α-vac perturbation theory

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Plan

- Present a simple class of initial states for inflation that encode effects of trans-Planckian physics
  - ➔ α-vacua
  - ➔ Potentially observable consequences on CMBR and cosmic ray observations
- Develop perturbation theory around α-vacua
  - ➔ Either coupling to gravity gives problems due to nonlocality
  - ➔ Or perturbative expansion fails due to IR divergent pinch singularites
- Cutoff α-vacua still possible if right strongly coupled and/or non-local, novel physics happens at UV cutoff

α–vacua

- Chernikov, Tagirov (1968) Mottola, Allen (1985) showed one-complex parameter family of vacua invariant under all symmetries of de Sitter group
- Any initial state should asymptote to one of these at late time if inflation sufficiently long-lived
- Unknown trans-Planckian physics can be encoded in nontrivial value for α
  - ➔Parameterization of new physics (Starobinsky,Tkachev, Danielsson, Goldstein,Lowe)
- Previous studies of inflation always start with Bunch-Davies vacuum state (α= –infinity)
  - ➔Rationale: free propagator matches with expected flat space limit as points coincide (Hadamard condition)
  - ➔Purely mathematical restriction, since this asymptotic limit not attainable in a cutoff effective field theory

- What happens if we give up this restriction to Hadamard states?
- General α–vacuum
  - ➔Non-equilibrium states
    - Unruh detector has non-thermal response
  - ➔Non-Hadamard
    - Interplay between curvature and short-distance physics
- Possible interesting consequences for ultra-high energy cosmic ray production and CMBR spectrum

Basic Setup

- Choose conformal metric

$$ds^2 = a(\eta)^2(-d\eta^2 + d\vec{x}^2)$$

- Make mode expansion of field

$$\phi = \frac{1}{a(\eta)}\sum_k e^{ikx}\chi_{Ek}(\eta)a_k + e^{-ikx}\chi^*_{Ek}(\eta)a_k^+$$

$$a_k|0\rangle = 0$$

χ's are Hankel functions of the second kind

➔Euclidean vacuum, aka Bunch-Davies

- General class obtained by taking linear combinations of + and – frequency modes $\chi_k = N_\alpha \left(\chi_{Ek} + e^\alpha \chi^*_{Ek} \right)$

$$N_\alpha = \frac{1}{\sqrt{1 - e^{\alpha + \alpha^*}}}$$

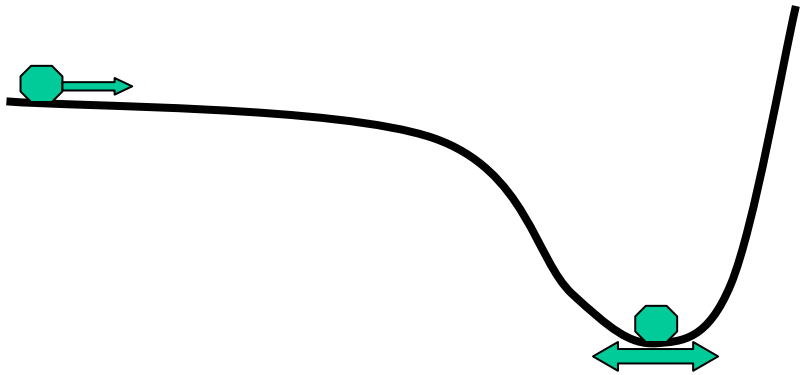
- MA showed these vacua give de Sitter invariant two-point functions
  - ➔local commutator
  - ➔Wightman function has peculiar anti-podal singularities

- Much studied in context of dS/CFT

  Bousso, Maloney and Strominger

  α marginal coupling in CFT

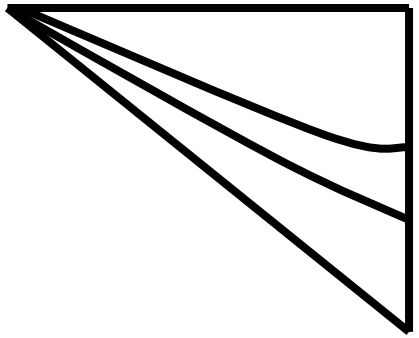
Consequences for CMBR



- Reheating, transition of vacuum energy to zero
- Power-law expansion begins, modes begin to re-enter the horizon
- α–vacuum generates huge back-reaction (k-independent)

$$n_k \propto \frac{e^{\alpha+\alpha^*}}{1-e^{\alpha+\alpha^*}}$$

Cutoff α-vacuum



- Work with proper wavenumber cutoff M_{cut} on set of "nice" timeslices (c.f. Corley and Jacobson)
- Assume trans-Planckian physics above cutoff scale M_{cut}, places modes in an α-vacuum by the time they expand out to cutoff scale

