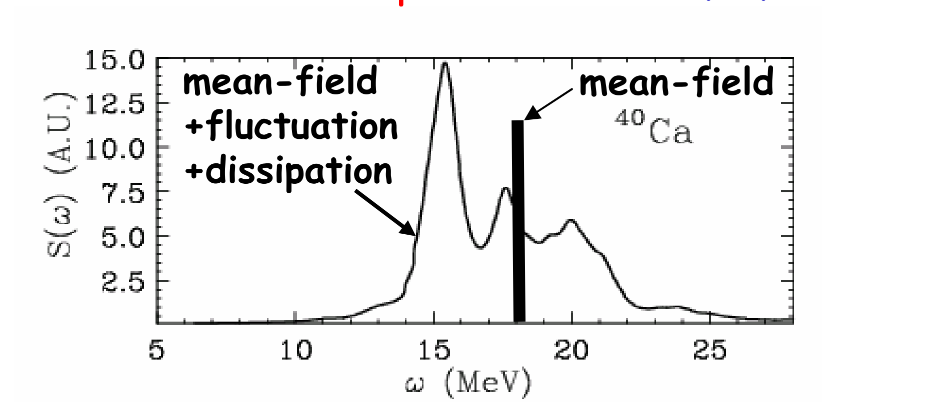
$$i\hbar\frac{\partial}{\partial t}\rho^{(n)} - [h(\rho^{(n)}),\rho^{(n)}] = K_I(\rho^{(n)}) + \delta K^{(n)}(t)$$

RPA

Coupling to 2p2h states

Coupling to ph-phonon





D. Lacroix et al, Progress in Part. and Nucl. Phys. (2004)

Alternative formulation with Stochastic Schroedinger equations

GOAL: Restarting from an uncorrelated state $D = |\Phi_0\rangle\langle\Phi_0|$ we should:

1-have an estimate of $D = |\Psi(t)\rangle\langle\Psi(t)|$

2-interpret it as an average over jumps between "simple" states

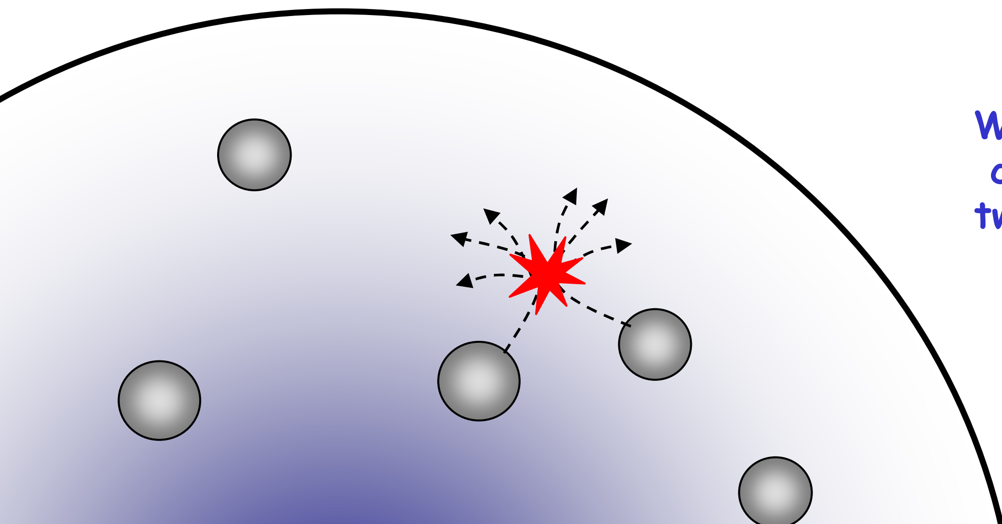
Weak coupling approximation : perturbative treatment

R.-G. Reinhard and E. Suraud, Ann. of Phys. 216, 98 (1992)

$$|\Psi(t')\rangle = |\Phi(t')\rangle - \frac{i}{\hbar}\int \delta v_{12}(s)|\Phi(s)\rangle\, ds - \frac{1}{2\hbar^2}T\left(\int\int \delta v_{12}(s)\delta v_{12}(s')dsds'\right)|\Phi(s)\rangle$$

