Local Oscillator Requirements for Chip-Scale Atomic Clocks

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This document outlines some requirements for the performance of local oscillators (LOs) being developed for DARPA's program on Chip Scale Atomic Clocks (CSAC). It is not intended to be a complete analysis but rather should be considered a starting point for discussion of this issue. As the CSAC program progresses, it is expected that the integration of moderately-stable compact local oscillators with physics packages will be examined in substantially more detail, particularly with regard to new devices never used before in atomic clocks. The LO requirements may change in light of this further examination. In addition, while I have attempted to keep the analysis fairly general, it almost certainly reflects some bias toward the NIST physics package design and other organizations may have different requirements.

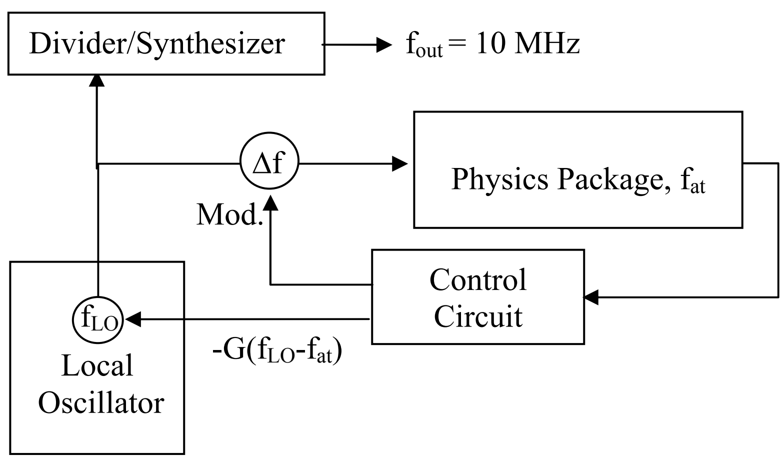

Figure 1. Schematic of the basic components of a passive frequency reference.

Passive frequency standards, such as the vapor-cell atomic clocks being considered to meet the DARPA program requirements, require local oscillators to interrogate the atoms and excite the atomic "clock transition" on which the frequency standard is based. A typical frequency reference geometry is shown in Figure 1. The reference is comprised of three main components: a physics package, a local oscillator and some control circuitry. The physics package takes as it's input, a microwave frequency compatible with the atomic transition frequency (9.2 GHz for Cs, 6.8 GHz for Rb^{87} and 3.0 GHz for Rb^{85}, or a sub-harmonic of these frequencies, depending on the design). The output of the physics package (after lock-in demodulation) is a voltage proportional to the frequency difference between the local oscillator and the atomic transition. This output voltage is used by the control system to correct the LO frequency and therefore to stabilize the output frequency of the frequency reference. If a high-frequency LO is used, a frequency divider, perhaps in combination with a synthesizer, might be required to generate the 10 MHz output needed by most applications.

Power

Several groups within the CSAC program intend on using the coherent population trapping (CPT) design with a modulated diode laser. With this method, the LO must be able to modulate the diode laser with sufficient modulation index to put a large fraction of the optical power into the first-order (or higher-order) optical sidebands. In initial experiments at NIST using vertical-cavity surface-emitting lasers (VCSELs), approximately 10 mW to 20 mW of RF power at 4.6 GHz was required to put roughly 60% of the optical power into the two first-order sidebands, although the coupling of the RF power into the laser was not optimized. It is anticipated that considerably less power (a few mW or less, perhaps) will be required for a laser optimally mounted and with an appropriate impedance-matching circuit. However, further testing will be required before this requirement can be clearly specified.

Tunability

The LO must produce a frequency compatible with the atomic transition. This transition frequency depends on which alkali atoms are being used for the frequency reference. It also depends on the exact pressure of the buffer gas (and/or composition of the wall coating) contained inside the cell with the alkali atoms and so probably cannot be defined precisely until after the cell is fabricated. Nevertheless, this frequency will very likely be a few tens of kHz above the frequency of a free atom: the free-atom frequencies for Rb^{87}, Rb^{85} and Cs are, 6,834,682,612 Hz, 3,035,732,440 Hz and 9,192,631,770 Hz respectively. In the NIST design, the frequency entering the physics package must be exactly equal to one-half of the buffer-gas-shifted frequency. Two possibilities to accomplish this would be a) to design the LO to have the capability to be coarsely tuned itself to a specific, predetermined frequency or b) combine the LO with a direct digital synthesis (DDS) chip to achieve the appropriate tuning.

