Errata for Multigrid Tutorial, Second Edition

- p. 3. The box should distinguish between weak and strict diagonal dominance and be consistent with the definition of positive definite.
- p. 11. The first sentence of the third paragraph should read " ... update all the even components first by <u>the</u> expression..."
- p. 17. The $(2,3)$ entry of the matrix near the bottom of the page should be -1.
- p. 22, last equation. Delete the second term involving $[\lambda_k(R_G)]^{j/2}$.
- p. 27, exercise 3d. Lead with the sentence "Assume that A is the matrix associated with the model problem."
- p. 28, exercise 14c. Replace $\lambda_k^{j/2}$ by $\cos(\frac{k\pi}{n})$.
- p. 29, exercise 15a. The first equation should be

$$\mathbf{v}^{(1)} = \mathbf{v}^{(0)} + \frac{s}{||A||_2}\mathbf{r}^{(0)} \quad \text{for} \quad 0 < s < 2,$$

  where $\mathbf{r}^{(0)} = \mathbf{f} - A\mathbf{v}^{(0)}$ is the residual. The weight for the Jacobi method is then $\omega = \frac{cs}{||A||_2}$, where c is the diagonal term of A.
- p. 29, exercise 15b. Replace ω by s.
- p. 29, exercise 16. Immediately following the title of exercise, add "Assume A is symmetric, positive definite."
- p. 29, exercise 16a. Replacement statement should be

$$v_j \leftarrow v_j + \frac{r_j}{a_{jj}}.$$

- p. 29, exercise 16b. Replacement statement should be have $+$ instead of $-$ to reflect the change in part (a).
- p. 29, exercise 16b and 16d. Change all indices to i.
- p. 52. In equation 4.5, the modes should have the form $A(m)e^{\iota(j\theta_1+k\theta_2)}$.
- p. 52. The last displayed equation should read:

$$A(m+1) = \left[1 - \omega\left(\sin^2\left(\frac{\theta_1}{2}\right) + \sin^2\left(\frac{\theta_2}{2}\right)\right)\right] A(m) \equiv G(\theta_1,\theta_2)A(m).$$

- p. 53. In Figure 4.4b, the units on the axes are wrong. The scales should be $0 \le \theta_i \le \pi$.
- p. 54. The second displayed equation should read

$$G(\theta_1,\theta_2) = \frac{e^{\iota\theta_1} + e^{\iota\theta_2}}{4 - e^{-\iota\theta_1} - e^{-\iota\theta_2}}.$$

- p. 70, Exercise 9. Part (b) should read

$$G(\theta_1,\theta_2) = \frac{e^{\iota\theta_1} + e^{\iota\theta_2}}{4 - e^{-\iota\theta_1} - e^{-\iota\theta_2}}.$$

- p. 71, exercise 13d. The third sentence should be replaced by:

  For example, for fixed k, take $f(x) = C\sin(k\pi x)$ on the interval $0 \le x \le 1$, where C is a constant. Then

  $$u(x) = \frac{C}{\pi^2 k^2 + \sigma}\sin(k\pi x)$$

  is an exact solution to the model problem (1.1).

- p. 72, exercise 15c. The third sentence should be replaced by:

  For example, for fixed k and ℓ, take $f(x,y) = C\sin(k\pi x)\sin(\ell\pi y)$ on the unit square ($0 \le x, y \le 1$), where C is a constant. Then

  $$u(x,y) = \frac{C}{\pi^2 k^2 + \pi^2\ell^2 + \sigma}\sin(k\pi x)\sin(\ell\pi y)$$

  is an exact solution to the model problem (1.4).

- p. 72, exercise 17. Let's start over completely on the statement of the boundary value problem. It should look like this:

  Consider the following convection-diffusion problem on the unit square $\Omega = \{(x,y) : 0 < x <, y < 1\}$:

  $$\begin{aligned} -\epsilon(u_{xx} + u_{yy}) + a u_x &= A\sin(\ell\pi y)(C_2 x^2 + C_1 x + C_0) \quad \text{on } \Omega, \\ u &= 0 \quad \text{on } \partial\Omega, \end{aligned}$$

  where $\epsilon > 0$, $A \in \mathbf{R}$, $a \in \mathbf{R}$, ℓ is an integer, $C_2 = -\epsilon\ell^2\pi^2$, $C_1 = \epsilon\ell^2\pi^2 - 2a$, and $C_0 = a + 2\epsilon$. It has the exact solution $u(x,y) = Ax(1-x)\sin(\ell\pi y)$. Apply the multigrid algorithms discussed in this chapter to this problem. Compare the algorithms ....

