TABLE-TOP MODEL FOR BLACK HOLE ELECTROMAGNETIC INSTABILITIES

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Abstract

Place two conducting rings, one of slightly smaller diameter than the other, coaxially in a plane. Spin the inner ring (relativistically) rapidly. If it is somewhat resistive, while the outer ring is a good conductor, a nontrivially radiating, *exponentially growing*, electromagnetic instability will set in at a finite rotation velocity, despite the time-invariance of the background system. This table-top configuration models a plausible electromagnetic instability of a rotating black hole in the presence of an accretion disk made from the last remnants of a swallowed neutron star. Such a situation would extract pure electromagnetic energy from the hole, building up a stored energy on the order of the gravitational binding energy of the disk material before explosively disassembling. In the resulting electromagnetic explosion, most lines of sight to infinity are uncontaminated by baryons, and thus capable of initiating the ultra-relativistic fireball needed to power gamma-ray bursts. However, there is an important reason that the talk on which this paper is based was titled, “NOT a Gamma-Ray Burst Model.”

1. Introduction

It has long been known [9, 17] that rotating black holes superradiantly scatter (i.e., amplify) certain modes of an incident electromagnetic wave, and that the condition for such amplification is simply $0 < \omega < \Omega m$, where ω is the angular frequency of the wave, m its axial mode number, and Ω is the rotation rate of the black-hole horizon as seen from infinity. Indeed, this bit of phenomenology applies more generally to any situation in which a wave interacts with a dissipative medium that is moving faster than the wave's local phase velocity, as recognized by Pierce [12] in the context of traveling wave tube amplifiers, and later by Zel'dovich [17] in the context of black holes, whose rotating event horizon provides the effective dissipation [15].

Superradiant scattering is an amplifier, not an instability. Like any amplifier, it can be turned into an instability by feeding output back to input, a point made explicit by Press and Teukolsky [14], who pointed out that a rotating black hole surrounded by a solid, reflecting mirror would explode electromagnetically. However, the required geometry and high reflectivity ($> 99.9\%$) rendered this suggestion astrophysically fanciful.

This paper reports an explicit example of a configuration of conductors that is (i) open to infinity in almost all 4π steradians, (ii) has a true exponential growing mode of the Zel'dovich-Pierce variety, and (iii) can be built on any scale, e.g., on a table top out of aluminum foil and carbon paper. (Spinning it up to the required relativistic velocity should not be tried at home.) We then argue that an analogous instability may be present when the inner edge of a thin, conducting accretion disk comes sufficiently close to a rapidly rotating black hole, and that this type could plausibly initiate the ultra-relativistic fireballs that are believed to power gamma-ray bursts. However, as we will see, this mechanism alone cannot be responsible for the full power output necessary for a gamma-ray burst.

2. The Gamma-Ray Burst Central Engine

With the cosmological distance of gamma-ray bursts confirmed [8], the ultrarelativistic fireball model [3, 10], an expanding electron-positron pair plasma singularly devoid of baryon content, has emerged as virtually the only possible mechanism for gamma ray bursts. (See [16, 2, 13] for further discussion and references.) Indeed, the fireball model seems capable of explaining both the wide structural variability of gamma-ray flux seen from burst to burst via internal shocks [6], and also the diversity that appears present in the X-ray, optical, and radio afterglows, via viewing angle and environmental effects [7].

By its posited nature, the origin of the fireball is not directly observable; it is shielded by $\sim 10^{12}$ optical depths to electron-positron pair production. What we can deduce [16] about the original fireball is only its total energy ($\sim 10^{52}$ erg, $\gtrsim 10^{-3} M_\odot c^2$), compactness ($\lesssim 100$ km), and – most remarkable by far – low baryonic content ($\lesssim 10^{-5} M_\odot c^2$), a requirement that follows directly from the observational necessity that the expanding fireball contains sufficient total energy *per rest mass* to accelerate to relativistic γ factors on the order $\gamma \sim 100$, at least along the lines of sight that we observe. However, as Piran [13] has emphasized, it is also true that almost *any* mechanism for producing a fireball that satisfies these constraints is a viable central engine.