Residual interaction in the mean-field interaction picture

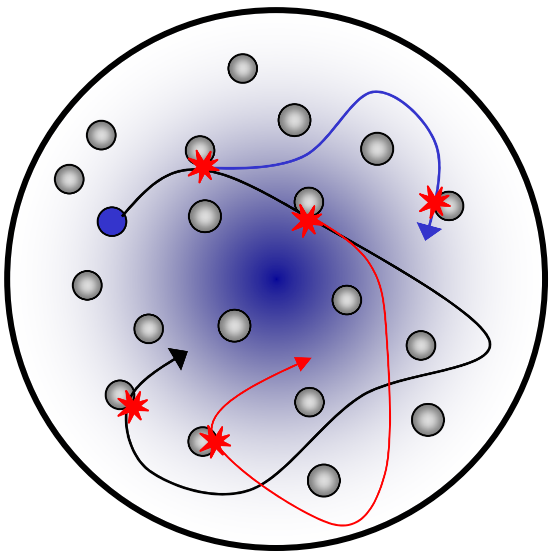
Statistical assumption in the Markovian limit :

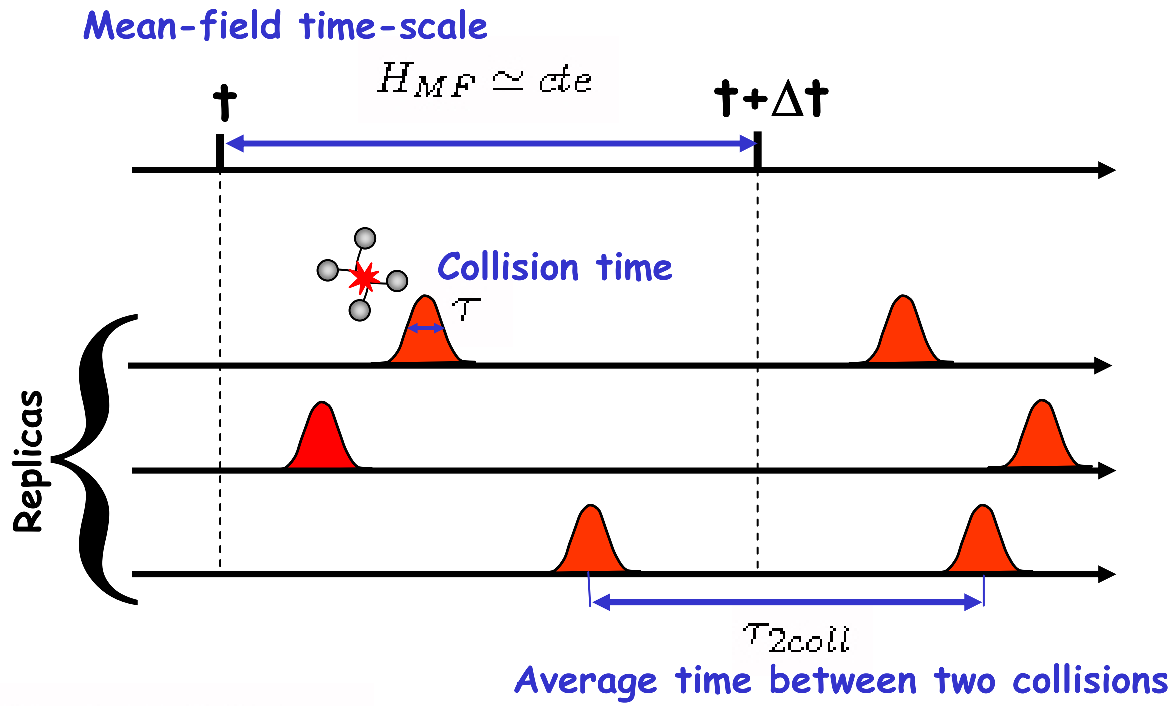


We assume that the residual interaction can be treated as an ensemble of two-body interaction:

$$\begin{cases} \overline{\delta v_{12}(s)} = 0 \\ \overline{\delta v_{12}(s)\delta v_{12}(s')} \propto \overline{\delta v_{12}^2(s)} e^{-(s-s')^2/2\tau^2} \end{cases}$$

