

# ***Cosmology and the origin of structure***

**Rocky I: Dark Energy**

**Rocky II: Dark matter and inflation**

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Fermilab & The University of Chicago***

viale  
dell'Astronomia

via  
della Fisica

# The universe observed

- **Cosmological parameters:**

$H_0$  → Hubble's constant

$q_0$  → deceleration parameter

$\Omega_i$  → the cosmic food chain

(  $\Omega_{\text{TOTAL}}$  ,  $\Omega_M$  ,  $\Omega_B$  ,  $\Omega_\Lambda$  ,  $\Omega_\gamma$  ,  $\Omega_\nu$  , ..... )



$t_0$  → age of the universe

$T_0$  → temperature of the universe

- **Power spectra—characterization of perturbations:**

$P(k)$ ,  $C_l$

- **“Standard model”: Dark Energy and Dark Matter**

$\Lambda$ CDM

# Big-Bang (theory)

Robertson-Walker metric

$a(t)$  = cosmic scale factor

$k = 0, \pm 1$

$$ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$$

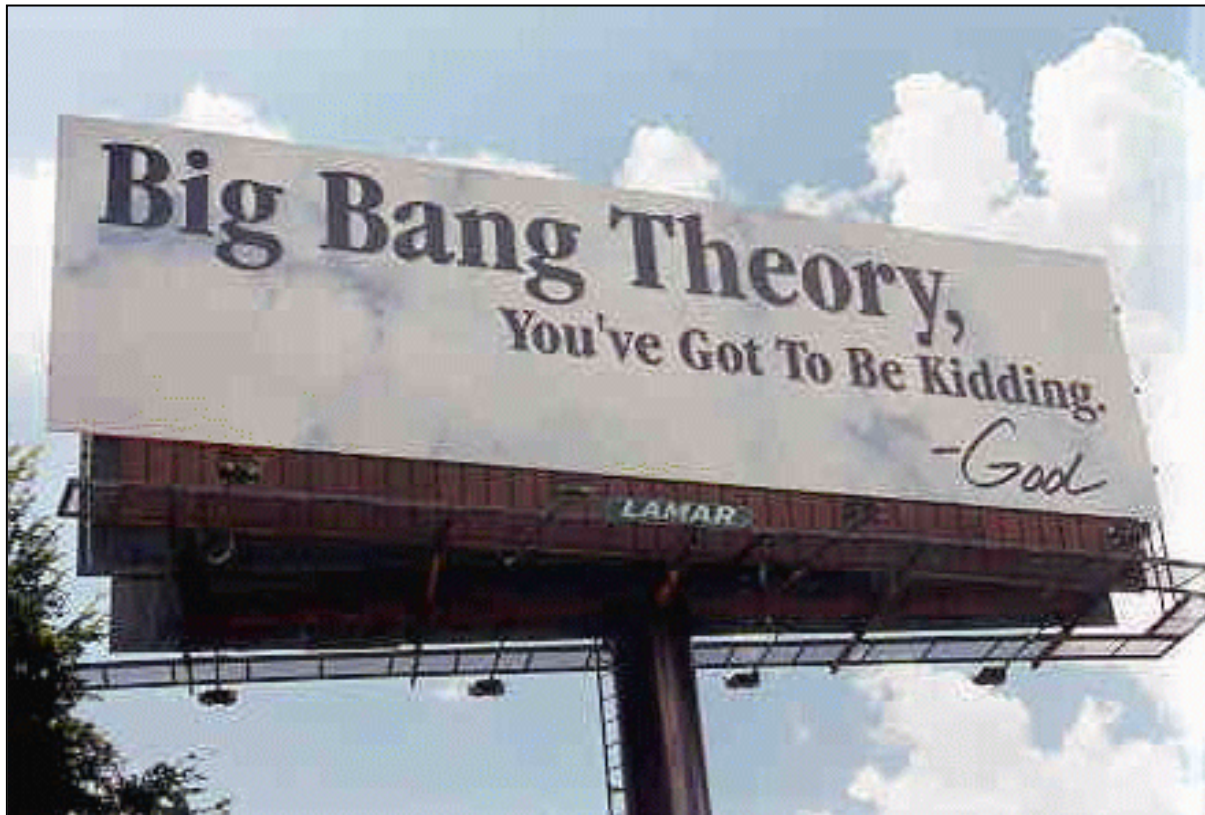
$$\leftarrow G_{\mu\nu} = 8\pi G T_{\mu\nu} \rightarrow$$

Perfect-fluid stress tensor

$\rho$  = energy density

$p$  = pressure

$T^\mu_\nu = \text{diag}(\rho, p, p, p)$



# Field equations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \rho \quad H \equiv \frac{\dot{a}}{a} = \text{expansion rate}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p) \quad q \equiv -\frac{\ddot{a}}{a} \frac{1}{H^2} = \text{deceleration parameter}$$

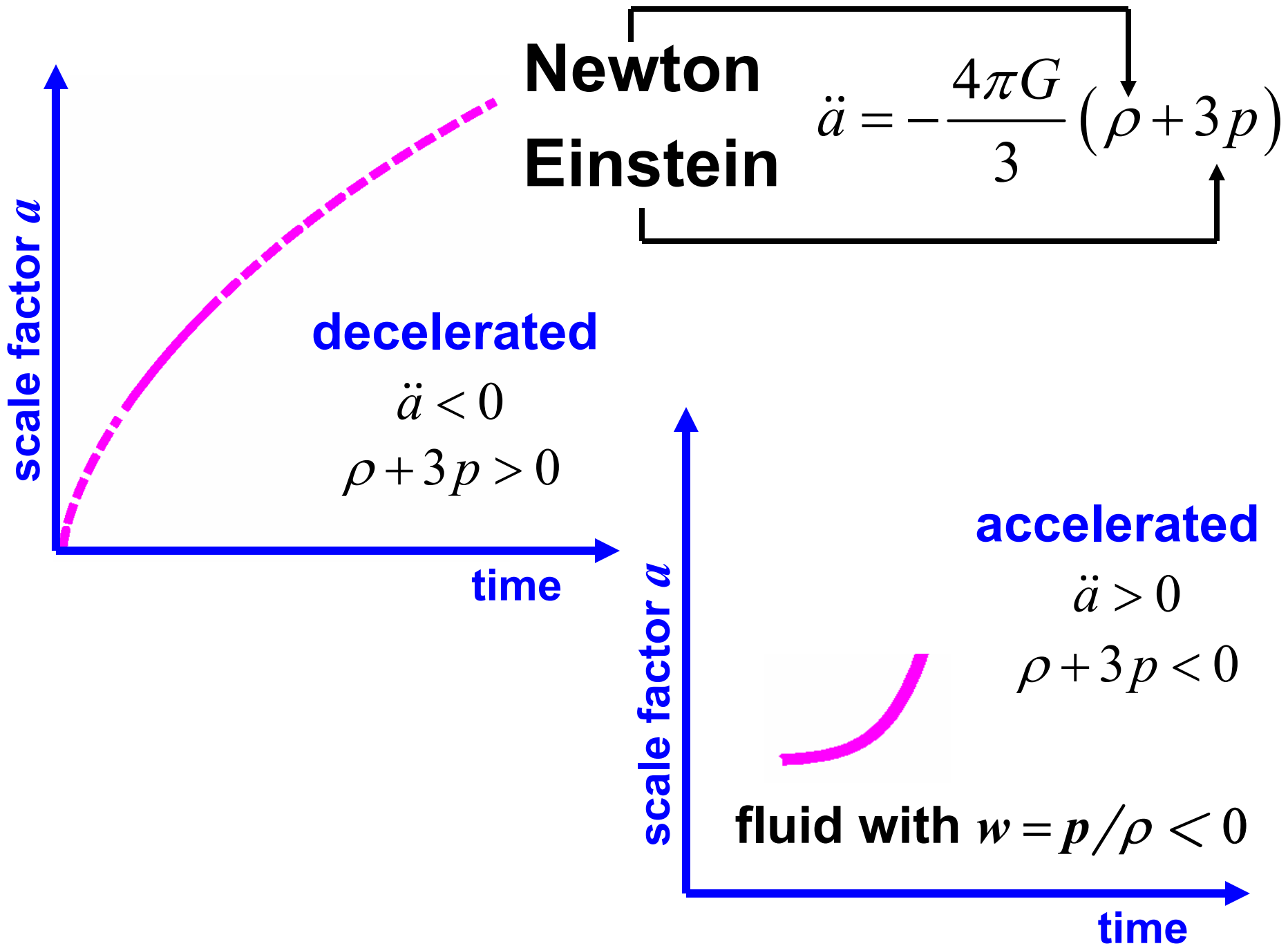
$$T^{\mu\nu}_{;\nu} = 0 \quad \rho \propto a^{-3(1+w)} \quad w = p/\rho$$

**$T^{\mu\nu}$ : fluids with different  $w$**

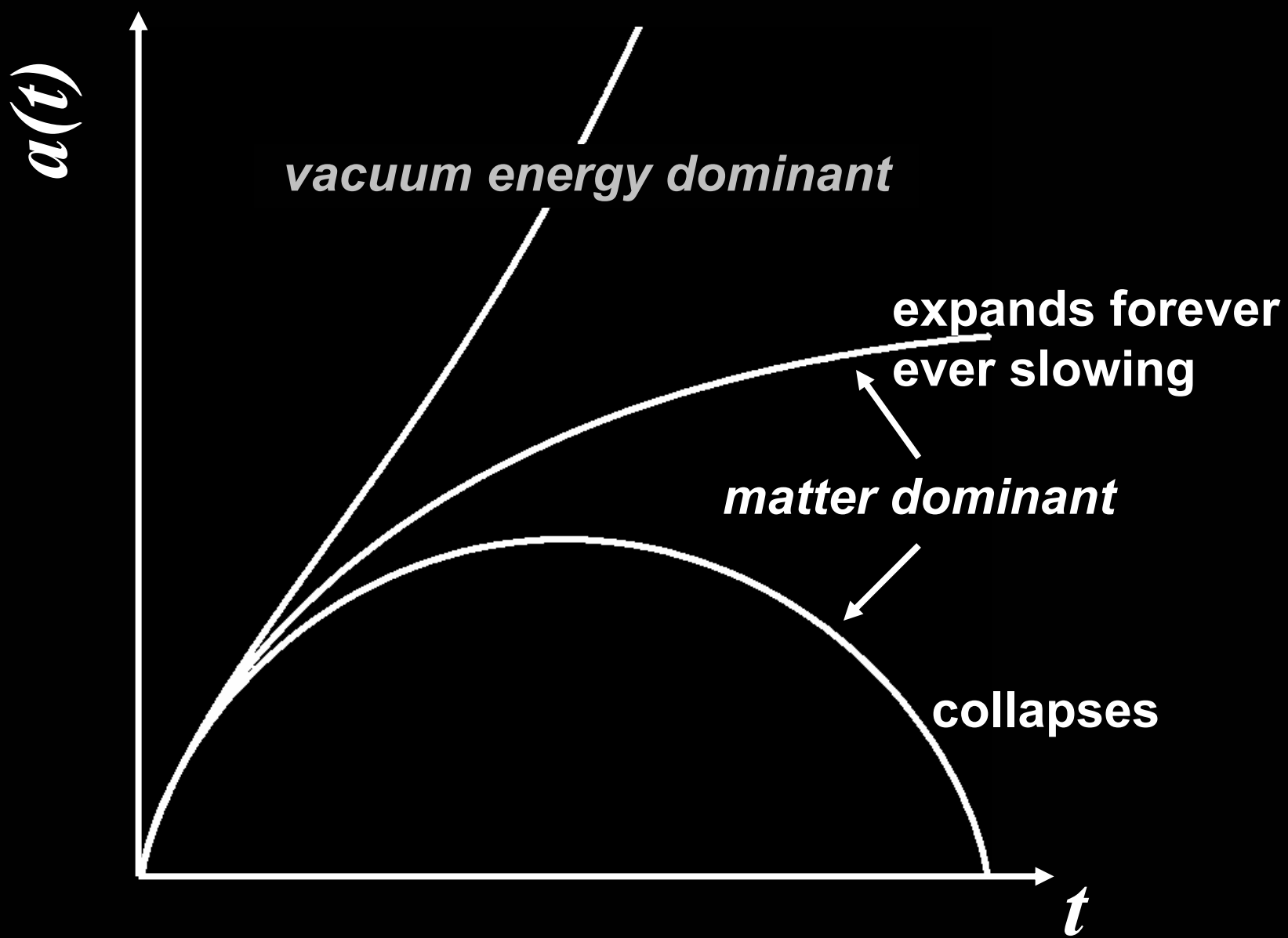
matter:  $p_M = 0$   $w = 0$   $\rho_M \propto a^{-3}$