Despite this seeming lack of observational constraint, there is a paucity of believable models for the central fireball engine. While the energy and size requirements are clearly reminiscent of collision or collapse events among solar-mass scale neutrons stars or black holes, there has been no credible mechanism (other than the highly exotic, for example [11]) for separating radiation from baryons so exquisitely cleanly. The disruption of a neutron star is, in plausible models, quite a dirty event!

The mechanism proposed in this paper has three things going for it. First, it requires (as we will show) only $\lesssim 10^{-2} M_\odot$ in the inner region of an accretion disk to produce $\gtrsim 10^{-3} M_\odot c^2$ of purely electromagnetic energy. Second, the required baryonic mass can all be restricted to the equatorial plane, allowing most of 4π steradians to be arbitrarily "clean" of baryons. Third, it is capable of providing bursts with both the millisecond timescale required of the fireball, and the ~ 1 second envelope structure required by some observed events. We envision the mechanism acting at a "late" time, on a black hole that has swallowed a neutron star (possibly during its own formation), when all the nearby debris has been swallowed – except for the *last* $\sim 10^{-2} M_\odot$ of degenerate neutron star material in a thin accretion disk.

What causes this model to fail as – by itself – a model for gamma-ray bursts is the quantum electrodynamic breakdown of the assumed hard vacuum: before the necessary energy density of electromagnetic field is reached, copious electron-positron pair production will always be initiated, "shorting out" the exponential instability posited here. Below, we will discuss this effect and its consequences.

3. The Table-Top Model: Setup

We now outline a toy electromagnetic calculation that any reader should readily be able to reproduce (perhaps with an old graduate electromagnetism text in hand [5]): Consider two conducting rings, one of slightly smaller major diameter than the other, coaxially in a plane. We write Maxwell's equations as

$$\Box A^\mu = -4\pi J^\mu \tag{1}$$

with the $3+1$ split

$$A^\mu \equiv (A^0, \mathbf{A}) \equiv (\varphi, \mathbf{A}) \tag{2}$$

$$J^\mu \equiv (J^0, \mathbf{J}) \equiv (\rho, \mathbf{J}) \tag{3}$$

The gauge condition and continuity equations are

$$A^\mu{}_{,\mu} = \varphi_{,t} + \nabla \cdot \mathbf{A} = 0 \tag{4}$$

$$J^\mu{}_{,\mu} = \rho_{,t} + \nabla \cdot \mathbf{J} = 0 \tag{5}$$

Now, the general retarded-potential solution that couples these fields to sources with an outgoing radiative boundary condition is

$$\varphi = \int \frac{\rho[t-R]_{retarded}}{R}\, d^3V \tag{6}$$

$$\mathbf{A} = \int \frac{\mathbf{J}[t-R]_{retarded}}{R}\, d^3V \tag{7}$$

Suppose a conducting ring of radius R_1 has circumferential 3-current (units of charge per time) and lineal charge density (units of charge per length) given by

$$J^{\hat{\phi}} = J_1 \exp(-i\omega t) \exp(im\phi) \tag{8}$$

$$\rho = \rho_1 \exp(-i\omega t) \exp(im\phi) \tag{9}$$

Note that $J^{\hat{\phi}}$ is a physical (orthonormal tetrad) component, not a covariant or contravariant component.

The continuity equation (5) implies

$$-\,i\omega\rho_1 = -\left(\frac{im}{R_1}\right) J_1, \qquad \text{i.e.,} \qquad \rho_1 = \frac{m}{\omega R_1}\, J_1 \tag{10}$$

Also, for the retardation calculation in equations (6) and (7), note that

$$J^{\hat{\phi}}(t-\tau) \propto J_1 \exp[-i\omega(t-\tau)] \propto J_1 \exp(i\omega\tau) \tag{11}$$

