DECAY CONSTANTS OF CHARGED PSEUDO-SCALAR MESONS

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Charged mesons formed from a quark and anti-quark can decay to a charged lepton pair when these objects annihilate via a virtual $W^\pm$ boson. Fig. 1 illustrates this process for the purely leptonic decay of a D^+ meson.

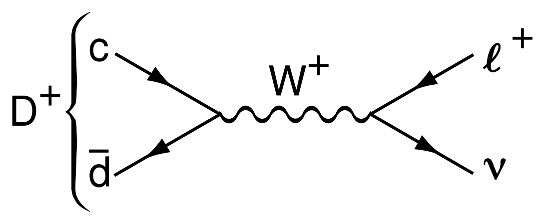

Figure 1: The annihilation process for pure D^+ leptonic decays in the Standard Model.

Similar quark-antiquark annihilations via a virtual W^+ (W^-) to the $\ell^+\nu$ ($\ell^-\bar{\nu}$) final states occur for the $\pi^\pm$, $K^\pm$, $D_s^\pm$, and $B^\pm$ mesons. Let P be any of these pseudoscalar mesons. To lowest order, the decay width is

$$\Gamma(P \to \ell\nu) = \frac{G_F^2}{8\pi} f_P^2 \, m_\ell^2 M_P \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2 |V_{q_1 q_2}|^2 \quad . \qquad (1)$$

Here M_P is the P mass, m_ℓ is the ℓ mass, $|V_{q_1 q_2}|$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element between the constituent quarks $q_1\bar{q}_2$ in P, and G_F is the Fermi coupling constant. The parameter f_P is the decay constant, and is related to the wave-function overlap of the quark and antiquark.

The decay $P^\pm$ starts with a spin-0 meson, and ends up with a left-handed neutrino or right-handed antineutrino. By angular momentum conservation, the $\ell^\pm$ must then also be left-handed or right-handed, respectively. In the $m_\ell = 0$ limit, the decay is forbidden, and can only occur as a result of the finite ℓ mass. This helicity suppression is the origin of the m_ℓ^2 dependence of the decay width.

There is a complication in measuring purely leptonic decay rates. The process $P \to \ell\nu\gamma$ is not simply a radiative correction, although radiative corrections contribute. The P can make a transition to a virtual P^*, emitting a real photon, and the P^* decays into $\ell\nu$, avoiding helicity suppression. The importance of this amplitude depends on the decaying particle and the detection technique. The $\ell\nu\gamma$ rate for a heavy particle such as B decaying into a light particle such as a muon can be larger than the width without photon emission [1]. On the other hand, for decays into a $\tau^{\pm}$, the helicity suppression is mostly broken and these effects appear to be small.

Measurements of purely leptonic decay branching fractions and lifetimes allow an experimental determination of the product $|V_{q_1q_2}|\, f_P$. If the CKM element is well known from other measurements, then f_P can be well measured. If, on the other hand, the CKM element is less well or poorly measured, having theoretical input on f_P can allow a determination of the CKM element. The importance of measuring $\Gamma(P \to \ell\nu)$ depends on the particle being considered. For the B system, f_B is crucial for using measurements of B^0-$\overline{B}^0$ mixing to extract information on the fundamental CKM parameters. Knowledge of f_{B_s} is also needed, but this parameter cannot be directly measured as the B_s is neutral, so the violation of the SU(3) relation $f_{B_s} = f_B$ must be estimated theoretically. This difficulty does not occur for D mesons as both the D^+ and D_s^+ are charged, allowing the direct measurement of SU(3) breaking and a direct comparison with theory. (In this note mention of specific particle charge also implies the use of the charge-conjugate partner.)

For B^- and D_s^+ decays, the existence of a charged Higgs boson (or any other charged object beyond the Standard Model) would modify the decay rates; however, this would not necessarily be true for the D^+ [2,3]. More generally, the ratio of $\mu\nu$ to $\tau\nu$ decays can serve as one probe of lepton universality [2,4].

As $|V_{ud}|$ has been quite accurately measured in super-allowed β decays [5], with a value of 0.97418(26), measurements of $\Gamma(\pi^+ \to \mu^+\nu)$ yield a value for f_π. Similarly, $|V_{us}|$ has been well measured in semileptonic kaon decays, so a value for

f_K from $\Gamma(K^- \to \mu^- \bar{\nu})$ can be compared to theoretical calculations. Recently, however, lattice gauge theory calculations have been claimed to be very accurate in determining f_K, and these have been used to predict $|V_{us}|$ [6].