radiation:  $p_R = \rho_R/3$   $w = 1/3$   $\rho_R \propto a^{-4}$

vacuum:  $p_\Lambda = -\rho_\Lambda$   $w = -1$   $\rho_\Lambda \propto a^0$



# Cosmological constant (dark energy)



# Evolution of $H(z)$ is a key quantity

Many observables based on the comoving distance  $r$

$$r(z) = \int_0^z \frac{dz'}{H(z')}$$

- Luminosity distance

$$d_L(z) \propto r(z)(1+z)$$

- Angular diameter distance

$$d_A(z) \propto \frac{r(z)}{(1+z)}$$

- Comoving volume element

$$\frac{dV(z)}{dz d\Omega} \propto \frac{r^2(z)}{H(z)}$$

- Age of the universe

$$t(z) \propto \int_0^z dz' \frac{1}{(1+z')H(z')}$$

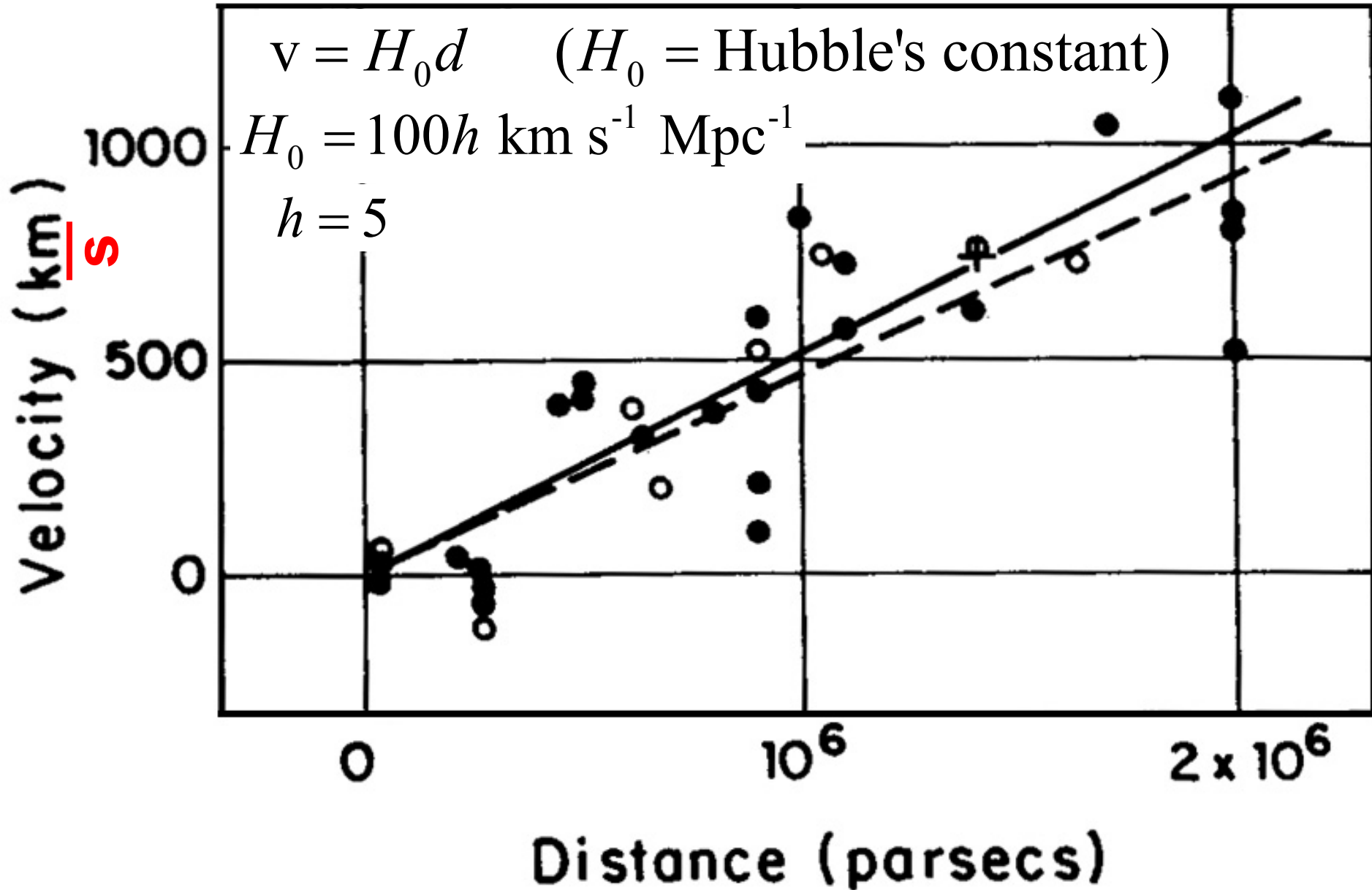


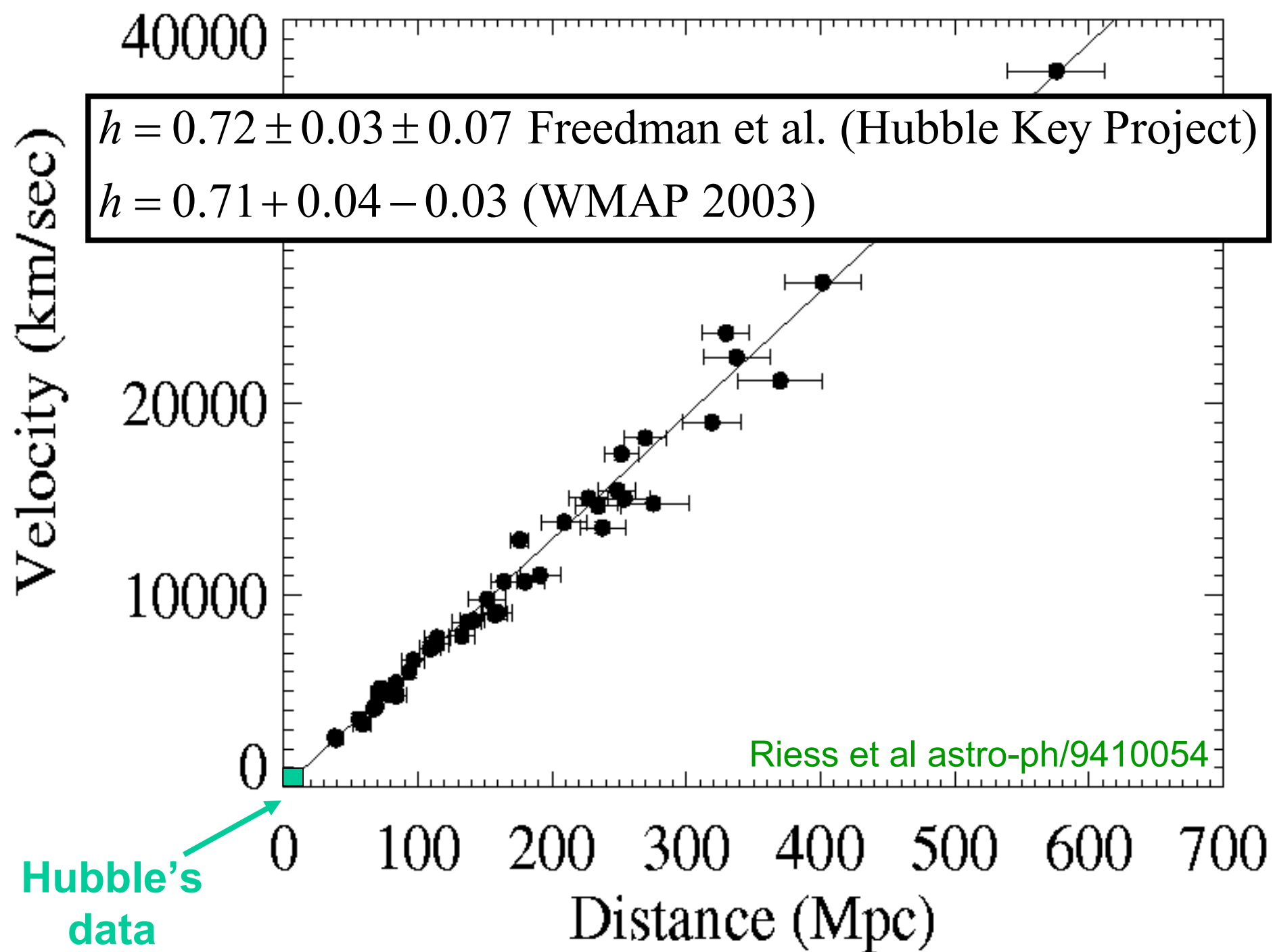
Hubble



**University of Chicago 1909 National Champions**

# Hubble's Discovery Paper - 1929





# Distance-redshift relation

$F = \frac{L}{4\pi d_L^2}$  defines **luminosity distance** - "know"  $L$ , measure  $F$

$4\pi d_L^2 =$  area of  $^2S$  centered on source at time of detection,  $t_0$

$$ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right) \Rightarrow \text{Area} = 4\pi a^2(t_0) r^2$$

Conservation of energy: flux redshifted:  $(1+z)^2 = \left[ a(t_0)/a(t_1) \right]^2$

redshift of energy  $\times$  redshift of time interval:  $(1+z)^2$

$$F = \frac{L}{4\pi a^2(t_0) r^2 (1+z)^2} \Rightarrow \boxed{d_L = a(t_0) r (1+z)}$$

light from comoving coordinate  $r$  at time  $t_1$  reaches us now **redshifted** by an amount  $(1+z) = a(t_0)/a(t)$