The constitutive relation ("Ohm's Law") $J = \sigma E_\phi$ must be applied in the matter rest frame of each ring element, so we now need to consider Lorentz transformations into the rotating ring frame. A ring of radius R_1 rotating with angular frequency Ω_1 has circumferential velocity and γ-factor

$$v_1 \equiv \Omega_1 R_1 \qquad \gamma_1 \equiv (1 - v_1^2)^{-1/2} \tag{12}$$

Using primes to denote the rotating frame, we have

$$J'^0 = \gamma J^0 - \gamma v J^{\hat{\phi}} \tag{13}$$

$$J'^{\hat{\phi}} = \gamma J^{\hat{\phi}} - \gamma v J^0 \tag{14}$$

(One can see that the signs are correct by noting that a rest charge J^0 turns into a *negative* current for positive v.) Now using equations (5) and (12), we get

$$J'^{\hat{\phi}} = \gamma_1 \left(1 - \frac{\Omega_1 m}{\omega}\right) J^{\hat{\phi}} \tag{15}$$

By contrast $E'^{\hat{\phi}}$ has the trivial Lorentz transformation

$$E'^{\hat{\phi}} = E^{\hat{\phi}} \tag{16}$$

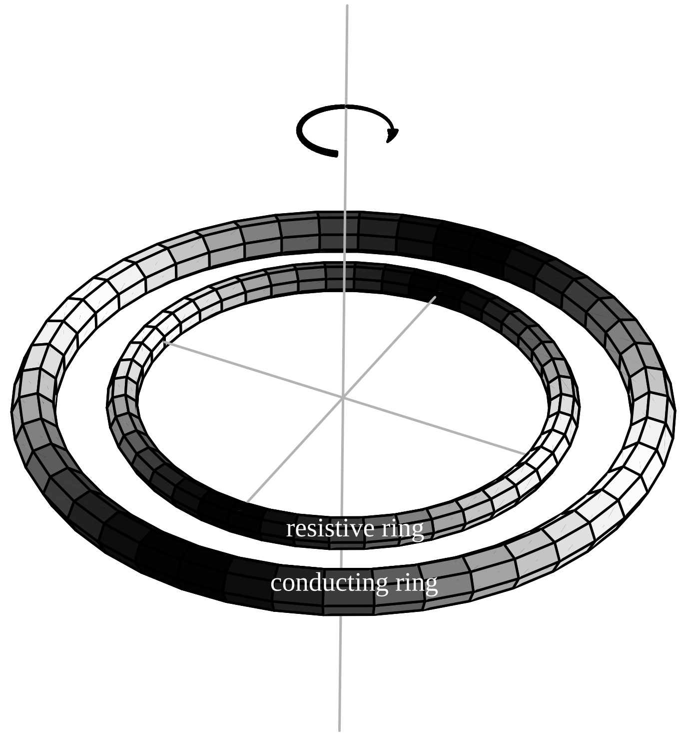

Fig. 1. Table-top model calculated in this paper. The inner resistive ring is spun at relativistic speed. The outer conductor may, but need not, be spinning. Shading shows schematically the location of positive and negative charge in an $m = 2$ growing mode. Rotational energy extracted from the resistive ring more than compensates radiative losses to infinity, yielding exponential growth of the stored field energy. Retardation effects are significant in the calculation.

because longitudinal E and B fields are invariant under boost. (One can also derive this the long way by using the Lorentz transformations of A^μ and taking the various partial derivatives.) Thus, the constitutive relation applied to each ring in its rest frame is

$$J_1'^{\hat\phi} = \sigma_1 E'^{\hat\phi} \qquad \text{or, equivalently,} \qquad \gamma_1 \left(1 - \frac{\Omega_1 m}{\omega}\right) J_1 = \sigma_1 E_1^{\hat\phi} \tag{17}$$

One sees the superradiance condition already emerging!

4. Field Elsewhere in the Plane Due to a Ring Current

The plan is to evaluate A^0 and $A^{\hat{\phi}}$ at a point $r = R_2$, $\phi = z = 0$. By separation of variables we will then know it at all values of ϕ. We can thus evaluate its time and ϕ derivatives and get $E_2^{\hat{\phi}}$.