Time-scale and Markovian dynamics





Hypothesis : $\tau \ll \Delta t \ll \tau_{2coll}$

Two strategies can be considered:

- Considering waves directly (philosophy of exact treatment)

$$\overline{\Delta |\Psi\rangle} = \frac{\Delta t}{i\hbar} H_{MF} |\Phi(t)\rangle - \frac{\tau \Delta t}{2\hbar^2} \overline{\delta v_{12}^2} |\Phi(t)\rangle$$

- Considering densities directly (philosophy of dissipative treatment)

$$\overline{\Delta D} = \frac{\Delta t}{i\hbar}[H_{MF}, D] - \frac{\tau \Delta t}{2\hbar^2}\overline{[\delta v_{12}, [\delta v_{12}, D]]}$$

Quantum jump with dissipation: link between Extended TDHF and Lindblad eq.

One-body density
Master equation
step by step

$$\overline{\Delta D} = \frac{\Delta t}{i\hbar}[H_{MF}, D] - \frac{\tau \Delta t}{2\hbar^2}\overline{[\delta v_{12}, [\delta v_{12}, D]]}$$

Initial simple state

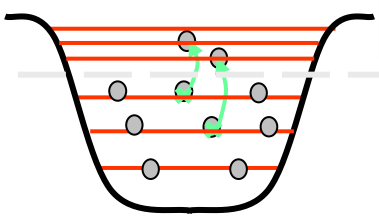
$$D = |\Phi\rangle\langle\Phi|$$

$$\rho = \sum_\alpha |\alpha\rangle\langle\alpha|$$

$$i\hbar\frac{d}{dt}\rho = [h_{MF}, \rho] - \frac{\tau}{2\hbar^2}\mathcal{D}(\rho)$$

with $\langle j|\mathcal{D}|i\rangle = \overline{\langle\left[\left[a_i^+ a_j, \delta v_{12}\right], \delta v_{12}\right]\rangle}$

2p-2h nature
of the interaction



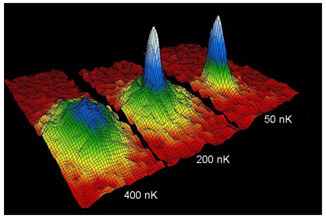
$$\mathcal{D}(\rho) = Tr_2\left[v_{12}, C_{12}\right]$$

with $C_{12} = (1-\rho_1)(1-\rho_2)v_{12}\rho_1\rho_2 - \rho_1\rho_2 v_{12}(1-\rho_1)(1-\rho_2)$

Separability of the interaction $v_{12} = \sum_\lambda O_\lambda(1)O_\lambda(2)$

$$\mathcal{D}(\rho) = \sum_k \gamma_k\left(A_k A_k \rho + \rho A_k A_k - 2A_k\rho A_k\right)$$

- Dissipation contained in Extended TDHF is included
- The master equation is a Lindblad equation
- Associated SSE *D. Lacroix, PRC73 (2006)*



Application to Bose condensate

1D bose condensate with gaussian two-body interaction

N-body density: $D = |N:\alpha\rangle\langle N:\alpha|$

SSE on single-particle state :

$$d|\alpha\rangle = \left\{ \frac{dt}{i\hbar} h_{MF}(\rho) + \sum_k dW_k (1-\rho) A_k - \frac{dt\tau}{2\hbar^2} \sum_k \gamma_k \left[A_k^2 \rho + \rho A_k \rho A_k - 2 A_k \rho A_k \right] \right\} |\alpha\rangle$$

