Transport treatment of DCC dynamics

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The field dynamics given by the linear σ model is recast in terms of medium-dependent quasi-particles moving consistently with the evolving local order parameter, yielding a transport framework that is more suitable for simulations of realistic collision scenarios.

1. MOTIVATION

The possibility of forming *disoriented chiral condensates* in high-energy collisions has gained significant attention in recent years as a means for probing the restoration of chiral symmetry [1]. The phase transition is expected to occur in the hot collision zone and the subsequent non-equilibrium relaxation towards the normal vacuum may then generate large-amplitude coherent oscillations of the pion field. The suggested observable consequences include an excess of soft pions, with an associated anomalous distribution of the neutral pion fraction, and a significant enhancement of dileptons and photons.

The most popular tool for DCC studies has been the linear σ model which describes the $O(4)$ chiral field $\boldsymbol{\phi} = (\sigma, \boldsymbol{\pi})$ by means of a simple effective quartic interaction,

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu \overrightarrow{\boldsymbol{\phi}} \circ \partial^\mu \overleftarrow{\boldsymbol{\phi}} - \tfrac{\lambda}{4}(\phi^2 - v^2)^2 + H\sigma \qquad \Rightarrow \qquad [\Box + \lambda(\phi^2 - v^2)]\, \overleftarrow{\boldsymbol{\phi}} = H\, \overleftarrow{e_\sigma} \ . \tag{1}$$

Invoking a mean-field approximation, and the associated Hartree factorization, it is possible to separate the field into a smooth part, $\underline{\boldsymbol{\phi}}$, identified with a local *order parameter*, and the fluctuations representing *quasi-particle* excitations, $\delta\boldsymbol{\phi}$, which are described approximately by Klein-Gordon dispersion relations containing an effective mass tensor depending on both temperature and order parameter [2,3],

$$\boldsymbol{\phi} \ = \ \underline{\boldsymbol{\phi}} \ + \ \delta\boldsymbol{\phi} \qquad \Rightarrow \qquad \begin{cases} \text{Order parameter :} & \Box\underline{\boldsymbol{\phi}} \ + \ \underline{\boldsymbol{M}} \circ \underline{\boldsymbol{\phi}} \ = \ He_\sigma \\ \text{Quasi-particles :} & \Box\delta\boldsymbol{\phi} + \boldsymbol{M} \circ \delta\boldsymbol{\phi} \ = \ 0 \end{cases} , \tag{2}$$

where $\boldsymbol{M} = \lambda(\langle\phi^2\rangle - v^2)\boldsymbol{I} + 2\lambda\langle\boldsymbol{\phi}\boldsymbol{\phi}\rangle = \underline{\boldsymbol{M}} + 2\lambda\langle\underline{\boldsymbol{\phi}\boldsymbol{\phi}}\rangle$. This framework has provided very instructive insight into the key feaures of the non-equilibrium DCC dynamics [3,4].

However, only rather idealized systems are tractable within the field framework, since neither expansion nor the coupling to the residual system is easy to treat. We therefore seek to recast the field formulation of the linear σ model into a particle description. This will make it possible to embed the chiral dynamics into microscopic transport codes so that more realistic simulations can be carried out.

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2. METHOD

This can be accomplished by considering the Wigner transform of the chiral field, $\boldsymbol{F}(x,p) = \int d^4s\ \delta\boldsymbol{\phi}(x+\frac{s}{2})\ \mathrm{e}^{ips}\ \delta\boldsymbol{\phi}(x-\frac{s}{2})$, which is governed by the Vlasov equation,

$$p^\mu \partial_\mu \overleftrightarrow{\boldsymbol{F}}(x,p) \approx -\frac{i}{2}\left[\overleftrightarrow{\boldsymbol{M}} \circ \overleftarrow{\boldsymbol{F}} - \overrightarrow{\boldsymbol{F}} \circ \overleftrightarrow{\boldsymbol{M}}\right] - \frac{1}{4}\left[\frac{\partial \overleftrightarrow{\boldsymbol{M}}}{\partial x^\mu} \circ \frac{\partial \overleftrightarrow{\boldsymbol{F}}}{\partial p_\mu} + \frac{\partial \overleftrightarrow{\boldsymbol{F}}}{\partial p_\mu} \circ \frac{\partial \overleftrightarrow{\boldsymbol{M}}}{\partial x^\mu}\right] . \tag{3}$$

