Application of the Kramers-Kronig relations to measurements of attenuation and dispersion in cancellous bone*

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***Abstract*—Ultrasonic (US) assessment of fracture risk in osteoporotic patients is based on measurements of broadband ultrasonic attenuation and speed of sound (SOS) of cancellous bone. Discrepancies in estimates of SOS can result from dispersion and frequency-dependent attenuation. An improved understanding of the relation between attenuation and dispersion in cancellous bone could benefit both numerical and experimental investigations of the effects of these properties on estimates of SOS. The Kramers-Kronig (K-K) dispersion relations are used to investigate the relation between attenuation and dispersion. We consider a finite-bandwidth form of the K-K relations that uses the US properties over only the experimental bandwidth. *In vitro* measurements of cancellous bone were performed to validate this technique. Cancellous bone specimens were obtained from the proximal end of four bovine tibia and prepared with the axis of measurement along the medial-lateral (ML) and superior-inferior (SI) directions. Measurements were performed by use of a through-transmission, immersion technique. The attenuation coefficient and phase velocity were determined for each specimen. We found that dispersion correlated well with apparent density for ML specimens (r = -0.95) but less so for SI specimens (r = 0.61). Finite-bandwidth K-K analysis of the attenuation coefficient predicted the corresponding phase velocity. We observed good agreement between the measured and K-K predicted dispersion (r = 0.99) for all specimens.**

Keywords-attenuation; cancellous bone; dispersion; fracture risk; Kramers-Kronig relations; osteoporosis; ultrasound characterization

I. INTRODUCTION

Ultrasonic (US) assessment of fracture risk in osteoporotic patients is based on measurements of broadband ultrasonic attenuation and speed of sound (SOS) of cancellous bone found at peripheral sites [1]. The estimated SOS can be affected by frequency-dependent attenuation and phase velocity. One approach for the study of attenuation and dispersion involves the Kramers-Kronig (K-K) dispersion relations, where the relations provide a means to explore the relationship between the two propagation properties of a material.

The objectives of this study include examination of dispersion for use in bone assessment and evaluation of the consistency between measurements and K-K analysis of attenuation and dispersion.

II. MATERIALS AND METHODS

A. Specimen Preparation

Cancellous bone specimens (n = 10) obtained from the proximal end of four bovine tibia were cut into cubes (15 mm sides) and prepared with the axis of measurement along the medial-lateral (ML, $n_{ML} = 5$) and superior-inferior (SI, $n_{SI} = 5$) directions. Marrow was removed prior to measurement. Apparent densities ranged from (96 to 534) kg/m^3. See [2] for further details of specimen preparation. For purposes of the present study, densities of approximately 250 kg/m^3 and lower are considered low density, between 250 kg/m^3 and 500 kg/m^3 are considered medium density, and above 500 kg/m^3 are considered high density.

B. Experimental Setup and Data Acquisition

Measurements were performed in a water bath (T ≈ 23 °C) using a pair of planar transducers (2.25 MHz, 12.7 mm diameter) aligned coaxially as shown in Fig. 1. A broadband

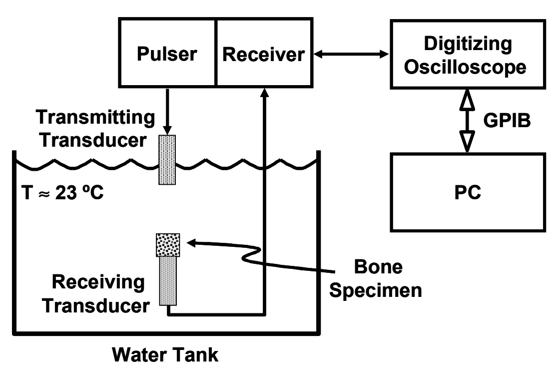

Figure 1. Schematic of experimental setup.

pulser/receiver was used to generate and detect electrical signals. Received signals were digitized at a sampling rate of 100 MHz with 8-bit resolution, temporally averaged 256 times, and then transferred to a personal computer for off-line analysis. Further setup details are available in [3].

