Neutrino-Mixing-Generated Lepton Asymmetry and the Primordial ^{4}He Abundance

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It has been proposed that an asymmetry in the electron neutrino sector may be generated by resonant active-sterile neutrino transformations during Big Bang Nucleosynthesis (BBN). We calculate the change in the primordial ^{4}He yield Y resulting from this asymmetry, taking into account both the time evolution of the ν_e and $\bar{\nu}_e$ distribution function and the spectral distortions in these. We calculate this change in two schemes: (1) a lepton asymmetry directly generated by ν_e mixing with a lighter right-handed sterile neutrino ν_s; and (2) a lepton asymmetry generated by a $\nu_\tau \leftrightarrow \nu_s$ or $\nu_\mu \leftrightarrow \nu_s$ transformation which is subsequently partially converted to an asymmetry in the $\nu_e\bar{\nu}_e$ sector by a matter-enhanced active-active neutrino transformation. In the first scheme, we find that the percentage change in Y is between -1% and 9% (with the sign depending on the sign of the asymmetry), bounded by the Majorana mass limit $m_{\nu_e} \lesssim 1$ eV. In the second scheme, the maximal percentage reduction in Y is 2%, if the lepton number asymmetry in neutrinos is positive; Otherwise, the percentage increase in Y is $\lesssim 5\%$ for $m^2_{\nu_\mu,\nu_\tau} - m^2_{\nu_s} \lesssim 10^4$ eV. We conclude that the change in the primordial ^{4}He yield induced by a neutrino-mixing-generated lepton number asymmetry can be substantial in the upward direction, but limited in the downward direction.

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I. INTRODUCTION

It has been known that a resonant active-sterile neutrino transformation during the Big Bang Nucleosynthesis (BBN) epoch can generate lepton number asymmetries in the active neutrino sectors [1–4]. The generated lepton number asymmetry L_{ν_α} (with ν_α being any of the three active neutrino species) has an order of magnitude

$$L_{\nu_\alpha} \equiv \frac{n_{\nu_\alpha} - n_{\bar{\nu}_\alpha}}{n_\gamma} \sim \pm \frac{|\delta m^2/\mathrm{eV}^2|}{10\,(T/\mathrm{MeV})^4} \quad \text{for } |L_{\nu_\alpha}| \ll 0.1. \tag{1}$$

In this equation, n is the particle proper number density, $\delta m^2 \equiv m^2_{\nu_s} - m^2_{\nu_\alpha}$, and T is the temperature of the universe.

The asymmetry may have an appreciable impact on the the primordial ^{4}He abundance Y if it is in the ν_e sector and if its magnitude at the weak freeze-out temperature $T \sim 1$ MeV is $|L_{\nu_e}| \gtrsim 0.01$. The resulting change of the primordial ^{4}He abundance, ΔY, however, is not easy to estimate. Not only is the generated asymmetry a function of time, but also the ν_e or $\bar{\nu}_e$ energy spectrum is distorted by mixing (in a time-dependent fashion as well). Therefore, previous attempts [5] to estimate ΔY in this lepton number generation scenario, employing BBN calculations based on a constant asymmetry and a thermal neutrino spectrum for electron-type neutrinos, is overly simplistic and may yield inaccurate results.

In this paper, we discuss in detail the time evolution of L_{ν_e} and the distortion in the ν_e or $\bar{\nu}_e$ spectrum in this neutrino-mixing-driven lepton number asymmetry generating scenario. We then calculate ΔY taking into account these time-dependent and energy-dependent effects by modifying the standard BBN code accordingly. We consider two schemes: a direct one and an indirect one.

The direct scheme involves a direct resonant $\nu_e \leftrightarrow \nu_s$ transformation which generates L_{ν_e} [1,2]. The indirect one first has a lepton asymmetry generated via a resonant $\nu_\mu \leftrightarrow \nu_s$ or $\nu_\tau \leftrightarrow \nu_s$ transformation and then has the asymmetry partially transferred into L_{ν_e} by an active-active $\nu_\mu \leftrightarrow \nu_e$ or $\nu_\tau \leftrightarrow \nu_e$ transformation [5]. In both cases, Eq. (1) indicates that the active-sterile channel requires $m^2_{\nu_\alpha} - m^2_{\nu_s} \gtrsim 0.1$ eV^2 to generate $|L_{\nu_\alpha}| \gtrsim 0.01$ at a temperature

$T \sim 1$ MeV. This implies $m_{\nu_\alpha} \gtrsim 0.3$ eV. In addition, the effective mixing angles associated with these neutrino oscillation channels have to be large enough to generate lepton number asymmetries efficiently, but not so large as to produce too many ν_s's before or during the onset of the process. The excess ν_s's may not only suppress the lepton number generation, but also yield a ^{4}He mass fraction that is too large [2,6]. These conditions can be quantified as:

$$|\delta m^2/\mathrm{eV}^2|^{1/6} \sin^2 2\theta \gtrsim 10^{-11}, \tag{2}$$

$$|\delta m^2/\mathrm{eV}^2| \sin^4 2\theta \lesssim 10^{-9}\,(10^{-7}) \quad \text{for } \nu_\alpha = \nu_e\,(\nu_\mu,\, \nu_\tau). \tag{3}$$

where θ is the vacuum mixing angle.

