Warped Fermions

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A daring alternative: Warped Extra Dimensions

- Warped extra Dimensions, offer a solution to the hierarchy problem,

  why $v \ll M_{Pl}$?

  without relying on unnatural cancellations, strong interactions or SUSY.

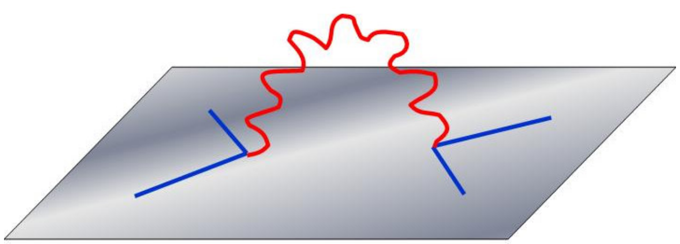
  ⇓

  All fundamental parameters are of the order of the Planck scale, and yet, due to the curvature of the extra-dimensional metric and the localization of the Higgs field, the Higgs v.e.v. is naturally of order of the TeV scale

4-D effective theory:

SM particles + Gravitons + tower of new particles: Kaluza Klein (KK) excited states with same quantum number as a gravition and/or the SM particles

Signatures:



• KK Gravitons, with masses of the order of the weak scale and couplings of order 1/TeV to SM particles → simplest mechanism

• Fermions and gauge boson KK modes of the order of the TeV scale may also exist, (strongly constrained by precision electroweak data.)

KK states produced as resonances or contribute to fermion pair production at colliders

Solution to the Hierarchy Problem

- Space is compact, of size 2 L, with orbifold conditions x, y $\longrightarrow$ x,-y
- Brane at y = 0 (Ultraviolet or Planck Brane)

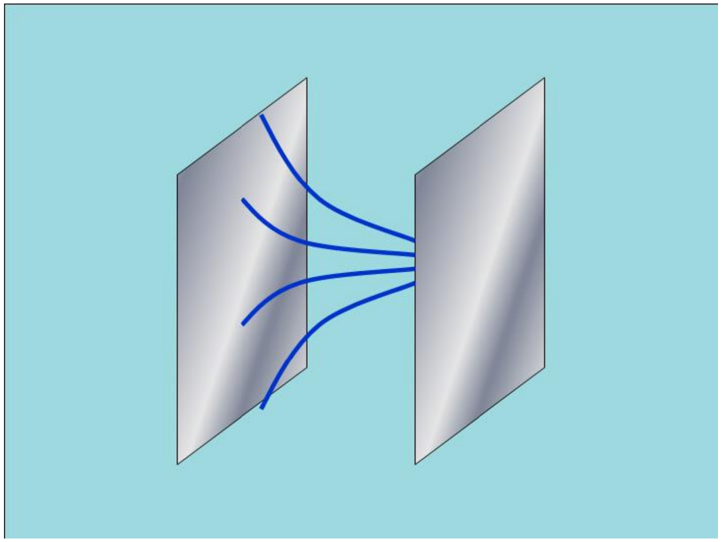
  Brane at y = L (Infrared or TeV Brane)
- Non-factorizable metric:

  $$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$ ➔ solution to 5d Einstein equations

- Newton's law modified: 5d Planck mass relates to M_{Pl} $M_{Pl}^2 = \frac{(M_{Pl}^{fund.})^3}{2k}(1 - e^{-2kL})$

➔ Natural energy scale at the UV brane: Fundamental Planck scale $\Rightarrow M_{Pl}^{fund.}$

At the TeV brane, all masses are affected by an exponential warp factor e^{-kL}



Assuming fundamental scales all of same order:

$$M_{Pl} \approx M_{Pl}^{fund.} \approx \mathrm{v} \approx \mathrm{k}$$

Solution to Hierarchy problem :

Higgs field lives on the TeV brane

$$\tilde{\mathrm{v}} \approx \mathrm{v}\, \mathrm{e}^{-\mathrm{kL}} \approx M_{Pl}\, \mathrm{e}^{-\mathrm{kL}}$$

with $kL \approx 30$

Bulk Fermions and Gauge Bosons

- Although the Higgs field must be located on the IR brane,
  ➔ fermions and gauge fields may live in the bulk.

- Fermions in the bulk may provide a solution to the flavor problem;
  the masses of fermions being related to the size of their zero mode wave functions at the IR brane.

- Since fermions are charged under the Standard interactions, gauge fields must also live in the bulk.

- Also, running of couplings in warped scenarios resemble the four dimensional case, and allow the possiblity of gauge coupling unification.

- KK mode wave functions are peaked towards the IR brane, inducing large corrections to zero mode gauge boson couplings and masses, due to the mixing induced by the Higgs field v.e.v.

Gauge Bosons: Brane Kinetic Terms

- Consider a 5d space with fermion field localized in 3-branes at y = 0,L and gauge bosons propagating in the bulk.
- Loops of charged fields in the brane induce rad. correc.→ brane gauge kinetic terms (the may also appear at tree level)

$$S = -\frac{1}{4g_5^2}\int d^4x\, dy \sqrt{-g}\left[F^{MN}F_{MN} + 2\delta(y)\, r_{UV} F^{\mu\nu}F_{\mu\nu} + 2\delta(y-L)\, r_{IR} F^{\mu\nu}F_{\mu\nu}\right]$$

$g \to$ determinant of the metric. $\sqrt{-g} = e^{-4ky}$

$M = 0, 1, 2, 3, 5;$ $\mu = 0, 1, 2, 3$ $g_5^{-2} \to$ dim. of mass,

local brane term coefficients: $r_i = g_5^2/g_i^2 \to$ dim. of length

$$\mathcal{F}^a_{MN} = \partial_M \mathcal{A}^a_N - \partial_N \mathcal{A}^a_M + f^{abc}\mathcal{A}^b_M \mathcal{A}^c_N$$

with the decomposition: $A^\lambda(x_\mu, y \equiv x_5) = \sum_n f_n(y) A^\lambda_n(x^\mu)$

and requiring A_n^λ to obey the EOM for a free massive gauge field,

$$\left[\partial_y^2 - 2k\partial_y + e^{2ky} m_n^2 \left(1 + 2r_{UV}\delta(y) + 2r_{IR}\delta(y-L)\right)\right] f_n(y) = 0$$

The solutions are Bessel Functions, same as in the transparent case brane

$$f_n(y) = \mathcal{N}_m e^{k|y|} \left\{ J_1\left(\frac{m_n}{k} e^{k|y|}\right) + b_n Y_1\left(\frac{m_n}{k} e^{k|y|}\right) \right\}$$

- Normalization factor determined by canonically normalized kinetic term condition

$$\frac{1}{g_5^2}\left[r_{UV} f_n^2(0) + r_{IR} f_n^2(L) + \int_0^L dy\, f_n^2(y)\right] = 1$$

- Boundary conditions at y =0,L should reflect discontinuity in the derivatives and define coefficients $b_n(y=0)$ and $b_n(y=L)$ as functions of Bessel Functions
- Equality of b's determines numerical solutions for the quantized masses m_n

KK mode couplings to brane fields localized at y $\Rightarrow g_n = f_n(y)$

Zero mode solution: $m_n = 0$ and constant $f_0(y) \Rightarrow$ constant coupling to all charged brane fields for any y, as required by gauge inv.

$$g_0 = f_0(y) = \frac{g_5}{\sqrt{L + r_{IR} + r_{UV}}}$$

KK mode masses and couplings significantly modified by brane kinetic terms!

Opaque IR Brane: Masses and Couplings

- Presence of IR kinetic terms expel the KK modes from the brane (decoupling):

From the 5d propagator :

For $p \ll k\,e^{-kL}$, $\quad G_p(L,L) \sim -g_5^2/\left[p^2(L+r_{IR})\right]$

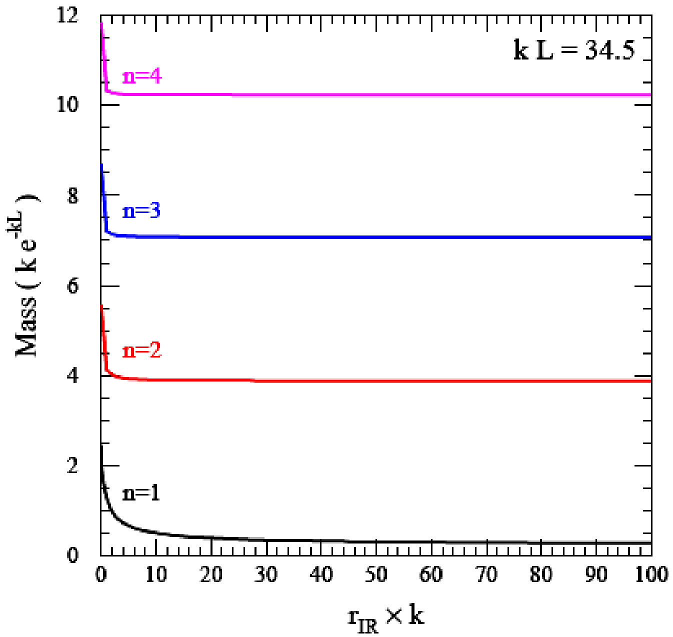
$\Longrightarrow$ zero mode with $g_0 = g_5/\sqrt{L+r_{IR}}$.