III. Data Analysis

A. Frequency-Domain Analysis

The attenuation coefficient and phase velocity were determined by spectral analysis [4] of the ultrasonic signals, as given by

$$\alpha_{bone}(\omega) = 20\log_{10}\left(\left|F_{ref}(\omega)\right| / \left|F_{bone}(\omega)\right|\right) / L \ , \tag{1}$$

and

$$c_{bone}(\omega) = v_w / \left(1 - v_w \Delta\varphi(\omega)/\omega L\right), \tag{2}$$

where α_{bone} is the attenuation coefficient, c_{bone} is the phase velocity, F_{ref} is the Fourier transform of the reference path signal, F_{bone} is the Fourier transform of the signal through bone, L is the thickness of the bone specimen, v_w is the speed of sound in water [5], $\Delta\varphi$ is the change in phase between the reference and bone measurements, and ω is angular frequency.

B. Kramers-Kronig Analysis

The K-K relations are based upon the fundamental notions of linearity and causality [6]. The relations can be used to connect the attenuation coefficient and phase velocity of a material and also provide a criterion for the causal consistency of a measurement or model of the propagation mechanisms. Concern has been raised regarding the applicability of the K-K relations to measurements of cancellous bone [7]. To avoid assumptions about the behavior of the propagation properties outside the measured bandwidth, we employ a finite-bandwidth form of the K-K relations. However, this technique introduces artifacts into the calculations due to the truncation of the K-K integrals. Compensation for these artifacts can be accomplished by use of appropriate subtraction frequencies that are determined empirically.

We start with the K-K relations with two subtractions, written as

$$k(\omega) - k(\omega_0) - (\omega-\omega_0)\frac{d}{d\omega}k(\omega)\Big|_{\omega=\omega_0} = -\frac{1}{\pi}\left[\mathcal{H}\{\alpha(\omega)\} - \mathcal{H}\{\alpha(\omega_0)\} - \mathcal{H}\left\{(\omega-\omega_0)\frac{d}{d\omega}\alpha(\omega)\Big|_{\omega=\omega_0}\right\}\right], \tag{3}$$

where k is the real wave number ($k(\omega) = \omega / c(\omega)$), $\mathcal{H}$ indicates the Hilbert transform, and ω_0 is the subtraction frequency. The method of subtractions requires that the complex wave number be known and well-behaved at ω_0. Further detail regarding restrictions on the behavior of the complex wave number is available elsewhere [6]. We modify the Hilbert transform to be applied over only the experimental bandwidth

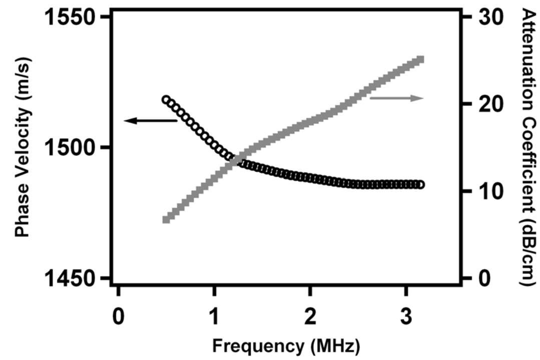

Figure 3. Phase velocity and attenuation coefficient for a low-density medial-lateral specimen.

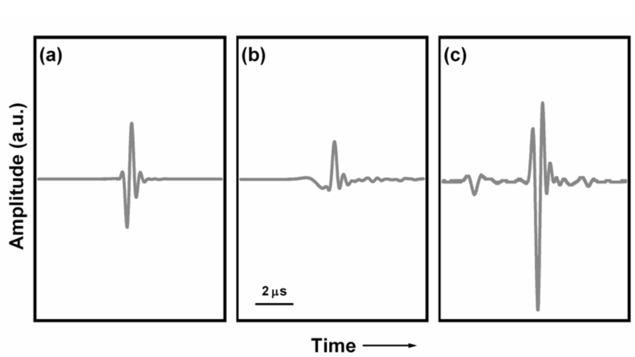

Figure 2. Ultrasonic waveforms propagated through (a) low-density medial-lateral, (b) medium-density medial-lateral, and (c) high-density superior-inferior specimens. The vertical scales are identical in each plot.

