# Non-forward BFKL kernel

**Fadin V.S.**
**Institute of Nuclear Physics**
**Novosibirsk**

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# Plan of the talk

- **Introduction**
- **Two-gluon contribution to the kernel**
  - **A general expression in terms of effective Reggeon vertices**
  - **Decomposition on "octet" and "symmetric" parts**
  - **Infrared safety of the "symmetric" part**
  - **Total "real" kernel through the "gluon" kernel and the "symmetric" part**
  - **Infrared properties of the kernel**
  - **Analysis of the two-dimensional integral in the "symmetric" part**
  - **Alternative representation of the "symmetric" part**
- **Summary**

# Introduction

Talking about the BFKL kernel one usually has in mind the case of the forward scattering, i.e. $t = 0$ and vacuum quantum numbers in the $t$-channel. However, the BFKL approach is not limited to this particular case and, what is more, from the beginning it was developed for arbitrary $t$ and for all possible $t$-channel colour states.

The forward BFKL kernel at NLO was found almost seven years ago.

V.S.F., L.N. Lipatov, 1998,
M. Ciafaloni, G. Camici, 1998.

The forward kernel can carry only restrictive information about the BFKL dynamics. Moreover, the non-forward case has an advantage of smaller sensitivity to large-distance contributions, since the diffusion in the infrared region is limited by $\sqrt{|t|}$. But the calculation of the non-forward kernel at NLO was completed only last year.

The reason was a complexity of the two-gluon contribution.

The kernel is given by the sum of "virtual" and "real" contributions.

$$\hat{\mathcal{K}} = \hat{\omega} + \hat{\mathcal{K}}_r$$

The "virtual" contribution is universal.

**It is expressed through the NLO gluon Regge trajectory $\omega(t)$ which is known**

**V.S.F., R. Fiore, M.I. Kotsky, 1995,**
**J. Blumlein, V. Ravindran, W. L. van Neerven, 1998,**
**V. Del Duca, E. W. N. Glover, 2001.**

**The "real" contribution**

$$\hat{\mathcal{K}}_r = \hat{\mathcal{K}}_G + \hat{\mathcal{K}}_{Q\bar{Q}} + \hat{\mathcal{K}}_{GG}$$

**is related to particle production in Reggeon-Reggeon collisions and consists of parts coming from one-gluon, two-gluon and quark-antiquark pair production. The first part is also universal, apart from a colour coefficient, and is also known in the NLO**

**V.S.F., D.A. Gorbachev, 2000.**

**The new contributions which appear in the NLO are $\hat{\mathcal{K}}_{Q\bar{Q}}$ and $\hat{\mathcal{K}}_{GG}$. Each of them is written as a sum of two terms with coefficients depending on a colour representation $R$ in the $t$-channel. For the $Q\bar{Q}$ case both these terms are known.**

**V.S.F., R. Fiore, A. Flachi, M.I. Kotsky, 1998,**
**V.S.F., R. Fiore, A. Papa, 1999.**

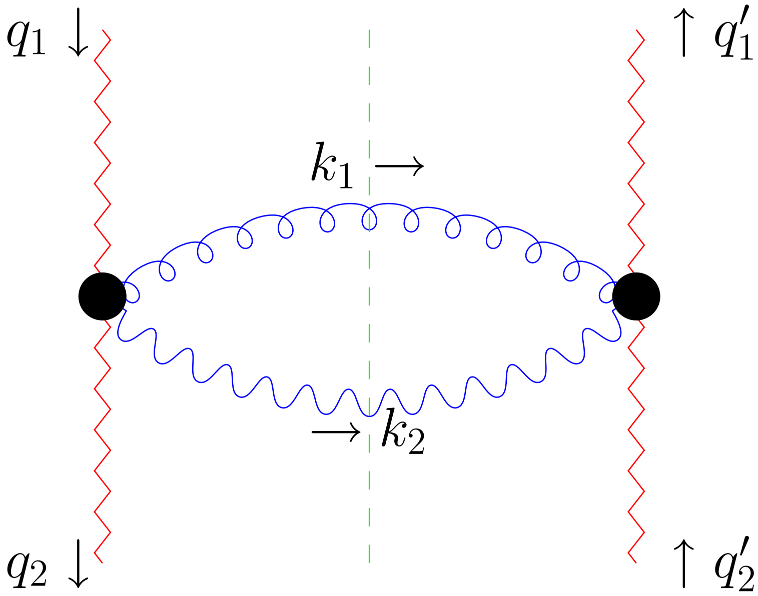
**Instead, only the piece related to the gluon channel was known for the $GG$ case.**