In a semi-classical approximation, the associated quasi-particle phase-space distribution, $\boldsymbol{f}(\mathbf{r},\mathbf{p};t) = \int d\omega\ \omega \boldsymbol{F}(\mathbf{r},t;\mathbf{p},\omega)$, can then be represented by a swarm of test particles,

$$\overleftrightarrow{\boldsymbol{f}}(\mathbf{r},\mathbf{p}) \ \approx\ \frac{1}{\mathcal{N}} \sum_n \overleftarrow{\boldsymbol{\chi}}_n \overrightarrow{\boldsymbol{\chi}}_n\ \frac{1}{(2\pi ab)^3} \exp[-\frac{(\mathbf{r}-\mathbf{r}_n)^2}{2a^2} - \frac{(\mathbf{p}-\mathbf{p}_n)^2}{2b^2}] , \tag{4}$$

which move on classical trajectories,

$$(\mathbf{r}_n, \mathbf{p}_n, \overleftarrow{\boldsymbol{\chi}}_n) : \qquad \left\{ \begin{array}{lll} \text{Position :} & \omega_n\ \dot{\mathbf{r}}_n & = \ \mathbf{p}_n \\ \text{Momentum :} & 2\omega_n\ \dot{\mathbf{p}}_n & = -\boldsymbol{\chi}_n \circ \boldsymbol{\nabla}\boldsymbol{M} \circ \boldsymbol{\chi}_n \\ \text{Orientation :} & 2\omega_n\ \dot{\boldsymbol{\chi}}_n & = -i\boldsymbol{M} \circ \boldsymbol{\chi}_n \end{array} \right. , \tag{5}$$

while obeying the in-medium dispersion relation,

$$\omega_n^2 \ = \ p_n^2 + \ \overrightarrow{\boldsymbol{\chi}}_n \circ\boldsymbol{M}\circ \overleftarrow{\boldsymbol{\chi}}_n \ . \tag{6}$$

The field fluctuations are thereby replaced by individual quasi-particles having a medium-modified mass tensor $\boldsymbol{M}(\mathbf{r})$ that depends self-consistently on the environment through the local order parameter $\underline{\boldsymbol{\phi}}(\mathbf{r})$, which is governed by the mean-field equation (top line of eq. (2)), and the local degree of agitation given in terms of the quasi-particles as

$$\langle \overleftarrow{\delta\boldsymbol{\phi}}\ \overrightarrow{\delta\boldsymbol{\phi}} \rangle \ = \ \frac{1}{\Delta\Omega}\frac{1}{\mathcal{N}} \sum_n{}'\ \overleftarrow{\boldsymbol{\chi}}_n\ \frac{1}{\omega_n}\ \overrightarrow{\boldsymbol{\chi}}_n , \tag{7}$$

where the sum includes those quasi-particles that are located in a local test volume $\Delta\Omega$. Finally, the $O(4)$ vector $\boldsymbol{\chi}_n(t)$ describes the chiral orientation of the test particle n.

3. TEST APPLICATION

In order to illustrate that the developed transport treatment is both numerically feasible and quantitatively reliable under quasi-realistic circumstances, we consider the evolution of a cylinder subjected to a longitudinal scaling expansion of the Bjorken type, similar to what was already considered in ref. [5]. This scenario provides a suitable test case because it exhibits some of the important features expected in real collision events. In particular, 1) it has a surface region through which the order parameter changes from its value in the bulk of the interior to its vaccum value outside and 2) it is in a state of rapid expansion. Due to the idealized nature of the Bjorken expansion, it is possible to solve the full field equation by transforming from t,z to τ,η, with $t=\tau\cosh\eta$ and $z=\tau\sinh\eta$. Moreover, in the present context, it is a special advantage that the one-dimensional expansion scenario keeps the system away from the unstable region of phase space (see ref. [4]), so that the present incomplete transport treatment is being tested only for situations where it is intended to apply (see later).

