" Extreme confinement of plasmas and Negative specific heat"

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Summary

When a system's temperature increases upon loosing heat, you have a recipe for dramatic and rare run-away/ collapse phenomena. To see how this works, consider a system with two components – a hot core and cooler envelope – where the core's temperature increases when heat is transferred out to its envelope. As the core continues to loose energy to the envelope, it gets hotter while the envelope gets cooler even as it gains energy. Because the increasingly hot core looses more energy to the cooling envelope at faster and faster rates, the temperature and other thermodynamic properties of the core can reach explosive levels in a system with negative specific heat. Density is moreover correlated with temperature in such systems, leading to runaway collapse of the core or extreme confinement. Lynden-Bell discovered negative specific heat for the first time in gravo-thermal collapse of stars and used it to explain the formation of red-giant stars. Tim Andersen and Chjan Lim found negative specific heat in electron magneto-hydrodynamics (EMH) and confirms the corollary that the core part of the plasma undergoes extreme confinement – a condition that could have profound impact on thermonuclear fusion.

In a paper [1] submitted on June 27th 2007 to Physical Review Letters (PRL) - the highest impact journal in physics – Tim A. and C. Lim reported on the discovery of negative specific heat in electron plasmas and discussed the profound implications of extreme confinement for fusion research. Recognizing the potentially high impact of this work, peer review of this paper was completed within one week – a record turn-around time - by the editors of PRL and the authors informed of reviewer's comments on July 5th 2007.

For the benefit of your readers, a very short excerpt of the paper's review is included here to highlight its impact: "This paper provides a new and very interesting theoretical investigation of a mysterious and important physical phenomenon, with wide possible applications. It should be published with very high priority. This work could have the same impact as Onsager's paper [2] on negative temperature states. …."

Onsager published a short note [2] in 1947 in which he introduced the concept of negative temperature states in 2D point vortex systems confined by angular momentum. His paper became a classic and is often referenced in the vortex dynamics, turbulence and statistical mechanics literature. However, point vortices in the plane cannot exhibit negative specific heat at any temperature, even if one uses a microcanonical thermodynamic formulation

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instead of the canonical ensemble proposed by Onsager in [2] – a microcanonical ensemble is one where the statistical properties of the system are studied at fixed energy. A system of wriggly vortex filaments is believed to be necessary in order for a vortex system to exhibit negative specific heat. Figure 2 shows a bundle of vortex filaments in a nearly parallel state.

The results in [1] show that the microcanonical formulation of a quasi-2D system of generalized vortex filaments supports negative specific heat. Numerical and other technical difficulties prevented this system from being simulated by Path-Integral Monte-Carlo (PIMC) methods. Instead, we used a closed-form mean field analysis of the partition function in order to derive a tractable expression for the free energy of the system. The partition function is a “huge” sum (path-integral) over the system’s states that encapsulates its thermodynamic behavior. Its free energy - F = U – TS where U is the energy, T is the temperature and S, the entropy measures disorder in the system – is a compact formula obtained from the system’s partition function that can give us the system’s thermodynamic properties including its specific heat. Specific heat is defined to be the rate of change of energy U with respect to temperature T – thus, a negative specific heat means that the system’s T will increase when its energy U decreases and vice-versa. Figure 1 depicts the specific heat of the electron plasma – generalized vortex filament system as a function of the temperature and shows clearly that it is negative over the whole range of temperatures.

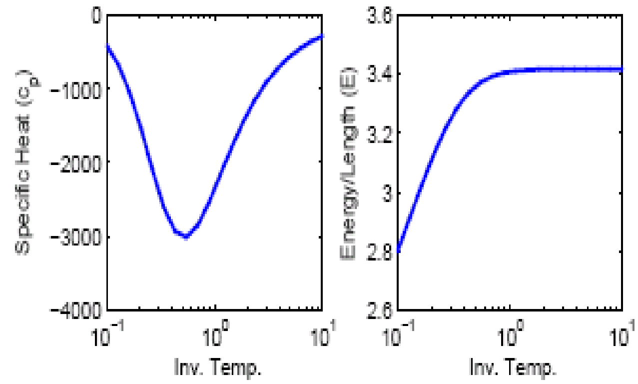

FIG. 1: The specific heat at constant pressure (Equation 22) for the thermally isolated system is negative, meaning that the constant pressure Enthalpy Per Length (Equation 20) decreases with increasing temperature. (Here $\alpha' = 5 \times 10^5$ and $p' = 8 \times 10^4$.)

Trapped slender vortex filaments 5

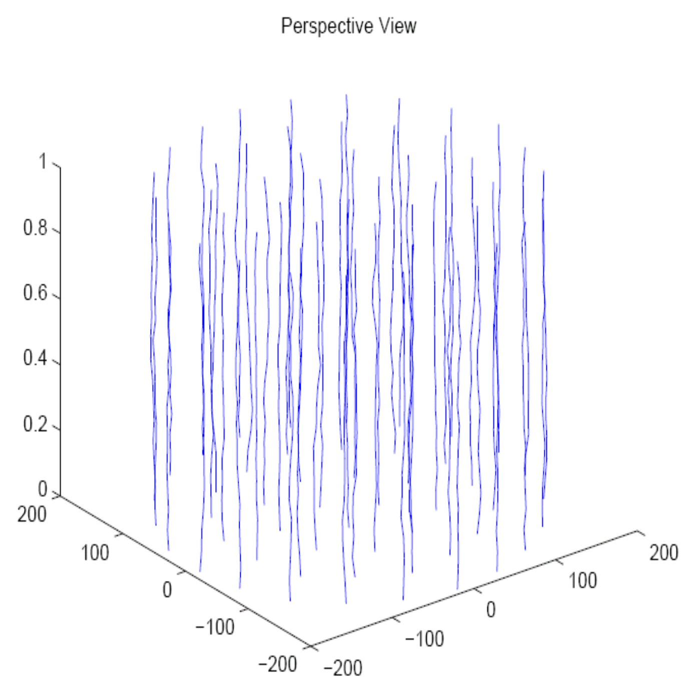

FIGURE 2. Nearly parallel vortex filaments are nearly parallel to the z-axis in an asymptotic sense that their deviation from straight is a small parameter. They are infinite in length but have a period L. Here there are 50 filaments.

Bundles of wriggly vortex filaments arise in many models [3] including cosmological strings [4] used in the study of topological defects in the early universe and their effects on cosmological phase transitions in the inflationary big bang theory. It is expected that the results in [1] and Andersen's thesis – on calculating entropic effects in vortex filaments - have deep implications for cosmological studies in addition to plasmas.

These accomplishments are announced in invited talks at several conferences including the ICIAM Zurich 2007, Yale Applied Math seminar 2006, GAMM Berlin 2006, the T. Jefferson Lab Virginia and Sandia National Lab 2007. They will form the strong foundations of a DOE proposal to be submitted in the fall 2007 by the PI.

[1] T. Andersen and C.C. Lim, Negative specific heat in a generalized vorticity model, Phys. Rev. Lett. Accepted 2007.

[2]L.Onsager, Statistical Hydrodynamics, Nuovo Cimento Suppl. 6 (1949) 279-289.

[3] P. Lions and A. Majda, Equilibrium statistical theory for nearly parallel vortex filaments, CPAM (2000), 76- 142.

[4] A. Vilenkin and E.P.S. Shellard, Cosmic Strings and Other Topological Defects, Cambridge U. Press, 1993.

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