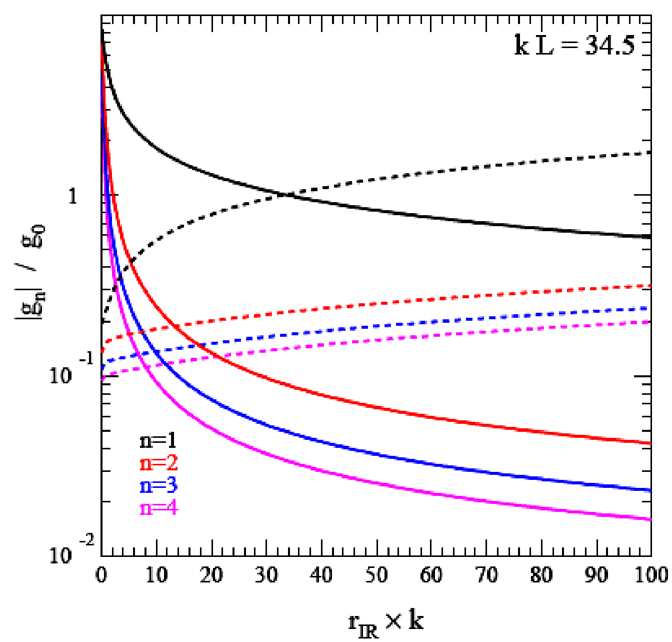
For $p \gg k\,e^{-kL}$, $\quad G_p(L,L) \simeq -g_5^2 e^{kL}/\left[p(1+p\,r_{IR}\,e^{kL})\right]$

if $k\,r_{IR} \gtrsim 1 \Longrightarrow$ 4d behavior $G_p(L,L) \simeq -g_5^2/\left(p^2\,r_{IR}\right)$

- For sufficiently large rk only the first KK mode couples to the brane with $g_1 = g_0\sqrt{L/r_{IR}}$

Other modes decouple

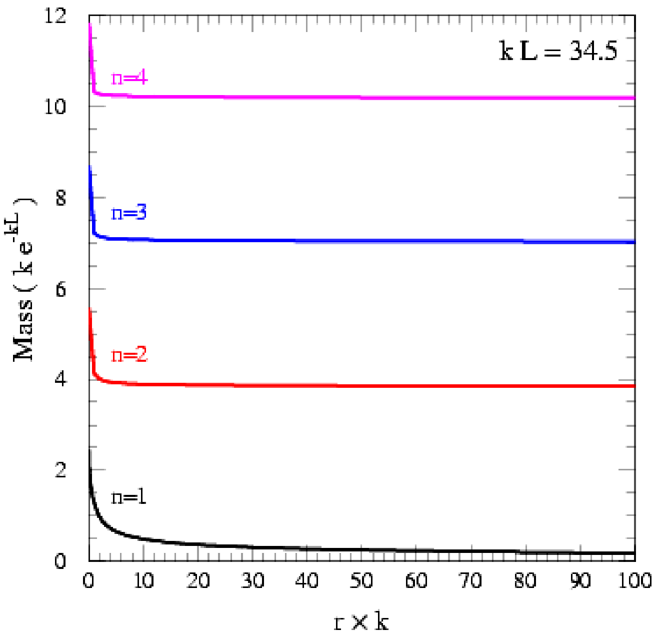




1st KK mode only relevant one for phenomenology

- Masses modified in a smooth way.

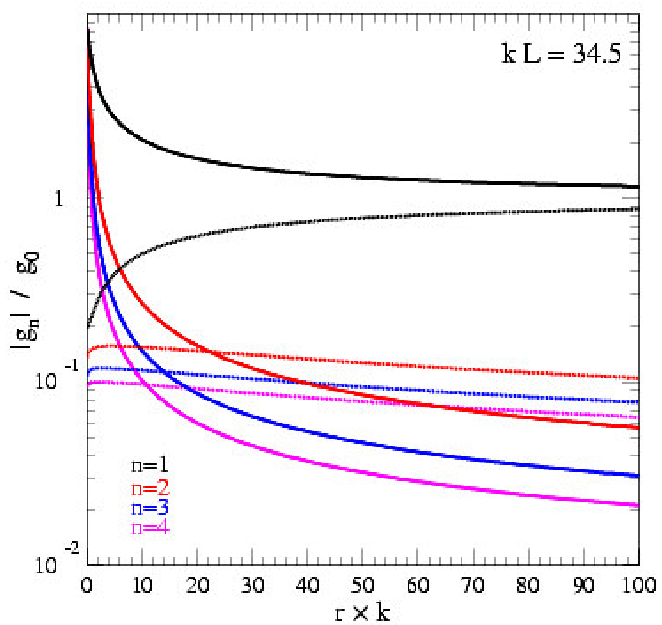
Opaque IR and UV Branes



$r_{IR}, r_{UV} \to \infty \longrightarrow$ an observer on either brane must be insensitive to the extra dimensions + other brane.

$\Longrightarrow$ physics at each brane determined by the local coupling

$\Longrightarrow$ two massless modes should appear, one l.c. of them couples to each brane with local brane coupling strength



$r_{IR} = r_{UV} = r$:

- large r: first KK-mode mass $\to 0$ and its coupling becomes equal (and opposite in sign for UV brane fields) to the zero mode one

- $r \to \infty$: bulk propagation switches off $\equiv$ two brane gauge theories which do not interact with each other. The higher modes decouple from both branes.

Electroweak Symmetry Breaking

Since the Higgs field lives on the IR brane, the effects of electroweak symmetry Breaking may be parametrized by the effective Lagrangian

$$-\int d^4x\, dy \sqrt{-g}\; 2\,\delta(y-L)\left\{(D_\mu H)^+ D^\mu H + \lambda\left(|H|^2 - \frac{\mathrm{v}^2}{2}\right)^2\right\}$$

In the low energy effective theory in four dimensions, after canonical normalization of the Higgs kinetic term:

$$-\int d^4x \left\{\eta^{\mu\nu}(D_\mu H)^+ D_\nu H + \lambda\left(|H|^2 - \frac{\tilde{\mathrm{v}}^2}{2}\right)^2\right\} \qquad \text{with } \tilde{\mathrm{v}} = \mathrm{v}\, \mathrm{e}^{-\mathrm{kL}}$$

This is the promised solution to the gauge hierarchy problem

The localized v.e.v. results in a gauge boson mass which is itself localized on the IR brane:

$$-\frac{1}{2}\int d^4x\, dy\; 2\,\delta(y-L)\, \tilde{\mathrm{v}}^2 \eta^{\mu\nu} A_\mu A_\nu$$

Observe that, once the gauge boson KK decomposition is inserted, this produces mixing between the different KK modes, proportional to the Higgs v.e.v., resulting in a deformation of the 5d wave functions, including the zero mode one, no longer flat.

This mixing has important phenomenological consequences.

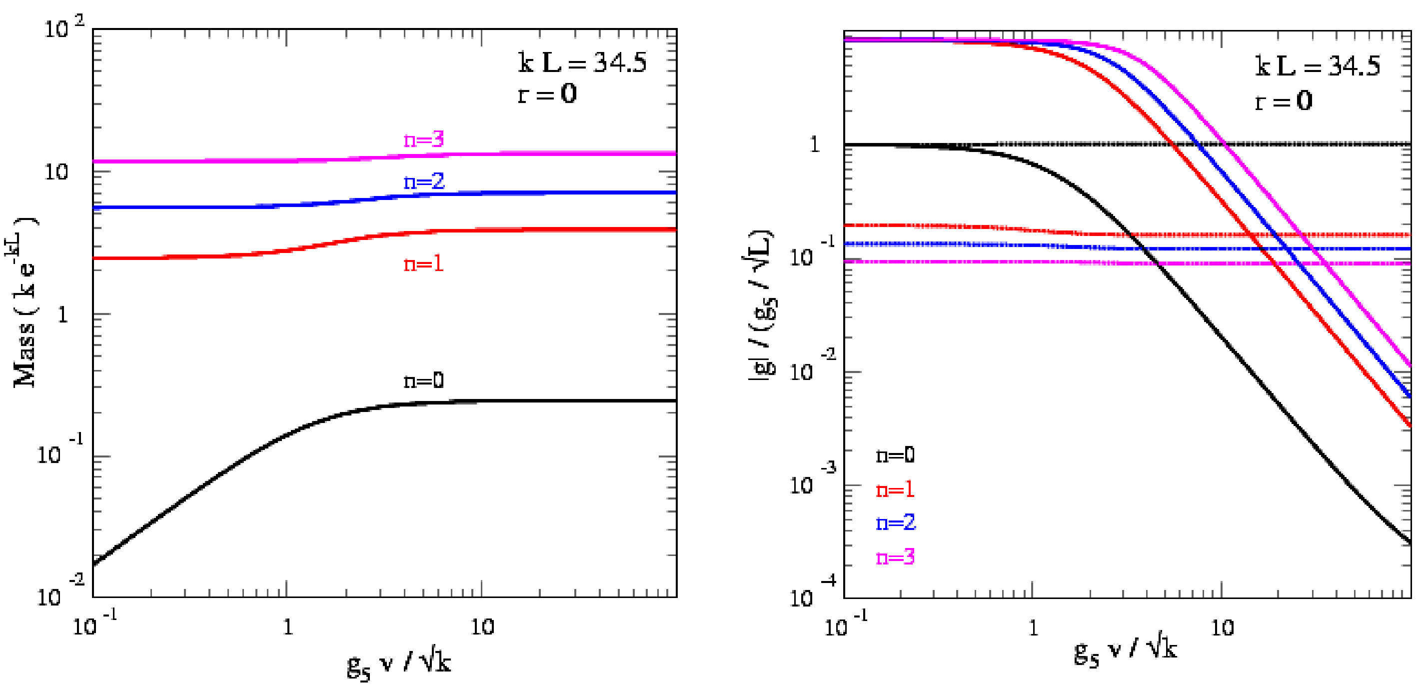
Effects of the Higgs v.e.v.

- Even in the absence of brane kinetic terms, the Higgs v.e.v. repel KK modes from the IR brane. Zero mode wave function also modified.