The change in the predicted primordial ^{4}He abundance can go either way, depending on whether L_{ν_e} is positive (decreasing Y) or negative (increasing Y). Our detailed BBN calculations show that in the direct scheme, we have $-0.002 \leq \Delta Y \leq 0.022$, bounded by the ν_e Majorana mass limit $m_{\nu_e} \lesssim 1$ eV. This possible change in Y is significant compared to the uncertainty involved in the Y measurements. (The measured primordial ^{4}He abundance from Olive *et al.* is $Y = 0.234 \pm 0.002\,(stat.) \pm 0.005\,(syst.)$ [7], while another group claims $Y = 0.244 \pm 0.002$ [8].) In fact, an increase in Y of magnitude $\gtrsim 0.01$ due to a large negative L_{ν_e} would already be inconsistent with observations. In the indirect scheme, we find that the maximal possible reduction in Y is 0.005. The expected increase of Y goes up with $m^2_{\nu_\mu,\nu_\tau} - m^2_{\nu_s}$, and can be as high as 0.013 for $m^2_{\nu_\mu,\nu_\tau} - m^2_{\nu_s} \sim 10^4$ eV^2. In both schemes, ΔY is rather limited in the negative direction. The possible reduction in Y, resulting from a positive L_{ν_e}, may to some degree narrow the gap between the lower Y measurement [7] and the standard BBN prediction ($Y = 0.246 \pm 0.001$ [9] assuming a primordial deuterium abundance D/H$\approx 3.4 \pm 0.3 \times 10^{-5}$ [10]). We note, however, that the maximal reduction of $\Delta Y \approx -0.005$ is achieved only in the indirect scheme when $m^2_{\nu_\mu,\nu_\tau} - m^2_{\nu_s} \sim 100$ to 300 eV^2, implying an unstable ν_μ or ν_τ with $m_{\nu_\mu,\nu_\tau} \gtrsim 15$ eV [4].

II. GENERATION OF LEPTON ASYMMETRY BY RESONANT ACTIVE-STERILE NEUTRINO TRANSFORMATION

The formalism of active-sterile neutrino transformation and associated amplification of lepton asymmetry when $\delta m^2 \equiv m^2_{\nu_s} - m^2_{\nu_\alpha} < 0$ has been discussed extensively elsewhere [1–3,6,11]. Here we summarize the main conclusions of these papers, and then concentrate our discussion on the final stage of the amplification process, when T approaches the weak freeze-out temperature $\sim$1 MeV.

For $\nu_\alpha \leftrightarrow \nu_s$ transformation with $\delta m^2 < 0$, a resonance occurs at

$$T_{\rm res} \approx T_0 \left(\frac{E}{T}\right)^{-1/3} \left|\frac{\delta m^2 \cos 2\theta}{1{\rm eV}^2}\right|^{1/6}, \tag{4}$$

where $T_0 \approx 19(22)$ MeV for $\alpha = e\,(\mu,\tau)$, and E/T is the neutrino energy normalized by the ambient temperature. Below $T_{\rm res}$, lepton asymmetry may be amplified to asymptotically approach one of the two values in eq. (1). Before this asymptotic value is reached, however, there is a brief "chaotic"phase in which L_{ν_α} oscillates around zero [2]. As a result, the sign of L_{ν_α} that emerges from the chaotic phase is unpredictable. We should point out that the detailed numerical evolution of the generated lepton number remains controversial. For example, whether or not the evolution of the lepton number represents true chaos is not precisely known. However, our BBN arguments simply reply on the sensitivity of the generated lepton number to the neutrino oscillation parameters. This sensitivity leads to a causality consideration on the sign of the generated lepton number. A discussion of the causality consideration can be found in Ref. [12].

It is not surprising that at these two asymptotic L_{ν_α} values, either $\nu_\alpha \leftrightarrow \nu_s$ (if $L_{\nu_\alpha} < 0$) or $\bar\nu_\alpha \leftrightarrow \bar\nu_s$ (if $L_{\nu_\alpha} > 0$) undergoes resonant transition due to matter effects. The system maintains the growth of L_{ν_α} by converting one of $\nu_\alpha/\bar\nu_\alpha$ resonantly but suppressing the transformation of the other. Of course, not all ν_α or $\bar\nu_\alpha$ undergo the resonant transformation, because neutrinos in the early universe have an energy distribution and the resonance condition is energy dependent. When L_{ν_α} is small, even the resonant conversion of a small fraction of either ν_α or $\bar{\nu}_\alpha$ with energy $E_{\rm res}$ will be enough to maintain the growth of L_{ν_α}. To quantify the above arguments, we note that the effective potential $\mathbf{V}= (V_x, V_y, V_z)$ of the $\nu_\alpha \leftrightarrow \nu_s$ transformation channel is

$$V_x = -\frac{\delta m^2}{2E}\sin 2\theta, \quad V_y = 0, \quad V_z = -\frac{\delta m^2}{2E}\cos 2\theta + V_\alpha^L + V_\alpha^T. \tag{5}$$