Next we review current measurements, starting with the charm system. The CLEO collaboration has measured the branching fraction for $D^+ \to \mu^+\nu$ and recently updated their published result [7]. By using the well measured D^+ lifetime of 1.040(7) ps and assuming $|V_{cd}| = |V_{us}| = 0.2255(19)$ [8], they report

$$f_{D^+} = (205.8 \pm 8.5 \pm 2.5)\ \text{MeV}\ . \tag{2}$$

This result includes a 1% correction for the radiative $\mu^+\nu\gamma$ final state based on the estimate by Dobrescu and Kronfeld [9].

Before we compare this result with theoretical predictions, we discuss the D_s^+. Measurements of f_{D_s} have been made by several groups and are listed in Table 1 [10–18]. Early measurements actually determined the ratio of the leptonic decay to some hadronic decay, usually $\Gamma(D_s^+ \to \ell^+\nu)/\Gamma(D_s^+ \to \phi\pi^+)$. This introduces a large additional source of error since the denominator is not well known. CLEO [10] has published absolute branching fractions for $\mu^+\nu$ and $\tau^+\nu$, $\tau^+ \to \pi^+\overline{\nu}$, and in a separate paper [11] for $\tau^+\nu$, $\tau^+ \to e^+\nu\overline{\nu}$; there is also an as-yet-unpublished result from Belle [12] for $\mu^+\nu$.

We extract the decay constant from the measured branching ratios using $|V_{cs}| = 0.9742$, and a D_s lifetime of 0.50 ps. Our experimental average,

$$f_{D_s} = (273 \pm 10)\ \text{MeV}, \tag{3}$$

uses only those results that are absolutely normalized [19]. We note that the experiments do not correct explicitly for any $\ell^+\nu\gamma$ that may have been included. We have included the radiative correction of 1% in the $\mu^+\nu$ rates [9] (the $\tau^+\nu$ rates need not be corrected). Other theoretical calculations show that this rate is a factor of 40–100 below the $\mu^+\nu$ rate for charm [20]

Table 2 compares the experimental f_{D_s} with theoretical calculations [21–30]. While most theories give values lower than the f_{D_s} measurement, the errors are sufficiently large,

Table 1: Experimental results for $\mathcal{B}(D_s^+ \to \mu^+\nu)$, $\mathcal{B}(D_s^+ \to \tau^+\nu)$, and $f_{D_s^+}$. Numbers have been updated using the D_s^+ lifetime of 0.50 ps. Results listed below the "Average" line have not been used in our average. The assumed value of $\mathcal{B}_{\phi\pi} \equiv \mathcal{B}(D_s^+ \to \phi\pi^+)$ is listed if evident. ALEPH averages their two results to obtain a value for $f_{D_s^+}$.

Experiment	Mode	$\mathcal{B}$	$\mathcal{B}_{\phi\pi}(\%)$	$f_{D_s^+}$ (MeV)
CLEO-c [10]	$\mu^+\nu$	$(5.94 \pm 0.66 \pm 0.31) \times 10^{-3}$		$264 \pm 15 \pm 7$
CLEO-c [10]	$\tau^+\nu$	$(8.0 \pm 1.3 \pm 0.4) \times 10^{-2}$		$310 \pm 25 \pm 8$
CLEO-c [11]	$\tau^+\nu$	$(6.17 \pm 0.71 \pm 0.36) \times 10^{-2}$		$273 \pm 16 \pm 8$
CLEO-c	combined			$274 \pm 10 \pm 5$
Belle [12]	$\mu^+\nu$	$(6.44 \pm 0.76 \pm 0.57) \times 10^{-3}$		$275 \pm 16 \pm 12$
Average		CLEO combined & Belle		275 ± 10
Average		With radiative correction		273 ± 10
CLEO [13]	$\mu^+\nu$	$(6.2 \pm 0.8 \pm 1.3 \pm 1.6) \times 10^{-3}$	3.6±0.9	$273 \pm 19 \pm 27 \pm 33$
BEATRICE [14]	$\mu^+\nu$	$(8.3 \pm 2.3 \pm 0.6 \pm 2.1) \times 10^{-3}$	3.6±0.9	$312 \pm 43 \pm 12 \pm 39$
ALEPH [15]	$\mu^+\nu$	$(6.8 \pm 1.1 \pm 1.8) \times 10^{-3}$	3.6±0.9	$282 \pm 19 \pm 40$
ALEPH [15]	$\tau^+\nu$	$(5.8 \pm 0.8 \pm 1.8) \times 10^{-2}$		
L3 [16]	$\tau^+\nu$	$(7.4 \pm 2.8 \pm 1.6 \pm 1.8) \times 10^{-2}$		$299 \pm 57 \pm 32 \pm 37$
OPAL [17]	$\tau^+\nu$	$(7.0 \pm 2.1 \pm 2.0) \times 10^{-2}$		$283 \pm 44 \pm 41$
BaBar [18]	$\mu^+\nu$	$(6.74 \pm 0.83 \pm 0.26 \pm 0.66) \times 10^{-3}$	4.71±0.46	$283 \pm 17 \pm 7 \pm 14$