For a source point at (R_1, ϕ), the law of cosines with $(R_2, 0)$ gives

$$\tau^2 = R_2^2 + R_1^2 - 2R_1R_2\cos\phi \tag{18}$$

So,

$$\begin{aligned}
\varphi &\equiv A^0 = \int \frac{\rho_{ret}d\ell}{\tau} = \int_0^{2\pi} \frac{\rho_1 \exp(i\omega\tau)\exp(im\phi)R_1 d\phi}{\tau} \\
&= \frac{mJ_1}{\omega R_1}\int_0^{2\pi} R_1 d\phi\, \frac{\exp(im\phi)\exp[i\omega(R_2^2+R_1^2-2R_1R_2\cos\phi)^{1/2}]}{(R_2^2+R_1^2-2R_1R_2\cos\phi)^{1/2}} \\
&= \frac{mJ_1}{\omega R_1}\int_0^{2\pi} d\phi\, \frac{\exp(im\phi)\exp[i\omega R_1(R^2+1-2R\cos\phi)^{1/2}]}{(R^2+1-2R\cos\phi)^{1/2}} \\
&\equiv \frac{mJ_1}{\omega R_1}\, H_t(\omega R_1, \frac{R_2}{R_1})
\end{aligned} \tag{19}$$

(where $R \equiv R_2/R_1$). We define an integral H_t by

$$\begin{aligned}
H_t(\varpi, r; m) &\equiv \int_0^{2\pi} d\phi\, \frac{\exp(im\phi)\exp[i\varpi(r^2-2r\cos\phi+1)^{1/2}]}{(r^2-2r\cos\phi+1)^{1/2}} \\
&= 2\int_0^{\pi} d\phi\, \frac{\cos(m\phi)e^{i\varpi(r^2-2r\cos\phi+1)^{1/2}}}{(r^2-2r\cos\phi+1)^{1/2}}
\end{aligned} \tag{20}$$

where the second equality uses the fact that $\cos\phi$ is even while $\sin(m\phi)$ is odd in ϕ.

Note that $H_t(\varpi, r; m)$ has a logarithmic singularity as $r \to 1$, due merely to the infinite potential of a loop of zero minor radius. We can regularize this by picking a small constant ϵ_0, approximating the minor radius of the ring, and redefining,

$$H_t(\varpi, r; m, \epsilon_0) \equiv 2\int_0^{\pi} d\phi\, \frac{\cos(m\phi)e^{i\varpi(r^2-2r\cos\phi+1)^{1/2}}}{(r^2-2r\cos\phi+1+\epsilon_0^2)^{1/2}} \tag{21}$$

Correspondingly for $A^{\hat{\phi}}$ we have

$$\begin{aligned}
A^{\hat{\phi}} &= \int \frac{J^{\hat{\phi}}_{ret,proj}d\ell}{\tau} = \int_0^{2\pi} \frac{J_1\exp(i\omega\tau)\exp(im\phi)R_1\cos\phi\, d\phi}{\tau} \\
&= J_1\int_0^{2\pi} R_1\cos\phi\, d\phi\, \frac{\exp(im\phi)\exp[i\omega(R_2^2+R_1^2-2R_1R_2\cos\phi)^{1/2}]}{(R_2^2+R_1^2-2R_1R_2\cos\phi)^{1/2}}
\end{aligned}$$

$$
\begin{aligned}
&= J_1 \int_0^{2\pi} \cos\phi \, d\phi \, \frac{\exp(im\phi)\exp[i\omega R_1(R^2+1-2R\cos\phi)^{1/2}]}{(R^2+1-2R\cos\phi)^{1/2}} \\
&\equiv J_1 \, H_\phi(\omega R_1, \frac{R_2}{R_1}) \qquad (22)
\end{aligned}
$$

with (regularizing the infinite self-inductance of a loop of zero minor radius),

$$
H_\phi(\varpi, r; m, \epsilon_0) \equiv 2\int_0^{\pi} d\phi \, \frac{\cos\phi\cos(m\phi)e^{i\varpi(r^2-2r\cos\phi+1)^{1/2}}}{(r^2-2r\cos\phi+1+\epsilon_0^2)^{1/2}} \qquad (23)
$$

Incidentally, we have the computationally useful relations

$$
H_\phi(\varpi, r; m, \epsilon_0) = \frac{1}{2}[H_t(\varpi, r; m+1, \epsilon_0) + H_t(\varpi, r; m-1, \epsilon_0)] \qquad (24)
$$

and

$$
H_t(\varpi, r; -m, \epsilon_0) = H_t(\varpi, r; m, \epsilon_0) \qquad (25)
$$

Also note that everything above allows ω and thus ϖ to be complex, so that this formalism can be used for exponentially growing or dying eigenmodes.