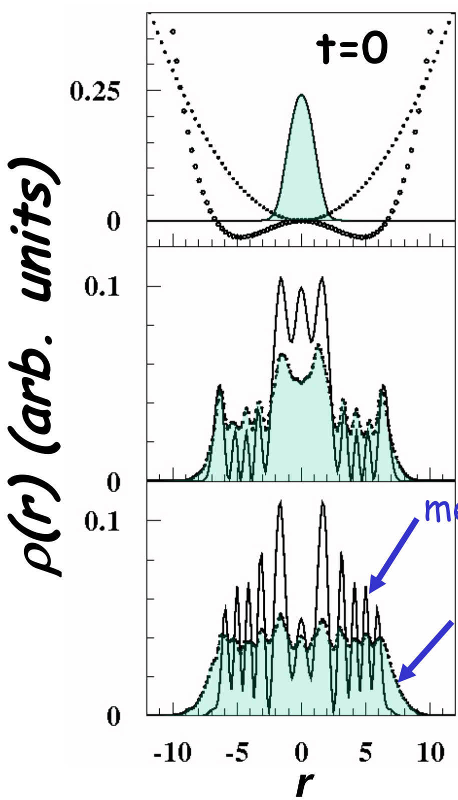
with $dW_k dW_{k'} = -\frac{dt\tau}{\hbar^2}\gamma_k \delta_{kk'}$

→ The numerical effort is fixed by the number of A_k

t>0

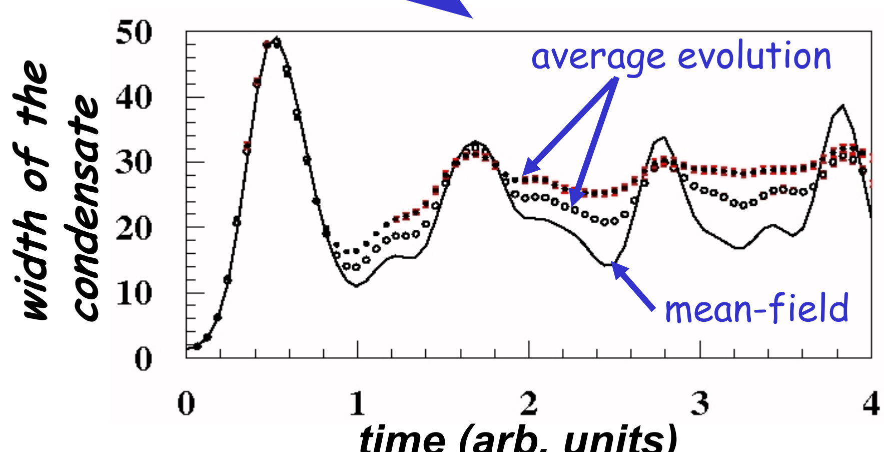
Density evolution

Condensate size



mean-field

average evolution



Summary

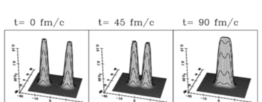
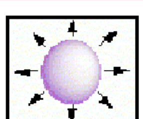
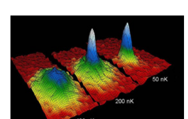
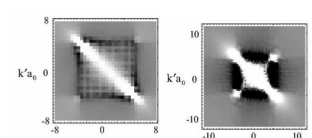
Quantum Jump (QJ) methods (or SSE) to extend mean-field