The contribution from matter-antimatter asymmetries (matter effect) is [13]

$$V_\alpha^L \approx \pm 0.35 G_F T^3 \Big[L_0 + 2L_{\nu_\alpha} + \sum_{\nu_\beta \neq \nu_\alpha} L_{\nu_\beta}\Big], \tag{6}$$

where G_F is the Fermi constant, and $L_0 \sim 10^{-9}$ represents the contributions from the baryonic asymmetry as well as the asymmetry in electron-positions. The "+" sign is for the neutrino oscillation channel, and the "−" sign is for the anti-neutrino oscillation channel. The contribution from the thermal neutrino background is V_α^T, whose value is [13]

$$V_\alpha^T \approx -A\frac{n_{\nu_\alpha} + n_{\bar{\nu}_\alpha}}{n_\gamma} G_F^2 E T^4, \tag{7}$$

where $A \approx 110(30)$ for $\alpha = e\,(\mu \text{ or } \tau)$. The effective matter mixing angle at temperature T is

$$\sin 2\theta_{\rm eff} = \frac{V_x}{(V_x^2 + V_z^2)^{1/2}}, \tag{8}$$

which reduces to vacuum mixing when V_α^L and V_α^T are zero.

Several physical processes with different times scales are involved in the resonant $\nu_\alpha \leftrightarrow \nu_s$ transformation process: (1) the local neutrino oscillation rate $|\mathbf{V}|$; (2) the weak interaction rate $\Gamma \sim 4G_F^2 T^5$; (3) the Hubble expansion rate $H = -\dot{T}/T = 5.5T^2/m_{\rm pl}$ where $m_{\rm pl} \approx 1.22 \times 10^{28}$ eV is the Planck mass; (4) the rate of change of $|\mathbf{V}|$, $|\dot{\mathbf{V}}|/|\mathbf{V}|$, caused by the change of lepton asymmetry and the Hubble expansion. If any one of the rates is much larger than the others, we may consider all the other processes as perturbations, which simplifies the picture greatly. For example, if $|\mathbf{V}|$ dominates we may consider the system as an ordinary neutrino transformation system with an effective mixing angle as in Eq. (8), and with all physical variables changing adiabatically. If the weak interaction dominates, each weak scattering acts as a "measurement" to the mixing system, effectively reducing the mixture to flavor eigenstates ν_α and ν_s. The neutrino transformation is hence suppressed, with a reduced $\sin 2\theta_{\rm eff} = V_x/(\Gamma/2)$. If the Hubble expansion dominates, the other processes are essentially "frozen out". This is the case at $T \lesssim 1$ MeV when the two-body weak interaction freezes out and the neutron-to-proton ratio becomes fixed (other than from the free neutron decay process). If $|\dot{\mathbf{V}}|$ dominates over $|\mathbf{V}|$, neutrino amplitude evolution becomes non-adiabatic.

The ratios of the first three rates are:

$$\left|\frac{V_x}{\Gamma}\right| \approx 5 \times 10^8 \left(\frac{|\delta m^2|}{\rm eV^2}\right) \left(\frac{T}{\rm MeV}\right)^{-6} \sin 2\theta; \tag{9}$$

$$\left|\frac{V_x}{H}\right| \approx 10^9 \left(\frac{|\delta m^2|}{\rm eV^2}\right) \left(\frac{T}{\rm MeV}\right)^{-3} \sin 2\theta; \tag{10}$$

$$\frac{\Gamma}{H} \approx \left(\frac{T}{\rm MeV}\right)^3. \tag{11}$$

In these ratios, an average $E = 3.15T$ is assumed. Since we are only concerned with $|\delta m^2| \gtrsim 0.1$ eV^2, as long as $\sin 2\theta \gtrsim 10^{-6}$ (the minimal mixing required to amplify L_{ν_α}, see Eq. (2)), the neutrino transformation rate easily dominates over the Hubble expansion and the weak interaction at $T \sim 1$ MeV.

We always have $\dot{V}_x = HV_x$. At $T \lesssim T_{\rm res}/2$ and away from resonances so that we can assume $V_\alpha^T \ll |\delta m^2|/2E \sim V_\alpha^L \sim V_z$, we have $|\dot{V}_z| \sim |(H - \dot{L}_{\nu_\alpha}/L_{\nu_\alpha})V_z| \sim |HV_z|$. (Note that L_{ν_α} is limited to the T^{-4} growth in Eq. (1).) Therefore at $T \lesssim T_{\rm res}/2$, the active-sterile neutrino transformation channels in our problem can be treated as ordinary oscillation channels with adiabatically varying mixing parameters except possibly at the resonances. We will discuss the question of adiabaticity at resonances later.