The local oscillator (or the LO/DDS combination) should also have the capability to be finely tuned with an external voltage in order to lock its frequency to the atomic transition. The tuning range must be large enough that any LO frequency changes caused by environment or aging over the lifetime of the frequency reference can be compensated for by changing the tuning voltage. The tunability should also not be too large, since then electronic noise from the control circuitry will create added noise on the output frequency. One option would be to have a coarse mechanical tuning capability that gets the LO close to the atomic resonance (within 10 kHz, say) and then provide the remaining tunability electronically.

Modulation of the frequency entering the physics package is also a requirement in order to generate the error signal necessary to lock the LO to the atomic resonance. Modulation frequencies are typically in the range of 100 Hz – 10 kHz, with a modulation index near unity. However, to avoid having this modulation frequency present on the output signal, it could be done externally, as shown in Figure 1. In this case the LO itself would not need to be modulated.

Short-Term Frequency Stability

The frequency stability of the local oscillator on short time scales is perhaps the most critical requirement. Normally, the LO is more stable than the atoms over short times but requires correcting at longer times because of 1/f frequency noise and frequency drifts associated with environmental changes, such as temperature, pressure, acceleration, and aging. A servo-system is used to correct the LO frequency and stabilize it to the atomic frequency. It is clear that the requirements on the stability of the LO will depend rather significantly on the design of the control circuit servo. We will assume for simplicity here that a simple integrator is used, providing high gain for corrections at low frequencies. However, if servos incorporating additional elements such as a proportional channel, derivative channel or second-stage integrator are used, the requirements on the LO would be expected to change. A circuit diagram of a typical integrating servo and its gain response curve is shown in Figure 2.

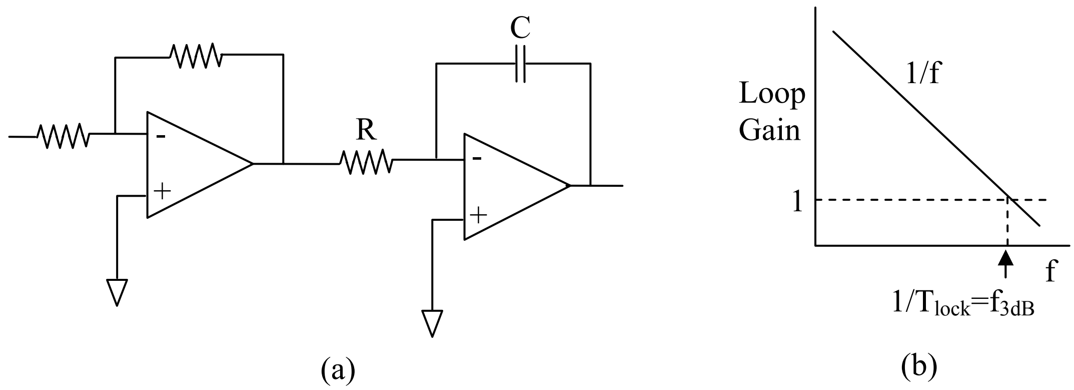

Figure 2. (a) Circuit diagram of a typical integrating servo control loop. The first stage provides overall gain while the second determines the time constant (τ=RC) of the integration. (b) The loop gain as a function of frequency. The unity gain point is determined not only by the control loop parameters but also by the input signal size and VCO tuning sensitivity.

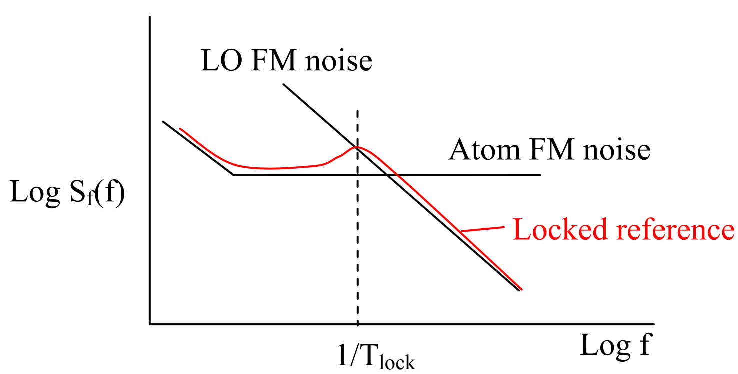

Figure 3. Frequency noise spectra typical of the atomic system, the free-running local oscillator and the final, locked local oscillator.