**V.S.F., D.A. Gorbachev, 2000.**

**For scattering of physical (colourless) particles only the Pomeron channel exists.**

**Thus, the two-gluon contribution was the only missing piece in the the non-forward BFKL kernel.**

# The two-gluon contribution




**The "non-subtracted" contribution to the kernel $\mathcal{K}_{GG}$ is**

$$\sum_{G_1 G_2} \int \gamma^{G_1 G_2} \left(\gamma'^{G_1 G_2}\right)^* d\phi_{G_1 G_2} \ ,$$

$\gamma^{G_1 G_2}$ **and** $\gamma'^{G_1 G_2}$ **– effective vertices for two-gluon production in collision of Reggeized gluons with momenta** $q_1, \ -q_2$ **and** $q_1', \ -q_2'$ **respectively;**

$$q_1 - q_1' = q_2 - q_2' = q,$$

$q$ **is the total momentum transfer,**

$$q_1 - q_2 = q_1' - q_2' = k_1 + k_2,$$

$k_i$ **– momenta of produced gluons,**
$d\phi_{G_1 G_2}$ **– their phase space element; the sum is over polarizations and colours of produced gluons.**

For two-gluon states (and only for them) the integral over their invariant mass $k^2$ is logarithmically divergent at large $k^2$, that requires subtraction of the region of large invariant mass. This region is taken into account in the leading terms.

The two-gluon vertex

L.N. Lipatov, V.S.F., 1989.

contains two colour structures:

$$\gamma^{G_1 G_2} = T^{G_1} T^{G_2} \gamma_{12} + T^{G_2} T^{G_1} \gamma_{21} \ ,$$

Accordingly, for any representation of $\mathcal{R}$ of the colour group the two-gluon contribution $\mathcal{K}_{GG}^{(R)}$ contains two terms:

"direct"

$$T^{G_1} T^{G_2} T^{G_2} T^{G_1}$$

and "interference"

$$T^{G_1} T^{G_2} T^{G_1} T^{G_2},$$

with different colour coefficients $a_R$ and $b_R$ and the functions $F_a$ and $F_b$,

$$F_a \propto \gamma_1 \gamma_1' + \gamma_2 \gamma_2',$$

$$F_b \propto \gamma_1 \gamma_2' + \gamma_2 \gamma_1',$$

**With account of the subtraction $\mathcal{K}_{GG}^{(R)}$ is presented in the form**

$$\frac{2g^4N_c^2}{(2\pi)^{D-1}}\hat{\mathcal{S}}\int_0^1 dx\int\frac{d^{2+2\epsilon}k_1}{(2\pi)^{D-1}}\left(\frac{a_RF_a(k_1,k_2)+b_RF_b(k_1,k_2)}{x(1-x)}\right)_+ ,$$

**where the operator $\hat{\mathcal{S}}$ symmetrizes with respect to exchange of the Reggeon momenta, $x$ is a fraction of longitudinal momenta of a produced gluon,**

$$\left(\frac{f(x)}{x(1-x)}\right)_+ \equiv \frac{1}{x}[f(x)-f(0)]+\frac{1}{(1-x)}[f(x)-f(1)],$$

**The group coefficients are expressed through the coefficients $c_R$ appearing in the leading order: $a_R=c_R^2$ and $b_R=c_R\left(c_R-\frac{1}{2}\right)$.**

**For the colour group $SU(N_c)$ with $N_c=3$ the possible representations $\mathcal{R}$ are**

$$\underline{1},\,\underline{8_a},\,\underline{8_s},\,\underline{10},\,\underline{\overline{10}},\,\underline{27}.$$

**Corresponding coefficients are**

$$c_1=1\,,\quad c_{8_a}=c_{8_s}=\frac{1}{2}\,,\quad c_{10}=c_{\overline{10}}=0\,,\quad c_{27}=-\frac{1}{4N_c}$$

**In particular,**

$$a_0=1\,,\quad a_{8_a}=a_{8_s}=\frac{1}{4},\quad b_1=1/2,\quad b_{8_a}=b_{8_s}=0.$$

**The last equality is especially important for the antisymmetric case, since the vanishing of $b_{8_a}$ is crucial for the gluon Reggeization.**

