Scalar and QCD String Confinement[1]

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Abstract. We compare scalar and string confinement mechanisms. Solutions for the massless quark case are found and discussed.

Although the concept of scalar confinement occupies a hallowed tradition in hadron physics, its origin from QCD has remained obscure. On the other hand, the QCD string provides a very plausible limit of QCD at large quark separation. In the first section we compare and contrast scalar and string confinement within a unified formalism [1]. In the first section we compare and contrast scalar and string confinement within a unified formalism. In the second part we examine the solutions for the ultra-relativistic limit of a massless quark confined by a scalar potential and by a QCD string [1].

FOUR-VECTOR INTERACTIONS: SCALAR AND QCD CONFINEMENT

QCD is intrinsically a vector-type theory and this is why it has been difficult to imagine scalar confinement in relation to QCD. Our first step will be to note that a four-vector interaction of a particular type is equivalent to a scalar potential. We begin with the invariant action for a mass m moving in a scalar potential and a four-vector potential,

$$S = -\int d\tau \left[m + \phi(x) - u^{\mu} A_{\mu}\right] , \tag{1}$$

where u^{μ} is the four-velocity of m. Next consider a special four-vector potential

$$A^{\mu'}(x) = \Big(\Phi(x), \mathbf{0}\Big) . \tag{2}$$

This four-vector is appropriate for an electric flux tube/string (in its rest frame). As pointed out by Buchmüller [2], this insures the Thomas-type spin-orbit interaction.

[1] Talk presented by M.G. Olsson

We then note that the invariant quantity $u^\mu A_\mu$ can be evaluated in the quark rest frame to give

$$-u^{\mu'} A_{\mu'} = -u^\mu A_\mu = \Phi(x)\,. \tag{3}$$

Thus this particularly "string-like" four-potential is exactly equivalent to scalar confinement if we identify $\Phi(x)$ with $\phi(x)$. As discussed in more detail in Reference [1], this four-potential is not exactly string-like in that the energy is concentrated at the quark and that it should not depend on the radial velocity.

MASSLESS QUARK SPECTROSCOPY

For one fixed (heavy) and one massless quark, the squared Hamiltonian with scalar confinement is [1]

$$H^2_{\mathrm{scalar}} = p^2 + (ar)^2\,. \tag{4}$$

This is a harmonic oscillator and hence the mass eigenstates are

$$M^2_{\mathrm{scalar}} = 2a(J + 2n + 3/2)\,, \tag{5}$$

where a is the "string tension," J and n are the angular and radial quantum numbers, respectively. An important aspect is the presence of "towers" of mass-degenerate states of the same parity. The spectroscopy of the scalar confinement result (5) is shown by the parallel lines in Fig. 1. The dots are exact numerical solutions [1,3] of the square root of Eq. (4).

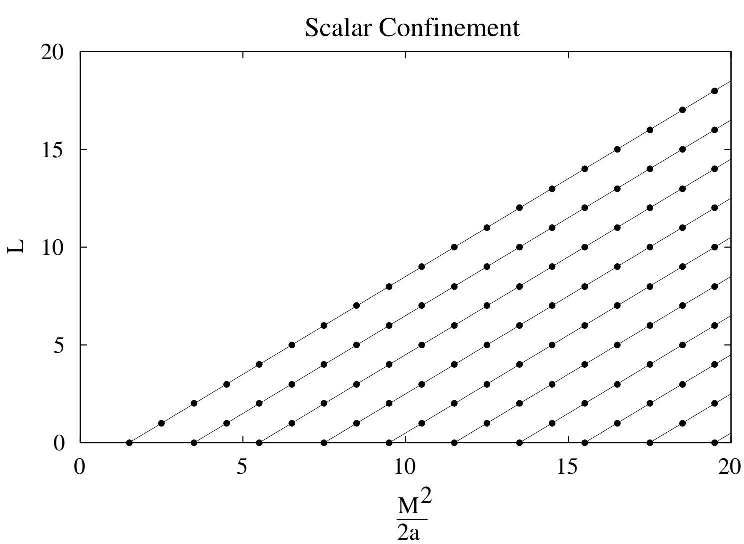

FIGURE 1. Regge structure and states in pure linear scalar confinement from numerical diagonization of the square root of H in (4). Solid lines are the harmonic oscillator result (5).