# Distance-redshift relation

$$d_L = a(t_0) r (1 + z)$$

$$ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$$

light travels on geodesics

$$ds^2 = 0 \quad \int \frac{dr}{\sqrt{1 - kr^2}} = \int \frac{dt}{a(t)} = \int \frac{da}{H(a)a^2}$$

$$H^2 = H_0^2 \left[ (1 - \Omega_{\text{TOTAL}})(1 + z)^2 + \Omega_M (1 + z)^3 + \dots \right]$$

$$\Omega_i = \rho_i / (3H_0^2 / 8\pi G)$$

$$\Omega_{\text{TOTAL}} = \Omega_M + \Omega_\Lambda + \Omega_R + \Omega_w + \dots \quad (1 - \Omega_{\text{TOTAL}}) \propto k$$

$$\int_0^r \frac{dr'}{\sqrt{1 - kr'^2}} = \int_0^z \frac{a^{-1}(t_0)H_0^{-1} dz'}{\sqrt{(1 - \Omega_0)(1 + z')^2 + \Omega_M (1 + z')^3 + \Omega_w (1 + z')^{3(1+w)} + \dots}}$$

# Distance-redshift relation

$$d_L = a(t_0) r (1 + z)$$

$$a(t_0) r \text{ from } \int_0^r \frac{dr'}{\sqrt{1 - kr'^2}} = \int_0^z \frac{a^{-1}(t_0) H_0^{-1} dz'}{\sqrt{(1 - \Omega_{\text{TOTAL}})(1 + z')^2 + \Omega_M (1 + z')^3 + \dots}}$$

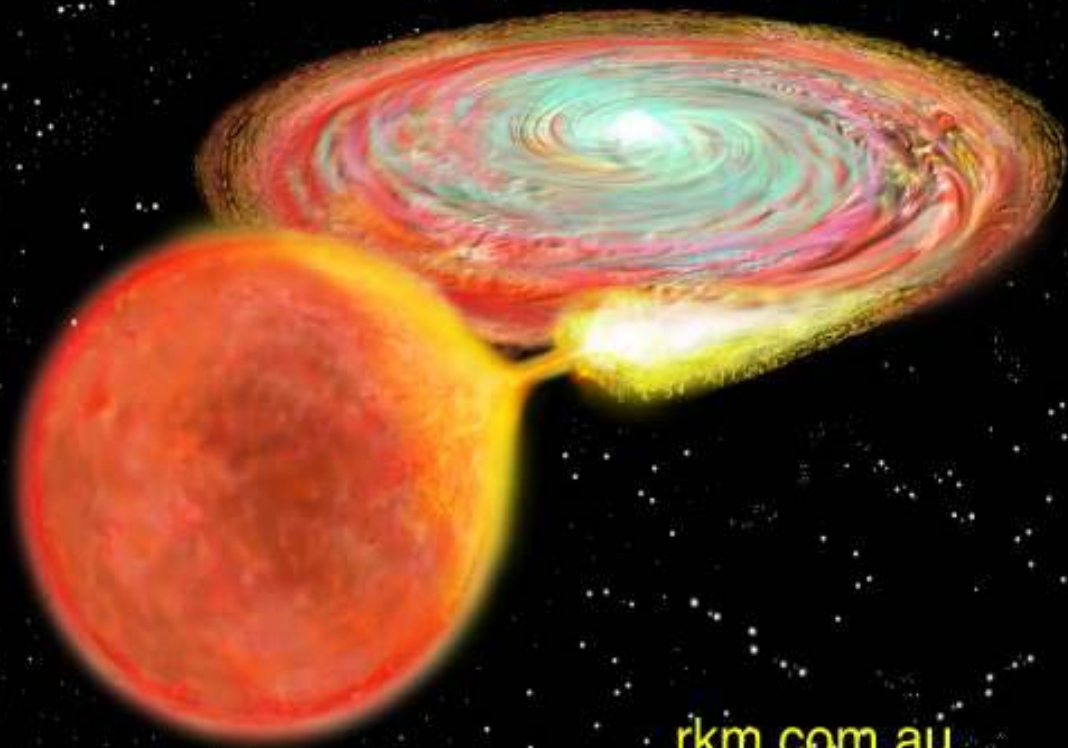
## Program:

- measure  $d_L$  (via  $d_L^2 = L/4\pi F$ ) and  $z$
- input  $\Omega_i$  (a model cosmology) and calculate  $a(t_0) r$
- compare to data  $F = L/4\pi d_L^2$
- need bright “standard candle”