**The equality $b_8 = 0$ extremely simplifies calculation of the octet kernel**

**V.S.F., D.A. Gorbachev, 2000.**

**Remarkably, that only planar diagrams contribute to $\mathcal{K}_{GG}^{(8)}$ due to the colour structure.**

**Instead of calculation of the second term in**

$$\frac{2g^4N_c^2}{(2\pi)^{D-1}}\hat{\mathcal{S}}\int_0^1 dx\int\frac{d^{2+2\epsilon}k_1}{(2\pi)^{D-1}}\left(\frac{a_R F_a(k_1,k_2)+b_R F_b(k_1,k_2)}{x(1-x)}\right)_+$$

**we have found more convenient to calculate the "symmetric" contribution**

$$\mathcal{K}_{GG}^{(s)}(\vec{q}_1,\vec{q}_2;\vec{q}) = \frac{2g^4N_c^2}{(2\pi)^{D-1}}\hat{\mathcal{S}}\int_0^1 dx\int\frac{d^{2+2\epsilon}k_1}{(2\pi)^{D-1}}\left(\frac{F_s(k_1,k_2)}{x(1-x)}\right)_+$$

**where**

$$F_s = F_a + F_b \propto (\gamma_1+\gamma_2)(\gamma_1'+\gamma_2').$$

**A marvellous feature of $\mathcal{K}_{GG}^{(s)}$ is absence of infrared singularities.**

**The disappearance of the singularities is rather tricky: it takes place due to independence of infrared singular terms in the $F_s$ from $x$. Because of this reason the singularities vanish after the substraction.**

**Relations between the colour coefficients $a_R$ and $b_R$ permits to write the two-gluon contribution to the kernel for any representation $R$ is the form**

$$\mathcal{K}_{GG}^{(R)} = 2c_R\mathcal{K}_{GG}^{(8)} + b_R\mathcal{K}_{GG}^{(s)}.$$

**Moreover, in pure gluodynamics an analogous relations is valid for total "real" parts of the kernel:**

$$\mathcal{K}^{(R)}{}_r = 2c_R\mathcal{K}^{(8)}{}_r + b_R\mathcal{K}_{GG}^{(s)}.$$

**Since $\mathcal{K}_{GG}^{(s)}$ is infrared safe, this relation greatly simplifies analysis of infrared singularities, especially because**

**The "real" part $\mathcal{K}^{(8)}{}_r$ for the gluon channel is rather simple**

$$\mathcal{K}_r^{(8)}(\vec{q}_1,\vec{q}_2;\vec{q}) = \frac{g^2N_c}{2(2\pi)^{D-1}}\left\{\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2}+\vec{q}_1^{\,\prime\,2}\vec{q}_2^{\,2}}{\vec{k}^{\,2}} - \vec{q}^{\,2}\right)\right.$$
$$\times\left(\frac{1}{2} + \frac{g^2N_c\Gamma(1-\epsilon)(\vec{k}^{\,2})^{\epsilon}}{(4\pi)^{2+\epsilon}}\left(-\frac{11}{6\epsilon}+\frac{67}{18}-\zeta(2)+\epsilon\left(-\frac{202}{27}+7\zeta(3)+\frac{11}{6}\zeta(2)\right)\right)\right)$$
$$+\frac{g^2N_c\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\left[\vec{q}^{\,2}\left(\frac{11}{6}\ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{q}^{\,2}\vec{k}^{\,2}}\right)+\frac{1}{4}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}^{\,2}}\right)\ln\left(\frac{\vec{q}_1^{\,\prime 2}}{\vec{q}^{\,2}}\right)+\frac{1}{4}\ln\left(\frac{\vec{q}_2^{\,2}}{\vec{q}^{\,2}}\right)\ln\left(\frac{\vec{q}_2^{\,\prime 2}}{\vec{q}^{\,2}}\right)\right.\right.$$
$$\left.+\frac{1}{4}\ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\right) - \frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2}+\vec{q}_2^{\,2}\vec{q}_1^{\,\prime\,2}}{2\vec{k}^{\,2}}\ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right) + \frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2}-\vec{q}_2^{\,2}\vec{q}_1^{\,\prime\,2}}{\vec{k}^{\,2}}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\left(\frac{11}{6}-\frac{1}{4}\ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{k}^{\,4}}\right)\right)$$
$$+\frac{1}{2}[\vec{q}^{\,2}(\vec{k}^{\,2}-\vec{q}_1^{\,2}-\vec{q}_2^{\,2})+2\vec{q}_1^{\,2}\vec{q}_2^{\,2}-\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2}-\vec{q}_2^{\,2}\vec{q}_1^{\,\prime\,2}+\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2}-\vec{q}_2^{\,2}\vec{q}_1^{\,\prime\,2}}{\vec{k}^{\,2}}(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})]$$
$$\left.\left.\times I(\vec{q}_1^{\,2},\vec{q}_2^{\,2},\vec{k}^{\,2})\right]\right\} + \frac{g^2N_c}{2(2\pi)^{D-1}}\left\{\vec{q}_i \longleftrightarrow \vec{q}_i^{\,\prime}\right\},$$