in most cases, to declare success. A recent unquenched lattice calculation [21], however, differs by more than three standard deviations [31]. Remarkably it agrees with f_{D^+} and consequently disagrees in the ratio $f_{D_s^+}/f_{D^+}$.

Akeroyd and Chen [32] first pointed out that leptonic decay widths are modified by new physics. Specifically, for the D^+ and D_s^+, in the case of the two-Higgs doublet model (2HDM), Eq. (1) is modified by a factor r_q multiplying the right-hand side:

Table 2: Theoretical predictions of $f_{D_s^+}$, f_{D^+}, and $f_{D_s^+}/f_{D^+}$. QL indicates a quenched-lattice calculation. (Only selected results having errors are included.)

Model	$f_{D_s^+}$(MeV)	f_{D^+}(MeV)	$f_{D_s^+}/f_{D^+}$
Experiment (our averages)	273 ± 10	205.8 ± 8.9	1.33 ± 0.07
Lattice (HPQCD+UKQCD) [21]	241 ± 3	208 ± 4	1.162 ± 0.009
Lattice (FNAL+MILC+HPQCD) [22]	$249 \pm 3 \pm 16$	$201 \pm 3 \pm 17$	$1.24 \pm 0.01 \pm 0.07$
QL (QCDSF) [23]	$220 \pm 6 \pm 5 \pm 11$	$206 \pm 6 \pm 3 \pm 22$	$1.07 \pm 0.02 \pm 0.02$
QL (Taiwan) [24]	$266 \pm 10 \pm 18$	$235 \pm 8 \pm 14$	$1.13 \pm 0.03 \pm 0.05$
QL (UKQCD) [25]	$236 \pm 8^{+17}_{-14}$	$210 \pm 10^{+17}_{-16}$	$1.13 \pm 0.02^{+0.04}_{-0.02}$
QL [26]	$231 \pm 12^{+6}_{-1}$	$211 \pm 14^{+2}_{-12}$	1.10 ± 0.02
QCD Sum Rules [27]	205 ± 22	177 ± 21	$1.16 \pm 0.01 \pm 0.03$
QCD Sum Rules [28]	235 ± 24	203 ± 20	1.15 ± 0.04
Field Correlators [29]	210 ± 10	260 ± 10	1.24 ± 0.03
Isospin Splittings [30]		262 ± 29	

$$r_q = \left[1 + \left(\frac{1}{m_c + m_q}\right)\left(\frac{M_{D_q}}{M_{H^+}}\right)^2 \left(m_c - m_q \tan^2\beta\right)\right]^2, \quad (4)$$

where m_{H^+} is the charged Higgs mass, M_{D_q} is the mass of the D meson (containing the light quark q), m_c is the charm quark mass, m_q is the light-quark mass, and $\tan\beta$ is the ratio of the vacuum expectation values of the two Higgs doublets. (Here we modified the original formula to take into account the charm quark coupling [33].) For the D^+, $m_d \ll m_c$, and the change due to the H^+ is very small. For the D_s^+, however, the effect can be substantial. One major concern is that we need to know the value of $f_{D_s^+}$ in the Standard Model (SM). We can take that from a theoretical model. Our most aggressive choice is that of the unquenched lattice calculation [21], because they claim the smallest error. Since the charged Higgs would lower the rate compared to the SM, in principle, experiment gives a lower limit on the charged Higgs mass. However, the value for the predicted decay constant using this model is more than 3 standard deviations *below* the measurement, implying that (a) either the model of Ref. 21 is not representative; or (b) no value of m_{H^+} in the two-Higgs doublet model will satisfy the constraint at 99.9% confidence level; or (c) there is new physics, different from the 2HDM, that interferes constructively with the SM amplitude [34].

Dobrescu and Kronfeld [9] emphasize that the discrepancy between the theoretical lattice calculation and the CLEO data is substantial and "is worth interpreting in terms of new physics." They give three possible examples of new physics models that might be responsible. These include a specific two-Higgs doublet model and two leptoquark models.