**Monastic Chronicles re: Supernova 1006:**



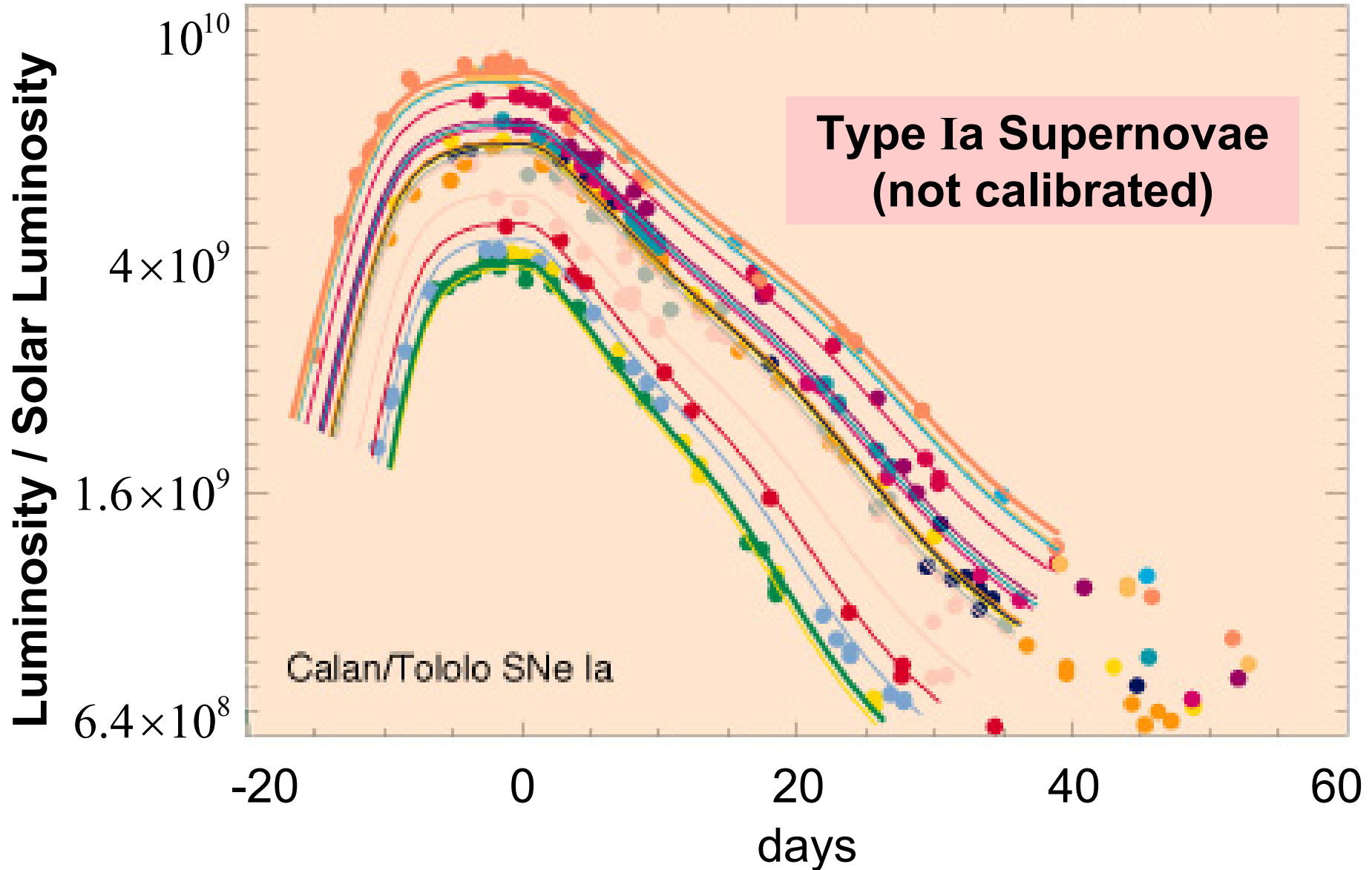
# Type Ia supernova





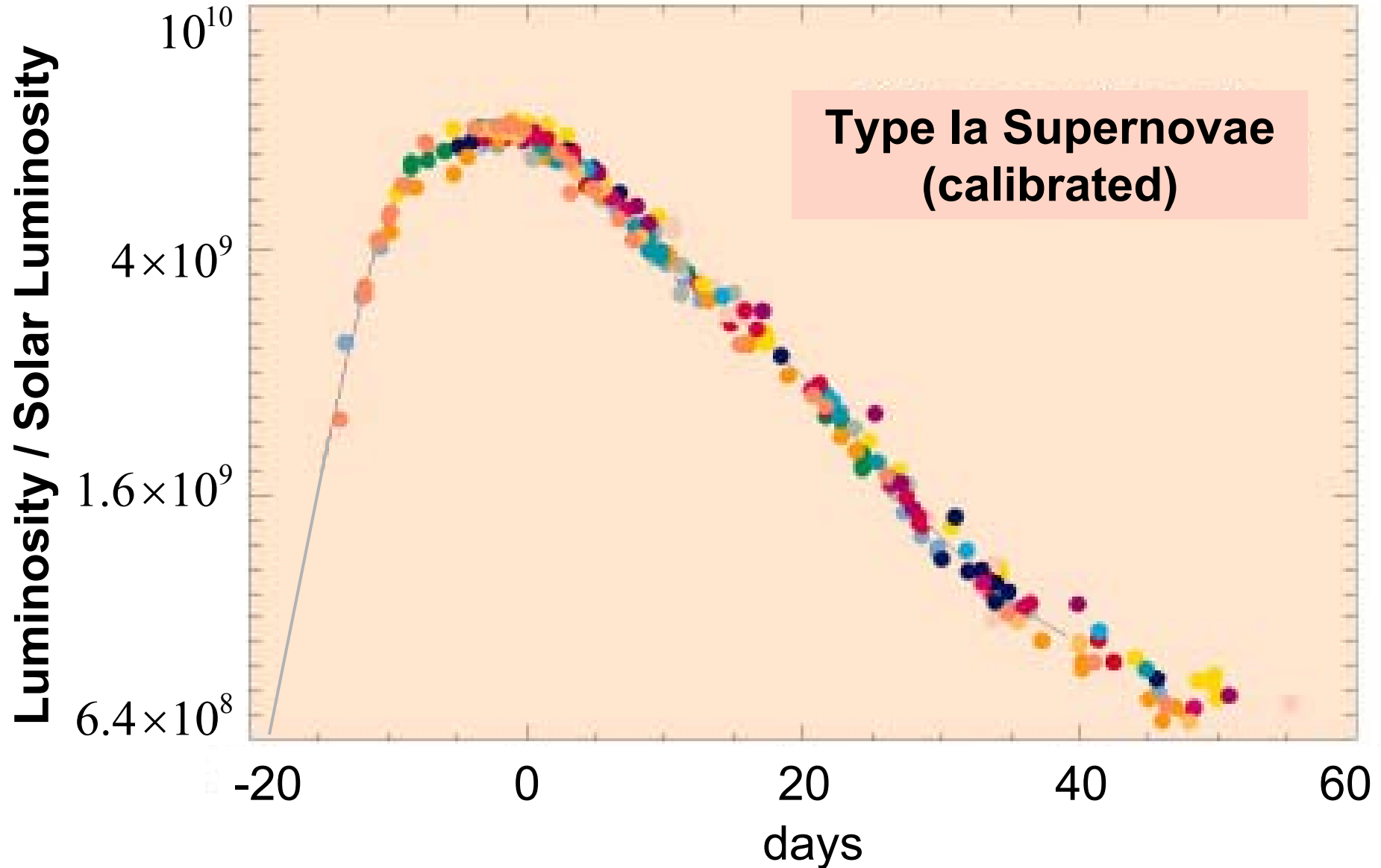
# Type Ia supernova

Supernova Cosmology Project

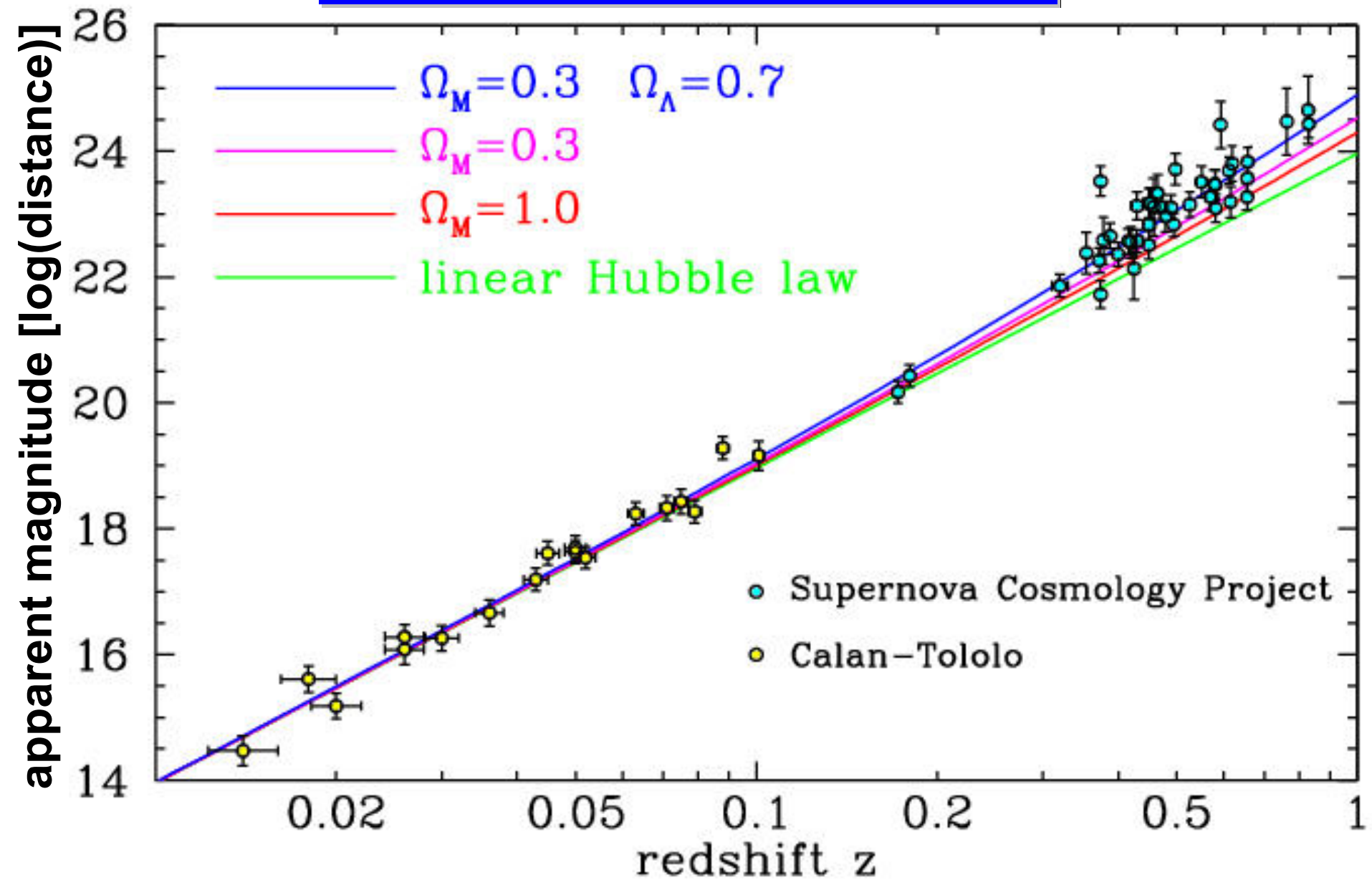


# Type Ia supernova

Supernova Cosmology Project



# Type Ia supernova



# *The mysterious language of astronomy*

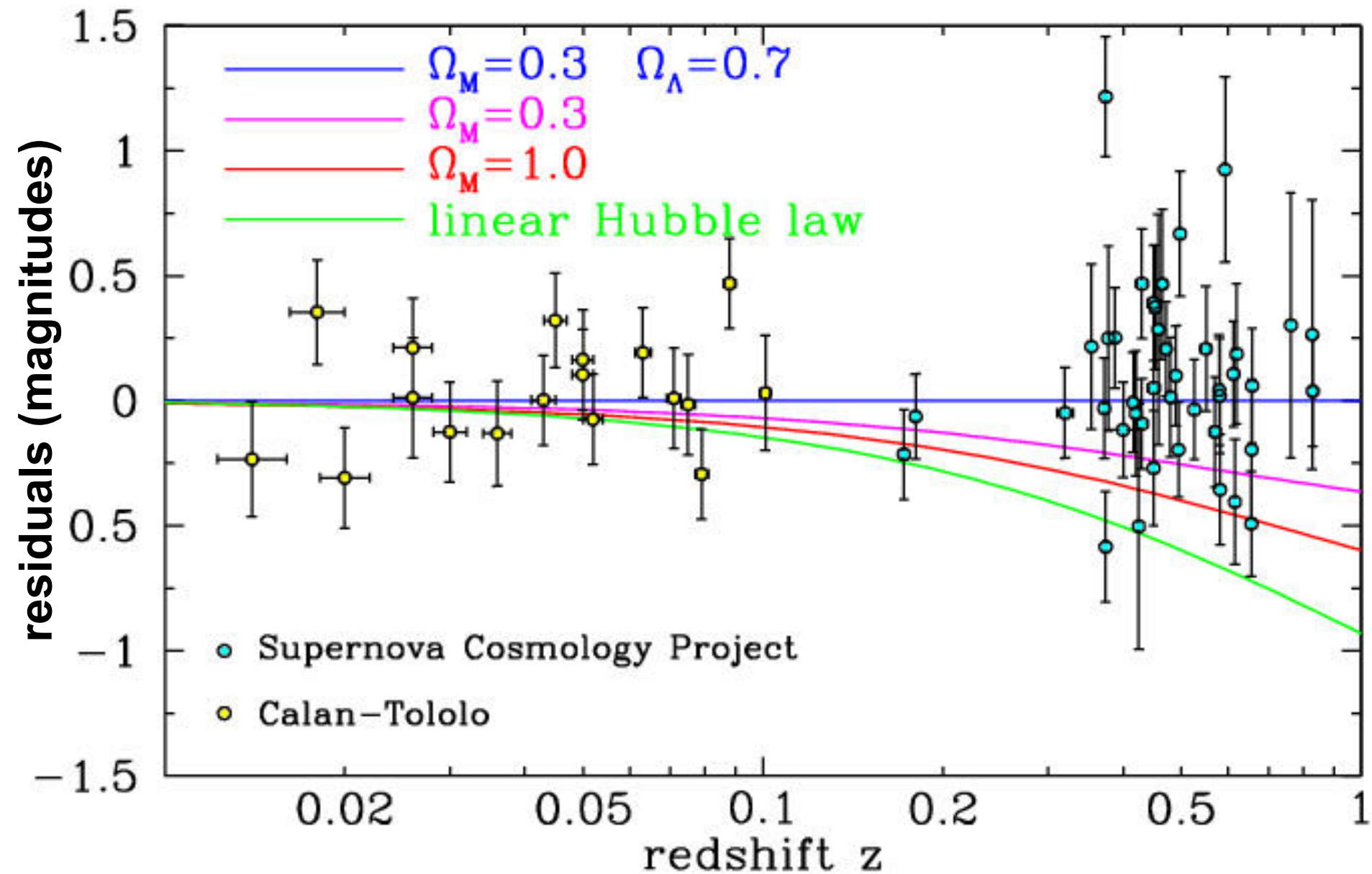
$$m_1 - m_2 = -2.5 \log \frac{I_1}{I_2} \quad \text{apparent magnitude}$$

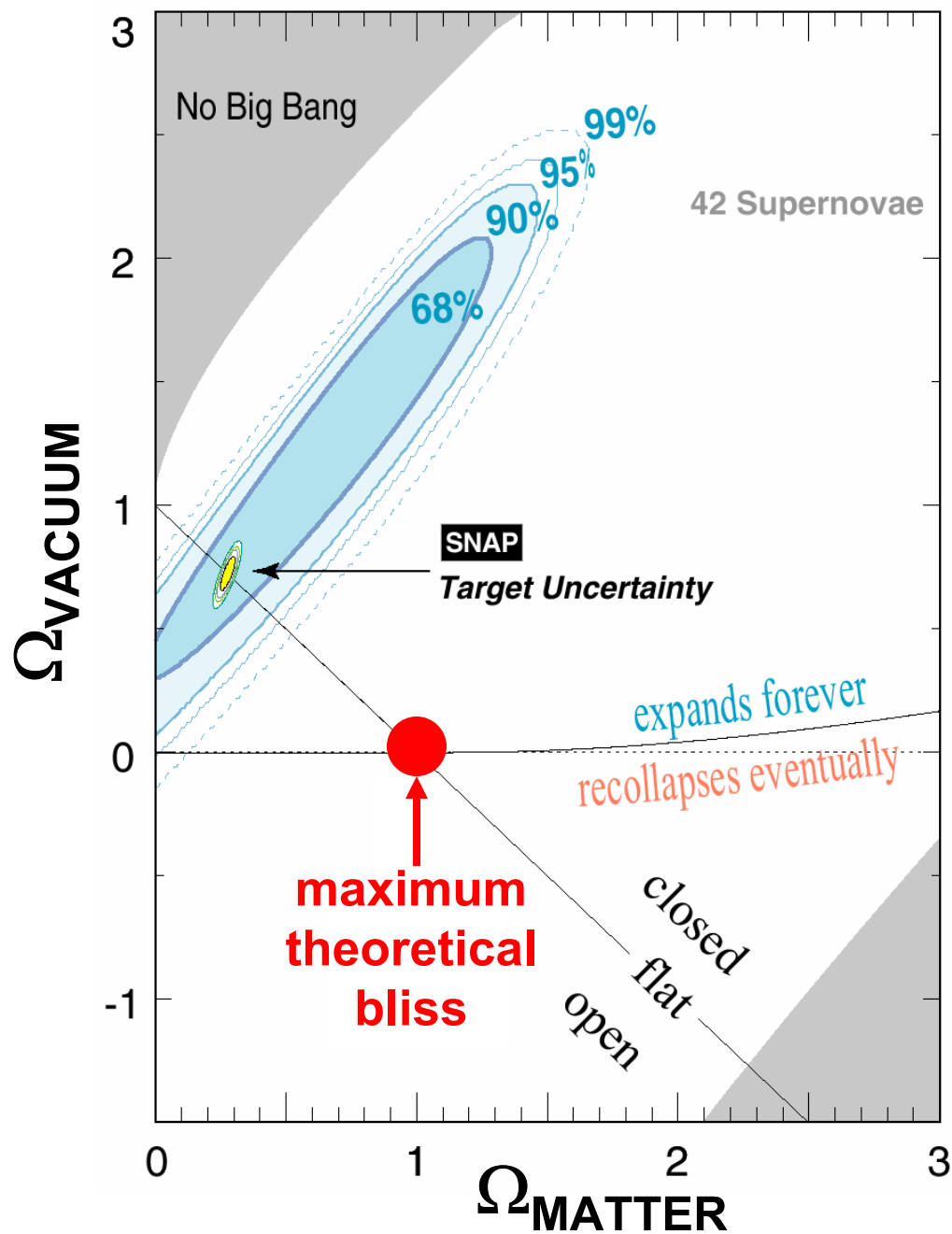
$$M_1 - M_2 = -2.5 \log \frac{L_1}{L_2} \quad \text{absolute magnitude}$$

$$I = \frac{L}{4\pi d_L^2} \quad \text{definition of luminosity distance}$$

$$m \propto 5 \log d_L$$

# Type Ia supernova





High-z SNeIa are fainter than expected in an Einstein-deSitter model

cosmological constant, or ...some changing non-zero vacuum energy, or ... or some unknown systematic effect(s)