At $T \lesssim T_{\rm res}/2$ when we can neglect V_α^T, the resonance condition $V_z = -\delta m^2/2E \cos 2\theta + V_\alpha^L = 0$ gives the asymptotic values of L_{ν_α} in Eq. (1). Note that for $L_{\nu_\alpha} < 0$ ($L_{\nu_\alpha} > 0$) the ν_α ($\bar{\nu}_\alpha$) transformation channel is matter-enhanced. The fraction F of resonantly converted ν_α ($\bar{\nu}_\alpha$) in the total ν_α ($\bar{\nu}_\alpha$) distribution is

$$F \sim 2 \left| V_x \frac{\mathrm{d}\epsilon}{\mathrm{d}V_z} \right|_{\epsilon_{\rm res}} f(\epsilon_{\rm res}) \approx 2\epsilon_{\rm res} \sin 2\theta f(\epsilon_{\rm res}) \tag{12}$$

where $\epsilon \equiv E/T$, $f(\epsilon) \approx 2\epsilon^2/3\zeta(3)(1+e^\epsilon)$ and $\epsilon_{\rm res}$ satisfies $V_z(\epsilon_{\rm res}) = 0$. The resonance $\epsilon_{\rm res}$ remains stationary (or varies very slowly) and $L_{\nu_\alpha} \propto T^{-4}$ if

$$|L_{\nu_\alpha}| \lesssim \frac{3}{8} F \sim \frac{3}{4} \epsilon_{\rm res} \sin 2\theta f(\epsilon_{\rm res}) \sim 0.1 \sin 2\theta. \tag{13}$$

In general, the stationary $\epsilon_{\rm res}$ has a value $\sim \mathcal{O}(0.1)$ [3,4]. When $|L_{\nu_\alpha}| > 0.1 \sin 2\theta$, $|L_{\nu_\alpha}|$ cannot keep up with the T^{-4} growth. Consequently $\epsilon_{\rm res}$ must increase as $\epsilon_{\rm res} \propto T^{-4}|L_{\nu_\alpha}|^{-1}$ at low temperatures when V^T can be safely neglected.

The resonance is adiabatic when the resonance region is characterized by slowly changing $\epsilon_{\rm res}$. As the resonance sweeps through the neutrino energy spectrum, a complete conversion of ν_α to ν_s at the resonance is possible only if the following adiabatic conditions are met:

$$\left| \frac{V_x^2}{\dot{V}_z'} \right| \sim \left| \frac{V_x \sin 2\theta'}{H} \right| \sim 10^9 \left(\frac{|\delta m^2|}{\mathrm{eV}^2} \right) \left(\frac{T}{\mathrm{MeV}} \right)^{-3} \sin^2 2\theta \gg 1, \tag{14}$$

and

$$2 \left| V_x \frac{\mathrm{d}\epsilon}{\mathrm{d}V_z} \right|_{\epsilon_{\rm res}} f(\epsilon_{\rm res}) \left| \frac{\mathrm{d}\epsilon_{\rm res}}{\mathrm{d}L_{\nu_\alpha}} \right| \gtrsim V_x \left| \frac{\mathrm{d}\epsilon}{\mathrm{d}V_z} \right|_{\epsilon_{\rm res}}. \tag{15}$$

The first condition simply implies that the timescale of completing the resonance has to be much longer than the neutrino oscillation period at resonance. This is satisfied if $|\delta m^2| \sin^2 2\theta \gtrsim 10^{-9}$ eV^2. The second condition requires that $\epsilon_{\rm res}$ move slowly through the spectrum. This amounts to

$$L_{\nu_\alpha} \leq \frac{3}{4} \epsilon_{\rm res} f(\epsilon_{\rm res}), \tag{16}$$

which is satisfied for $\epsilon_{\rm res}$'s that cover the bulk of the neutrino energy spectrum. Adiabaticity at resonances is therefore a valid assumption for $\nu_\alpha \leftrightarrow \nu_s$ mixing with vacuum mixing angles which are not too small but which are still within the BBN bound Eq. (3).

Given adiabaticity, the growth of L_{ν_α} (assumed to be positive for simplicity and no loss of generality) as $\epsilon_{\rm res}$ sweeps through the neutrino energy spectrum can be easily estimated by solving

$$L_{\nu_\alpha}(T) \approx \frac{3}{8}\int_0^{\epsilon_{\rm res}(T)} \beta f(\epsilon) {\rm d}\epsilon \tag{17}$$

and the resonance condition $V_z(\epsilon_{\rm res}) = 0$ which at low temperatures is equivalent to

$$\epsilon_{\rm res}(T) \approx \frac{|\delta m^2/{\rm eV}^2|}{16(T/{\rm MeV})^4 L_{\nu_\alpha}}. \tag{18}$$

In Eq. (17) β takes account of the effect of collisions that redistribute energy among neutrinos.