Next we examine the comparable solution for a QCD string [1,3]

$$H = W_r\gamma_\perp + ar\frac{\arcsin v_\perp}{v_\perp}\,, \tag{6}$$

$$\frac{J}{r} = W_r\gamma_\perp v_\perp + \frac{ar}{2v_\perp}\left(\frac{\arcsin v_\perp}{v_\perp} - \sqrt{1-v_\perp^2}\right)\,, \tag{7}$$

where $v_\perp$ is the quark velocity perpendicular to the string and $W_r = \sqrt{p_r^2+m^2}$. The massless quark limit is tricky since $W_r \to 0$ but $\gamma_\perp \to \infty$ for circular orbits. Considering the combination $H^2 - J^2/r^2$ and then setting $v_\perp \to 1$ in the interaction (since this is a smooth limit) we obtain

$$M^2 - \frac{a\pi J}{2} = p^2 + \left(\frac{a\pi r}{4}\right)^2\,, \tag{8}$$

where $p^2 = p_r^2 + J^2/r^2$. This would appear to be just a shifted harmonic oscillator but one must carefully examine the classical turning points. From (8) with $p_r = 0$ (and hence $v_\perp = 1$) we get

$$\frac{a\pi}{2}\,r_\pm = M \pm \sqrt{M^2 - a\pi J}\,. \tag{9}$$

These would seem to be the classical turning points. At these turning points [from (6)] we have (since $W_r\gamma_\perp \geq 0$)

$$M \geq \frac{a\pi r_\pm}{2} \tag{10}$$

or

$$M \geq M \pm \sqrt{M^2 - a\pi J}\,, \tag{11}$$

which is only satisfied for r_-. The larger turning point hence must be smaller than r_+. This larger turning point occurs at a finite value of p_r and hence is a “bounce.” Its precise value depends on a more careful approximation, but if we take it as the circular orbit radius

$$\frac{a\pi}{2}\,r_0 = M\,, \tag{12}$$

we have a “half oscillator” which just doubles the radial excitation energies (by eliminating half the excitations). The half oscillator solution is obtained from (8) by replacing n by $2n+1$, yielding

$$M^2 = a\pi\left(J + 2n + \frac{7}{4}\right)\,. \tag{13}$$

This is very close to the scalar confinement result (5) although the physical process seems quite different. In Fig. 2, the solution $M^2 = a\pi(J + 2n + \frac{3}{2})$ is represented by parallel lines. The dots are the exact numerical solution [1,3] of the string equation (7).

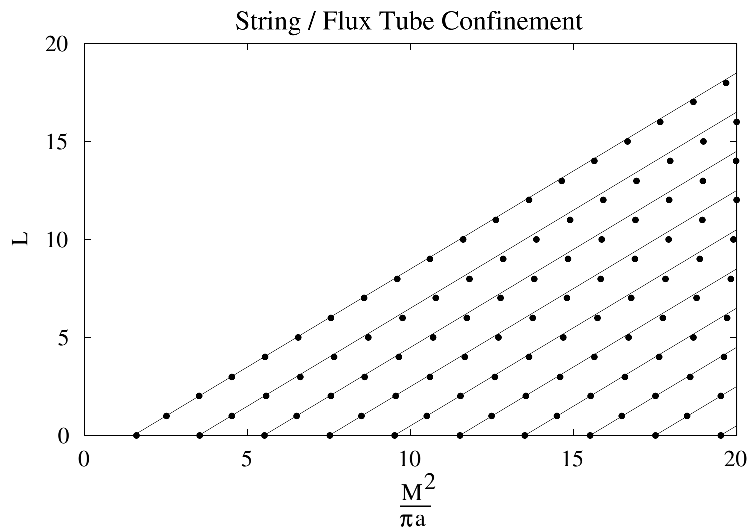

FIGURE 2. Regge structure of string confinement from numerical quantization of Eqs. (6) and (7). The lines are the solution (13) with intercept 3/2.

REFERENCES

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