**where**

$$I(a,b,c) = \int_0^1 \frac{dx}{a(1-x)+bx-cx(1-x)}\ln\left(\frac{a(1-x)+bx}{cx(1-x)}\right).$$

Due to infrared safety of $\mathcal{K}^{(s)}_{GG}$ the singularities are the same for all colour states in the $t$-channels, apart from colour factors.

Actually the singularities are the same as for the forward case, since they are proportional to the LO kernel.

The total kernel $\hat{\mathcal{K}} = \hat{\omega} + \hat{\mathcal{K}}_r$
must be infrared safe for the Pomeron channel.

In this case the singularities of $\mathcal{K}_r$ are cancelled by the singularities of the gluon trajectory.

The infrared safety is explicitly demonstrated and forms free from the singularities are found.

V.S.F., R.Fiore, 2005.

The "symmetric" contribution is rather complicated.

The complexity is related to the non-planar diagrams.

It is known since the calculation of the

non-forward kernel for the QED Pomeron

V.N. Gribov, L.N. Lipatov, G.V. Frolov, 1970

H. Cheng, T.T. Wu, 1970

where only box and cross-box diagrams are relevant.

The kernel was found only in the form of two-dimensional integral.

**In QCD the situation is greatly worse because of the existence of cross-pentagon and cross-hexagon diagrams in addition to QED-type cross-box diagrams.**

**It requires the use of additional Feynman parameters. At arbitrary $D$ no integration over these parameters at all can be done in elementary functions. It occurs, however, that**

**in the limit $\epsilon \to 0$ the integration over additional Feynman parameters can be performed, so that the result can be written as two-dimensional integral, as well as in QED.**

**The result can be written as**

$$\mathcal{K}^{(s)}_{GG}(\vec{q}_1, \vec{q}_2; \vec{q} = \frac{\alpha_s^2 N_c^2}{4\pi^3}\left\{\left(\left[(\vec{q}^{\,2} - 2\vec{q}_1^{\,2})(\frac{25}{9} - \frac{\pi^2}{12})\right.\right.\right.$$
$$-\frac{11}{12}\left(2\vec{q}_1^{\,2}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{k}^{\,2}}\right) - \vec{q}^{\,2}\ln\left(\frac{\vec{q}^{\,2}}{\vec{k}^{\,2}}\right)\right) + \frac{\vec{q}^{\,2}}{4}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}^{\,2}}\right)\ln\left(\frac{\vec{q}_1^{\,\prime\,2}}{\vec{q}^{\,2}}\right)$$
$$-\frac{\vec{q}_1^{\,\prime\ 2}}{2}\left(\frac{(\vec{k}^{\,2} - \vec{q}_1^{\,2} - \vec{q}_2^{\,2})^2 - 4\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{2\vec{k}^{\,2}} I(\vec{k}^{\,2}, \vec{q}_2^{\,2}, \vec{q}_1^{\,2})\right.$$
$$\left.\left.+\frac{\vec{k}^{\,2} + \vec{q}_2^{\,2} - \vec{q}_1^{\,2}}{2\vec{k}^{\,2}}\ln\left(\frac{\vec{k}^{\,2}}{\vec{q}_2^{\,2}}\right)\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\right) - J(\vec{q}_1, \vec{q}_2; \vec{q})\right]$$
$$\left.+\left[\vec{q}_i \leftrightarrow -\vec{q}_i^{\,\prime}\right]\right) + \left(\vec{q}_1 \leftrightarrow -\vec{q}_2^{\,\prime}\right)\Big\},$$