The case for  $\Lambda$ :

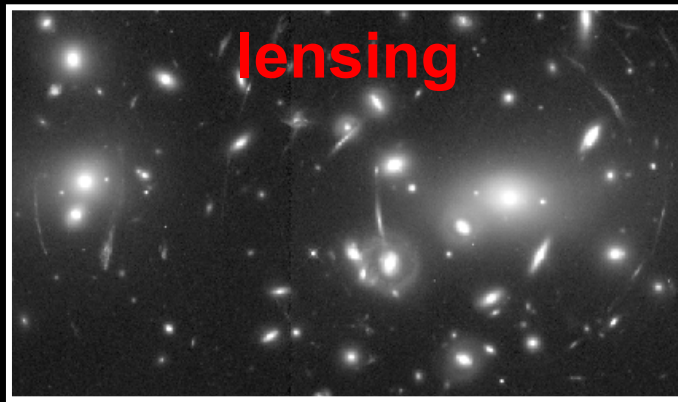
- 1) Hubble diagram
- 2) subtraction

# Matter $\Omega_M \sim 0.3$

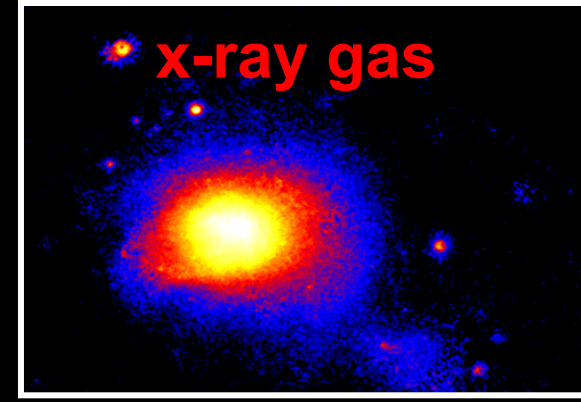
dynamics



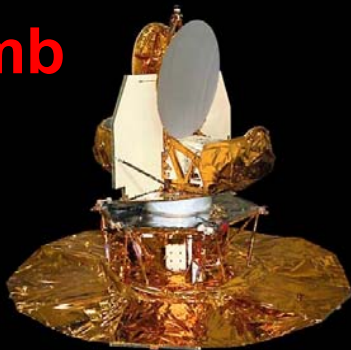
lensing



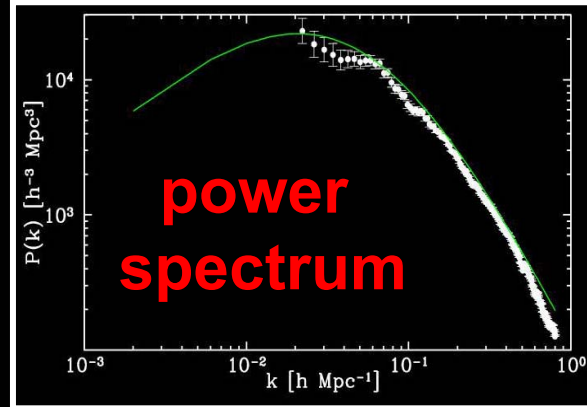
x-ray gas



cmb



simulations



$$\Omega_{\text{TOTAL}} = 1, \quad \Omega_M = 0.3$$

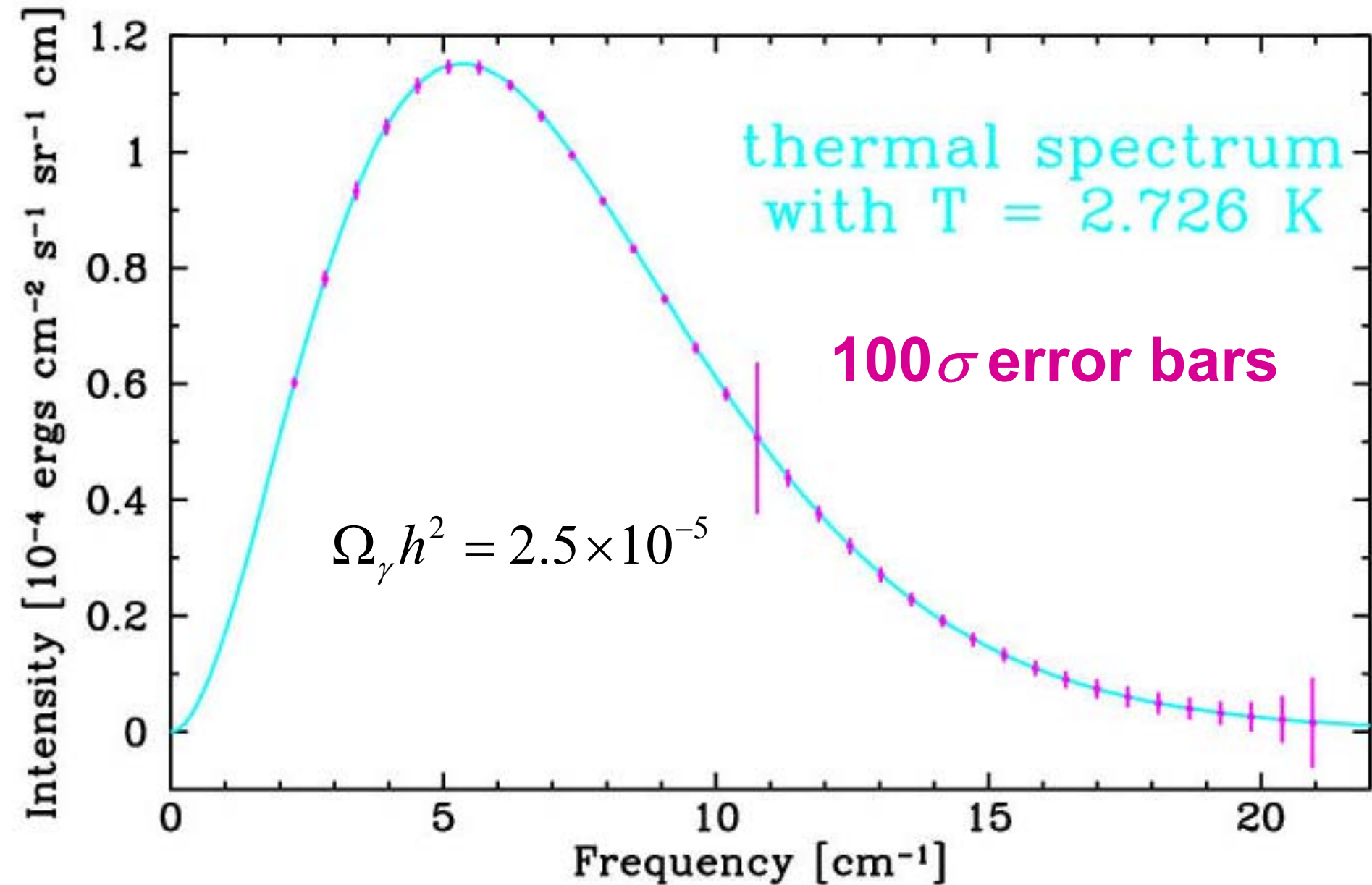
$$1 - 0.3 = 0.7$$

**Theorist's  
view of the  
universe  
(isotropic)**

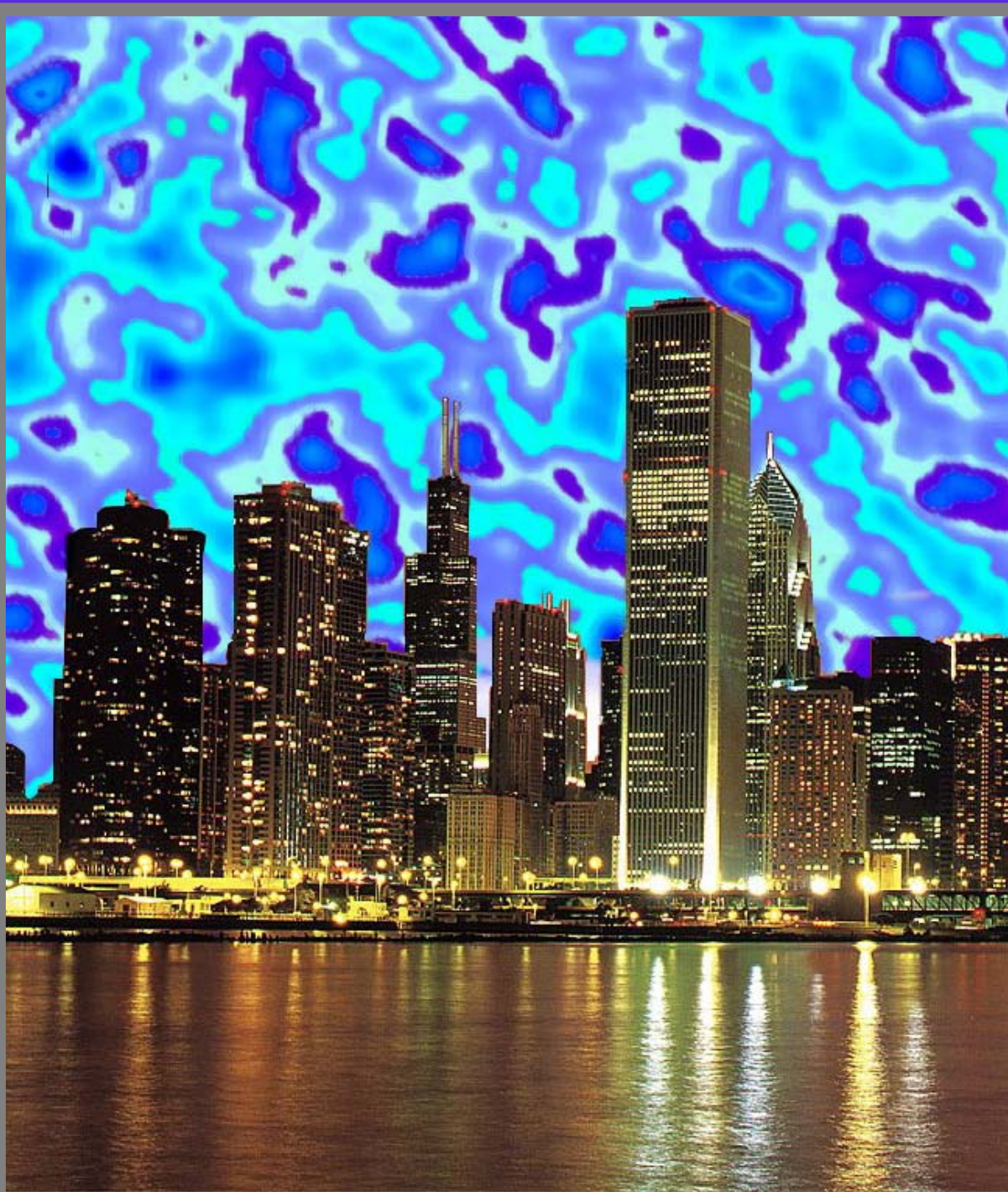




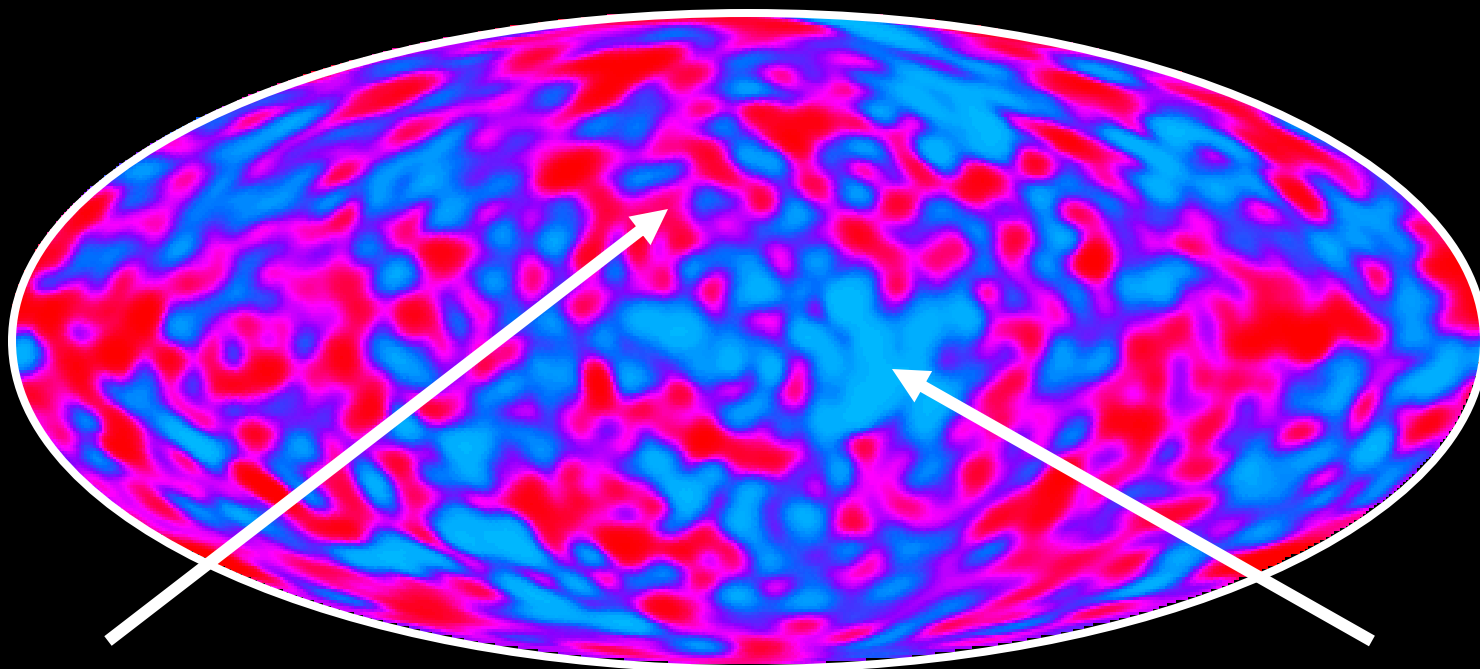
# Temperature of the universe



**Observer's  
view of the  
universe  
(fluctuations)**



# Angular power spectrum



$$\delta T(\theta_1, \phi_1)$$