If the LO is locked to the atomic resonance with a servo with attack time $T_{lock}=1/f_{3dB}$, the FM noise of the LO at low-frequencies (i.e. frequencies < $1/T_{lock}$) will be controlled by the atoms while the higher-frequency FM noise will not. The higher-frequency noise will therefore follow the intrinsic noise of the LO. As a result, the frequency noise spectrum of the final frequency reference, as compared to the frequency noise of the atoms and LO individually, will look something like that in Figure 3. It is assumed in this analysis that the equivalent FM noise of the atomic system is white (at least down to mHz frequencies) while the local oscillator FM noise behaves as 1/f or $1/f^2$ at low frequencies. Hence the LO FM noise exceeds the atom FM noise in some low-frequency range.

In this low-frequency range, an integrating servo reduces the LO FM noise spectral density by a factor proportional to f^2, to a minimum determined by the atom FM noise spectral density if the loop gain is sufficient. This reduction can be described analytically as:

$$S_f^{out} = \frac{1}{(1+G)^2} S_f^{LO} + \frac{G^2}{(1+G)^2} S_f^{at} = \frac{f^2 T_{lock}^2}{(1+fT_{lock})^2} S_f^{LO} + \frac{1}{(1+fT_{lock})^2} S_f^{at} \qquad (1)$$

where $S_f^{out}(f)$ is the FM spectral density of the final, locked frequency reference output, $S_f^{LO}(f)$ is the free-running local oscillator frequency noise and $S_f^{at}(f)$ is the atomic frequency noise. If $S_f^{LO}(f) \propto 1/f^{\alpha}$ with $\alpha<2$, the LO FM noise will eventually be brought down to the level of the atomic FM noise at low enough frequencies, as shown in Figure 2. If however, $\alpha \geq 2$, then a simple, integrating servo will be unable to reduce the LO FM noise down to the atom FM noise level if the inverse of the lock time is below the frequency at which the LO FM noise and the atom FM noise are equal. This case is shown in Figure 3 with $\alpha=2$.

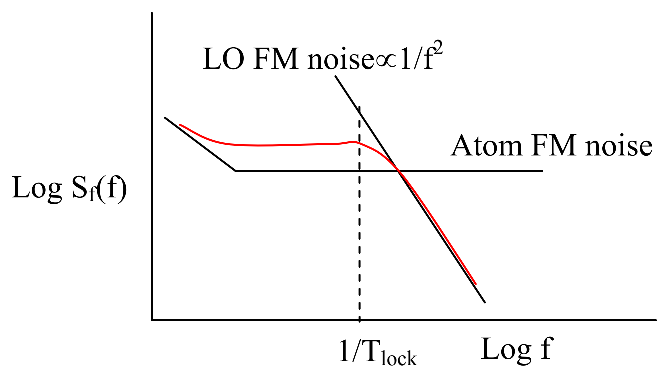

Figure 4. Stabilization of the free-running LO FM noise with an integrating servo for α=2. The FM noise spectral density of the locked LO is shown in red.

<u>Time-Domain Stability</u>

The FM noise can be translated into a time-domain measure of the frequency stability, the Allan variance, $\sigma_y^2(\tau)$, using the following formula[1]:

$$\sigma_y^2(\tau) = \frac{2}{\left(\pi \nu_0^2 \tau\right)^2} \int_0^{\infty} S_f^{out}(f) \sin^4(\pi f \tau) df = \frac{2}{\left(\pi \nu_0^2 \tau\right)^2} \int_0^{\infty} S_{\phi}^{out}(f) f^2 \sin^4(\pi f \tau) df \qquad (2)$$

where $S_\phi^{out}(f)$ is the double-sided phase noise spectrum of the locked LO output and ν_0 is oscillation frequency of the LO. A typical Allan deviation is shown in Figure 5, where the stability is plotted as a function of the integration time, τ. The final stability of the locked LO is shown in red.

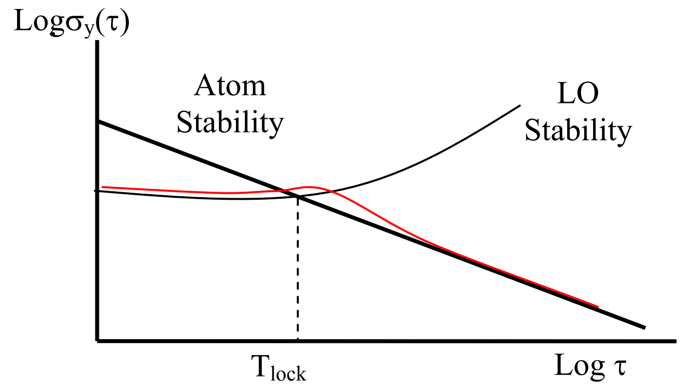

Figure 5. The stability of the LO compared with the stability of the atomic resonance, as measured by the Allan deviation. The stability of the locked LO is shown in red.