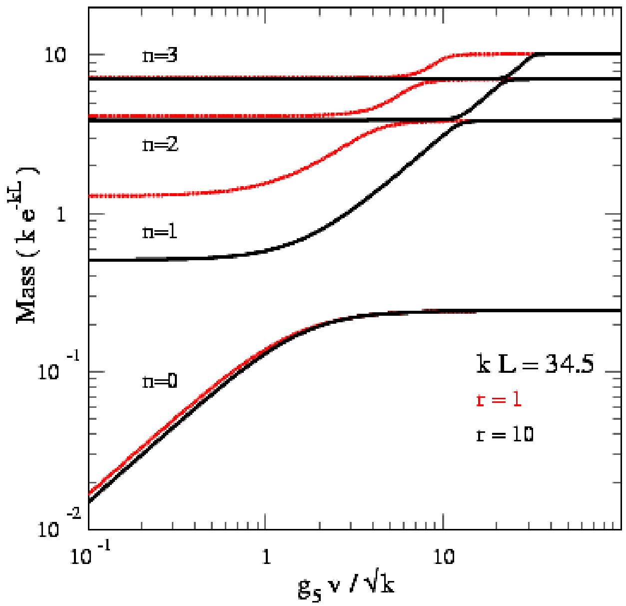
- For v << k ➔ zero mode approx. flat with mass $g_5\, \tilde{\mathrm{v}}/\sqrt{L}$

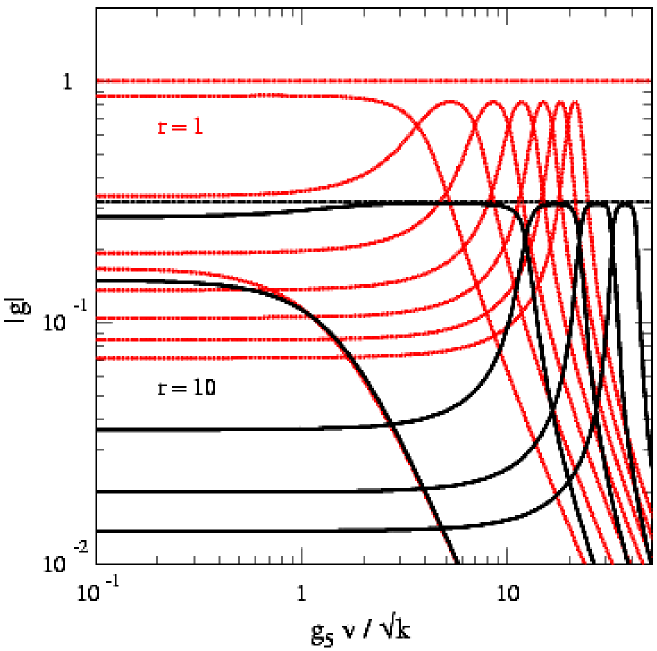
- For v >> k ➔ zero mode no longer flat, mass insensitive to v
  ➔ KK modes and zero mode bend away from the IR brane with a ratio of KK coupling /zero coupling larger than $\sqrt{2kL}$

(v = 0 values)



Gauge Modes : Opaque IR Brane and Higgs v.e.v.





From the propagator with end points in the IR brane:

$$G_p(L,L) \sim -g_5^2 / \left(p^2\, r_{IR} + g_5^2 \tilde{v}^2\right) \qquad p \gg k\, e^{-kL} \quad (r_{IR} k > 1)$$

$$G_p(L,L) \sim -g_5^2 / \left[p^2\,(L + r_{IR}) + g_5^2 \tilde{v}^2\right] \qquad p \ll k\, e^{-kL}$$

- For $v \ll k$ and $p \ll k\, e^{-kL}$, $\rightarrow$ single state with $g_0 \simeq g_5/\sqrt{L + r_{IR}}$ and $m_0 \simeq g_0 \tilde{v}$, up to correc. of order $g_5^2 v^2/k \Longrightarrow$ zero mode grows linearly with $\tilde{v}$

- If $r_{IR} \gg L$, both propagators describe a single 4d state with mass $g_{IR}\, \tilde{v}$ and coupling g_{IR}.
  All other modes decouple.

- For $v/k \gtrsim 1 \rightarrow$ zero mode mass becomes insensitive to v; other modes successively have masses prop. to v. and couplings $g_{IR} = g_5/\sqrt{r_{IR}}$

Behavior for non-vanishing brane kinetic term is similar as before, but now there is always a mode which couples strongly to the Higgs, with coupling of order of the zero mode coupling

Mass and couplings of guage zero mode

For phenomenological applications: interesting case is $\tilde{\mathrm{v}} \ll k \exp^{-kL}$
Useful to define next order corrections for the mode which couples strongly to the Higgs

$$m_0 = \frac{g_5 \tilde{\mathrm{v}}}{\sqrt{\mathrm{L} + \mathrm{r}_{\mathrm{IR}}}} \left[1 - \eta \left(\frac{g_5^2 \mathrm{v}^2}{\mathrm{k}} \right) + O\left(\frac{g_5^2 \mathrm{v}^2}{\mathrm{k}} \right)^2 \right]$$

$$g_0 = \frac{g_5}{\sqrt{\mathrm{L} + \mathrm{r}_{\mathrm{IR}}}} \left[1 - 2\eta \left(\frac{g_5^2 \mathrm{v}^2}{\mathrm{k}} \right) + O\left(\frac{g_5^2 \mathrm{v}^2}{\mathrm{k}} \right)^2 \right]$$

$$\text{with} \qquad \eta = \frac{2k^2 L^2 - 2kL + 1}{8k^2 (L + r_{IR})^2}$$

Next order corrections will be crucial when considering the phenomenology of the AdS5SM and fitting its parameters to precision electroweak the data

Electroweak Theory

- Higgs and Fermions in the IR brane, Guage fields in the bulk
- IR brane kinetic terms. After EWSB:

$$\mathcal{L}^5_{EW} = \sqrt{-g}\left\{-\frac{1}{4g_5^2}\mathcal{W}_{MN}\mathcal{W}^{MN}\left(1+2r_2\delta(y-L)\right) - \frac{1}{4g_5'^2}\mathcal{B}_{MN}\mathcal{B}^{MN}\left(1+2r_1\delta(y-L)\right)\right.$$
$$\left. -v^2\delta(y-L)\left[\mathcal{W}^1_M\mathcal{W}^M_1 + \mathcal{W}^2_M\mathcal{W}^M_2 + \left(\mathcal{W}^3_M - \mathcal{B}_M\right)\left(\mathcal{W}^M_3 - \mathcal{B}^M\right)\right]\right\}$$

After gauge rotations to diagonalize masses in terms of bulk couplings:
For the simple case $r_1=r_2$, and in the bases:

$$s = g_5^{'}/\sqrt{g_5^2 + g_5^{'2}}$$

$W^3_\mu = c^2 Z_\mu + A_\mu \qquad B_\mu = -s^2 Z_\mu + A_\mu \Rightarrow$ photon and Z boson KK towers decouple

$$\mathcal{L}^5_{EW} = \sqrt{-g}\left\{-\frac{s^2}{2e_5^2}\mathcal{W}^+_{MN}\mathcal{W}^{MN}_-\left[1+2r\delta(y-L)\right] - \frac{1}{4e_5^2}\mathcal{F}_{MN}\mathcal{F}^{MN}\left[1+2r\delta(y-L)\right]\right.$$
$$\left. -\frac{s^2c^2}{4e_5^2}\mathcal{Z}_{MN}\mathcal{Z}^{MN}\left[1+2r\delta(y-L)\right] - 2v^2\delta(y-L)\left(\mathcal{W}^+_M\mathcal{W}^M_- + \frac{1}{2}\mathcal{Z}_M\mathcal{Z}^M\right)\right\}$$

5d photon coupling ➔ $1/e_5^2 = 1/g_5^2 + 1/g_5^{'2}$

"Zero Modes" ➔ weak gauge bosons observed in experiments

Considering gauge field decomposition:

$$A^\lambda = \sum_n f_A^n(y) A_n^\lambda(x^\mu), \quad Z^\lambda = \sum_n f_Z^n(y) Z_n^\lambda(x^\mu), \quad W^{\pm\lambda} = \sum_n f_W^n(y) W_n^{\pm\lambda}(x^\mu)$$

the effective 4D Lagrangian for the zero modes reads:

$$\mathcal{L} = -\frac{1}{2} W^+_{\mu\nu} W^{\mu\nu}_- - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - m_W^2 W^+_\mu W^\mu_- - \frac{m_Z^2}{2} Z_\mu Z^\mu + \frac{1}{\sqrt{2}} f_W \left(\overline{\psi}\gamma^\mu T_+ \psi W^+_\mu + \overline{\psi}\gamma^\mu T_- \psi W^-_\mu \right) + f_Z \overline{\psi}\gamma^\mu (T_3 - s^2 Q)\psi Z_\mu + f_A \overline{\psi}\gamma^\mu Q \psi A_\mu \ ,$$

- m_W and m_Z are given in terms of the param. of the model; e_5, s, v, r, L and k

$$m_Z = \frac{e\tilde{\mathrm{v}}}{\mathrm{sc}}(1 - \eta\varepsilon + \ldots) \qquad m_W = \frac{e\tilde{\mathrm{v}}}{\mathrm{s}}(1 - c^2\eta\varepsilon + \ldots) \qquad \text{with} \quad \eta\varepsilon = \frac{2k^2L^2 - 2kL + 1}{8k(L + r_{IR})} \frac{e^2 \mathrm{v}^2}{\mathrm{s}^2 c^2 k^2}$$

- $f_W, f_Z, f_A \rightarrow$ zero mode wave functions at y=L

$$f_A = e_5/\sqrt{L+r} \equiv e$$

$$f_Z = \frac{\sqrt{g_5'^2 + g_5^2}}{\sqrt{L+r}}(1 - 2\eta\epsilon) \equiv (e/sc)\hat{f}_Z = \sqrt{g^2 + g'^2}\,\hat{f}_Z$$

$$f_W = \frac{g_5}{\sqrt{L+r}}(1 - 2c^2\eta\epsilon) \equiv (e/s)\hat{f}_W = g\hat{f}_W$$

•The photon experience no symmetry breaking ➔ zero mode is flat

Matching to the Effective Theory

- Using precisely measured quantities: α_Z, M_Z, G_μ

determine SM electroweak parameters $e, \sin^2\theta_W = s_0$ and $\tilde{v}$

- Determine e_5, s and v from data and leave r and L as free parameters in the fit to precision observables

- $\alpha_Z^{-1} = 128.92(3)$. yields $e = e_5/\sqrt{L+r}$

- W boson at zero momentum transfer with entire KK tower effects:

$$1/\tilde{v}^2 = 4\sqrt{2}G_\mu \equiv f_W^2/m_W^2 + \sum_{n\neq 0} f_{W_n}^2/m_{W_n}^2$$

with $G_\mu = 1.16639(1)\times 10^{-5}$ GeV^{-2} yields $\tilde{v} \simeq 123$ GeV.