We can identify two extreme cases. When the collisions are too inefficient to change the neutrino distribution at $\epsilon > \epsilon_{\rm res}$ (such as when $T \lesssim 1$ MeV), $\beta = 1$. In another limit, $\beta \approx 1 - 8L_{\nu_\alpha}/3$ which obtains when the collisions are highly efficient (such as when $T \gtrsim 1$ MeV) and neutrinos are always distributed thermally. Eqs. (17) and (18) give a solution in fair agreement with the results obtained by solving Eq. (18) of Foot and Volkas [5]. In Figure 1 we plot our results in terms of L_{ν_α} vs. $m^2_{\nu_\alpha} - m^2_{\nu_s}$ at various temperatures. From Figure 1 we can deduce a power law relation applicable to $|L_{\nu_\alpha}| \lesssim 0.1$,

$$|L_{\nu_\alpha}| \approx 0.05|\delta m^2/{\rm eV}^2|^{2/3}(T/{\rm MeV})^{-8/3}. \tag{19}$$

We note that this power law applies only in the stage when the resonance sweeps through the neutrino energy spectrum. When the resonance is stationary (Eq. (13)), the dependence is $|L_{\nu_\alpha}| \propto |\delta m^2|T^{-4}$ instead.

The asymmetry L_{ν_α} is generated as the resonance conversion region moves up through the neutrino energy spectrum. This suggests a distortion of the ν_α (or $\bar{\nu}_\alpha$) energy spectrum (see also Ref. [14]). Indeed, when $L_{\nu_\alpha} \lesssim 0.1$, most of the L_{ν_α} is generated at the lowest temperatures (Eq. [19]), when the neutrino scattering processes that tend to thermalize the neutrino spectrum are the most inefficient. The fact that the resonant transformation of ν_α to ν_s (or $\bar{\nu}_\alpha \to \bar{\nu}_s$) starts at lower neutrino energies only further deepens the inefficiency of neutrino re-thermalization, as neutrino interaction cross sections scale roughly linearly with neutrino energies. In Figure 2, we plot a semi-analytical calculation of the ν_α spectrum at $T = 1$ MeV for a $\nu_\alpha \to \nu_s$ resonant transformation (which generates a negative L_{ν_α}) with

$m^2_{\nu_\alpha} - m^2_{\nu_s} = 1$ eV^2. In the calculation, the thermalization process is approximated as a relaxation process (with a rate Γ) driving the system toward a thermal distribution. As a result of the inefficiency of this process, for cases with $|\delta m^2| \lesssim 1$ eV^2 (so that $L_{\nu_\alpha} \lesssim 0.1$ at $T \sim 1$ MeV), the ν_α neutrino spectrum at and below its thermal decoupling temperature $T \sim 1$ MeV can be well approximated by a thermal spectrum with a low energy cut-off. The ν_α deficit below the cut-off energy results in the L_{ν_α} asymmetry. In the mean time, $\bar{\nu}_\alpha$ is not subject to resonant transformation. Its spectrum is therefore not significantly changed, due to the inefficiency of neutrino pair production. (The opposite is true if $L_{\nu_\alpha} > 0$: the $\bar{\nu}_\alpha$ distribution will have its lower energy region truncated but the ν_α distribution will remain intact.)

III. DIRECT GENERATION OF ELECTRON-NEUTRINO ASYMMETRY BY RESONANT $\nu_E \leftrightarrow \nu_S$ TRANSFORMATIONS

If $\alpha = e$ (the direct scheme to generate an asymmetry in the $\nu_e\bar{\nu}_e$ sector), the ν_e or $\bar{\nu}_e$ spectral distortion will directly impact the neutron-to-proton ratio at the weak freeze-out temperature, and hence the ^{4}He yield. Figure 3 shows the changes in n$\leftrightarrow$p rates due to the neutrino spectral distortion in the case $m^2_{\nu_e} - m^2_{\nu_s} = 1$ eV^2. When $L_{\nu_e} > 0$ (i.e., a deficit of low energy $\bar{\nu}_e$), the major effect is an enhanced neutron decay rate at low temperatures due to the reduced Pauli-blocking of $\bar{\nu}_e$. For reaction p+$\bar{\nu}_e \rightarrow$n+e^+, the low energy deficit in $\bar{\nu}_e$ is of little significance because only $\bar{\nu}_e$ with $E > 1.9$ MeV can participate in the reaction. Conversely, its reverse reaction mostly generates $\bar{\nu}_e$ at the higher end of the energy spectrum. Its rate is therefore insensitive to the spectral distortion at the lower end. When $L_{\nu_e} > 0$ (a deficit of low energy ν_e), the rate for n+$\nu_e \rightarrow$p+e^- is significantly reduced while the reverse rate is slightly increased. Figure 4 shows the resultant ΔY from the spectral distortion as a function of $m^2_{\nu_e} - m^2_{\nu_s}$ for both $L_{\nu_e} > 0$ and $L_{\nu_e} < 0$. The disparity between the two cases of opposite L_{ν_e} is transparent from Figure 3: the change in n$\leftrightarrow$p rates is much larger when $L_{\nu_e} < 0$.