In terms of the integrals defined by equations (21) and (23), which are easy to compute numerically and may thus be thought of as "tabulated" functions, the circumferential electric field at R_2 is

$$
E^{\hat{\phi}} \equiv E_2 \exp(-i\omega t)\exp(im\phi) \qquad (26)
$$

with

$$
\begin{aligned}
E_2 &= -A^{\hat{\phi}}{}_{,t} - \nabla_{\hat{\phi}} A^0 \\
&= i\omega J_1 \left[H_\phi(\omega R_1, \frac{R_2}{R_1}) - \frac{m^2}{\omega^2 R_1 R_2} H_t(\omega R_1, \frac{R_2}{R_1}) \right] \\
&\equiv i\omega J_1 L_{21}(\omega, m) \qquad (27)
\end{aligned}
$$

Here we have introduced the abbreviation $L_{21}(\omega, m)$ for what amounts to the "dynamic and retarded mutual inductance" between the two ring positions R_1 and R_2. (Because of retardation, it is *not* symmetrical on the indices 1 and 2.)

5. Finessing the Complex Eigenproblem

Consider now the mutual interaction of the two rings shown in Figure 1. To simplify our problem in a way that will later model the astrophysically interesting situation, let us suppose that the outer ring 2 is a perfect conductor ($\sigma \to \infty$), while the inner ring 1 (which models the electromagnetic properties of a black hole's event horizon) has finite conductivity. Then, in terms of the mutual inductances defined by equation (27), we have

$$\begin{aligned} \gamma_1 \left(1 - \frac{\Omega_1 m}{\omega}\right) J_1 = \sigma_1 E_1 &= i\omega\sigma(L_{11}J_1 + L_{12}J_2) \\ 0 = E_2 &= (L_{21}J_1 + L_{22}J_2) \end{aligned} \tag{28}$$

This is a homogeneous set of equations for J_1 and J_2, admitting a solution only with the vanishing of the determinant

$$\begin{vmatrix} L_{11} - \frac{\gamma_1}{i\omega\sigma}\left(1 - \frac{\Omega m}{\omega}\right) & L_{12} \\ L_{21} & L_{22} \end{vmatrix} = 0 \tag{29}$$

For fixed σ, m, Ω, equation (29) is a transcendental equation in ω whose solutions are the complex eigenfrequencies of the two-ring system. (Note that the L's are implicitly quite complicated transcendental functions of ω.)

Now a nice trick becomes possible: While it would be a challenging numerical problem to solve equation (29) for the eigenfrequencies ω for a fixed σ, it is quite trivial to solve for σ at any fixed complex ω, because σ occurs in only one term, as shown. Of course most of the resulting σ's are complex, or negative, and therefore unphysical. Discarding these, one is left, in the complex ω-plane, with a set of one-dimensional segments whose ω's yield real and positive values of σ. (Each segment could in principle be labeled along its length by the value of σ, generally varying from zero at one end to infinity at the other.)

Figure 2 shows a specific example, calculated with the parameters $m = 2$, $v_1 = 0.99$, $R_2/R_1 = 1.2$, and $\epsilon^2 = .003$. One sees that almost all the physical solutions are in the lower-half (stable) complex plane. However, in the region close to $\omega = \Omega_1 m$ (shown shaded and also shown, magnified, in Figure 3), there are physical solutions extending into the upper-half (unstable) complex plane. Indeed, with these parameters, the value of σ corresponding to the most unstable solution, converted to physical units, is about 6 Ohms.