- Back-reaction problem solved

$$M_{cut}^4 \frac{e^{\alpha+\alpha^*}}{1-e^{\alpha+\alpha^*}} << \Lambda = M_{planck}^2 H^2$$

Easy to satisfy

- Correction to CMBR spectrum

$$\left\langle \delta\phi_k{}^2 \right\rangle = \frac{H^2}{2k^3} N_\alpha^2 \left|1-e^{\alpha}\right|^2,$$

- Contribution largely unconstrained

- Expect trans-planckian physics to tie α to M_{cut}
- E.g. Danielsson, Easther, et al.

$$e^{\alpha} = \frac{H}{M_{cut}}$$

- Back-reaction bound becomes

$$H^2 M_{cut}^2 << H^2 M_{planck}^2$$

- Main conclusion: if new physics places modes in an α–vacuum below some UV cutoff, there are no obvious problems, and observable effects are produced
  - ➔ Danielsson; Goldstein,Lowe, : rate of thermalization to Bunch-Davies too slow
  - ➔ See also (Starobinsky,Tkachev; Alberghi, Goldstein, Lowe) for studies of ultra-high energy cosmic ray production

Can local perturbative physics place modes in an α-vacuum?

- Define vacuum in an interacting theory
- Basic assumptions:
  - ➔Theory must be de Sitter invariant
  - ➔Suitably defined two-point correlators should reduce to MA result
- Try to send cutoff to infinity and look for problems…

Two approaches

- Do everything in real time formalism
  - ➔Can think of α-vacuum as squeezed state on Bunch-Davies vacuum
  - ➔Real time ordered propagators match those of MA (Einhorn,Larsen; Collins,Holman,Martin)
- Start with theory on S^4 then continue to real-time
  - ➔Imaginary time propagators match continuation of MA
  - ➔Analyticity leads to new form of real-time ordering (Goldstein, Lowe)

Real-time approach

- Can represent α-vac as a squeezed state on asymptotic future or past infinity

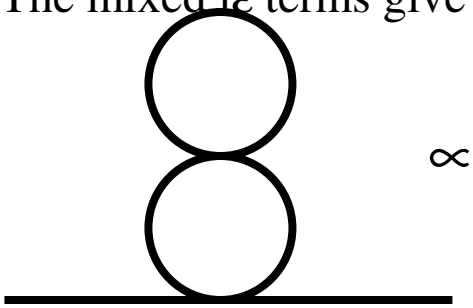
$$|\alpha\rangle = e^{\sum_n c a_n^{+2} - c^* a_n^2} |E\rangle$$

- In position space, time-ordered propagator contains a linear combination of terms with differing iε prescriptions

$$\langle \alpha | T(\phi(x)\phi(y)) | \alpha \rangle \xrightarrow{(x-y)^2 \to 0} \frac{a}{(x-y)^2 + i\varepsilon} + \frac{b}{(x-y)^2 - i\varepsilon}$$

Pinch singularities

- The mixed iε terms give pinch singularities in loops (Einhorn, Larsen)



$$\propto \int \frac{1}{\left((x-y)^2 - i\varepsilon\right)\left((x-y)^2 + i\varepsilon\right)} \propto \frac{1}{\varepsilon}$$

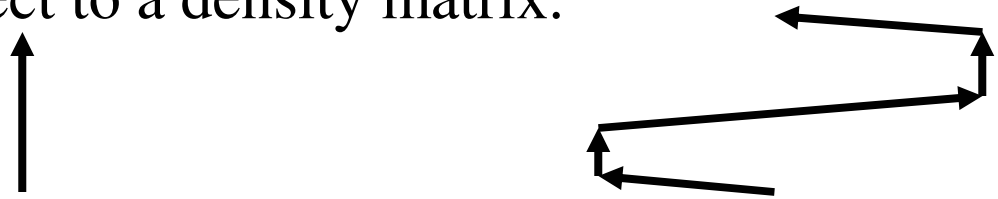
- Note: not a UV divergence, but an IR divergence arising when points are null separated
- Can be regulated
  - ➔ Finite time cutoff
  - ➔ Define resummed propagator

$$\varepsilon \to \Gamma$$

    - Perturbation theory difficult, c.f. non-equilibrium statistical field theory
- IR divergence is a signal of breaking of dS symmetry
- This approach with cutoffs (and subsequent slow decay to Bunch-Davies) still viable phenomenologically (Danielsson; Collin and Martin) but requires fine-tuning of initial state

Imaginary time approach

- Follow similar approach, but for imaginary time, then continue back at end (Goldstein, Lowe)
- In general this approach computes expectation values with respect to a density matrix.