- Having e_5 and v as a function of r, $L \to$ adjust s so that the Z zero mode mass, $m_Z \equiv M_Z = \frac{e\tilde{v}}{s_0\, c_0} = 91.1875(21)$

This defines s as a function of s_0

$$s = s_0\left(1 - \frac{c_0^2}{c_0^2 - s_0^2}\eta\,\epsilon + \cdots\right)$$

Precision Measurements:
parametrization in terms of Oblique corrections

- Once α_Z, M_Z, G_μ are fixed, all corrections to the tree-level zero mode Lagrangian may be absorbed in corrections to M_w and gauge boson wave functions

- Tree-level modifications of masses and couplings depend on the strength of the five dimensional gauge couplings and the couplings to the Higgs.
  ➔ they are different for the photon and the Z and W bosons.

  ⬇

  These modifications may be parametrized as a function of the same oblique parameters S, T and U, in a way similar as the one done to parametrize similar effects at the quantum level in 4-dimensions.

- There are also non-oblique corrections (not related to a change of mass or a flavor independent change of couplings) associated with the interchange of a tower of KK mode gauge bosons.

Effective S, T and U parameters

- Non-oblique corrections affect the definition of the four-Fermi coupling constant, ➔ have an impact on the determination of Z-peak data and on the ratio of the W-mass to the Z-mass.

- The computation of the five dimensional gauge boson propagator on the brane serves to determine these non-oblique corrections :

$$-G(p^2=0;L,L)=\frac{f_W^2}{m_W^2}+\underbrace{\sum_{n\neq 0}\frac{f_{W_n}^2}{m_{W_n}^2}}_{\delta G_\mu}=\frac{1}{\tilde{v}^2}=4\sqrt{2}G_\mu$$

δG_μ is the non-oblique contribution due to the exchange of KK modes and the f-terms are their couplings to fields localized on the IR brane.

- Using this expression and the modification of the gauge boson masses and couplings presented before, one may compute the effective S, T and U parameters describing the precision electroweak data.

Effective S, T, U parametrization:

➔ serves to describe all Z-pole observables as well as mw

- Tree level 5D contributions, including non-oblique corrections to G_μ

$$\overline{S}_{eff} = -\frac{16 s_0^2 c_0^2}{\alpha}\eta\varepsilon + \dots. \approx -366\eta\varepsilon \qquad \overline{T}_{eff} = -\frac{2}{\alpha}\eta\varepsilon + \dots. \approx -258\eta\varepsilon$$

$$\overline{U}_{eff} = \frac{8 s_0^2 c_0^2}{\alpha}\eta\varepsilon + \dots. \approx 183\eta\varepsilon \qquad \text{recall:} \quad \eta\varepsilon = \frac{2k^2L^2 - 2kL + 1}{8k(L + r_{IR})}\frac{e^2 \mathrm{v}^2}{s_0^2 c_0^2 k^2}$$

e is the electromagnetic coupling, and

$$\boxed{s_0^2\, c_0^2 = \frac{\pi\alpha_Z}{\sqrt{2}G_\mu m_Z^2}}$$

The full Effective S, T, U are given as a sum of the extra dimensional and the Higgs contributions

$$S_H \simeq \frac{1}{12\pi}\log\left(\frac{m_h^2}{m_{ref}^2}\right)$$

$$T_H \simeq -\frac{3}{16\pi c_0^2}\log\left(\frac{m_h^2}{m_{ref}^2}\right)$$

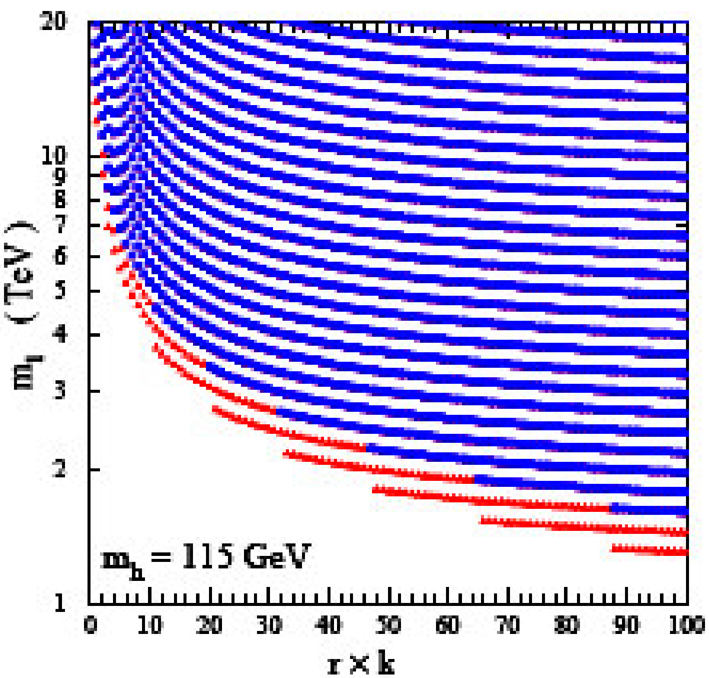
$$U_H \simeq 0$$

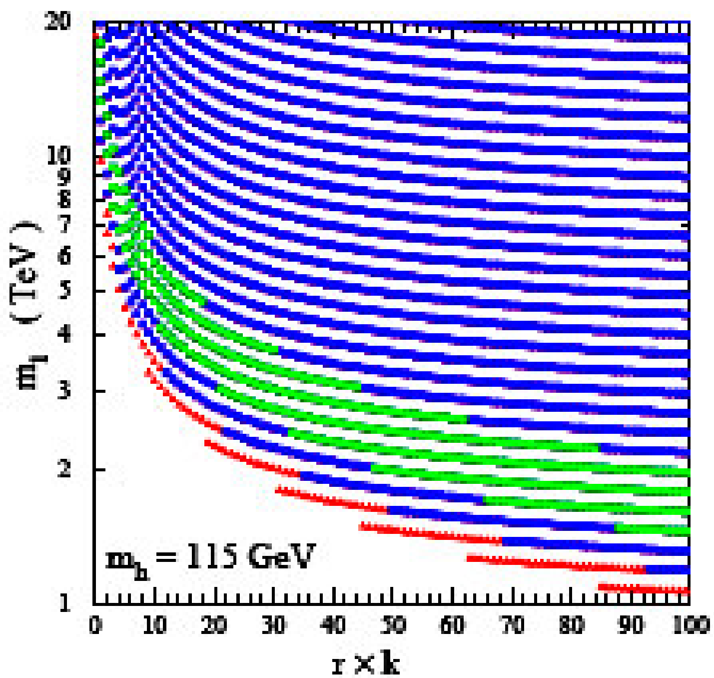
Fit to the data. Fermions on the IR Brane

Large corrections to the precision electroweak data are reduced for larger brane kinetic terms.

- For r → 0 ➔ lightest mode above 20 TeV
- As r increases → a first KK photon with fermionic couplings of order of the zero mode coupling and a mass of a few TeV appears

For large r, light KK modes couple weakly to fermions (apart from lighter one), but still couple to Z, photon and W's.