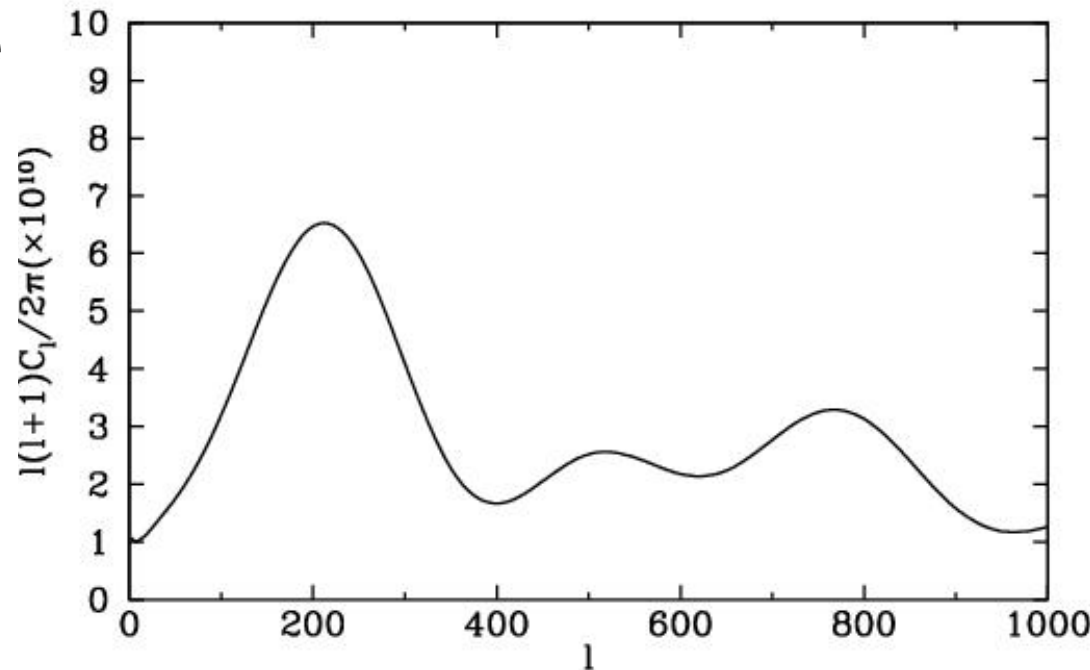
$$\delta T(\theta_2, \phi_2)$$

$$\delta T(\theta, \phi) = \sum a_{lm} Y_{lm}(\theta, \phi)$$

$$C_l \equiv \left\langle |a_{lm}|^2 \right\rangle$$

# Acoustic peaks

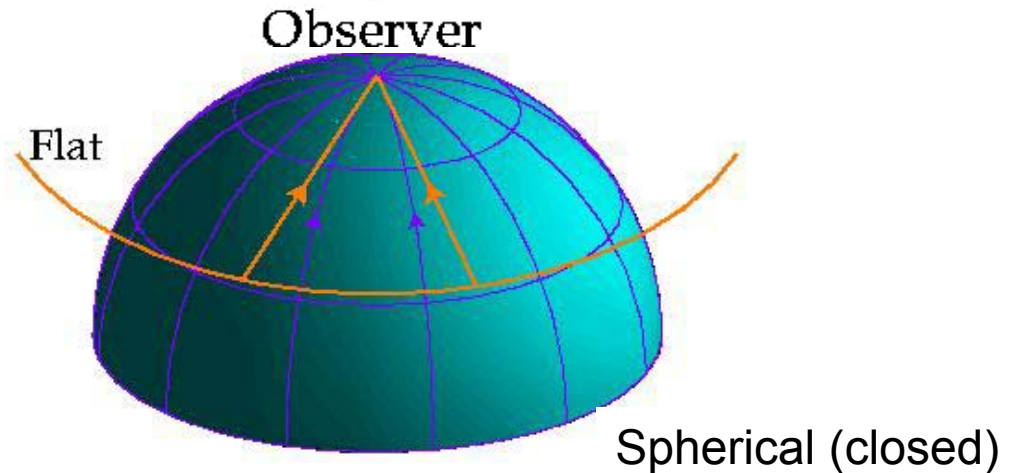
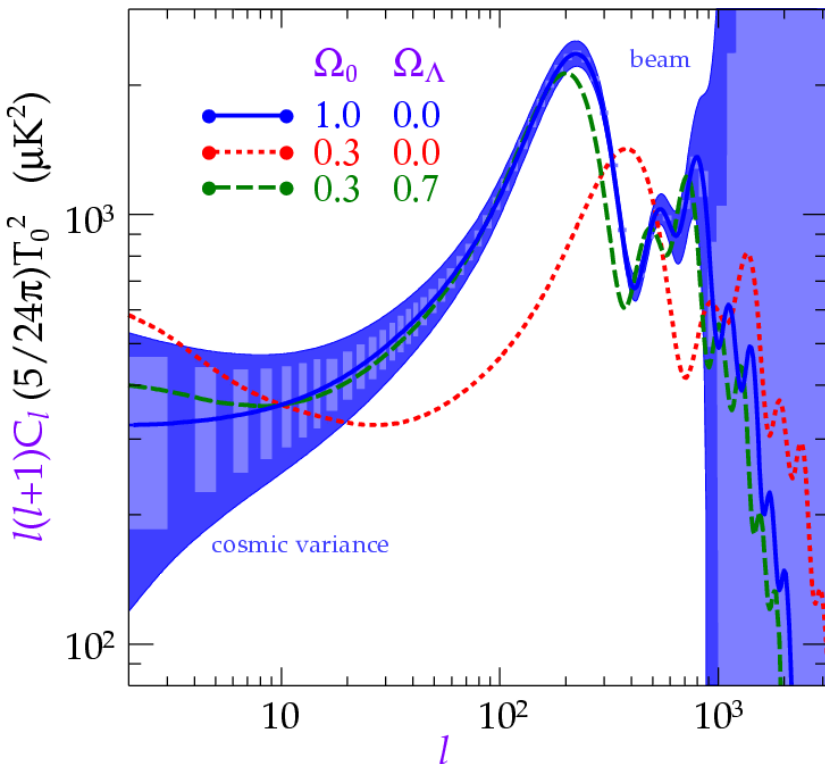
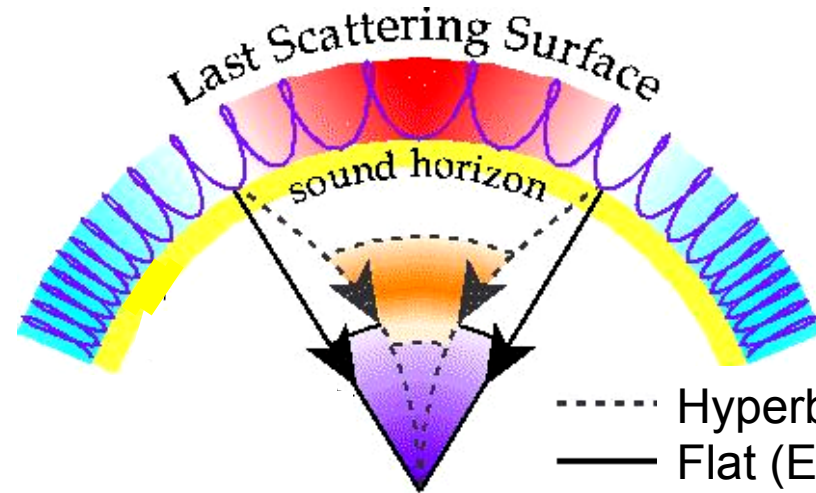
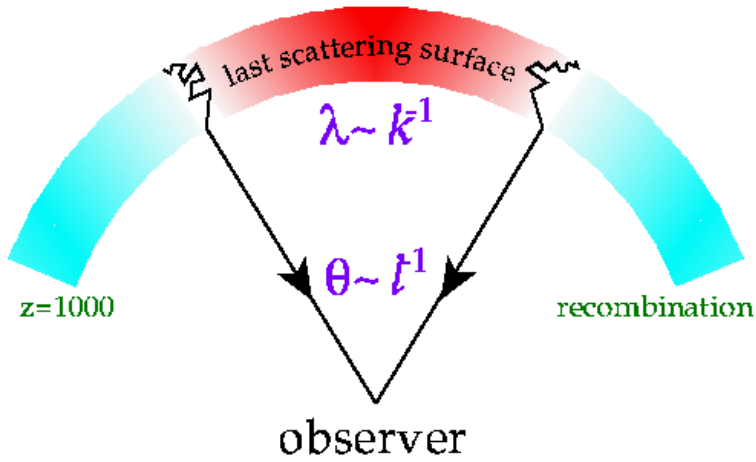
- At recombination, baryon–photon fluid undergoes “acoustic oscillations”  $A \cos kt + B \sin kt$
- Compressions and rarefactions change  $T_\gamma$
- Peaks in  $\Delta T_\gamma$  correspond to extrema of compressions and rarefactions
- Multipole number corresponds to a physical length scale