We have found similar unstable modes with $m = 2, 3, 4$ when the ratio of major diameters is in the range of ~ 1.1 to ~ 1.4 (with ratio of minor to major diameters ~ 0.03 to ~ 0.1). The unstable modes are always in the vicinity of $\omega \approx m\Omega$. They occur when the tangential velocity of the inner ring is sufficiently large (say, $\gtrsim 0.98c$), and when the conductance is *not* too large. These modes are the analytical continuation of the uninteresting (real ω) solution with $\omega = m\Omega$ and $\sigma = 0$, representing a spinning charge distribution on an insulator. For the parameters already mentioned, this mode gets perturbed into the upper, unstable, complex half-plane. The maximum growth rate is never large, with a ratio Im $\omega/$Re $\omega \lesssim 0.02$.

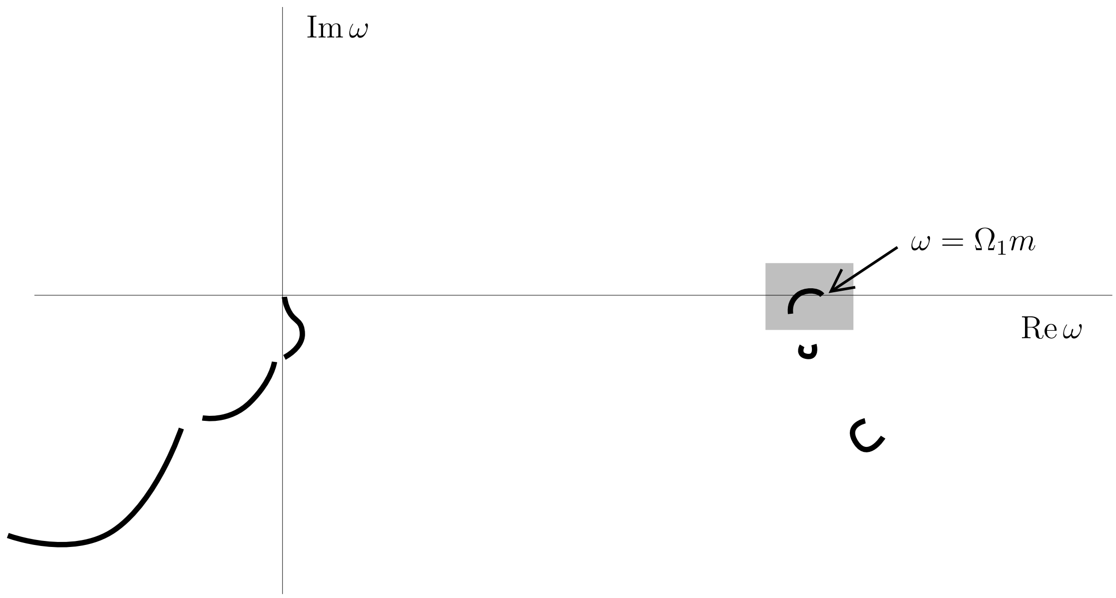

Fig. 2. Complex ω plane for a sample two-ring system, showing the locus of allowed eigenfrequencies for all (real) positive values of the inner ring conductivity σ. All of the modes outside of the shaded region are stable. A magnified view of the shaded region is in Figure 3. See text for details.

Incidentally, the trick of solving for σ instead of ω generalizes to the case of one ring with finite conductance, and any number of perfectly conducting rings. In that case the determinant is larger than 2×2, but can be evaluated by partitioning so that an explicit formula for σ can still be written.

6. From Table Top Model to Black Holes

A large literature (see, e.g., [15]) details the quantitative analogy between the rotating event horizon of a black hole and a relativistically spinning, partially conducting surface. Figure 4 shows schematically the charge, and image charge, distribution for a (conjectural) $m = 2$ growing mode in the black hole case, exactly analogous to the calculated 2-ring case shown in Figure 1.

The actual calculation of the modes of a black hole in the presence of moving conductors in plausible geometries is a technically challenging project in computational relativity; it is hoped that this paper will stimulate such calculations. For the remainder of this paper, let us assume that an instability analogous to the table-top model exists, and discuss possibly relevant astrophysical scenarios.