Regions of 1,2,3 sigma agreement with EW data:

for all precision data (left) and without leptonic determination of $\sin^2\theta_W$ (right)

Fermions in the bulk

The five dimensional covariant action for fermions in the bulk may be written as,

$$\boxed{S = -\int d^4x \int_0^L dy \sqrt{-G}\,\{ i\bar{\Psi}\Gamma^A e_A^M D_M \Psi + iM(y)\bar{\Psi}\Psi + 2\alpha_f \delta(y-L) i\bar{\Psi}_L \gamma^a e_a^\mu \partial_\mu \Psi_L \}\ .}$$

where the bulk mass term may be written as

$$M(y) = c_f \sigma'$$

and only left-handed field are even under orbifold transformations, and have a zero mode with properly normalized wave functions given by

$$\boxed{f_L^{0}(y) \;=\; \sqrt{\frac{k(1-2c_f)}{e^{(1-2c_f)kL}\,[1+(1-2c_f)\alpha_f k]-1}}\, e^{(1/2-c_f)\sigma}}$$

As happens with the gauge bosons, brane kinetic terms repel the KK wave functions from the IR brane

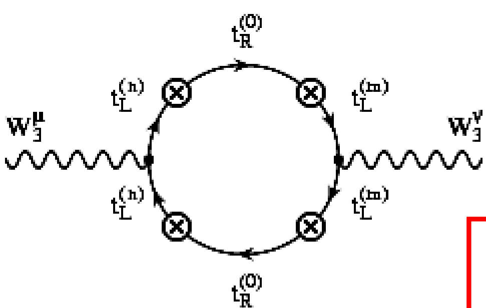
Dominant loop corrections

Dominant loop corrections come from the top-quark sector. The four dimensional Yukawa coupling is given by,

$$\boxed{\lambda_t = a_Q a_t \frac{\lambda_5}{L}}$$

$$a_f = \sqrt{\frac{(1-2c_f)kL e^{(1-2c_f)kL}}{e^{(1-2c_f)kL}[1+(1-2c_f)\alpha_f k]-1}} \approx \begin{cases} \sqrt{(2c_f-1)kL}\, e^{-(c_f-1/2)kL} & c_f - 1/2 \gtrsim 1/2kL \\ \sqrt{\frac{L}{L+\alpha_f}} & c_f = 1/2 \\ \sqrt{\frac{(1-2c_f)kL}{1+(1-2c_f)\alpha_f k}} & 1/2 - c_f \gtrsim 1/2kL \end{cases}$$

The loop corrections are induced by the interchange and mixing of KK modes



$$\boxed{\lambda_{t_L^{(n)}, t_R^{(0)}} \equiv \lambda_{n0} = \sqrt{\frac{2kL}{1+(1-2c_Q)\alpha_Q k + \alpha_Q^2 k^2 x_{Qn}^2}}\, a_t \frac{\lambda_5}{L}}$$

$$\boxed{\lambda_{t_L^{(n)}, t_R^{(m)}} \equiv \lambda_{nm} = \sqrt{\frac{2kL}{1+(1-2c_Q)\alpha_Q k + \alpha_Q^2 k^2 x_{Qn}^2}} \sqrt{\frac{2kL}{1+(1-2c_t)\alpha_t k + \alpha_t^2 k^2 x_{tm}^2}}\, \frac{\lambda_5}{L}}$$

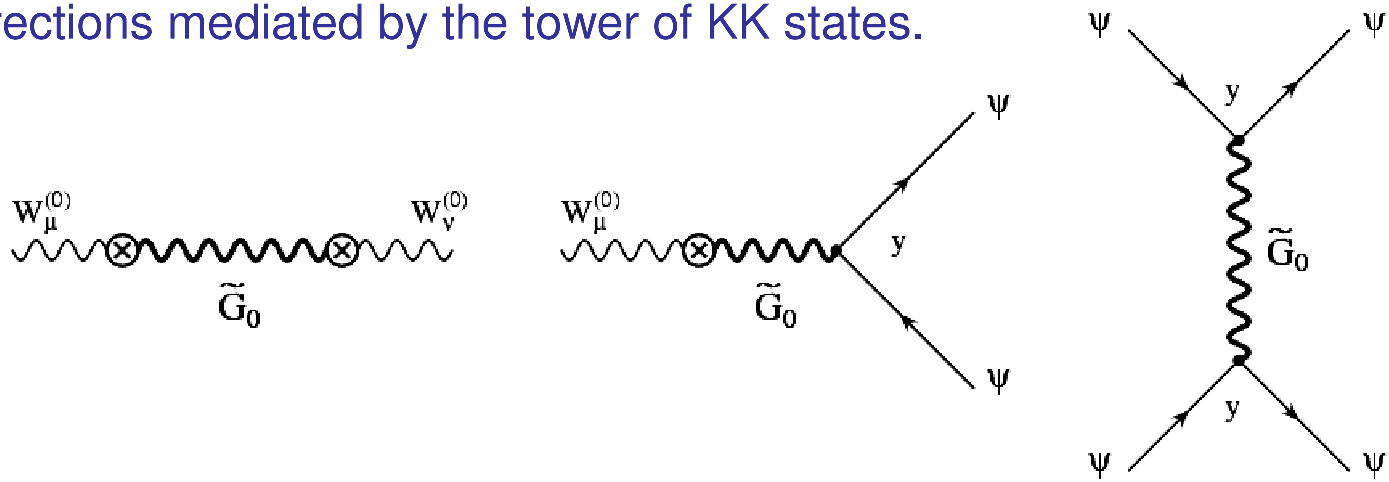
For $c_Q = ½$ and $c_U = 0$, and values of the brane kinetic terms k alpha larger than a few, we get

$$\boxed{\Delta T_t \approx \left(\frac{\lambda_{10}}{\lambda_t}\right)^2 \left(\frac{m_t}{m_{t_L}^{(1)}}\right)^2 \left[\frac{N_c}{16\pi s^2 c^2}\left(\frac{m_t}{m_Z}\right)^2\right] \left\{\frac{4}{3}\left(\frac{\lambda_{10}}{\lambda_t}\right)^2 + 4\left[2\log\left(\frac{m_{t_L}^{(1)}}{m_t}\right) - \frac{3}{2}\right]\right\}}$$

$$\boxed{m_{t_L}^{(1)} \simeq k e^{-kL} \sqrt{8\frac{(1+\alpha/L)}{(1+4k\alpha)}}}$$

Tree-level corrections

Important tree-level corrections are induced, as before, via the mixing of KK gauge boson modes with the zero mode. There are also non-oblique corrections mediated by the tower of KK states.



The parameters that characterize the modification of the zero mode couplings and the non-oblique corrections at the lowest order in v/k are given by

$$
\begin{aligned}
G'_f &\equiv \int_0^L dy\, \tilde{G}'_0(L,y)\,|f^{(0)}(y)|^2 \;\left(1+2\alpha_f\delta(y-L)\right) \\
G'_{ff} &\equiv \int_0^L dy\,dy'\, |f^{(0)}(y)|^2 \tilde{G}'_0(y,y')\,|f^{(0)}(y')|^2 \;\left(1+2\alpha_f\delta(y-L)\right)\;\left(1+2\alpha_f\delta(y'-L)\right)
\end{aligned}
$$

where we have used the five dimensional propagator $\tilde{G}_p(y,y')$, where the zero mode contribution has been substracted.

The dominant corrections to the mass parameters are parametrized by $\tilde{G}_0(L,L)$

Effective Precision Electroweak parameters

- Apart from the loop-corrections given before, the effective electroweak parameters are given by an extension of the ones we obtained for the gauge field.

- We shall work at c = ½ for all fermions, apart from the right-handed top quark, since this choice allows to avoid a strongly coupled top-Yukawa sector, while preventing dangerous FCNC.

- For c = ½, the fermion zero-mode and KK mode wave functions are similar to the one of gauge bosons, implying that if alpha is equal to r, orthogonality relations will ensure that G_f and G_{ff} are zero.

- For the aim of this presentation, we shall set all UV brane kinetic terms to zero. UV brane kinetic terms different from zero modify the numerical results by small amounts and do not have an impact on the KK mode masses.

Propagator Expressions

One can obtain an analytical expression for the propagators,

$$\bar{G}_0(L,L) = -\frac{e^{2kL} g^2}{k^2} \frac{2k^2L^2 - 2kL + 1}{4k(L + r_{IR})} .$$

$$G_f = \frac{e^{2kL} g^2}{k} \frac{(2k^2L^2 - 2kL + 1)(r_{IR} - \alpha_f)}{4k^2(L + r_{IR})(L + \alpha_f)} ,$$

$$G_{ff} = -e^{2kL} g^2 \frac{(2k^2L^2 - 2kL + 1)(r_{IR} - \alpha_f)^2}{4k^2(L + r_{IR})(L + \alpha_f)^2}$$

For values of r of order a few, only the first KK mode of the gauge bosons contribute to the propagators. These are normalized in such a way that the first expression is the square of the coupling of the first KK mode to fields localized in the IR brane divided by its mass squared, while the last expression is the same but the coupling is the coupling of the zero mode at c = ½. Then, the coupling of the first KK mode to the Higgs and to the zero mode fermions is

$$g_1(L) = g\sqrt{\frac{L}{r_{IR}}} \quad , \qquad g_{10} = g_1(L)\,\frac{r - \alpha}{L + \alpha}$$

Observe that for moderate values of the IR brane kinetic terms the gauge KK mode couples strongly to the IR brane, while it couples weakly to zero mode fermions.

Expressions of the effective parameters

The contributions to the effective parameters S and U are proportional to the change in effective couplings and the non-oblique corrections. They vanish for r = alpha.

$$T_{\rm eff} \simeq \frac{\pi}{c^2}\left(\frac{\tilde{v}}{\tilde{k}}\right)^2\left[\frac{k(L+2r_{IR}-\alpha)}{(1+r_{IR}/L)(1+\alpha/L)}\right] - \frac{U_{\rm eff}}{4s^2},$$

$$S_{\rm eff} \simeq 8\pi\left(\frac{\tilde{v}}{\tilde{k}}\right)^2\left[\frac{k(r_{IR}-\alpha)}{(1+r_{IR}/L)(1+\alpha/L)}\right],$$

$$U_{\rm eff} \simeq \frac{S_{\rm eff}}{2}\left[\frac{r_{IR}/L-\alpha/L}{1+\alpha/L}\right],$$

The parameter T is the dominant one, and receives also the important top-quark KK mode contributions. For moderate values of the brane kinetic terms, the U parameter tends to be negligible, and we shall neglect it in the fit.

Higgs Contributions

- Contrary to the case of low energy supersymmetry, there is no constraint on the Higgs mass.