Approximate evolution

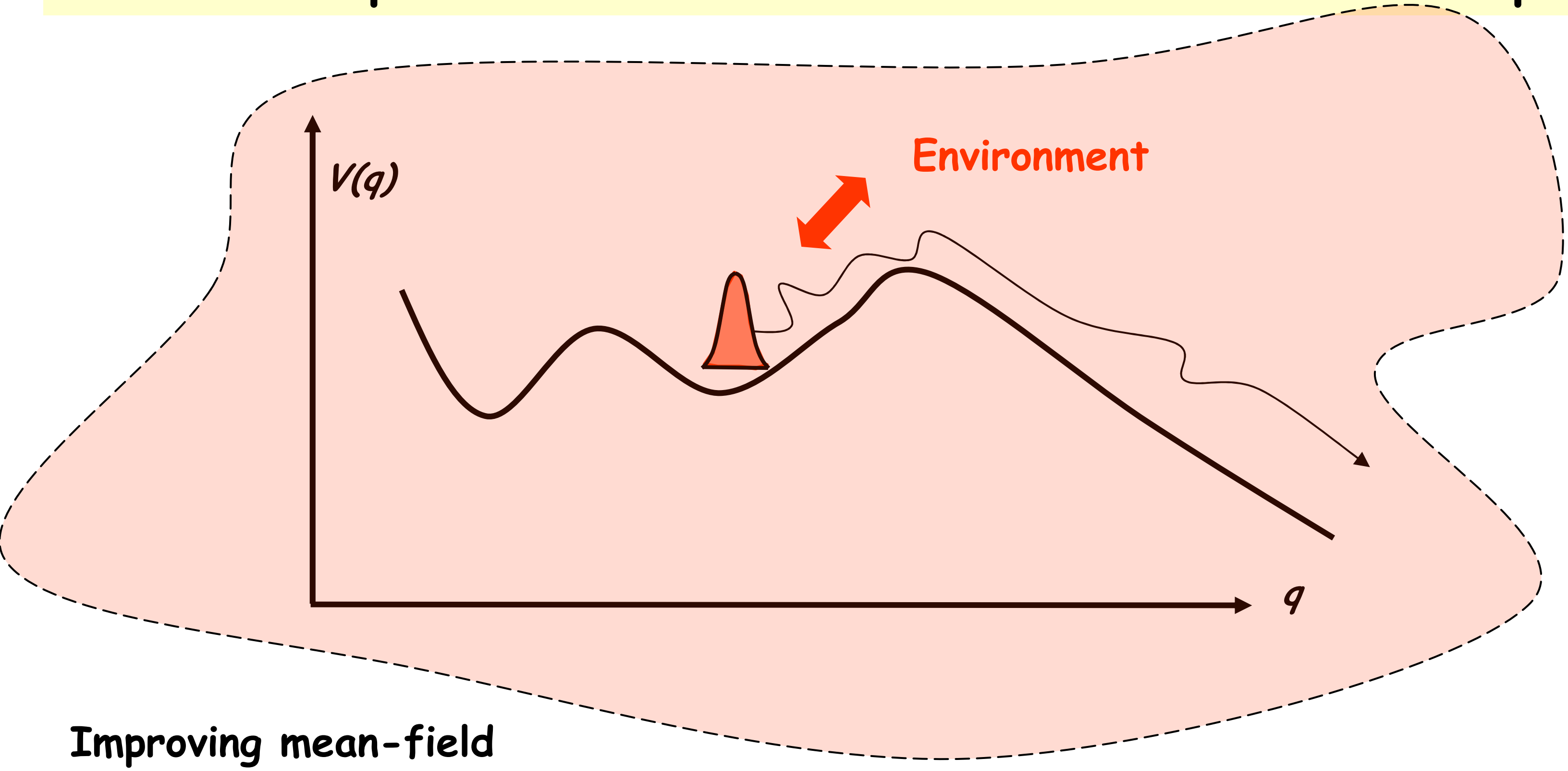
Mean-field	Simplified QJ	Generalized QJ	Exact QJ	variational QJ
$D = \|\Phi\rangle\langle\Phi\|$	$D = \overline{\|\Phi_1\rangle\langle\Phi_2\|}$	$D = \overline{\|\Phi\rangle\langle\Phi\|}$	$D = \overline{\|\Phi_1\rangle\langle\Phi_2\|}$	$D = \overline{\|Q_1\rangle\langle Q_2\|}$ $\|Q_1\rangle = \|q_1, \cdots, q_N\rangle$
~~Fluctuation~~ ~~Dissipation~~	Fluctuation ✓ ~~Dissipation~~	Fluctuation ✓ Dissipation ✓	Everything ✓	Partially everything ✓

Numerical issues

	Flexible	Fixed	Fixed	Flexible

Some developments: stochastic mechanics in a collective space



Improving mean-field

➡ Stochastic mean-field as an alternative to Generator Coordinate Method?