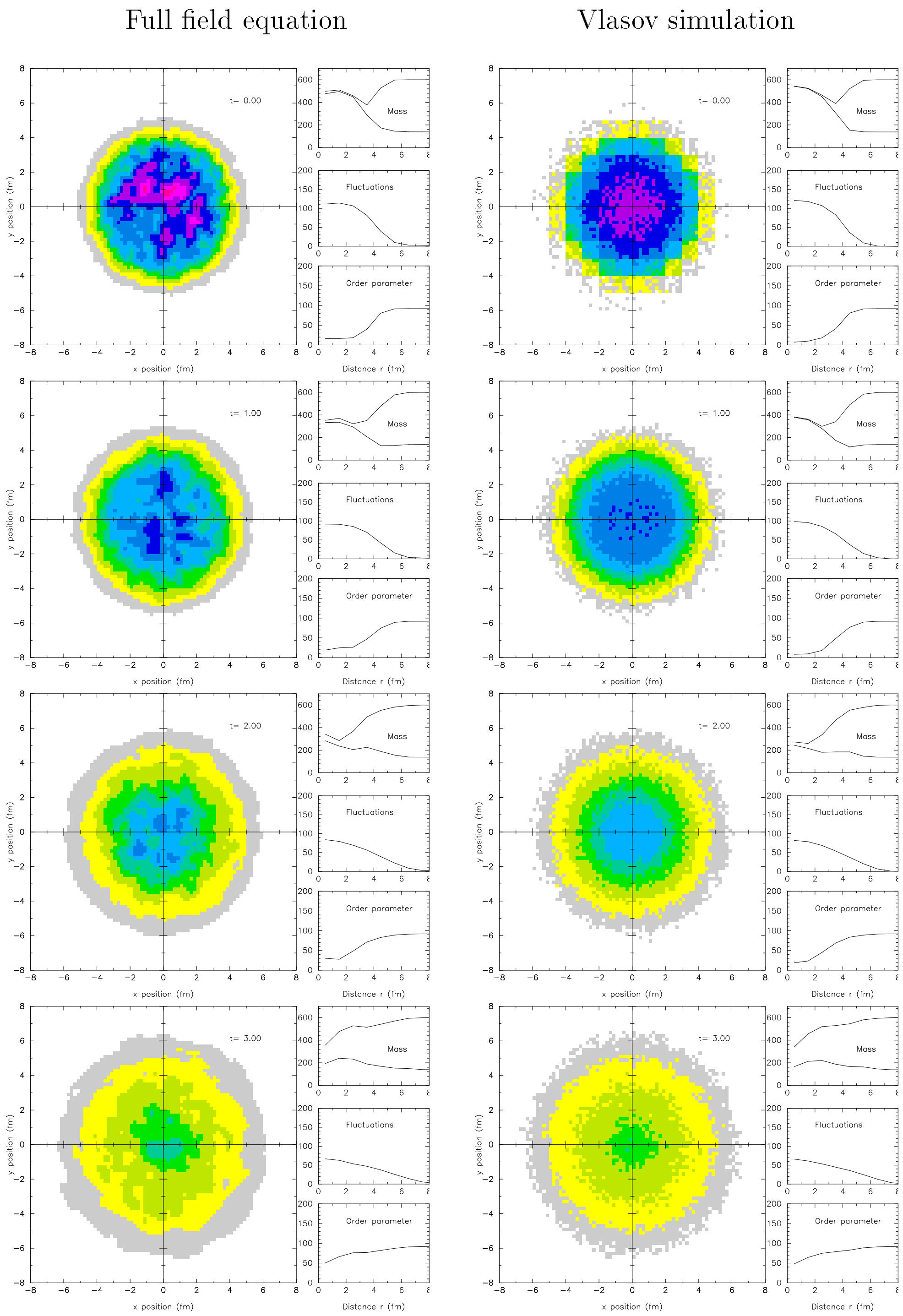

Figure 1. Projections of the *field fluctuation* (the square root of the trace of (7)) for the expanding cylinder, at times $\tau = 0, 1, 2, 3$ fm/c after initialization. On the right are the results obtained with the full field equation (1) and on the left are the results obtained when the Vlasov equation (3) is solved by test-particle simulation (5). The auxiliary graphs show the evolving transverse profiles of the *order parameter* (bottom), the *field fluctuation* (middle), and the *effective masses* (top) μ_π (lower) and μ_σ (upper).

The cylinder is prepared in a manner intended to approximate local thermal equilibrium, the temperature profile $T(\mathbf{r})$ being of Saxon-Woods form with a width parameter of 0.8 fm and having a bulk value of T_0=400 MeV. The evolution is treated by both the full field equation (1) and the transport treatment, in which individual test particles are moving according to (5) in the effective field provided by the self-consistently evolving order parameter $\underline{\phi}(\mathbf{r})$. The initial field is constructed from equilibrium matter at T_0 by means of a local scaling (see ref. [3]) based on the initial temperature profile, while the test-particle distribution is sampled locally from the corresponding Bose-Einstein thermal gas. In this application, we have used $\mathcal{N}$=100 test particles per physical quantum and we calculate the mass tensor on a macro lattice with a spacing of 1 fm, so the test volumes are $\Delta\Omega = 1$ fm^3 (as is visible in the initial contour plot in fig. 1).

An inspection of the contour plots in fig. 1 shows that the two calculations lead to quite similar overall evolutions. More detailed comparisons can be made by means of the auxiliary displays and it is seen that the radial profiles of both the order parameter and the local field fluctuation evolve very similarly in the two calculations. As a consequence, the effective quasi-particle masses are also similar. From such test comparisons, it appears that the developed transport treatment may in fact be quantatively useful.

4. OUTLOOK

The above transport treatment is directly applicable to scenarios where the quasi-particle trajectories remain within the classically allowed region of phase space. However, special considerations are needed whenever a quasi-particle mode grows unstable, as one hopes will happen in the *DCC* context, since spontaneous pair creation then becomes possible and a purely classical framework is inadequate. Preliminary studies based on quantum-field theory suggest that it is possible to extend the transport treatment in a quantatively satisfacory manner to such scenarios as well [6].

The resulting reformulation of the linear σ model in terms of medium-modified quasi-particles will make it practically easier to simulate *DCC* dynamics in the complicated dynamical environments of high-energy collisions, thus allowing a more realistic assessment of the prospect for the emergence of measurable signals of the *DCC* phenomenon.

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