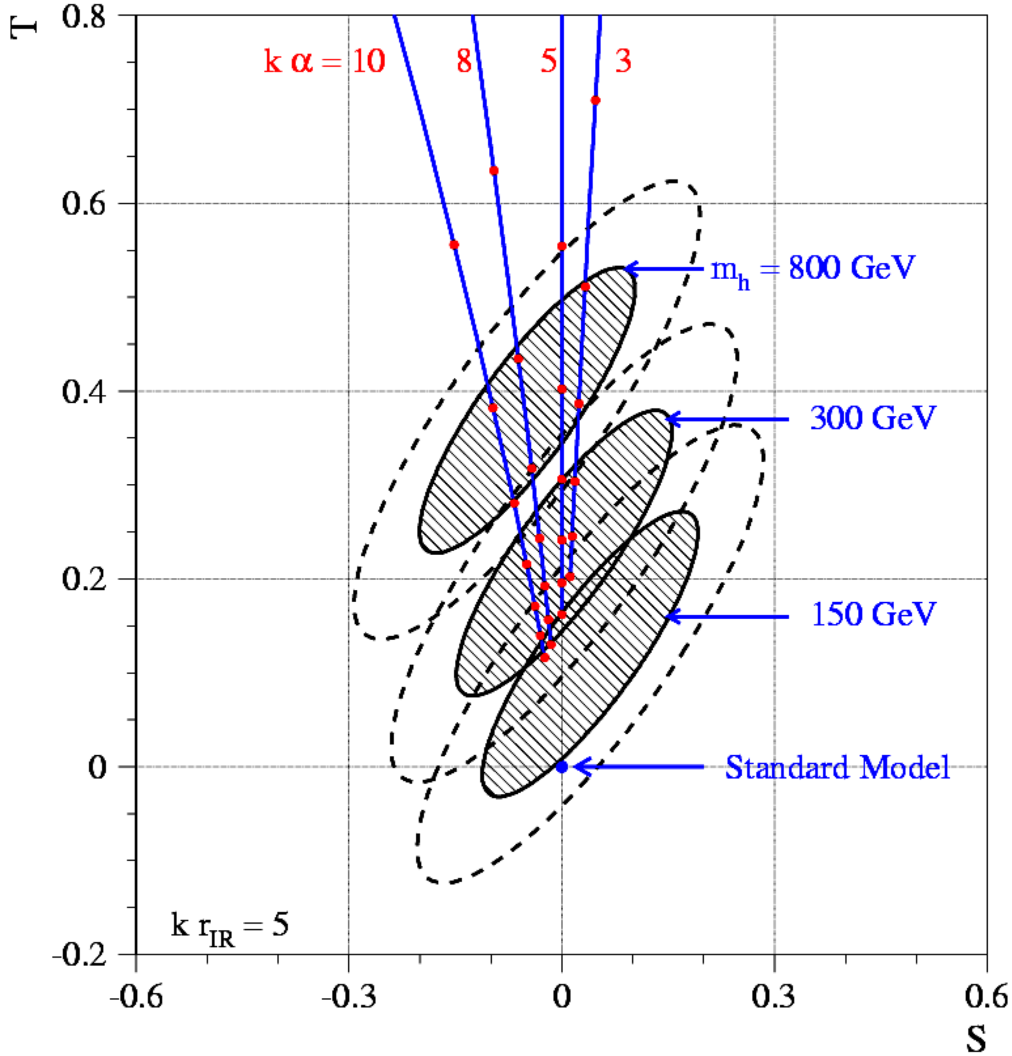
- Contribution of the Higgs mass to the S and T parameters are given by

$$S_H = \frac{1}{12\pi} \ln\left(\frac{m_H^2}{m_{H,ref}^2}\right) ,$$

$$T_H = -\frac{3}{16\pi c^2} \ln\left(\frac{m_H^2}{m_{H,ref}^2}\right) .$$

where $m_{H,ref}$ is a reference value.

Electroweak Fit



The ellipses represent the allowed values of S and T parameters consistent with new physics. For each value of the Higgs bosons mass, the origin of coordinates represents the Standard Model contribution. The lowest dots represent values of k exp(-kL) = 10 TeV, and successive dots correspond to an increase of k exp(-kL) by 1 TeV.

Results and interpretation

- The results of the electroweak fit suggest that, so far the Higgs is heaavy, values of k exp(-kL) of about 4 to 5 TeV are consistent with data.

- As said before, the masses of the first KK modes of the fermions for values of c = ½ are given by (same expression is obtained for first KK gauge boson mass as a function of r).

$$\boxed{m_{t_L}^{(1)} \simeq k e^{-kL} \sqrt{8 \frac{(1+\alpha/L)}{(1+4k\alpha)}}}$$

- Hence, values of the KK masses of fermions and gauge boson fields of about 2 to 3 TeV are consistent with data for the range of parameters analyzed here, and may be searched for at the LHC. Coupling of KK gauge bosons to fermions suppressed, but produced in association with zero mode gauge bosons.

- Second KK mode is out of reach of the LHC collider.

Conclusions

- Warped Extra Dimensions solve the hierarchy problem by means of a non-trivial metric in the extra dimensions.
- Higgs field confined to the IR brane, but gauge bosons and fermions may propagate in the bulk, open new possibilities for the solution of the flavor problem and unification of courplings.
- Bulk gauge bosons and fermions present a challenge for precision electroweak data, which in general demand KK masses of about 10 TeV, out of the reach of even the LHC.
- Brane kinetic terms of fermions and gauge bosons tend to improve this situation. We have shown that even moderate ones allow for the existence of KK modes of the order of a few TeV and hence at the reach of the LHC.

What is the Dimensionality of our universe?

- Three (spatial) plus one (time) dimensions are readily apparent to our senses

- The laws of physics are sensitive ot the dimensionality of spacetime:

  Guass Law in 4+D dimensions: $\rightarrow F \propto 1/r^{2+D}$

  → the $1/r^2$ behaviour of gravity and electrostatics are a good test that we live in 4d,

  but, Newton's law has only been tested to distances of order $200\mu\mathrm{m}$.

- SM successful description of EM, weak and strong interactions among fundamental particles depends crucially on three spacial dimensions,

  but, SM forces only tested down to distance scales of order $10^{-18}\,\mathrm{m}$.

A daring alternative: Extra spatial Dimensions

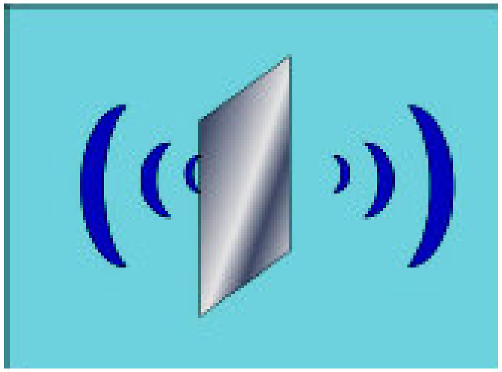
- String theory framework suggests ED (10 or 11) with no guidance about their size/s

A daring alternative:Extra Dimensions

- If seen by SM particles, they should be quite small: $\mathrm{R} \leq 10^{-17} cm \approx 1\ \mathrm{TeV}^{-1}$
- If seen only by gravity $\longrightarrow$ they can be larger: $\mathrm{R} \leq 1mm$

Gravity in ED $\Longrightarrow$ fundamental scale, pushed down to electroweak scale by geometry

Metric: $ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$ $\Longrightarrow$Solution to 5d Einstein eqs.



k=0 (flat)

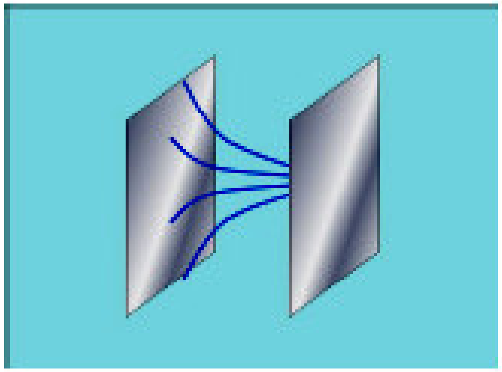
gravity flux in ED $\Longrightarrow$ Newton's law modified:

$$M_{Pl}^2 = (M_{Pl}^{\mathrm{fund.}})^{2+d}\, R^d$$

this lowers the fundamental Planck scale,

$\Longrightarrow$ depending on the size & number of ED.

$M_{Pl}^{\mathrm{fund.}} \simeq 1$ TeV $\Longrightarrow$ R= 1 mm, 10^{-12} cm if d = 2,6



k$\neq$0 (warped ED)

$$M_{Pl}^2 = \frac{(M_{Pl}^{\mathrm{fund.}})^3}{2k}(1 - e^{-2kL})$$

fundamental scales: $M_{Pl} \sim M_{Pl}^{\mathrm{fund.}} \sim v \sim k$

$\Longrightarrow$ Physical Higgs v.e.v. suppressed by e^{-kL}

$\Longrightarrow$ $\tilde{v} = v\, e^{-kL} \simeq m_Z$ if $kL \approx 34$

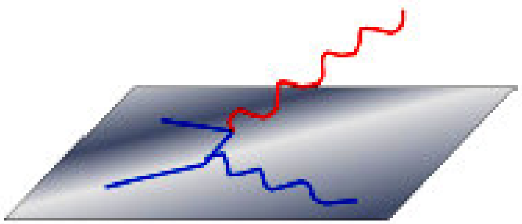
How can we probe ED from our 4D wall (brane)?