The Belle [35] and BaBar [36] collaborations have found evidence for $B^- \to \tau^- \bar{\nu}$ decays. The measurements are

$$\begin{aligned}
\mathcal{B}(B^- \to \tau^- \bar{\nu}) &= (1.79^{+0.56}_{-0.49}\ {}^{+0.46}_{-0.51}) \times 10^{-4}\ \text{(Belle)};\\
&= (1.2 \pm 0.4 \pm 0.3 \pm 0.2) \times 10^{-4}\ \text{(BaBar)};\\
&= (1.42 \pm 0.43) \times 10^{-4}\ \text{(our average)}. \qquad (5)
\end{aligned}$$

The Belle and BaBar values have 3.5 and 2.6 standard-deviation significances. More data are needed, and the average can only be provisional. Here the effect of a charged Higgs is different as it can either increase or decrease the expected SM branching ratio. The factor r is given in terms of the B meson mass, M_B, by [2]

$$r = \left(1 - \tan^2\beta\, \frac{M_B^2}{m_{H^+}^2}\right)^2 . \qquad (6)$$

In principle, we can get a limit in the $\tan\beta$–m_{H^+} plane even with this statistically limited set of data. Again, we need to know the SM prediction of this decay rate. We ascertain this value using Eq. (1). Here theory provides a value of $f_B = (216 \pm 22)$ MeV [37]. The subject of the value of $|V_{ub}|$ is addressed elsewhere [38]. Taking an average over inclusive and exclusive determinations, and enlarging the error using the PDG prescription because the results differ, we find $|V_{ub}| = (3.9 \pm 0.5) \times 10^{-3}$, where the error is dominantly theoretical. We thus arrive at the SM prediction for the $\tau^- \bar{\nu}$ branching fraction of $(1.25 \pm 0.41) \times 10^{-4}$. Taking the ratio of the experimental value to the predicted branching ratio at its 90% c.l. *upper* limit and using Eq. (6), we find that we can limit $M_{H^+}\ /\tan\beta > 3.5$ GeV. The 90% c.l. *lower* limit also permits us to exclude the region 4.1 GeV $< M_{H^+}\ /\tan\beta <$ 9.6 GeV [39].

We now discuss the determination of charged pion and kaon decay constants. The sum of branching fractions for $\pi^- \to \mu^-\bar{\nu}$ and $\pi^- \to \mu^-\bar{\nu}\gamma$ is 99.98770(4)%. The two modes are difficult to separate experimentally, so we use this sum, with Eq. (1) modified to include photon emission and radiative corrections [40]. The branching fraction together with the lifetime 26.033(5) ns gives

$$f_{\pi^-} = (130.4 \pm 0.04 \pm 0.2)\ \text{MeV}\ . \tag{7}$$

The first error is due to the error on $|V_{ud}|$, 0.97418(26) [5]; the second is due to the higher-order corrections.

Similarly, the sum of branching fractions for $K^- \to \mu^-\bar{\nu}$ and $K^- \to \mu^-\bar{\nu}\gamma$ is 63.57(11)%, and the lifetime is 12.3840(193) ns [41]. We use a value for $f_+(0)|V_{us}|$ obtained from the average of semileptonic kaon decays of 0.21661(46). The $f_+(0)$ must be determined theoretically. We follow Blucher and Marciano [8] in using the Leutwyler-Roos calculation $f_+(0) = 0.961 \pm 0.008$, that gives $|V_{us}| = 0.2255 \pm 0.0019$, yielding

$$f_{K^-} = (155.5 \pm 0.2 \pm 0.8 \pm 0.2)\ \text{MeV}\ . \tag{8}$$

The first error is due to the error on Γ, the second is due to the CKM factor $|V_{us}|$, and the third is due to the higher-order corrections. The largest source of error in these corrections depends on the QCD part, which is based on one calculation in the large N_c framework. We have doubled the quoted error here; this would probably be unnecessary if other calculations were to come to similar conclusions. A large part of the additional uncertainty vanishes in the ratio of the K^- and π^- decay constants, which is

$$f_{K^-}/f_{\pi^-} = 1.193 \pm 0.002 \pm 0.006 \pm 0.001\ . \tag{9}$$

The first error is due to the measured decay rates; the second is due to the uncertainties on the CKM factors; the third is due to the uncertainties in the radiative correction ratio.

These measurements have been used in conjunction with lattice calculations that predict f_K/f_π in order to find a value for $|V_{us}|/|V_{ud}|$. Together with the precisely measured $|V_{ud}|$, this gives an independent measure of $|V_{us}|$ [6,41].

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