Oscillating elastic defects

A fundamental question in elasticity theory is to understand the patterns of deformation and strains created by stress, defects or inhomogeneities in a material. In a celebrated 1957 paper [1], Eshelby derived an exact solution for the strain fields created by an inhomogeneous elliptic inclusion in an isotropic elastic medium. This result is a cornerstone of the theory of inhomogeneous elastic media, nowadays routinely used in engineering sciences. From a physicist's point of view, it demonstrates that a local perturbation induces long range strain fields, slowly decaying as $1/r^d$ in d dimensions ($d \geq 2$, and r being the distance from the perturbation). These long-range and anisotropic interactions between strain fields induce, for instance, pattern formation in martensitic materials [3].

Eshelby's work considered static inhomogeneities only; in a number of applications in physics and material science, it is actually important to consider the *dynamics* of the strain fields created by an elastic inclusion. The inhomogeneities themselves can be oscillating: for instance, polarons tunneling between two positions in high T_c superconductors can be viewed as locally oscillating defects. Other aplications include the design of micro electromechanical devices (MEMs). Although the dynamical generalizations of the Eshelby problem have been recently investigated in the context of engineering sciences [2], no effort has been made in that direction from a physicist's point of view. In this work [4], we consider the dynamical Eshelby problem for localized oscillating defects, and focus on the general and qualitative features of the resulting strain fields. In particular, we ask the following specific questions: how do the strain fields depend on the oscillating frequency? How do defects interact with each other through the strain fields they create?

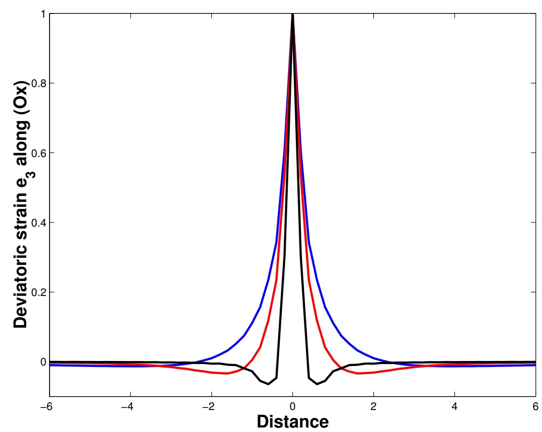

Figure 1: Profile along (Ox) of the deviatoric strain e_3, for different frequencies $\omega_0 = 1$ (blue), $\omega_0 = 10$ (red) and $\omega_0 = 100$ (black). The higher the frequency ω_0, the more localized is the perturbation.

<u>One defect case</u>

We consider a two-dimensional elastic medium, and we impose the presence of an oscillating defect, represented by a locally oscillating deviatoric strain e_3 (e_3 essentially measures locally how much a square is deformed into a rectangle): $e_3(\vec{r} = \vec{0}) = e_0 \sin \omega_0 t$. At the level of linear elasticity, the equations can be solved exactly, and expressions are obtained for e_3, as well as for the dilatation strain e_1 and the shear strain e_2. Fig. 1 shows such strain profiles for different frequencies ω_0, and Fig. 2 shows a surface plot of the e_3 field.

This study allows us to demonstrate two important results:

• The strain fields created by the defect decay at large distances as $1/r^2$, as in the static case.

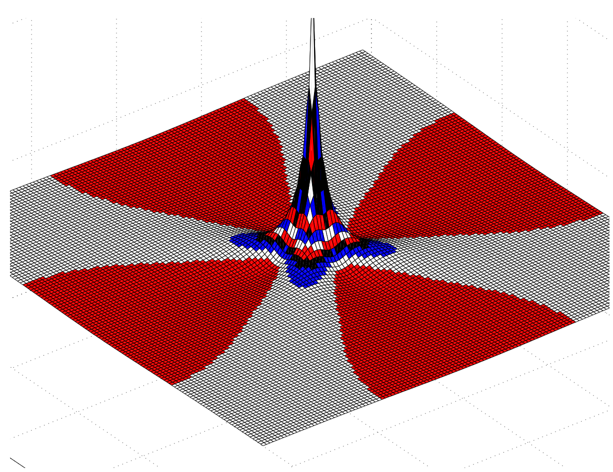

Figure 2: Surface plot of the deviatoric strain e_3, for $\omega_0 = 1$. Notice the anisotropy of the strain field.