- p. 91. The last set of equations in exercise 3 is missing a superscript h on u_{i-1}.

- p. 92, exercise 3c. Replace the first sentence by the following:

  Recall that if A is symmetric, then $||A||_2 = \rho(A)$, and that if $A\mathbf{w} = \lambda\mathbf{w}$, then $A^{-1}\mathbf{w} = \lambda^{-1}\mathbf{w}$. Use the eigenvalues of A^h to show that ...

- p. 92, exercise 3d. It should read:

  Show that if f'' is bounded on $[0,1]$ and $v_i = f''(\xi_i)$, then $||\mathbf{v}||_h$ is bounded. Is this result true for the (unscaled) Euclidean norm?

- p. 96. In the first equation, the term f_j should not be bold.

- p. 98. Change the paragraph in the middle of the page to

  Less obvious is the fact that an exact solution of the fine-grid problem is a fixed point of the FAS iteration (Exercise 2). This fixed point property, which is a desirable attribute of most iterative methods, means that the process stalls at the exact solution.

- p. 108. The right side of the last equation should have $2j$ rather than j for subscripts.

- p. 109. The right sides of the first two equations should read $(I_h^{2h} r^h)_j$.

- p. 109-110, exercise 2. Change the exercise to

  2. Assume that relaxation has the fixed point property, namely that if the exact solution $\mathbf{u}^h$ of equation (6.4) is used as the initial guess $\mathbf{v}^h$, then the result is $\mathbf{u}^h$. Assume also that the coarse-grid equation has a unique solution.

  a. Assume that the coarse-grid equation is solved exactly. Show that FAS also has this fixed point property, namely that if the exact solution $\mathbf{u}^h$ of equation (6.4) is used as the initial guess $\mathbf{v}^h$, then the result is $\mathbf{u}^h$.

b. Now show that this fixed point property holds even if the exact solver of the coarse-grid equations is replaced by an appropriate approximate solver. Assume only that the coarse-grid solver itself exhibits this fixed point property.

Note that part (b) has been deleted.

- p. 116, equation 7.8, last line. The sum should run from $i = 0$ to $i = n + 1$.
- p. 116, equation 7.10. Replace round brackets with angle brackets.
- p. 117, equation 7.12, second line. The first term to the right of the arrow should be v_1^h; that is,

$$v_1^h \longleftarrow v_1^h + \frac{v_0^{2h} + v_1^{2h}}{2}.$$

- p. 119. In Table 7.2, the convergence factor for the $N = 511$ case should be 0.100, not 0.010.
- p. 123, second sentence of first paragraph. Change it to read:

  In the case of (7.14), if we order the unknowns along lines of constant y (because strong coupling is in the x-direction), the matrix A^h can be written in block form as ...
- p. 123, second sentence of second paragraph. Change to "each line of constant y."
- p. 124, caption to Figure 7.4. "Upper left" should be "left."
- p. 129, equation (7.20), second equation. The numerator should be

$$h_{2j+\frac{3}{2}} v_j^{2h} + h_{2j+\frac{1}{2}} v_{j+1}^{2h}$$

- p. 139. Note for next printing (not errata). At the top of the page, state that weighted Jacobi converges for $\omega = \alpha ||D^{-1/2}AD^{-1/2}||^{-1}$ for some $0 < \alpha < 2$ and refer to new exercise.
- p. 139. The sentence in the middle of the page, "This translates ..." should be replaced by

  Assuming that $\omega = \alpha ||D^{-1/2}AD^{-1/2}||^{-1}$ for some fixed $\alpha \in (0, 2)$ and that $||D^{-1/2}AD^{-1/2}||$ is $O(1)$ (for the model problem, it is bounded by 2), it can be shown (Exercise 1) that

$$(D^{-1}A\mathbf{e}, A\mathbf{e}) \ll (\mathbf{e}, A\mathbf{e}).$$

- p. 159, exercise 1. Replace the first sentence by

  Under the assumptions that $\omega = ||D^{-1/2}AD^{-1/2}||^{-1}$ and $||D^{-1/2}AD^{-1/2}||$ is $O(1)$, show that an implication of the smooth error condition ...
- p. 159, exercise 2. The assumption $||r||_{D^{-1}} \ll ||e||_A$ should appear in the exercise opening, so it applies to all three parts.