The ν_e Majorana mass limit (which is uncertain by a factor of 2, ranging from $m_{\nu_e} \leq 0.45$ eV to $m_{\nu_e} \lesssim 1$ eV [15–17]) implies an upper limit $m^2_{\nu_e} - m^2_{\nu_s} \leq 1$ eV^2. [1] Figure 4 shows that the maximally allowed reduction in Y is only ≈ 0.0021, about 1% of the standard prediction. But the maximally allowed increase of Y can be as high as ≈ 0.022, a 9% effect. An increase this large in the predicted primordial ^{4}He abundance would have already been too large to accommodate observations [7,8].

IV. INDIRECT GENERATION OF ELECTRON-NEUTRINO ASYMMETRY BY NEUTRINO TRANSFORMATIONS

For convenience in the indirect scheme, we assume that a L_{ν_τ} is first generated by a $\nu_\tau \leftrightarrow \nu_s$ transformation process. This asymmetry may then be transferred to L_{ν_e} by a resonant $\nu_\tau \leftrightarrow \nu_e$ oscillation [5]. (Ordinary oscillations without resonance cannot transfer the asymmetry efficiently.) In fact we are likely to have $\delta m^2_{(\tau s)} \equiv m^2_{\nu_s} - m^2_{\nu_\tau} \approx \delta m^2_{(\tau e)} \equiv m^2_{\nu_e} - m^2_{\nu_\tau}$ if $m_{\nu_\tau} \gg m_{\nu_e}, m_{\nu_s}$ (which will be the case in order to have an appreciable impact on the primordial ^{4}He abundance). Such a neutrino mass spectrum is consistent with a simultaneous solution of the solar neutrino problem [18] and the atmospheric neutrino problem [19]. For the moment, we will simply assume $\delta m^2_{(\tau s)} \approx \delta m^2_{(\tau e)}$.

The matter asymmetry contributions to the effective potentials of the two neutrino transformation channels become

$$V^L_{(\tau s)} \approx \pm 0.35 G_F T^3 (2L_{\nu_\tau} - L_{\nu_e}), \quad V^L_{(\tau e)} \approx \pm 0.35 G_F T^3 (L_{\nu_\tau} - L_{\nu_e}). \tag{20}$$

Apparently, the $\bar{\nu}_\tau \leftrightarrow \bar{\nu}_s$ and $\bar{\nu}_\tau \leftrightarrow \bar{\nu}_e$ resonances when $L_{\nu_\tau} > 0$ (or the $\nu_\tau \leftrightarrow \nu_s$ and $\nu_\tau \leftrightarrow \nu_e$ resonances when $L_{\nu_\tau} < 0$) do not simultaneously share the same part of the neutrino energy spectrum.

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[1] We point out that it might be possible to construct mixings between left-handed electron neutrinos and *left-handed* sterile neutrinos. In such a case the Majorana mass limit will not apply but the standard weak interaction theory will then need to be reconsidered.

Guaranteed adiabaticity (i.e., satisfying Eq. [14]), the efficiency of resonant neutrino conversion is still determined by whether or not the neutrino collision time scale dominates over the timescale for a complete ν_τ to ν_e (or $\bar{\nu}_\tau$ to $\bar{\nu}_e$) transition at resonance (the resonance width). The collision timescale is important because the two resonances in Eq. (20) do not overlap, so for example, any deficit in $\bar{\nu}_\tau$ caused by the $\bar{\nu}_\tau \leftrightarrow \bar{\nu}_s$ resonant transition relies on neutrino scattering to redistribute neutrinos into the energy region where the $\bar{\nu}_\tau \leftrightarrow \bar{\nu}_e$ resonance occurs. In previous work [5] this redistribution has been assumed to be instant. However, this is not a good approximation at $T \lesssim 5$ MeV as we will show below.