## with the two-dimensional integral $J(\vec{q}_1,\vec{q}_2;\vec{q})$:

$$J(\vec{q}_1,\vec{q}_2;\vec{q}) = \int_0^1 dx \int_0^1 dz \left\{ \vec{q}_1\vec{q}_1^{\,\prime} \left( (2-x_1x_2)\ln\left(\frac{Q^2}{\vec{k}^{\,2}}\right) - \frac{2}{x_1}\ln\left(\frac{Q^2}{Q_0^2}\right)\right) \right.$$

$$-\frac{1}{2Q^2}x_1x_2(\vec{q}_1^{\,2}-2\vec{q}_1\vec{p}_1)(\vec{q}_1^{\,\prime 2}-2\vec{q}_1^{\,\prime}\ \vec{p}_2) + \frac{2}{x_1}\left[ \left(x_2\vec{q}_1\vec{q}_1^{\,\prime}\ (\vec{p}_1(\vec{q}_1^{\,\prime}-\vec{p}_2)) - \vec{q}_1^{\,\prime\,2}\vec{q}_1\vec{p}_2\right)\frac{1}{Q^2}\right.$$

$$\left. + \left(z(1-z)\vec{q}_2^{\,\prime\,2}\vec{q}_1\vec{q}_1^{\,\prime}\ + \vec{q}_1^{\,\prime\,2}(z\vec{q}_1\vec{k}+(1-z)\vec{q}_1\vec{q}_1^{\,\prime})\right)\frac{1}{Q_0^2}\right] - \frac{1}{Q^2}\left(\vec{q}_1^{\,\prime\,2}\vec{q}_1\,(\vec{p}_1-2\vec{q}_1^{\,\prime})\right.$$

$$\left. +4x_1\vec{q}_1^{\,2}(\vec{q}_1^{\,\prime}\vec{p}_2) + \vec{q}_1^{\,\prime}\vec{q}_1(\vec{q}_1^{\,\prime}\vec{q}_1 - \vec{q}_1^{\,\prime}\vec{p}_1 - \vec{q}_1\vec{p}_2) + 2(\vec{q}_1^{\,\prime}\vec{p}_1)(\vec{q}_1\vec{p}_2) - 2(\vec{q}_1^{\,\prime}\vec{p}_2)(\vec{q}_1\vec{p}_1)\right)$$

$$+\vec{q}_1^{\,\prime\,2}\left[\frac{-1}{\mu_2^2Q^2}\left(2\frac{x_2}{x_1}(\vec{q}_1\vec{p}_2)\vec{q}_1^{\,\prime}\vec{k} + x_2(\vec{q}_1^{\,\prime}\vec{p}_2)(\vec{q}_2^{\,2}-\vec{k}^{\,2}) + 2(\vec{q}_2\vec{p}_2)\vec{q}_1\vec{q}\right)\right.$$

$$+\frac{2}{\mu_0^2Q_0^2}\frac{1}{x_1}(\vec{q}_1\vec{p}_0)\vec{q}_1^{\,\prime}\vec{k} - \frac{\vec{q}_1(\vec{q}_1^{\,\prime}+\vec{k})}{x_1}\left(\frac{x_2}{\vec{p}_2^{\,2}}\ln\left(\frac{Q^2}{\mu_2^2}\right) - \frac{1}{\vec{p}_0^{\,2}}\ln\left(\frac{Q_0^2}{\mu_0^2}\right)\right)$$

$$+\frac{1}{\vec{p}_2^{\,2}}\left(\frac{1}{\vec{p}_2^{\,2}}\ln\left(\frac{Q^2}{\mu_2^2}\right) + \frac{1}{Q^2}\right)\left(2\frac{x_2}{x_1}(\vec{q}_1\vec{p}_2)(\vec{q}_1^{\,\prime}+\vec{k})\vec{p}_2 - 2((x_2\vec{q}_1^{\,\prime}+\vec{q}_2)\vec{p}_2)\vec{q}_1\vec{p}_2\right)$$

$$-\frac{1}{\vec{p}_0^{\,2}}\left(\frac{1}{\vec{p}_0^{\,2}}\ln\left(\frac{Q_0^2}{\mu_0^2}\right) + \frac{1}{Q_0^2}\right)\left(2\frac{1}{x_1}(\vec{q}_1\vec{p}_0)(\vec{q}_1^{\,\prime}+\vec{k})\vec{p}_0\right) + \frac{(x_2\vec{q}_1^{\,\prime}+\vec{q}_2)\vec{q}_1}{\vec{p}_2^{\,2}}\ln\left(\frac{Q^2}{\mu_2^2}\right)$$