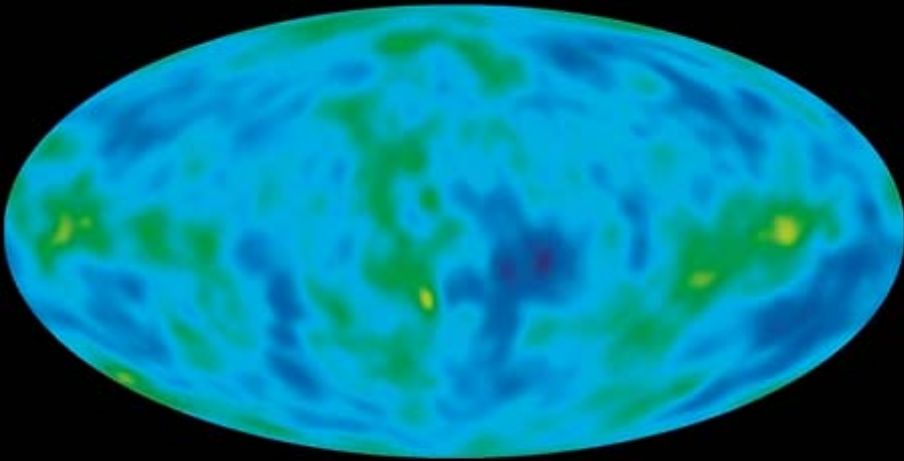
# Acoustic peaks

Sound travel distance known

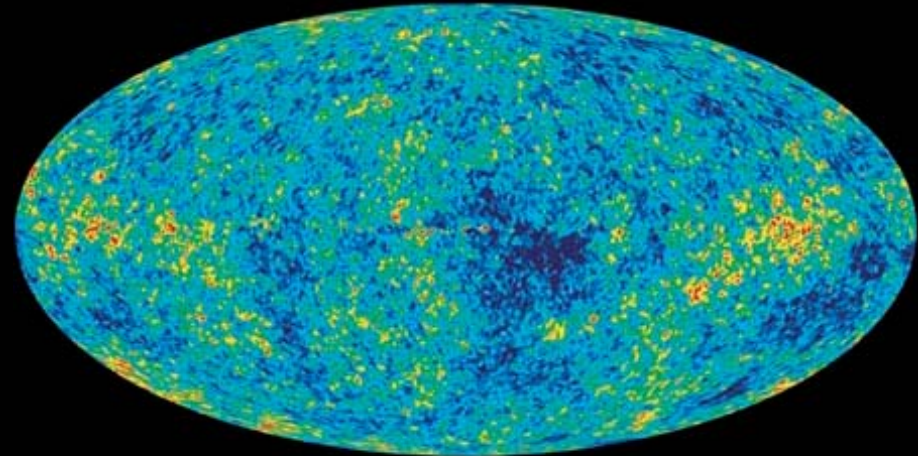
Observed  $l_{\text{peak}} \sim$  geometry