The ratio of the resonance timescale to the collision timescale is

$$\left|\frac{V_x^{(\tau e)}}{\dot{V}_z^{(\tau e)}}\right| \Gamma \sim \frac{\Gamma}{H} \sin 2\theta_{(\tau e)} \sim \left(\frac{T}{\mathrm{MeV}}\right)^3 \sin 2\theta_{(\tau e)}. \tag{21}$$

Calculations based on active-active neutrino transformation in Type II supernova nucleosynthesis suggest that $\sin^2 2\theta_{(\tau e)} \lesssim 10^{-4}$ for $m_{\nu_\tau}^2 - m_{\nu_e}^2 \gtrsim 1$ eV^2 [20], and the Bugey experiment constrains $\sin^2 2\theta_{(\tau e)} \leq 0.04$ for $m_{\nu_\tau}^2 - m_{\nu_e}^2 \lesssim 1$ eV^2 [21]. Therefore, the collision timescale dictates the growth of L_{ν_e} at $T \lesssim 5$ MeV for $|\delta m^2_{(\tau s)}| \approx |\delta m^2_{(\tau e)}| \gtrsim 1$ eV^2.

Above 5 MeV, the collisions may be deemed instantaneous, and the equations of ref. [5] become valid. But a side effect of generating a large L_{ν_τ} above ~ 5 MeV by the $\nu_\tau \leftrightarrow \nu_s$ mixing (by having $m_{\nu_\tau}^2 - m_{\nu_s}^2 \gtrsim 10^4$ eV^2) is bringing sterile neutrinos into chemical equilibrium [6]. As a result, even though a reduction as large as $\Delta Y = -0.006$ may result from a positive L_{ν_e} alone in the indirect scheme [5], the extra sterile neutrinos produced will increase Y by at least as much by bringing $\bar{\nu}_s$ into chemical equilibrium through the $\bar{\nu}_\tau \leftrightarrow \bar{\nu}_s$ resonant transformation. The net effect is an increase of Y instead of a reduction, even if L_{ν_e} is positive.

We again model the transfer of L_{ν_τ} to L_{ν_e} by assuming that active neutrinos tend to their equilibrium distributions with a rate Γ. Figure 5 shows the growth of L_{ν_τ}, L_{ν_e} and the accompanying increase in the total neutrino energy density (including that of sterile neutrinos) in the units of that of one unperturbed active neutrino flavor, for one particular set of mixing parameters. Figure 5 shows that L_{ν_τ} indeed grows initially according to

Eq. (19), and tapers off at $L_{\nu_\tau} \approx 0.22$ when most of ν_τ or $\bar{\nu}_\tau$ have undergone resonances. The transfer of L_{ν_τ} to L_{ν_e} is efficient at $T \gtrsim 2$ MeV, but freezes out below ~ 2 MeV. It freezes out at a higher temperature than L_{ν_τ} because the ν_τ-ν_e (or $\bar{\nu}_\tau$-$\bar{\nu}_e$) resonance occurs at a higher energy than the ν_τ-ν_s (or $\bar{\nu}_\tau$-$\bar{\nu}_s$) resonance (Eq. [20]). The resonance therefore sweeps through the ν_e (or $\bar{\nu}_e$) spectrum faster. The figure also shows that increase in the total neutrino energy density in this case (about 2%, or $\Delta N_\nu \sim 0.07$) is moderate.

The spectra of ν_e and $\bar{\nu}_e$ in the indirect scheme is only slightly distorted. Figure 6 shows the modified $\bar{\nu}_e$ spectrum when L_{ν_τ}, $L_{\nu_e} > 0$ for $m^2_{\nu_\tau} - m^2_{\nu_e} = m^2_{\nu_\tau} - m^2_{\nu_s} = 100\ \mathrm{eV}^2$, compared to an unperturbed active neutrino spectrum with zero chemical potential. The distortion is small because the transfer of L_{ν_τ} into L_{ν_e} occurs in the entire energy distribution (albeit at different temperatures), unlike in the direct scheme when the generation of L_{ν_e} occurs only in low energies. The distortion in ν_e and $\bar{\nu}_e$ spectra can be well approximated by an overall multiplication factor $1 + \delta_\pm$. The net asymmetry is therefore $L_{\nu_e} = 3(\delta_+ - \delta_-)/8$, and the percentage increase in the $\nu_e\bar{\nu}_e$ number density due to pair production is $(\delta_+ + \delta_-)/2$.

By modifying the standard BBN code with the new ν_e and $\bar{\nu}_e$ spectra, and with the increased total neutrino energy density, we obtain their effects on Y in Figure 7. At $m^2_{\nu_\tau} - m^2_{\nu_e} = m^2_{\nu_\tau} - m^2_{\nu_s} \ll 100\ \mathrm{eV}^2$ the effect on Y is dominated by the asymmetry in the $\nu_e\bar{\nu}_e$ sector. But as $m^2_{\nu_\tau} - m^2_{\nu_e} = m^2_{\nu_\tau} - m^2_{\nu_s}$ increases, the increased total neutrino energy density gradually becomes significant. The increased energy density causes Y to increase, regardless of the sign of the neutrino asymmetry. As a result of these two factors, a maximal reduction $\Delta Y \approx -0.005$ is achieved in cases of positive lepton number asymmetries when $m^2_{\nu_\tau} - m^2_{\nu_e} = m^2_{\nu_\tau} - m^2_{\nu_s} \sim 100$ to $300\ \mathrm{eV}^2$. This mass-squared-difference, however, implies that tau neutrinos are unstable, based on cosmological structure formation considerations [4].