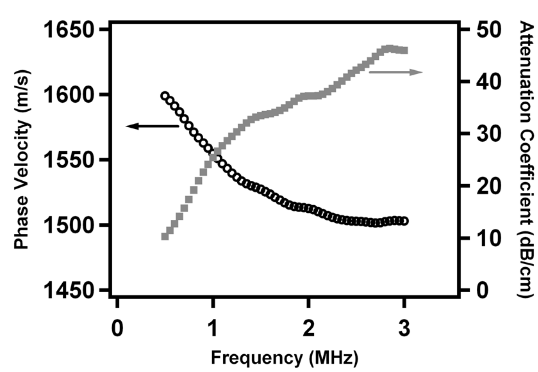

Figure 4. Phase velocity and attenuation coefficient for a medium-density medial-lateral specimen.

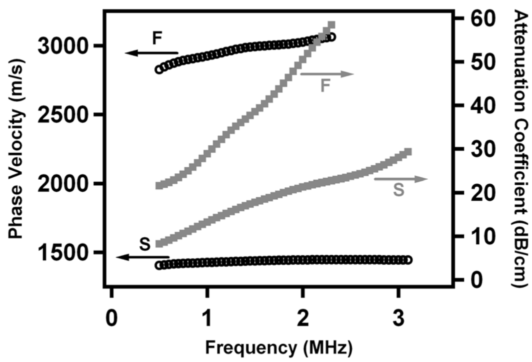

Figure 5. Phase velocity and attenuation coefficient for fast and slow waves of high-density superior-inferior specimen.

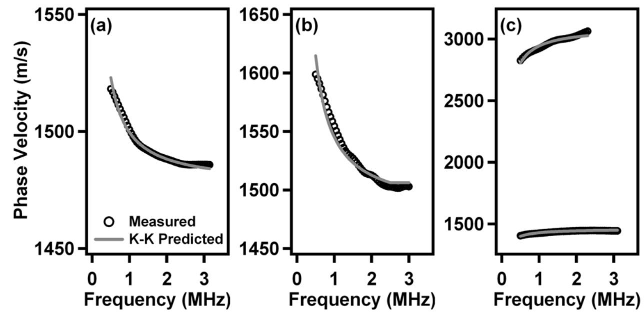

Figure 7. Measured and Kramers-Kronig predicted phase velocities for (a) low-density medial-lateral, (b) medium-density medial-lateral, and (c) high-density superior-inferior specimens.

$$\int_{0}^{\infty}\frac{\alpha(\omega')-\alpha(\omega)}{\omega'-\omega}d\omega' \rightarrow \int_{\omega_l}^{\omega_h}\frac{\alpha(\omega')-\alpha(\omega)}{\omega'-\omega}d\omega' , \qquad (4)$$

where ω_l and ω_h are the low and high end of the experimental bandwidth, respectively. (The left hand side of (4) is the half of the Hilbert transform over only positive frequencies. A similar operation is performed over the negative frequencies as well.) Given the measured attenuation coefficient, we empirically determine the subtraction frequency that minimizes the discrepancy between the measured and predicted phase velocity. Further details of this technique are described elsewhere [8].

IV. RESULTS AND DISCUSSION

A. Time-Domain Waveforms

Representative time-domain waveforms of the ultrasonic signals through cancellous bone specimens are shown in Fig. 2. The ultrasound propagates with modest loss and dispersion in the low-density ML specimen (Fig. 2a), but with moderate loss and dispersion in the medium-density ML specimen (Fig. 2b). For the high-density SI specimen (Fig. 2c), we observe a fast and slow wave. A reference signal through a water path is similar in shape to that through the low-density ML specimen, but has a larger amplitude. The existence of both a fast and slow compressional wave is consistent with the Biot theory applied to acoustic propagation in cancellous bone. The fast wave develops when the fluid (water) moves in phase with the solid (cancellous bone). The slow wave results from the fluid and solid moving out of phase with one another [9]. In a clinical setting, marrow and blood would replace the water.