• The deformation induced by the defect is progressively more localized as ω_0 is increased; this has an important consequence for the interaction between defects: it can be suppressed by tuning the frequency. This point is developed in the next section.

Two defects case

Next, we consider the case of two defects, to probe the interaction between them. We suppose that the two defects, located at $\vec{r} = \vec{0}$ and $\vec{r} = \vec{r_0}$, oscillate at the same frequency ω_0, and have a phase difference φ. Again, at the level of linear elasticity, the equations can be solved exactly, giving access to the strain profiles. An example is given in Fig. 3.

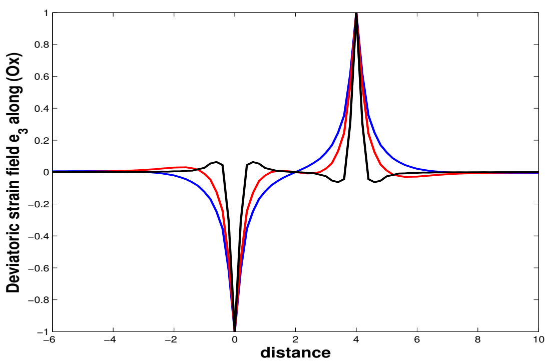

Figure 3: Profiles of the deviatoric strain e_3 with two defects, for $\omega_0 = 1$ (blue), $\omega_0 = 10$ (red) and $\omega_0 = 100$ (black). The phase difference between the two defects is $\varphi = \pi$.

From a given configuration of the strain fields, it is easy to calculate the associated elastic energy; this energy obviously oscillates with frequency ω_0. To probe the interaction between the two defects, we average this energy over one period, and obtain the average energy $U(\varphi, \vec{r}, \omega_0$ as a function of the phase difference between the defects φ, the distance between them $\vec{r_0}$ and their common frequency ω_0. We focus on a situation where the defects are pinned, and interact through their relative phase; the relevant quantity is then $U(\varphi)$, at fixed $\vec{r_0}$ and ω_0. These energies are well described by the "XY-like" function:

$$U(\varphi, \vec{r}, \omega_0) = C(\vec{r}, \omega_0) + J(\vec{r}, \omega_0) \cos\varphi \ . \qquad (1)$$

A positive J corresponds to a ferromagnetic, or phase locking, interaction, whereas a negative J leads to an antiferromagnetic interaction.

Fig. 4 shows some representativeenergy profiles, and emphasizes the effects of varying the frequency or the distance:

• As expected, J goes to zero for large distances and/or large frequencies.

• More dramatically, J changes sign: at low enough ω_0 and short enough distances, the interaction is ferromagnetic; and antiferromagnetic otherwise.

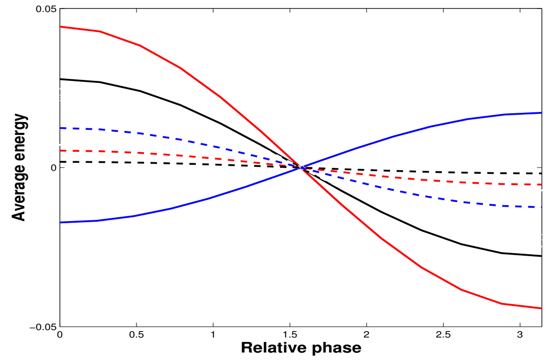

Figure 4: $U(\varphi, \vec{r_0}, \omega_0)$ for $\omega_0 = 1$ (blue), $\omega_0 = 10$ (red) and $\omega_0 = 100$, and for $\vec{r_0} = 2\vec{x}$ (solid lines) and $\vec{r_0} = 8\vec{x}$ (dashed lines). The interaction is always antiferromagnetic, except for $\omega_0 = 1$, $\vec{r_0} = 2\vec{x}$.

This last property may have important consequences for the collective behavior of an assembly of defects: one can imagine inducing phase transitions between phase locked and disordered states by tuning the defect oscillation frequency.

Conclusion

Summarizing, we have extended the classical study by Eshelby of static defects in the 50's to dynamical oscillating defects . As in the static case, local perturbation of the strain fields propagates to large distances and decays away only as $1/r^2$, r being the distance from the perturbation. However, the strain profile strongly depends on the oscillation frequency: the higher the frequency, the more localized the strain field. This has important consequences for the interaction between several such oscillating defects, opening the door to collective effects controlled by the frequency. This may also have applications for the interactions between polarons in high T_c superconductors, and for the design of micromechanical devices.

References

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