Requirement A:
One requirement that may be considered for the LO is that it does not degrade the intrinsic stability of the atoms. This requirement is more stringent than is needed to satisfy the DARPA CSAC program goals but is useful in the sense that the overall stability is then optimized for a given physics package stability. For this to be the case, we require

$$\sigma_y^{LO}(\tau) < \sigma_y^{at}(\tau) \quad \forall \tau . \qquad (3)$$

The atom stability, $\sigma_y^{at}(\tau)$, is in turn determined by the atomic vapor cell geometry and ultimately by the DARPA stability goal of 10^{-11} at one hour of integration. Numerical integration of Eq. (2) for 1/f FM noise gives the result that the condition on the LO phase noise corresponding to Eq. (3) is[2] $S_\phi^{LO}(1/T_{lock}) < S_\phi^{at}(1/T_{lock}) = 2T_{lock}^2 \nu_0^2 {\sigma_y^{at}}^2 \tau$. Analyses for other noise types (white FM, $1/f^2$ FM) give similar results. The requirement on the LO is therefore $S_\phi^{LO}(100Hz) < -25\ dBc/Hz$, assuming T_{lock}=10 ms. From Eq. (4), it is clear that short lock times relax the constraint on the LO stability requirement (see Figure 5). However the lock time cannot be shorter than the inverse of width of the atomic resonance, which will most likely be of the order of a few kHz for CSACs.

Requirement B:
A considerably less stringent limit on the LO frequency instability can be obtained if the fractional frequency instability of the final frequency reference is required to satisfy only the DARPA program goal of 10^{-11} at one hour of integration. In this case it is possible to allow the atom stability to be degraded substantially at integration times below one hour,

as long as the instability eventually drops below 10^{-11} at one hour of integration. This situation is shown schematically in Figure 6.

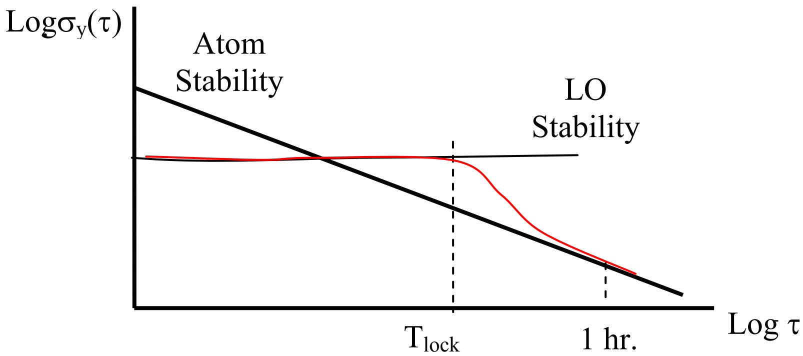

Figure 6. The Allan deviation of the locked LO in the case that the free-running LO stability is significantly worse than the atom stability at the lock time. A degradation of the stability of the frequency reference compared to the atom stability is observed at intermediate integration times, but the atom stability is eventually reached at long integration times. The Allan deviation of the locked LO is shown in red.

It is found through numerical integration of Eq. (2), that the way in which the locked frequency reference stability approaches the atom stability at integration times larger than T_{lock} depends on the character of the LO noise and the method of locking. We assume again a simple integrator servo. If the LO FM noise is 1/f in character (phase noise $\propto 1/f^3$) the Allan deviation for $\tau > T_{lock}$ will be proportional to $1/\tau$ (as shown in Figure 6) and the frequency reference stability will eventually approach the atom stability at long integration times. In this case the requirement on the LO stability to reach the DARPA goal is roughly $S_f^{LO}(100Hz) < +23\ dBc/Hz$. If the character of the LO FM noise is instead $1/f^2$, which is not uncommon in LOs at very low Fourier frequencies, a simple integrator will cause the Allan deviation to drop as $1/\sqrt{\tau}$ for $\tau > T_{lock}$. Hence the Allan deviation will never catch up with the atom stability if the LO stability is worse than the atom stability at the $\tau = T_{lock}$. The requirement on the LO phase noise in this case is therefore substantially more stringent at $S_f^{LO}(100Hz) < -36\ dBc/Hz$. It should be noted, however, that the use of additional integrator stages could relax this requirement.