Figure 7 is very different from the previous estimates of Foot and Volkas [5]. For example, Foot and Volkas have argued for a possible reduction $\Delta Y \approx -0.006$ across the mixing parameter range $10 \lesssim m^2_{\nu_\tau} - m^2_{\nu_e} = m^2_{\nu_\tau} - m^2_{\nu_s} \lesssim 3000\ \mathrm{eV}^2$. While our result clearly shows a concave feature of ΔY in this range, with a maximum at ≈ -0.005. Foot and Volkas' result also indicated that ΔY is smaller in the positive direction (when L_{ν_τ}, $L_{\nu_e} < 0$) than in the negative direction (when L_{ν_τ}, $L_{\nu_e} > 0$). Our analysis indicates the opposite: when L_{ν_τ}, $L_{\nu_e} < 0$, the changes in Y due to the spectral asymmetry and the extra neutrino energy add constructively; while L_{ν_τ}, $L_{\nu_e} < 0$, these two effects add destructively. ΔY, therefore, is larger in the positive direction than in the negative direction.

These differences, we believe, stem from our detailed consideration of neutrino spectrum distortion and its time dependence. These factors are crucial to the neutron-to-proton freeze-out process, and in turn the primordial ^{4}He yield.

V. SUMMARY

We have calculated the spectral distortions for neutrinos and the time dependence of the neutrino distribution function during the lepton asymmetry generation via resonant active-sterile neutrino transformation. We have included these crucial effects in our BBN calculation assessing the effect on the primordial ^{4}He abundance of the possible lepton number asymmetry in the $\nu_e\bar{\nu}_e$ sector. We conclude that the possible increase in the primordial ^{4}He yield, as a result of a negative lepton number asymmetry, can be substantial. The maximal increase can be as high as ~ 0.01 to 0.02 (or 5 to 9%) for mixing parameters that are consistent with neutrino mass constraints. The possible decrease due to a positive lepton number asymmetry, however, is limited to $\lesssim 0.002$ (or $\lesssim 1\%$) if the asymmetry is generated by a resonant $\nu_e \leftrightarrow \nu_s$ mixing, or $\lesssim 0.005$ (or $\lesssim 2\%$) if the asymmetry is generated by a three-family resonant mixing scheme. The magnitude of these possible changes in the primordial ^{4}He abundance induced by the neutrino-mixing-generated lepton number asymmetry is comparable to or greater than the uncertainty of current primordial ^{4}He measurements. Therefore, the role of resonant active-sterile neutrino mixing in Big Bang Nucleosynthesis cannot be underestimated.

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Figure Captions:

Figure 1. The magnitude of lepton asymmetry as a function of δm^2 at various temperatures. The bands denote the range of the asymmetry enclosed by the two extreme cases: (1) collisions are completely inefficient (upper limits); (2) collisions are completely efficient (lower limits).

Figure 2. The solid curve: the calculated ν_α ($\alpha = e$, μ, τ) distribution function. The dashed curve: an unperturbed thermal neutrino distribution function with zero chemical potential.

Figure 3. The change in n$\leftrightarrow$p rates due to $\bar{\nu}_e$ (if $L_{\nu_e} > 0$) or ν_e (if $L_{\nu_e} < 0$) spectral distortion for $m^2_{\nu_e} - m^2_{\nu_s} = 1$ eV^2.

Figure 4. The impact on the primordial ^{4}He abundance Y if an asymmetry in $\nu_e\bar{\nu}_e$ is generated by a resonant $\nu_e \leftrightarrow \nu_s$ mixing in BBN. Baryon number density to photon number density ratio is set to $\eta = 5.1 \times 10^{-10}$.

Figure 5. The solid curves: $|L_{\nu_\tau}|$ and $|L_{\nu_e}|$ as a function of temperature for $m^2_{\nu_\tau} - m^2_{\nu_e} = m^2_{\nu_\tau} - m^2_{\nu_s} = 100$ eV^2. The dashed curve: the increase in the total neutrino energy density as a function of temperature, normalized by the energy density of one thermalized active neutrino flavor with zero chemical potential.

Figure 6. The solid curve: the calculated ν_e distribution function in the indirect mixing scheme. The dashed curve: an unperturbed thermal neutrino distribution function with zero chemical potential. Inset: the ratio of the two distribution functions vs. neutrino energy.

Figure 7. The impact on the primordial ^{4}He abundance Y in the indirect neutrino mixing scheme, as a function of $m^2_{\nu_\tau} - m^2_{\nu_e} = m^2_{\nu_\tau} - m^2_{\nu_s}$. Baryon number density to photon number density ratio is set to $\eta = 5.1 \times 10^{-10}$.