➡ Introduction of correlations

➡ Quantize (A)TDHF

Include Thermal effects

➡ System + Environment

Simplified scenario for introducing fluctuations beyond Mean-field

Interpretation of the equation on waves as an average over jumps:

$$\overline{\Delta\,|\Psi\rangle} = \frac{\Delta t}{i\hbar} H_{MF}\,|\Phi(t)\rangle - \frac{\tau \Delta t}{2\hbar^2}\,\overline{\delta v_{12}^2}\,|\Phi(t)\rangle \quad \Longleftrightarrow \quad \Delta\,|\Psi\rangle = \left\{ \frac{\Delta t}{i\hbar} H_{MF} + \Delta B \delta v_{12} + \frac{1}{2}\left(\Delta B \delta v_{12}\right)^2 \right\} |\Phi(t)\rangle$$

Let us simply assume that $\delta v_{12} \longrightarrow \sigma \delta v_{12}$ **with** $\Delta B = i\sigma \frac{\sqrt{\tau \Delta t}}{\hbar}$

SSE in one-body space

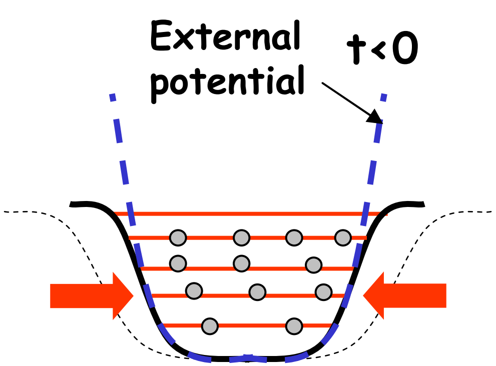
Assuming $D_{ab} = |\Phi_a\rangle \langle \Phi_b|$ with $\langle \Phi_b \mid \Phi_a \rangle = 1$

and $\langle a_i^+ a_j \delta v_{12}^2 \rangle \simeq \langle a_i^+ a_j \rangle \langle \delta v_{12}^2 \rangle + 2 \langle a_i^+ a_j \delta v_{12} \rangle \langle \delta v_{12} \rangle - 2 \langle a_i^+ a_j \rangle \langle \delta v_{12} \rangle^2$

$$d\rho = \frac{dt}{i\hbar}\left[h_{MF}, \rho\right] + dB_a (1-\rho) U_\delta(\rho) \rho + dB_b^* \rho U_\delta(\rho)(1-\rho)$$

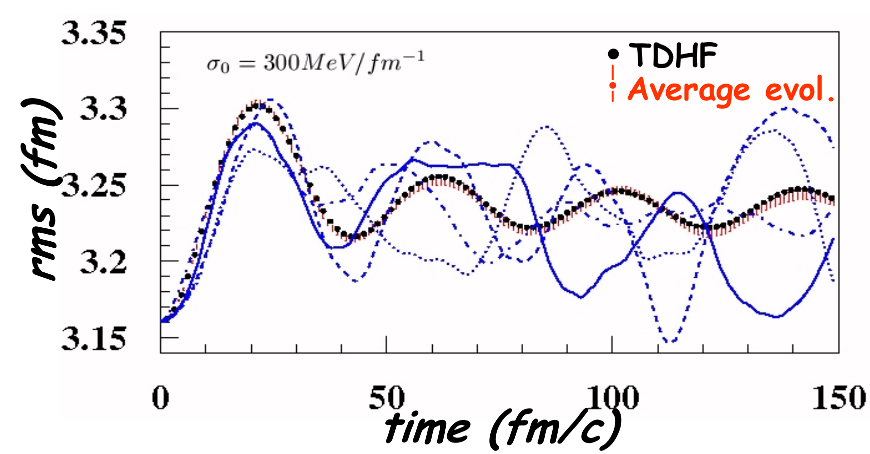
D. Lacroix, PRC73 (2006)