4-D effective theory:

SM particles + gravitons + tower of new particles: Kaluza Klein (KK) excited states with the same quantum numbers as the graviton and/or the SM particles

mass of the KK modes $\Longrightarrow E^2 - \vec{p}^2 = p_d^2 = m_{KK}^2$
imbalance between measured energies and momentum in 4-D = momentum in ED

Signatures

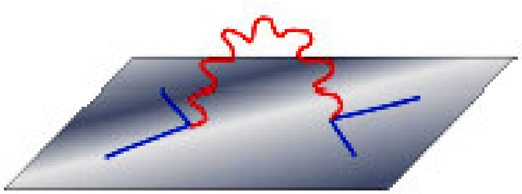


flat

• Coupling of gravitons to matter with $1/M_{Pl}$ strength
$M_{G_1} \simeq 10^{-2}$ GeV ($d = 6$); $M_{G_1} \simeq 10^{-4}$ eV ($d = 2$);

(a) Emission of KK graviton tower states: $G_n \Longleftrightarrow \not{E}_T$
(emitted gravitons appear as continuous mass distribution)

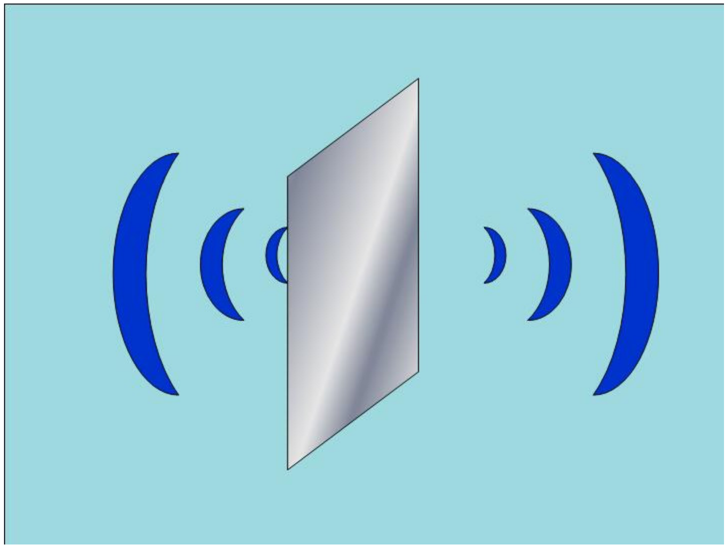
(b) Graviton exchange in $2 \rightarrow 2$ scattering – deviations for SM cross sections or new decays

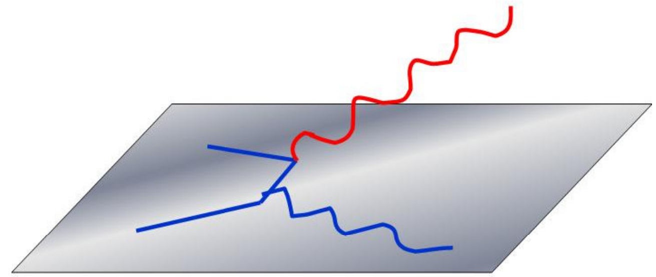


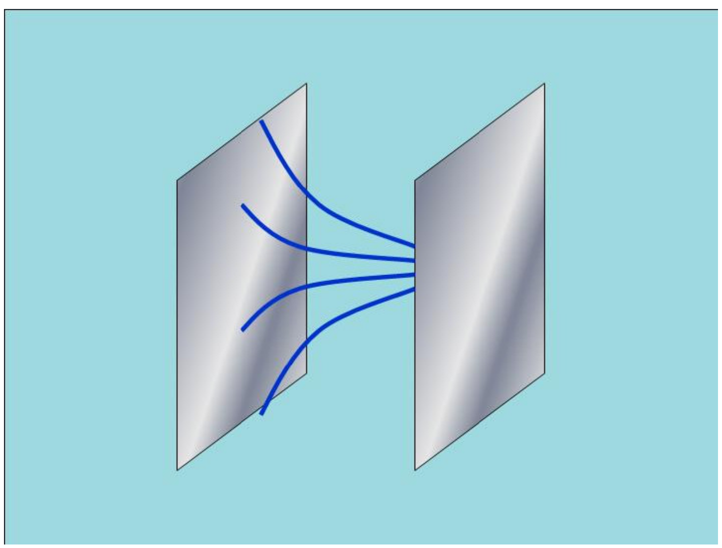
warped

• Graviton KK modes have 1/TeV coupling strength to SM fields and masses starting with a few hundred GeV.

KK graviton states produced as resonances or may contribute to $f\bar{f}$ production.







SM fields propagating in ED

$\Longrightarrow$ TeV-scale Extra dimensions or warped extra dimensions

- Gauge bosons and/or fermions in the bulk

$\Longrightarrow$ new particles may be within reach of LHC.

Universal Extra Dimensions (flat ED):

All fields in the bulk – no wall or branes

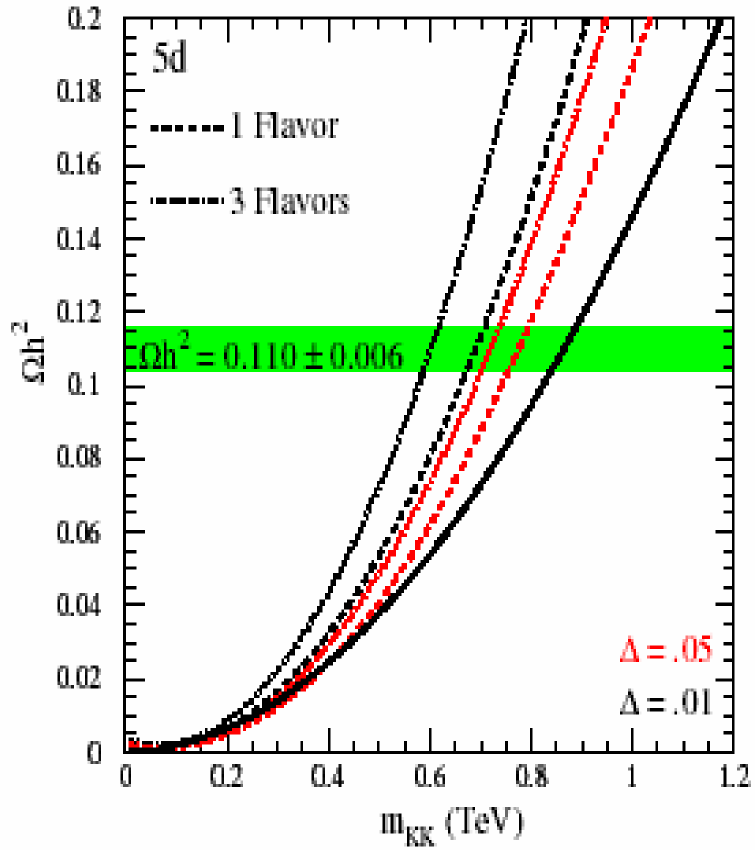
$\Longrightarrow$ momentum conserved in ED.

- KK modes produced by pairs

- no big corrections to EW observables

- Lightest Kaluza-Klein Particle (LKP)

  $\longrightarrow$ **good dark matter candidate**



Extra Dimensions

- Flat Extra Dimensions

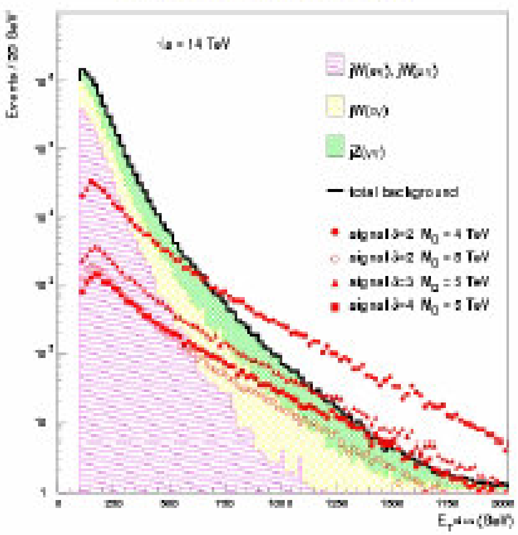
– emission of KK graviton tower states

$$p\bar{p} \to g\, G_N \;\; (G_N \to \not{E}_T) \longrightarrow \text{jet}+\not{E}_T$$

cross section summed over full KK towers

$$\Longrightarrow \qquad \sigma \propto (\sqrt{s}/M_{\rm Pl}^{\rm fund})^{2+d}$$

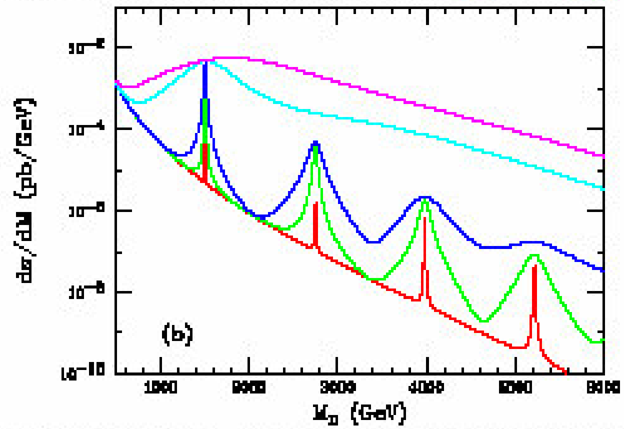
emitted graviton appears as a continuous mass distribution



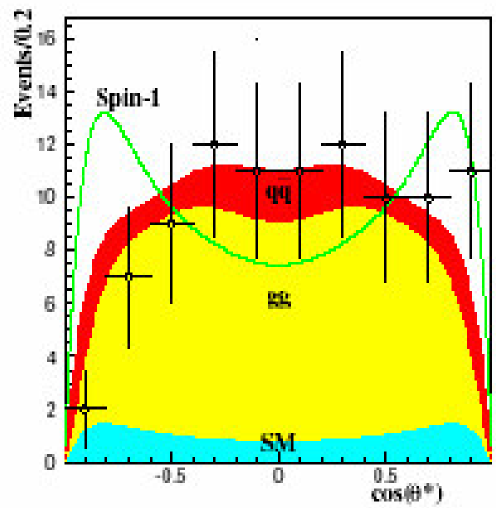
Discovery reach for fundamental Planck scales in the order of 5-10 TeV (depending on d = 4,3,2)