$$+\frac{\vec{q}_1^{\,2}}{d}\left((\vec{q}_2\vec{k})(\vec{q}_2^{\,\prime}\vec{k})\left(\frac{Q^2}{d}\mathcal{L} - \frac{1}{\vec{k}^{\,2}}\right) + (\vec{q}_2\vec{p}_2)(\vec{q}_2^{\,\prime}\vec{k})\left(\frac{1}{\mu_2^2} - \frac{\mu_1^2}{d}\mathcal{L}\right) + (\vec{q}_2\vec{k})(\vec{q}_2^{\,\prime}\vec{p}_1)\left(\frac{1}{\mu_1^2} - \frac{\mu_2^2}{d}\mathcal{L}\right)\right.$$

$$\left.\left.\left. +(\vec{q}_2\vec{p}_2)(\vec{q}_2^{\,\prime}\vec{p}_1)\left(\frac{\vec{k}^{\,2}}{d}\mathcal{L} - \frac{1}{Q^2}\right) + \frac{(\vec{q}_2\vec{q}_2^{\,\prime})}{2}\mathcal{L}\right)\right]\right\} .$$

**Here**

$$\vec{p}_1 = zx\vec{q}_1 + (1-z)(x\vec{k}-(1-x)\vec{q}_2^{\,\prime}),\ \ \vec{p}_2 = z((1-x)\vec{k}-x\vec{q}_2) + (1-z)(1-x)\vec{q}_1^{\,\prime};$$

$$Q^2 = x(1-x)(\vec{q}_1^{\,2}z + \vec{q}_1^{\,\prime 2}(1-z)) + z(1-z)(\vec{q}_2^{\,2}x + \vec{q}_2^{\,\prime 2}(1-x) - \vec{q}^{\,2}x(1-x)),$$

$$\mu_i^2 = Q^2 + \vec{p}_i^{\,2},\quad d = \mu_1^2\mu_2^2 - \vec{k}^{\,2}Q^2\ ,\quad \mathcal{L} = \ln\left(\frac{\mu_1^2\mu_2^2}{\vec{k}^{\,2}Q^2}\right)\ ,$$

$$\vec{p}_0 = z\vec{k} + (1-z)\vec{q}_1^{\,\prime};\ \ Q_0^2 = z(1-z)\vec{q}_2^{\,\prime\,2},\quad \mu_0^2 = z\vec{k}^{\,2} + (1-z)\vec{q}_1^{\,\prime\,2}.$$

**Presence of the two-dimensional integral $J(\vec{q}_1,\vec{q}_2;\vec{q})$ in the kernel makes difficult its use both for analytical investigation and for numerical calculations.**

**Unfortunately, the integrand $INT(x,z)$ of the two-dimensional integral $J(\vec{q}_1,\vec{q}_2;\vec{q})$ is too complicated. Moreover, it's behaviour near the board of the integration region is not smooth. Analytical analysis of the integrand near the board can facilitate the numerical calculations.**

**The most important are the corner regions:**

$x \ll 1, \quad z \ll 1$:

$$INT(x,z) \simeq \frac{2}{x}\left(\frac{(\vec{q}_1\vec{q}_1{}')\vec{q}_1{}'\vec{q}_2{}'}{\vec{q}_1{}'^{\,2}} - \frac{(\vec{q}_1\vec{q}_2{}')}{2}\right)\ln\left(\frac{x\vec{q}_1{}'^{\,2}+z\vec{q}_2{}'^{\,2}}{z\vec{q}_2{}'^{\,2}}\right) + \frac{1}{x\vec{q}_1{}'^{\,2}+z\vec{q}_2{}'^{\,2}}$$
$$\times\left[\vec{q}_1{}'^{\,2}(\vec{q}_2^{\,2}-\vec{k}^{\,2}+\vec{q}_1\vec{q}_2{}') - \vec{q}_1^{\,2}(\vec{q}_1{}'\vec{q}_2) + (\vec{q}_1\vec{q}_1{}')(2\vec{q}_1\vec{q}_1{}'+\vec{q}_1{}'\vec{q}_2{}') + 2(\vec{q}_1{}'\vec{q}_2)(\vec{q}_1\vec{q})\right].$$