Application *Monopole vibration in ^{40}Ca*



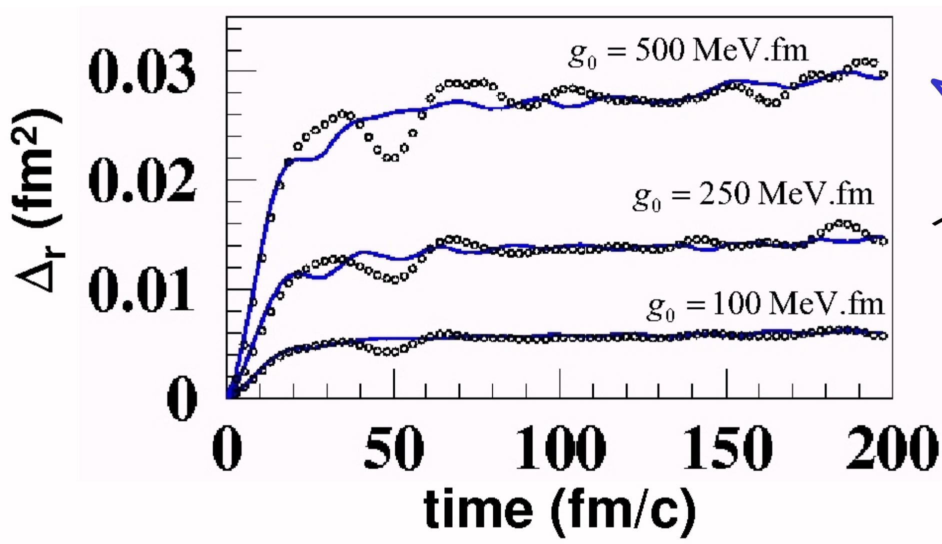
Stochastic part:

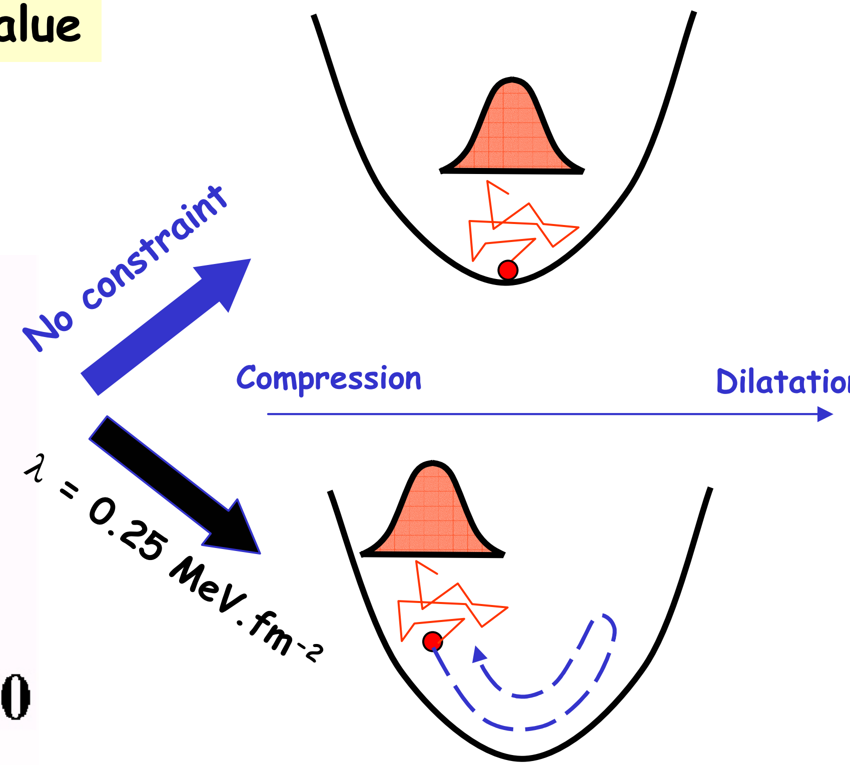
$$\delta v_{12} = \sigma_0 \delta(\mathbf{r}_1 - \mathbf{r}_2)$$