Requirement C:

The phase noise of the LO at Fourier frequencies equal to multiples of the LO modulation frequency can also affect the stability of the locked LO at long integration times[3]. A rough rule of thumb given in Ref. [3] is

$$\sigma_y^{mult}(\tau) = \sqrt{\frac{S_f^{LO}(2f_m)}{4\tau\nu_0^2}}$$

where $\sigma_y^{mult}(\tau)$ is the lower limit on the locked LO Allan deviation and f_m is the frequency at which the LO is modulated to produce the error signal for the servo. In order to obtain the DARPA fractional frequency stability goal of 10^{-11} at one hour of integration time, $S_\phi^{LO}(2f_m) < 16\ Hz/f_m^2$. This limit corresponds to – 48 dBc/Hz at 2 kHz offset (for $f_m = 1$ kHz) or -28 dBc/Hz at 200 Hz offset (for $f_m = 100$ Hz). This requirement is very close to that of Requirement A, but may be easier to meet since the LO modulation frequency will always be higher than the servo unity gain frequency and the LO noise tends to improve at high Fourier frequencies. Filtering and modulation techniques can also be used to reduce this effect[4], although the extent to which this can be done in a miniature package with modest power remains quantitatively unclear.

Summary of LO stability requirements:
It is assumed here that single-stage integrator servo is used to lock the LO to the atoms with unity gain at $1/T_{lock}$.

If $S_\phi^{LO}(f) = \frac{A}{f^2} + \frac{B}{f^3} + \frac{C}{f^4}$, **(4)**

then Requirement A above implies

$$A < 33\ Hz; \quad B < \frac{66\ Hz}{T_{lock}}; \quad C < \frac{66 Hz}{T_{lock}^2}, \qquad \textbf{(5)}$$

and Requirement B implies

$$A < \frac{1.2\times10^5}{T_{lock}}; \quad B < \frac{4.3\times10^4}{T_{lock}^2}; \quad C < \frac{66\ Hz}{T_{lock}^2}. \qquad \textbf{(6)}$$

Requirement C implies
$S_\phi^{LO}(2f_m) < 16\ Hz / f_m^2$.

If the goal is simply to achieve the DARPA goal of a fractional frequency stability of 10^{-11} at 3600 s, then Requirement A does not apply. Requirement C is more stringent than Requirement B for white FM noise and 1/f FM noise. Requirements B and C together are summarized in the graph in Figure 7. The most difficult goal to meet is probably Requirement C of -48 dBc/Hz at 2 kHz offset for both $1/f^2$ and $1/f^3$ phase noise components. In terms of an Allan deviation, the requirements are shown in Figure 8.

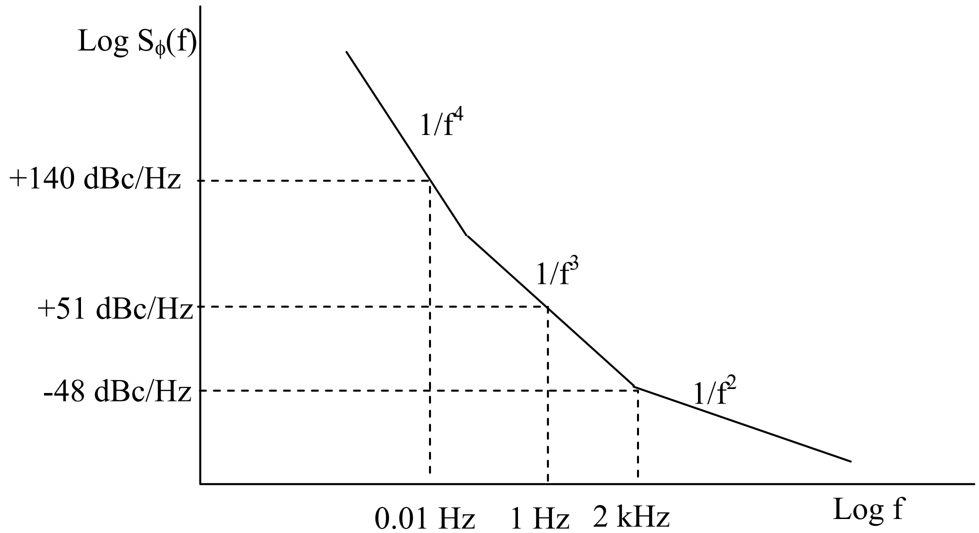

Figure 7. Phase noise of an LO (at 6.8 GHz) that is likely (based on this analysis) to meet the CSAC requirement of a fractional frequency instability of 10^{-11} at one hour of integration when locked to the atomic resonance, assuming a simple integrator servo with T_{lock} = 10 ms and f_m = 1 kHz.