- Truncation of real time contour gives transition amplitudes
- Pinch singularities cannot arise in Euclidean signature
- All divergences take form of UV divergences that can be regulated with local counter-terms

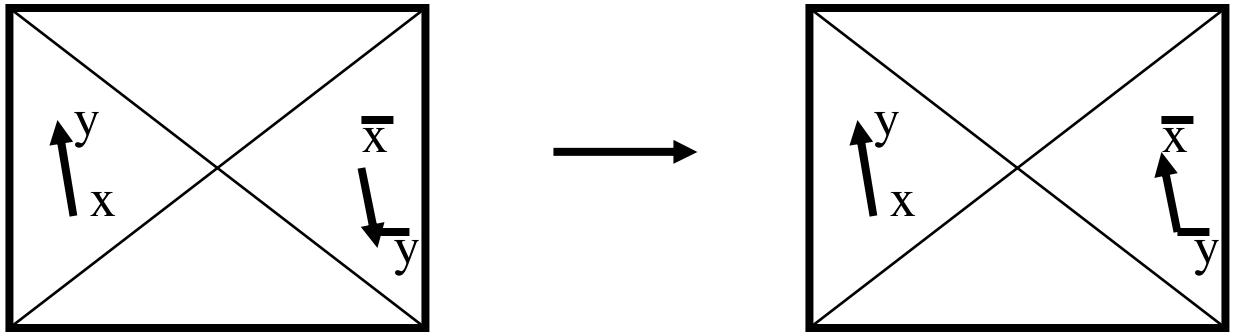
Analytic continuation back to real time

- Can think of field as a bi-local linear combination of Bunch-Davies vacuum fields: $\bar{x}$ is antipodal to x

$$\tilde{\phi}(x) = U^\dagger \phi(x) U$$

$$= N_\alpha \sum_n \left(\phi_n^E(x) + e^\alpha \phi_n^E(\bar{x})\right) a_n + \left(\phi_n^E(x) + e^\alpha \phi_n^E(\bar{x})\right)^* a_n^\dagger$$

- Real-time propagator has new time-ordering prescription



Path integral

- Can represent transition amplitudes with a path integral with action

$$S = \frac{1}{2}\int d^4x\sqrt{-g(x)}\; d^4y\sqrt{-g(y)}\tilde{\phi}(x)K(x,y)\tilde{\phi}(y) - \int d^4x\sqrt{-g(x)}\left(V(\tilde{\phi}) + j(x)\tilde{\phi}(x)\right)$$

- Only sign of non-locality in kinetic term

$$K(x,y) = \left(a\frac{\delta^4(x-y)}{\sqrt{-g(x)}} + b\frac{\delta^4(x-\bar{y})}{\sqrt{-g(x)}}\right)(\Box_x - m^2)$$

- In **fixed** de Sitter background
  - ➔Commutators still local due to local interactions
  - ➔Set of finite renormalizable transition amplitudes
- However always need to couple to gravity at end
  - ➔Stress energy tensor non-local
  - ➔Fluctuations in geometry allow for observable violations of causality
  - ➔Expect strong quantum gravity effects to kick in as in chronology protection conjecture (Hawking)
- Conclude imaginary time approach cannot be consistently coupled to gravity at non-perturbative level

Exotic solutions

- Place graviton in α-vacuum
  - ➔End up with extremely non-local interactions
- Orbifold by antipodal map (Parikh et al.)
  - ➔Solves non-locality problem $x \sim \bar{x}$
  - ➔No longer time orientable, conventional global quantization techniques fail
- Some other non-local and/or non-perturbative solution

Conclusions

- Constructed a useful parameterization of trans-Planckian effects in slow-roll inflation
- No go theorem for producing an α-vacuum via local, perturbative means
  - ➔Have gained a more physical understanding of uniqueness of Bunch-Davies vacuum and Hadamard condition
  - ➔Exotic effects and/or fine tuning required to "pump up" the α-vacua
- If UV "pump" can be explained, decay to Bunch-Davies can be very slow
  - ➔Potentially interesting effects in the CMBR spectrum and production of ultra-high energy cosmic rays