$x \ll 1, \quad 1-z \ll 1$:

$$INT(x,z) \simeq \frac{2}{x}\left[\vec{q}_1\vec{q}_1{}' - \vec{q}_1{}'^{\,2}\left(\frac{(\vec{q}_1\vec{k})(\vec{q}_1{}'\vec{k})}{\vec{k}^{\,4}} - \frac{(\vec{q}_1\vec{q}_2{}')}{2\vec{k}^{\,2}}\right)\right]\ln\left(\frac{x\vec{q}_1^{\,2}+(1-z)\vec{q}_2{}'^{\,2}}{(1-z)\vec{q}_2{}'^{\,2}}\right)$$
$$+\frac{1}{x\vec{q}_1^{\,2}+(1-z)\vec{q}_2{}'^{\,2}}\left[-2\vec{q}_1{}'^{\,2}(\vec{q}_1\vec{q}) - (\vec{q}_1\vec{q}_1{}')(2\vec{q}_1{}'^{\,2}+2\vec{q}_1^{\,2}+\vec{q}_1\vec{q}_2{}')\right.$$
$$+\frac{\vec{q}_1{}'^{\,2}}{\vec{k}^{\,2}}\left(2(\vec{q}_1\vec{k})^2+2(\vec{q}_1\vec{k})(\vec{q}_1^{\,2}+\vec{q}_1\vec{q}_2)+\vec{q}_1^{\,2}(\vec{q}_1{}'\vec{k})+\frac{\vec{q}_1^{\,2}(\vec{q}_2\vec{k})(x\vec{q}_1\vec{q}_2{}'-(1-z)\vec{q}_2{}'^{\,2})}{x\vec{q}_1^{\,2}+(1-z)\vec{q}_2{}'^{\,2}}\right)$$
$$\left.+\frac{\vec{q}_1^{\,2}\vec{q}_1{}'^{\,2}}{\vec{k}^{\,2}}\left(\frac{(\vec{q}_2\vec{k})(\vec{q}_2{}'\vec{k})}{\vec{k}'^{\,2}} - \frac{(\vec{q}_2\vec{q}_2{}')}{2}\right)\right].$$

$1 - x \ll 1, \quad z \ll 1$:

$$INT(x,z) \simeq \frac{1}{(1-x)\vec{q}_1^{\,\prime\,2} + z\vec{q}_2^{\,2}} \Big[ (\vec{q}_1\vec{q}_1^{\,\prime})(\vec{q}_2\vec{q}_1^{\,\prime}) + \vec{q}_1^{\,\prime\,2}(\vec{q}_1^{\,2} - 2\vec{q}_1\vec{q}_1^{\,\prime})$$

$$+\vec{q}_1^{\,2}\left(\frac{(\vec{q}_2\vec{k})(\vec{q}_2^{\,\prime}\vec{k})}{\vec{k}^4} - \frac{(\vec{q}_2\vec{q}_2^{\,\prime})}{2\vec{k}^2}\right)\Big] + \frac{(1-x)\vec{q}_1^{\,\prime}\vec{q}_2 - z\vec{q}_2^{\,2}}{((1-x)\vec{q}_1^{\,\prime\,2} + z\vec{q}_2^{\,2})^2}\vec{q}_1^{\,\prime\,2}\left(2\vec{q}_1\vec{q} + \vec{q}_1^{\,2}\frac{\vec{q}_2^{\,\prime}\vec{k}}{\vec{k}^{\,2}}\right).$$

$1 - x \ll 1, \quad 1 - z \ll 1$:

$$INT(x,z) \simeq \frac{\left[-\vec{q}_1^{\,\prime\,2}(\vec{q}_1(\vec{q}_1^{\,\prime} + \vec{q}_2)) - 2\vec{q}_1^{\,2}(q_1^{\prime}\vec{q}_2) - (\vec{q}_1\vec{q}_1^{\,\prime})(\vec{q}_1\vec{q}_2)\right]}{(1-x)\vec{q}_1^{\,2} + (1-z)\vec{q}_2^{\,2}}.$$

**Contributions of the corner regions to the kernel are calculated analytically.**

**Unfortunately, it is practically impossible to do for the regions where one of the variables is closed to the board, whereas another is arbitrary, although the integrand in these regions is also found.**

**Another form of the "symmetric" contribution: the integral in transverse momentum space:**

$$\mathcal{K}^{(s)}_{GG}(\vec q_1,\vec q_2;\vec q) = \frac{\alpha_s^2 N_c^2}{4\pi^3}\left([J^s(\vec q_1,\vec q_2;\vec q) + J^s(-\vec q_2,-\vec q_1;-\vec q)] \right.$$
$$\left. +[\vec q_n \leftrightarrow \vec q - \vec q_n]\right),$$