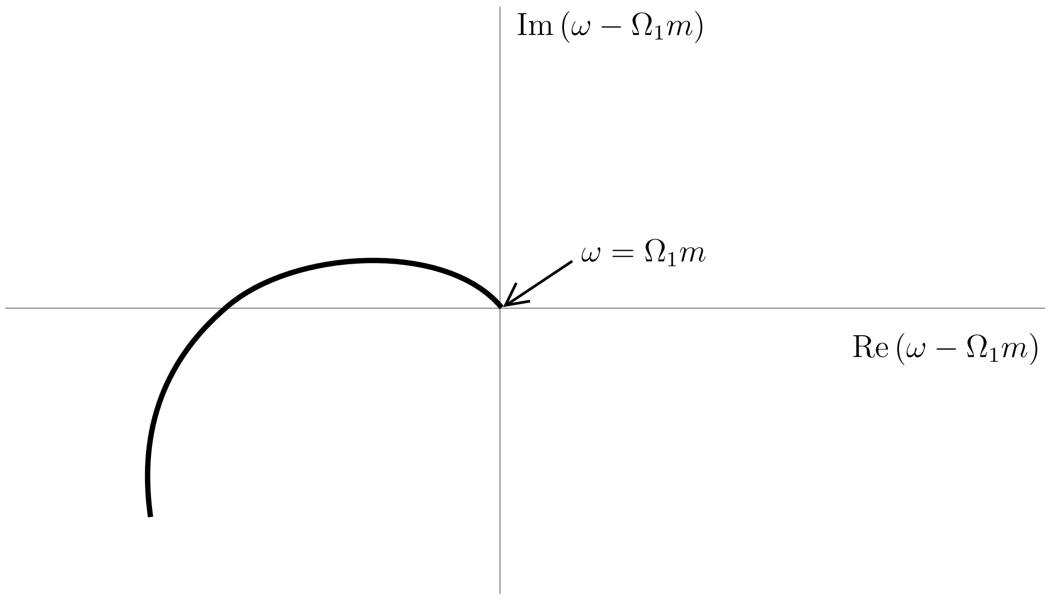

Fig. 3. Magnified view of the shaded region in Figure 2. For some values of conductivity σ, there are modes with frequencies in the upper-half complex plane, corresponding to exponentially growing electromagnetic modes. See text for details.

The table-top calculation suggests that a black hole may need to be spun up by accretion to near-maximal rotation before an instability appears. It is important that this be done in an environment that does not permit the electromagnetic instability to grow; otherwise the hole would simply "sputter" at the edge of instability, converting incrementally added spin to outgoing low-frequency electromagnetic pulses. This might be astrophysically interesting in its own right, but it is not what we have in mind here. We therefore imagine a "dirty" accretion phase during which the plasma density surrounding the hole is sufficient to blanket or "short-out" any growing electromagnetic mode. For cases of interest, only after the hole is spun-up well into the unstable zone does its environment clear up.

What might the situation be, say when only $\sim 10^{-2} M_\odot$ of original neutron star material remain in the inner portion of the accretion disk, and when the environment clears? Two possibilities are interesting: (1) We might be left with a conducting inner accretion disk already inside the (supposed) radius for an electromagnetic instability, in which case the instability begins to grow immediately. (2) We might be left with a degenerate, electrically neutral, disk of neutron material (the ratio of whose thickness to radius can be calculated as $\ll 1$), in which