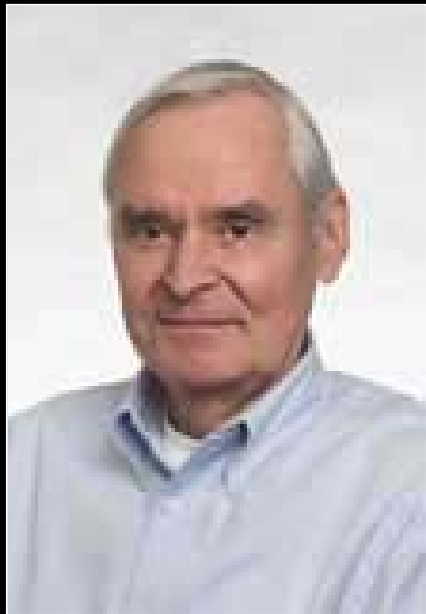
# WMAP



COBE



WMAP



David T. Wilkinson  
1935-2002



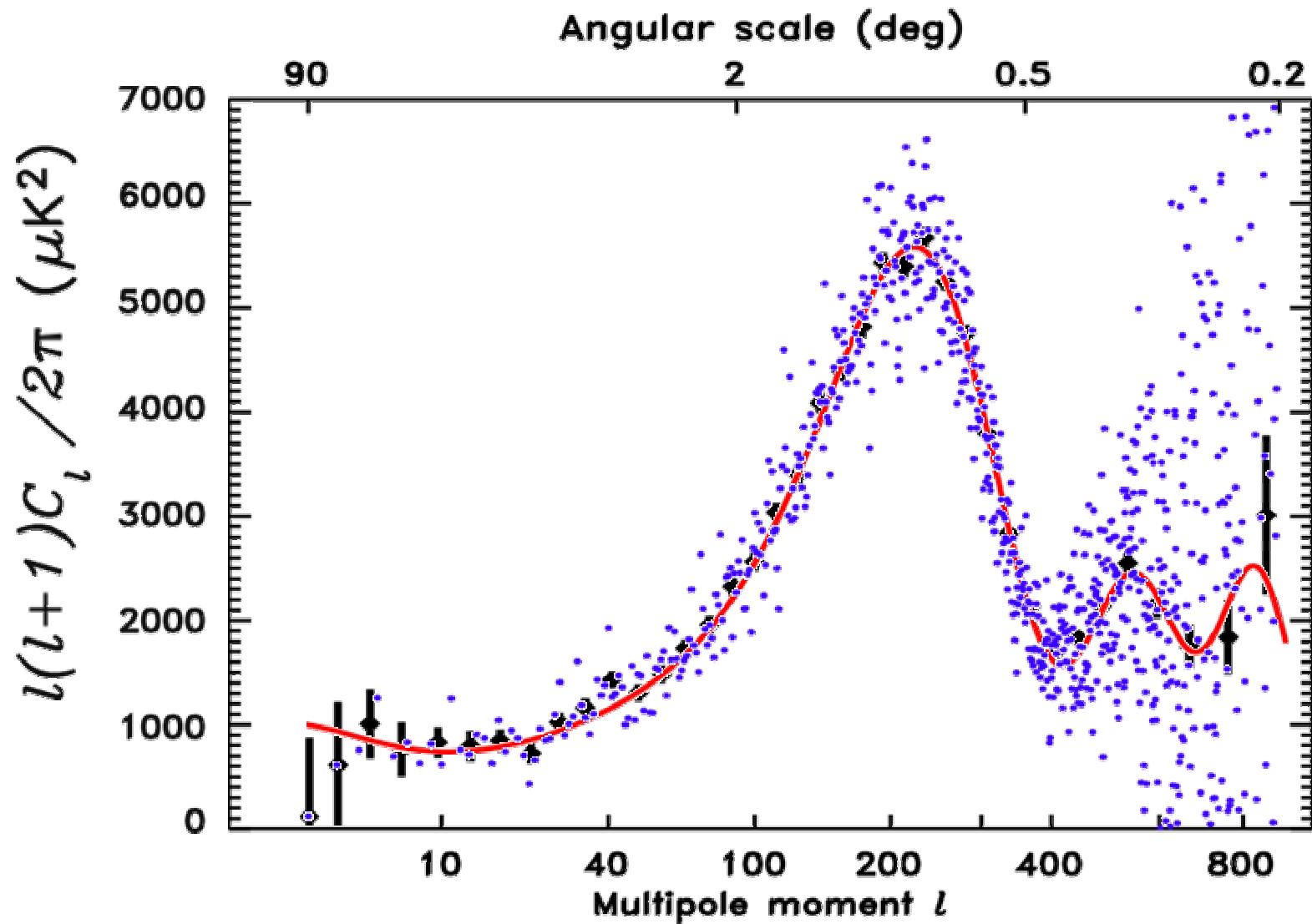
WMAP model

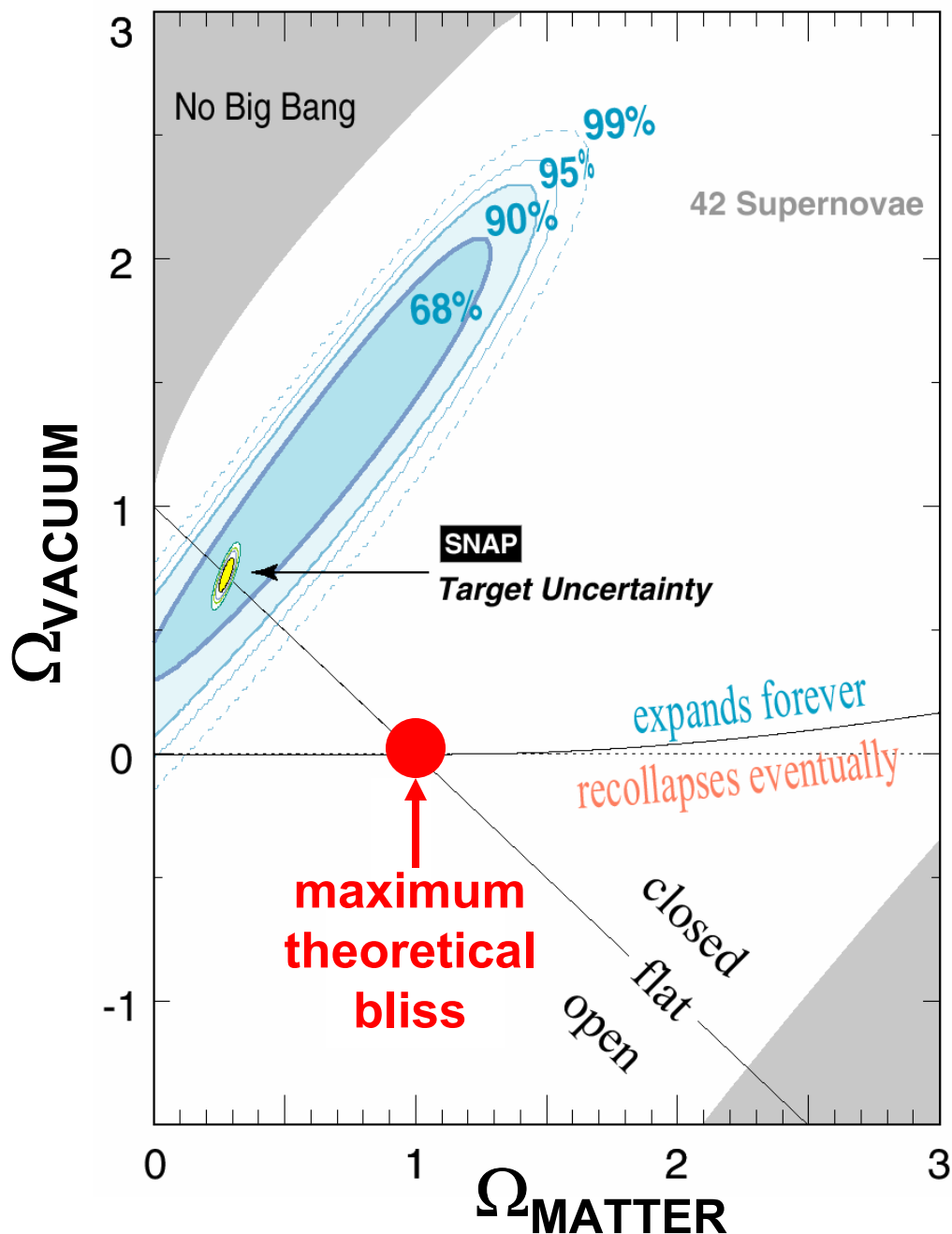


WMAP science team

# WMAP Angular Power Spectrum

$$\Omega_{\text{TOTAL}} = 1.02 \pm 0.02$$





High- $z$  SNeIa are fainter than expected in an Einstein-deSitter model

cosmological constant, or ...some changing non-zero vacuum energy, or ... or some unknown systematic effect(s)

The case for  $\Lambda$ :

- 1) Hubble diagram
- 2) subtraction

$$\Omega_{\text{TOTAL}} = 1$$

$$\Omega_M = 0.3$$

$$1 - 0.3 = 0.7$$

- 3) age of the universe



# $t_0$ : age of the universe

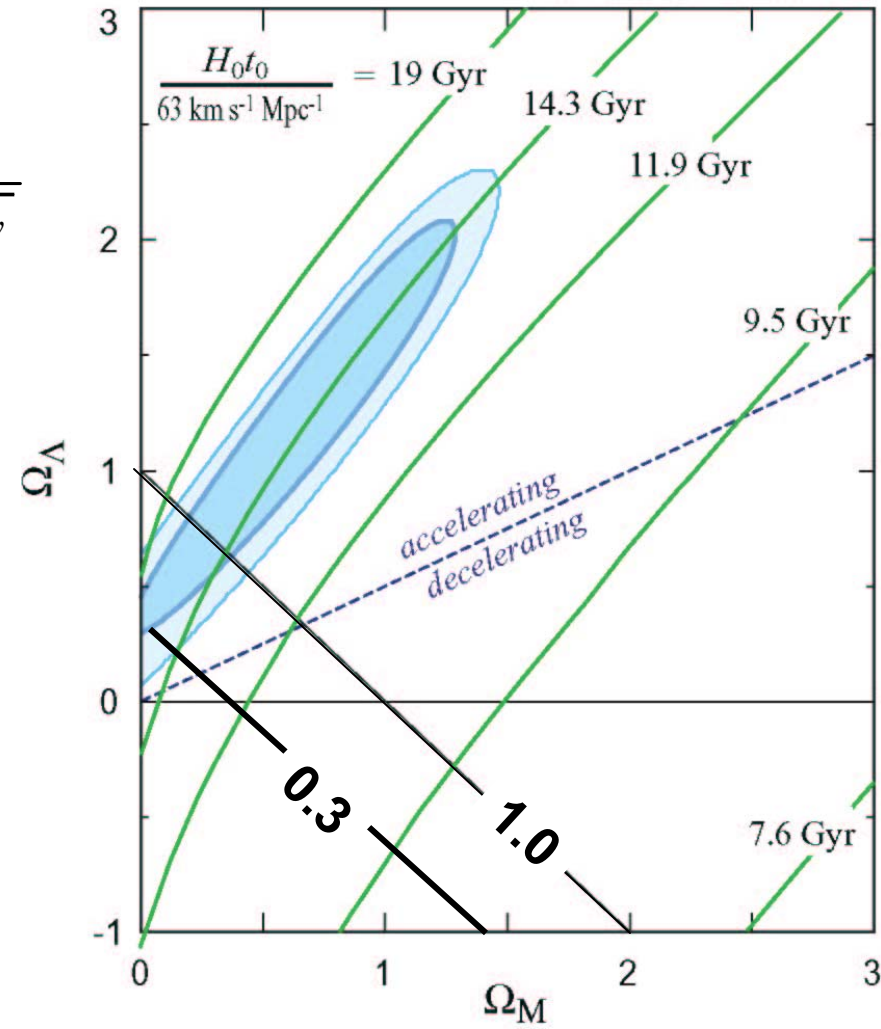
Example: matter + lambda  $\rightarrow \Omega_{\text{TOTAL}} = \Omega_M + \Omega_\Lambda$

Integrate Friedmann equation:

$$H_0 t = \int_0^1 \frac{dx}{\sqrt{1 - \Omega_{\text{TOTAL}} + \sum_w \Omega_w x^{-1+3w}}}$$

$$\Omega_{\text{TOTAL}} = \Omega_M + \Omega_\Lambda$$

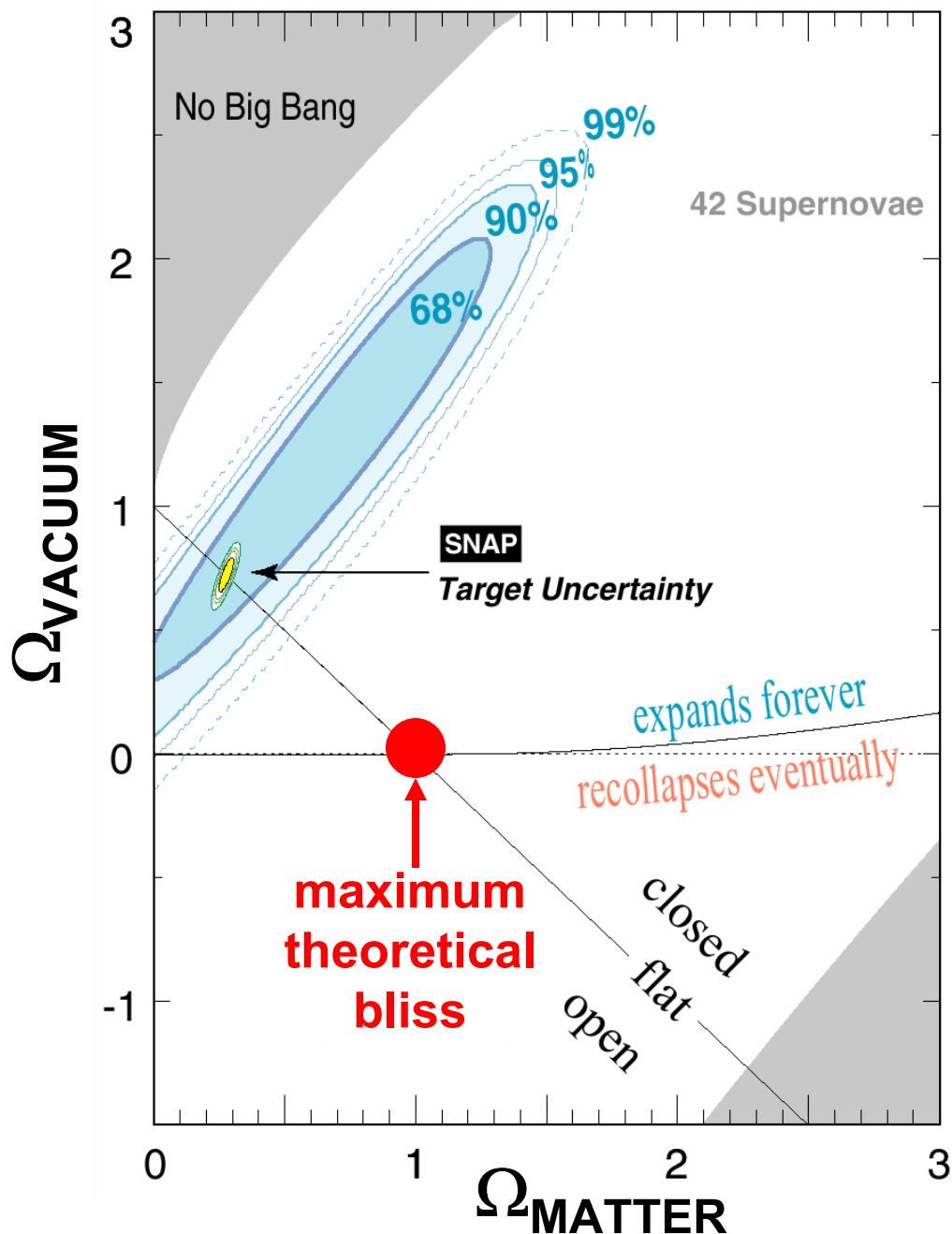
Supernova Cosmology Project  
Perlmutter et al. (1998)



# $t_0$ : age of the universe

- white dwarf cooling  $11 \pm 2$  Gyr
- nucleocosmochronology  $12.6 \pm 3$  Gyr
- globular cluster evolution  $13.5 \pm 2$  Gyr

$H_0 = 70$	$\Omega_M$	$\Omega_\Lambda$	$t_0$ (Gyr)
Flat	1.0	0	9.3
Open	0.3	0	12
Open	0.2	0	14
Flat	0.3	0.7	13.5
Flat	0.2	0.8	15



High- $z$  SNeIa are fainter than expected in an Einstein-deSitter model

cosmological constant, or ...some changing non-zero vacuum energy, or ... or some unknown systematic effect(s)

The case for  $\Lambda$ :

- 1) Hubble diagram
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$$\Omega_{\text{TOTAL}} = 1$$

$$\Omega_M = 0.3$$

$$1 - 0.3 = 0.7$$

- 3) age of the universe
- 4) structure formation

# ***Cosmology and the origin of structure***

**Rocky I: Dark Energy**

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# *Einstein's* *Biggest Blunder?*

1917 Einstein proposes  
cosmological constant

1929 Hubble discovers  
Expansion of the universe

1934 Einstein calls it  
“my biggest blunder”

1998 Astronomers find  
evidence for it