**where** $\vec q_n$ **and** $\vec q - \vec q_n \equiv -\vec q\,'_n$ $(n = 1,2)$ **are the** $t$**–channel Reggeized gluon momenta,**

$$J^s(\vec q_1,\vec q_2;\vec q) = \frac{\vec k^{\,2}}{2} + \frac{5}{2}(\vec q_1\vec q_2) + \frac{\vec q^{\,2}}{2}\left(\frac{13}{18} - \zeta(2)\right) - \frac{(\vec q_1^{\,2} - \vec q_2^{\,2})(\vec q_1^{\,\prime\,2} - \vec q_2^{\,\prime\,2})}{2\vec k^{\,2}}$$
$$+2\left(\vec q_1\vec q_2 - \vec q_1^{\,2}\frac{\vec k\vec q_1^{\,\prime}}{\vec k^{\,2}}\right)\ln\left(\frac{\vec q_1^{\,2}}{\vec k^{\,2}}\right) - \vec q^{\,2}\left(\frac{11}{12}\ln\left(\frac{\vec q^{\,2}}{\vec k^{\,2}}\right) + \frac{5}{6}\ln 2\right)$$
$$-\frac{\vec k^{\,2}}{2}\ln\left(\frac{\vec q_1^{\,2}}{\vec k^{\,2}}\right)\ln\left(\frac{\vec q_2^{\,2}}{\vec k^{\,2}}\right) + \frac{\vec q^{\,2}}{4}\ln\left(\frac{\vec q_1^{\,2}}{\vec q^{\,2}}\right)\ln\left(\frac{\vec q_1^{\,\prime\,2}}{\vec q^{\,2}}\right) + (\vec q_1\vec q$$
$$+\frac{\vec q_1^{\,2}(\vec k\vec q_2^{\,\prime}) - \vec q_1^{\,\prime\,2}(\vec k\vec q_2)}{\vec k^{\,2}})\left(\frac{1}{2}\ln\left(\frac{\vec q_2^{\,2}}{\vec k^{\,2}}\right)\ln\left(\frac{\vec q_1^{\,2}}{\vec q_2^{\,2}}\right) + (\vec q_2\vec k)I(\vec q_1^{\,2},\vec q_2^{\,2},\vec k^{\,2})\right)\Bigg)$$
$$-\vec q_2^{\,2}(\vec q_1\vec q)I(\vec q_1^{\,2},\vec q_2^{\,2},\vec k^{\,2}) + \int\frac{d^2k_1}{\pi}\Bigg[\Bigg(-\frac{\vec k_1^{\,2}\vec k_2^{\,2}}{2} + (\vec k_1\vec k_2)^2$$
$$+(Q_1^i\Omega_1^{ij}Q_2^{\prime j})(Q_1^i\Omega_2^{ij}Q_2^{\prime j}) - \vec Q_2^{\,\prime\,2}(Q_1^i\Omega_1^{ij}\Omega_2^{jl}Q_1^l)\Bigg)$$
$$\times\frac{1}{\vec Q_1^{\,2}\vec Q_2^{\,\prime\,2} - \vec k_1^{\,2}\vec k_2^{\,2}}\ln\left(\frac{\vec Q_1^{\,2}\vec Q_2^{\,\prime\,2}}{\vec k_1^{\,2}\vec k_2^{\,2}}\right) - \frac{1}{2} + \frac{5}{6}\frac{\vec q^{\,2}}{\vec k_1^{\,2} + \vec k_2^{\,2}}\Bigg]\,.$$

**Here** $\vec k = \vec q_1 - \vec q_2 = \vec q_1^{\,\prime} - \vec q_2^{\,\prime},\ \vec k_2 = \vec k - \vec k_1,\ \vec Q_n = \vec q_1 - \vec k_n,\ \vec Q_n^{\,\prime} = \vec q_1^{\,\prime} - \vec k_n,\ \Omega_n^{ij} = \delta^{ij} - 2k_n^i k_n^i/\vec k_n^{\,2}$

# Summary

- The BFKL kernel is known now for $t \neq 0$ and all possible $t$-channel colour states $\mathcal{R}$
- It is expressed in terms of the gluon trajectory, the kernel in the octet channel and the "symmetric" contribution of two-gluon production
- The "symmetric" contribution is infrared safe
- It makes simple the infrared structure of the kernel for any $\mathcal{R}$ and evident the infrared safety of the singlet kernel
- However "symmetric" contribution contains two-dimensional integral with a complicated integrand
- Analysis of the integrand in the near-board region is performed
- Alternative representation of the "symmetric" contribution is found
- Work on search of suitable representations for the kernel and on investigation of its properties is continuing