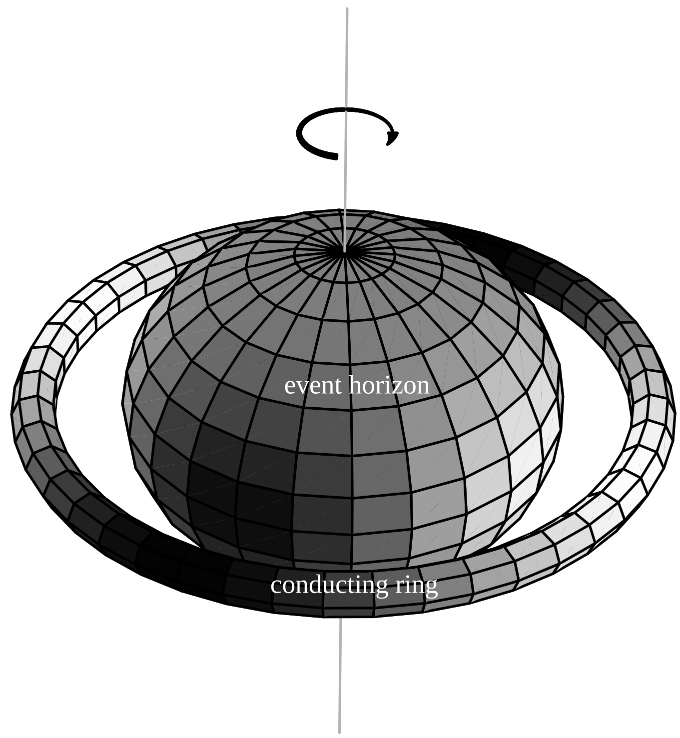

Fig. 4. Conjectural analog of the table-top model to the case of a rotating black hole. The spinning event horizon plays the role of the inner ring. Shading shows the charge density on the ring, and the image charge density on the horizon. Radiative losses to infinity are partially suppressed by non-flat geometry effects, particularly so for a rapidly rotating black hole.

case the system remains quiescent until some other event (late-time accretion? free decay of neutrons?) causes the disk to become conducting, in turn triggering the instability.

For a maximally rotating Kerr black hole, we have $\Omega \approx 10^5(M_\odot/M)$ Hz, so the e-folding time of the instability should be $\sim 1(M/M_\odot)$ ms. Energetically, all the action is of course in the *last* e-fold. The important astrophysical question is: with what total stored energy does this last e-fold occur? That is, what processes limit the exponential growth?

7. Quantum Electrodynamic Vacuum Breakdown

If no other limiting process first intervened, the stored energy would grow to become comparable to the gravitational binding energy of the ring of material that is confining the growing electromagnetic mode. At that point, the system would disassemble, the ring blowing off in the equatorial plane while electromagnetic energy is released in all other directions. For a ring at ~ 10 Schwarzchild radii, this total energy release is $\sim 10^{-3} M_\odot c^2$, and the implied peak luminosity is $\sim 10^{54}$ erg/s, both very interesting numbers in the context of gamma-ray bursts.

However, for black holes in the solar mass range, the quantum electrodynamic breakdown of the vacuum (see, e.g., [4]) occurs long before the system disassembles. The critical electric field strength for vacuum breakdown is

$$E_{crit} \sim \frac{m_e c^2}{e} \left(\frac{\hbar}{m_e c} \right)^{-1} = 5 \times 10^{13} \text{ statvolt (cgs)} \tag{30}$$

(Indeed, the critical value may be a factor of 10 or 100 smaller due to tunneling and cascading, but we'll use the above more favorable value here.) Thus, in a sphere of radius R, the maximum stored energy is

$$\mathcal{E} = \frac{E_{crit}^2}{8\pi} \frac{4\pi R^3}{3} \tag{31}$$

The most favorable assumption about luminosity is that the stored energy is cleared out, and then replaced by a newly growing instability, at the light travel time c/R. This gives a luminosity

$$L = \mathcal{E} \frac{c}{R} = 1 \times 10^{49} \left(\frac{R}{10\text{km}} \right)^2 \text{ erg/s} \tag{32}$$

which is about three orders of magnitude too small to be interesting as a gamma-ray burst. Furthermore, the above most favorable assumption about luminosity is almost surely wrong by a factor of 100 or more due to the fact the black hole instability growth times, if they are anything like our table top model, are likely ~ 100 times the light travel time.

What happens after vacuum breakdown occurs? Certainly the vacuum mode analysis of this paper breaks down. It is by no means obvious, however, that extraction of rotational energy from the black hole ceases. Because breakdown occurs only where the electric field dominates the magnetic field (in the sense of $E^2 - B^2 > 0$), the region near the hole is momentarily divided into regions of high current density, and other regions with high magnetic field strength. Instead of itself being a model for gamma ray bursts, the phenomenology discussed here might then be the initial “sparkplug” that initiates energy extraction that more resembles the stationary force-free regime discussed by Blandford and Znajek [1].

About all we can be certain is that there is a need, after a long historical hiatus, for further work on the complicated details of black hole electrodynamics.

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