B. Attenuation and Phase Velocity

The phase velocity and attenuation coefficient corresponding to the waveforms in Fig. 2(a,b,c) are shown in Figs. 3-5, respectively. As expected from the time-domain waveforms, the medium-density ML specimen exhibits attenuation larger than that of the low-density ML specimen. The fast wave of the high-density SI specimen is, however, the most lossy and dispersive. Negative dispersion is observed for the low- and medium-density specimens, whereas positive dispersion is observed for the high-density SI specimen. Only the medium-density ML specimen exhibits attenuation that would not be described as linear with frequency. Dispersion as a function of density is shown for all bone specimens in Fig. 6. Linear trends indicate a good correlation between density and dispersion for the ML specimens (r = -0.95), but a modest correlation for the SI (r = 0.61) specimens.

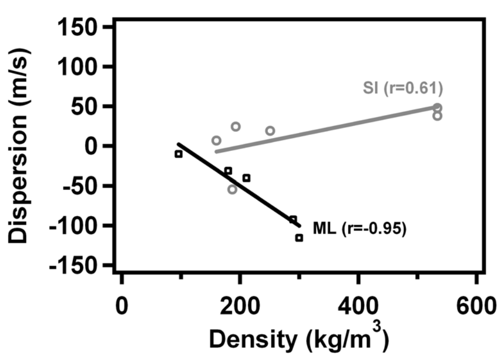

Figure 6. Dependence of dispersion on density.

V. KRAMERS-KRONIG ANALYSIS

The measured and K-K predicted phase velocities for each representative density specimen are shown in Fig. 7. We

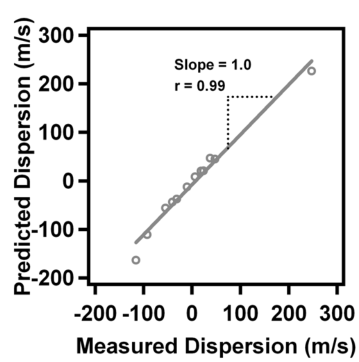

Figure 8. Comparison of measured and Kramers-Kronig predicted dispersions.

observe good agreement in all cases. The empirically determined subtraction frequencies for the low-, medium-, and (fast wave and slow wave) high-density specimens were respectively 2.05 MHz, 1.25 MHz, 1.25 MHz, and 1.85 MHz. The measured and K-K predicted dispersions across the experimental bandwidth for all specimens are compared in Fig. 8. We observe good agreement with a high correlation.

The K-K relations provide a useful tool for evaluating the causal consistency of measurements of attenuation and dispersion in cancellous bone. The finite-bandwidth form of the K-K relations permits one to avoid assuming the ultrasonic behavior outside the measured bandwidth, even though the role of the subtraction constant is not yet completely understood. Furthermore, these same techniques can be used to evaluate the consistency between the attenuation and phase velocity in analytic and numerical models.

VI. Conclusion

We have performed *in vitro* ultrasonic measurements of cancellous bone from bovine tibia in the medial-lateral and superior-inferior directions. The attenuation coefficient and phase velocity were determined. Dispersion was found to correlate well with bone density and consequently may be a useful indicator for prediction of fracture risk in osteoporotic patients. Dispersion predictions from a finite-bandwidth form of the Kramers-Kronig relations were found to be consistent with the experimentally measured dispersion. The application of these relations may potentially provide insight into the ultrasonic propagation properties of cancellous bone. The role of the subtraction frequency of the finite-bandwidth Kramers-Kronig relations is a topic of continuing research.

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