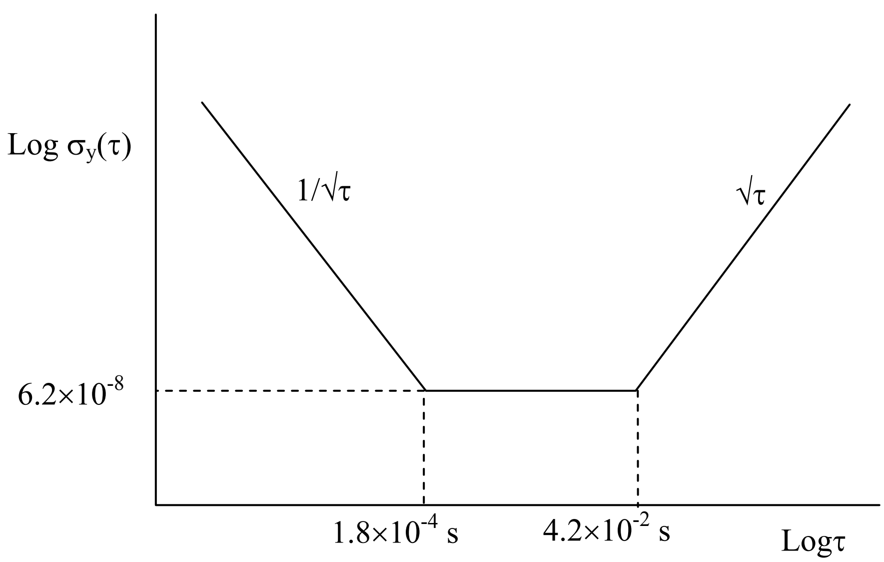

Figure 8. LO stability requirements in Figure 7 as expressed by the Allan deviation, assuming a simple integrator servo with T_{lock} = 10 ms and f_m = 1 kHz.

LO Drift

The frequency drift of the free-running LO over longer time periods can cause difficulties in maintaining the servo lock and also cause shifts in the frequency of the locked LO. One important constraint for the LO drift comes from the simple need that the LO cannot come out of lock. This means that the LO cannot drift by more than roughly one atomic resonance linewidth during the time T_{lock}. For a resonance linewidth of 1 kHz and a lock time of 10 ms, this puts a limit of $(\Delta f/f)/\Delta t < 10^{-5}$/s on the LO drift. A more stringent requirement is based on the frequency offset a drifting LO produces in the final, locked output frequency. If an integrating servo is used to lock up the LO, the frequency offset will be given in terms of the drift rate as

$$\Delta f = \frac{df}{dt} T_{lock} \tag{7}$$

where *df/dt* is the drift rate of the LO. The maximum drift rate can therefore be set at

$$\frac{df}{dt} \le \frac{\Delta f}{T_{lock}}. \tag{8}$$

For a long-term stability of 10^{-11}, as specified by the DARPA requirements, and a lock time of 10 ms, this results in a limit on the drift of 10^{-9}/sec. If temperature is causing the drift, and the sensitivity of the LO frequency to temperature changes is 1 ppm/C, the temperature drift would need to be below 1 mK/s or about 1 K/h, which does not seem unreasonable. The addition of more complicated servo elements (such as a second integration stage, for example) would relax this requirement further.

<u>Other LO-related issues not addressed here but of some importance to a variety of applications:</u>

1. Thermal transient effects
2. Shock
3. Resonator Q-factor in relation to the phase noise requirements presented above.
4. Experimental verification of the limits outlined above.
5. LO modulation implementation
6. Divider/synthesizer stage to generate the desired output frequency

[1] J. A. Barnes, et al., IEEE Trans. Instrum. Meas., **IM-20**, 105-120, 1971.
[2] See, for example, P. Lesage and C. Audoin, Radio Science, **14**, 521-39, 1979.
[3] C, Audoin, V. Candelier and N. Dimarq, IEEE Trans. Instrum. Meas., **40**, 121-5, 1991.
[4] R. Barillet, F. Hamouda, D. Venot and C. Audoin, IEEE Trans. Ultra. Ferr. Freq. Control, **47**, 1152-8, 2000.