Diffusion of the rms around the mean value

Standard deviation $\Delta_r = \sqrt{\overline{\langle r^2 \rangle^2} - \overline{\langle r^2 \rangle}^2}$





Similar to Nelson stochastic theory

Nelson, Phys. Rev. 150, 1079 (1966).

Ruggiero and Zannetti, PRL 48, 963 (1982).

Summary and Critical discussion on the simplified scenario

- The stochastic method is directly applicable to nuclei
- It provide an easy way to introduce fluctuations beyond mean-field
- It does not account for dissipation.
- In nuclear physics the two particle-two-hole components dominates the residual interaction, but $U_{\delta_{2p2h}}(\rho) = 0$!!!

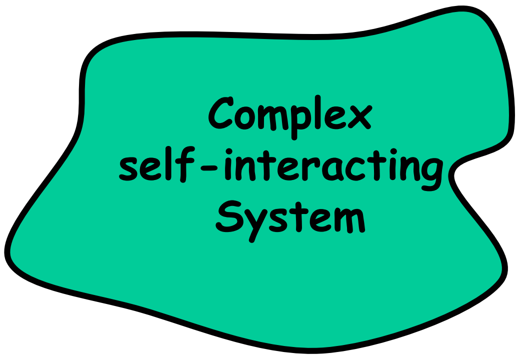
Coming back to the system+environment picture

The instantaneous Hamiltonian

$$H = \mathcal{P}_1 H + (1-\mathcal{P}_1) H$$

System: relevant degrees of freedom

Environment: other degrees of freedom



Introduction of SSE : what do we gain ?

Exact state → Trial states

$$|\Psi(t)\rangle \rightarrow \begin{cases} |Q(t)\rangle \\ |Q+\delta Q\rangle = e^{\sum_\alpha \delta q_\alpha A_\alpha}|Q\rangle \end{cases}$$

But now $\delta q_\alpha \rightarrow \delta q_\alpha + \delta\xi_\alpha^{[2]} + \delta\xi_\alpha^{[3]} + \cdots$

mean-field alone:

$$|\delta Q\rangle = \sum_\alpha \delta q_\alpha A_\alpha |Q\rangle$$

mean-field + noise:

$$\delta q_\alpha \rightarrow \delta q_\alpha + \delta\xi_\alpha^{[2]}$$

$$\overline{\delta\xi_\alpha^{[2]}\delta\xi_\beta^{[2]}} \propto \delta t$$

$$\overline{|\delta Q\rangle} = \Big(\sum_\alpha \delta q_\alpha A_\alpha + \sum_{\alpha\beta} \overline{\delta\xi_\alpha^{[2]}\delta\xi_\beta^{[2]}} A_\alpha A_\beta\Big)|Q\rangle$$

mean-field + noise + noise :

$$\delta q_\alpha \rightarrow \delta q_\alpha + \delta\xi_\alpha^{[2]} + \delta\xi_\alpha^{[3]}$$

$$\overline{\delta\xi_\alpha^{[3]}\delta\xi_\beta^{[3]}\delta\xi_\gamma^{[3]}} \propto \delta t$$

$$\overline{|\delta Q\rangle} = \Big(\sum_\alpha \delta q_\alpha A_\alpha + \sum_{\alpha\beta} \overline{\delta\xi_\alpha^{[2]}\delta\xi_\beta^{[2]}} A_\alpha A_\beta + \sum_{\alpha\beta\gamma} \overline{\delta\xi_\alpha^{[3]}\delta\xi_\beta^{[3]}\xi_\gamma^{[3]}} A_\alpha A_\beta A_\gamma\Big)|Q\rangle$$