- Warped Extra Dimensions

narrow graviton resonances: $pp \to G_N \to e^+e^-$



from top to bottom: $k/M_{Pl} = 1, 0.5, 0.1, 0.05, 0.01$



⋆ angular distributions reveal spin of resonance

Extra Dimensions

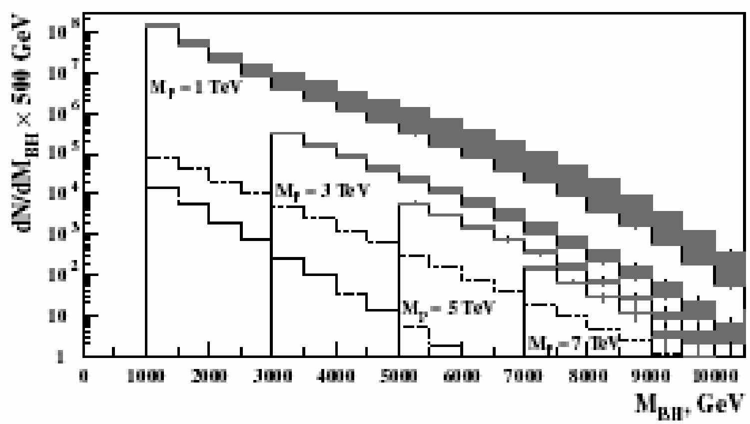
Exciting Possibility: **TeV-scale Production of Black Holes**

If $M_{BH} \gg M_{\rm Pl}^{\rm fund} \Longrightarrow$ BH properties understood:

- Two partons with center of mass energy: $\sqrt{\hat{s}} \equiv M_{BH}$ moving in opposite direction
If impact parameter smaller than the Schwarzschild radius $\Longrightarrow$ BH forms

- If $M_{\rm Pl}^{\rm fund} \sim 1$ TeV $\Longrightarrow$ more than 10^7 BH per year at the LHC !!

- Signal: sprays of SM particles in equal abundances
$\longrightarrow$ look for hard, prompt leptons & photons;



May be the first signal of TeV-scale Quantum Gravity!

- At LHC, limited space for trans-Planckian region and quantum gravity pollution

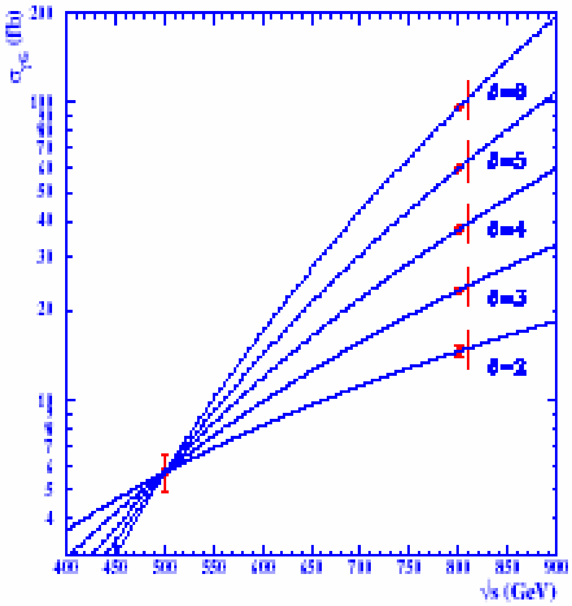
- At a VLHC ($\sqrt{s} \geq 100$ TeV), perfect conditions

Extra Dimensions

Flat ED:

graviton emission: $e^+e^- \to \gamma\, G_N$

- if signal observed, reach on $M_{\rm Pl}^{\rm fund}$ comparable to LHC if beams partially polarized
- varying $\sqrt{s}$ one can determine values of fundamental parameters: $M_{\rm Pl}^{\rm fund}$ & δ

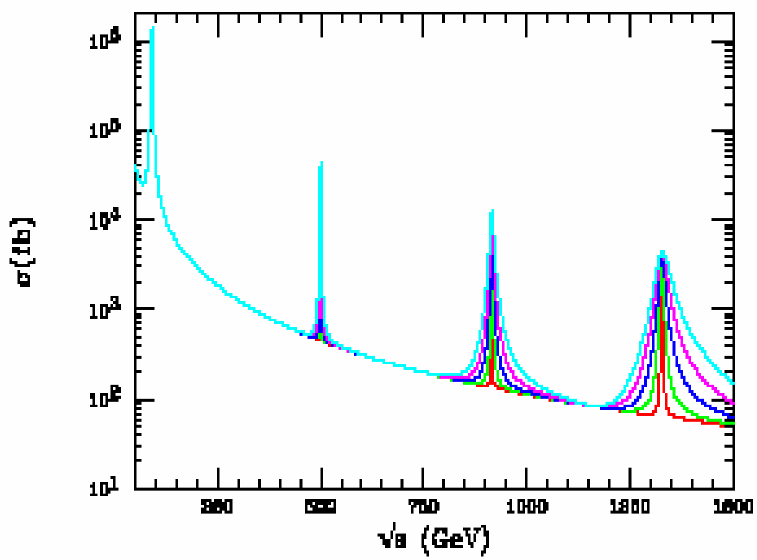


graviton exchange in $2 \to 2$ processes:

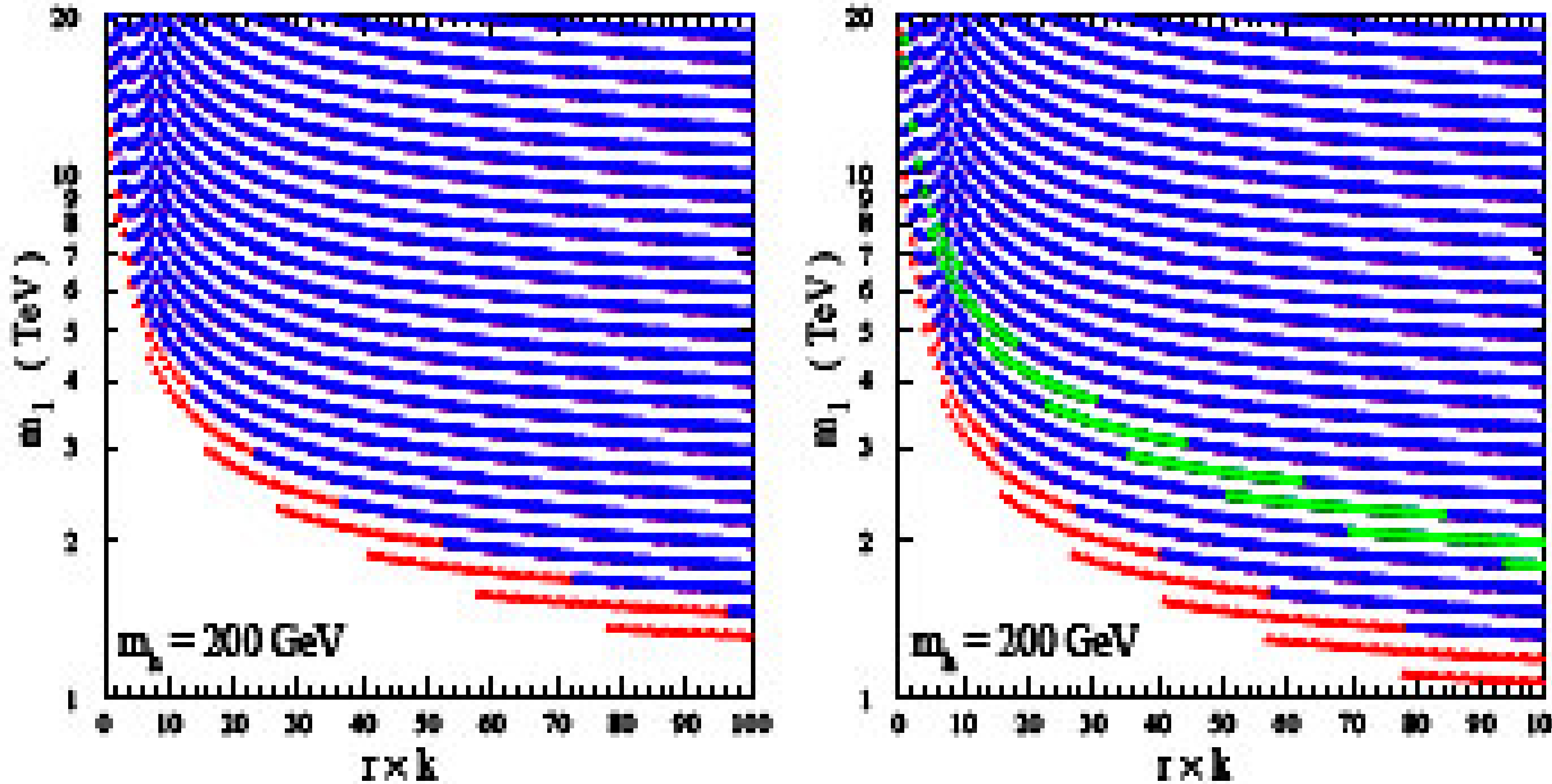
- deviations for $e^+e^- \to f\bar{f}$ or new decays with hh or $\gamma\gamma$

Warped ED:

- Given sufficient center-of-mass energy, KK graviton states produced as resonances:



$\sigma(e^+e^- \to \mu^+\mu^-)$ as a function of $\sqrt{s}$, including KK graviton exchange, $m_1 = 500$ GeV, $k/M_{Pl} = 0.01$–0.05 range.



20
10
9
8
7
6
5
4
3
2
1
m_1 (TeV)
0
10
20
30
40
50
60
70
80
90
100
$r \times k$
$m_h = 200$ GeV