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Present and Future Electroweak Precision Measurements and the Indirect Determination of the Mass of the Higgs Boson[‡]

The Snowmass Working Group on Precision Electroweak Measurements
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Abstract

We discuss the experimental and theoretical uncertainties on precision electroweak observables and their relationship to the indirect constraints on the Higgs boson mass, M_H, in the Standard Model (SM). The critical experimental measurements (M_W, $\sin^2\theta_{\rm eff}$, m_t, ...) are evaluated in terms of their present uncertainties and their prospects for improved precision at future colliders, and their contribution to the constraints on M_H. In addition, the current uncertainties of the theoretical predictions for M_W and $\sin^2\theta_{\rm eff}$ due to missing higher order corrections are estimated and expectations and necessary theoretical improvements for future colliders are explored. The constraints from rare B decays are also discussed. Analysis of the present experimental and theoretical precisions yield a current upper bound on M_H of ~ 200 GeV. Including anticipated improvements corresponding to the prospective situation at future colliders (Tevatron Run II, LHC, LC/GigaZ), we find a relative precision of about 25% to 8% (or better) is achievable in the indirect determination of M_H.

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The Precision Electroweak Working Group of Snowmass 2001

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We discuss the experimental and theoretical uncertainties on precision electroweak observables and their relationship to the indirect constraints on the Higgs-boson mass, M_H, in the Standard Model (SM). The critical experimental measurements (M_W, $\sin^2\theta_{\rm eff}$, m_t, ...) are evaluated in terms of their present uncertainties and their prospects for improved precision at future colliders, and their contribution to the constraints on M_H. In addition, the current uncertainties of the theoretical predictions for M_W and $\sin^2\theta_{\rm eff}$ due to missing higher order corrections are estimated and expectations and necessary theoretical improvements for future colliders are explored. The constraints from rare B decays are also discussed. Analysis of the present experimental and theoretical precisions yield a current upper bound on M_H of ~ 200 GeV. Including anticipated improvements corresponding to the prospective situation at future colliders (Tevatron Run II, LHC, LC/GigaZ), we find a relative precision of about 25% to 8% (or better) is achievable in the indirect determination of M_H.

I. INTRODUCTION

In this contribution we address the status and possible future developments in the measurements of and the theoretical predictions for the most important electroweak precision observables. We estimate their precision from upcoming and proposed accelerator experiments. In all cases we quote uncertainties which we believe to be realistically achievable, not excluding even greater precisions. As a result of imposing similar standards in all cases, our quoted uncertainties should be directly comparable. Similarly, we attempt to anticipate which improvements can be expected in the theoretical predictions for the observables.

Within the SM, the mass of the Higgs boson, M_H, can be constrained indirectly with the help of electroweak precision observables (EWPO). Among the experimental measurements of EWPO which are used in global fits, the W-boson mass, M_W, and the effective leptonic weak mixing angle, $\sin^2\theta_{\rm eff}$, have the largest impact on the extracted value of M_H. Although the current relative precision of M_W is better by a factor of 1.8 compared to $\sin^2\theta_{\rm eff}$, the latter is the most relevant parameter for the indirect M_H determination due to its more pronounced dependence on the Higgs mass. For equal relative experimental precisions, it yields a 3.1 times higher sensitivity (for M_H around 115 GeV). Other observables include the leptonic Z-boson width, Γ_l; the mass and width of

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the Z boson, M_Z and Γ_Z; the peak hadronic cross section of the Z boson, $\sigma^0_{\rm had}$; EWPO from deep inelastic neutrino scattering; and others. Furthermore the top quark mass, m_t, enters in the global fit; its value and its error have a strong impact on the extracted M_H value. The results from rare B decays do not enter the fit directly, but they are important for constraining certain non-SM extensions of the Higgs sector.

In this article, we begin by summarizing the performance parameters we have assumed for future colliders. We then discuss several critical measurements and the anticipated experimental and theoretical uncertainties. Finally, we consider the precision of future indirect determination of the Higgs-boson mass from precision electroweak fits. A related discussion of these fits can be found in Ref. [1].

II. COLLIDER PARAMETERS

At the time of the present study, LEP has finished running, and nearly all of the relevant analyses are complete. The final results from Tevatron Run I are all available, and no results are yet available from Tevatron Run II. The b-factories at SLAC and KEK are running and have some results already available.

A proton-antiproton center-of-mass energy of 2 TeV is assumed for Run II of the Tevatron at Fermilab. Run IIA is expected to deliver an integrated luminosity of 2 fb^{-1} to each of the two experiments. The next phase of operations, Run IIB, is expected to deliver 15 fb^{-1} according to the plan currently adopted by Fermilab, but this could go as high as 30 fb^{-1} if additional upgrades are made to the accelerator complex. We refer to these two scenarios as Run IIB and Run IIB*.

The Large Hadron Collider is assumed to deliver 100 fb^{-1} to each experiment each year, with a proton-proton center-of-mass energy of 14 TeV.

For a future Linear Collider (LC), we assume an electron-positron (or electron-electron) center-of-mass energy of 0.5 TeV and 500 fb^{-1} delivered to a single experiment. For those measurements which require polarized beams, we make this assumption explicit. "GigaZ" collectively denotes an LC operating at $\sqrt{s} = M_Z$ or $\sqrt{s} \approx 2M_W$ with a luminosity of $\mathcal{L} \approx 5 \times 10^{33}\text{cm}^{-2}\text{s}^{-1}$.

III. THE MASS OF THE W BOSON

We begin by describing the experimental uncertainties in the measurement of the W-boson mass, first at hadron colliders, then at lepton colliders. We then discuss the theoretical issues. In the theoretical discussion, we use the phrase *intrinsic* uncertainties for the ones arising from unknown higher-order corrections in the perturbative expansion, as well as for other uncertainties arising from other computational limitations. These are to be distinguished from *primordial* uncertainties, which are those uncertainties in relating a primordial quantity (like the actual mass of the W boson) to the observables that experiments use to estimate the primordial quantity. More details can be found in Ref. [1].

A. Experimental Prospects

The current precision of M_W is dominated by the direct mass reconstruction of W-pair events at LEP2. Transverse-mass fits from Run I at the Tevatron and data from threshold scans at LEP2 also contribute significantly but carry less statistical weight.

<u>Hadron Colliders:</u>

The next opportunity to improve the measurement of M_W will be at Run II of the Tevatron. The measurement of the W-boson mass from Tevatron Run I data achieved a precision of 68 MeV[2, 3]. A variety of methods can be used to measure the W-boson mass with different tradeoffs between statistical and systematic uncertainties. These include fits of the transverse mass and lepton p_T spectra to templates from Monte Carlo simulations. Most systematics, such as the detector calibration and the recoil model, are driven by the number of Z-boson decays observed[2, 3]. Measurements of the charge asymmetry in $W \to \ell\nu$ decays will help constrain the parton distribution functions. In the ratio method, developed by DØ[4], the Z-boson data are rescaled to fit the W-boson data. This reduces most experimental and some theoretical uncertainties at the cost of statistical sensitivity. In all cases, systematic uncertainties are expected to dominate. Since the main systematics differ, these methods can be used to check the results for consistency at the 10 MeV level.

Table I shows the expected precision of the W mass measurement from the transverse mass fit, extrapolated from the Run Ib measurement by DØ[5]. The calorimeter scale and linearity assume constraints from Z data only, but not the J/ψ and π^0 data used in Run I. By about 30 fb^{-1}, the determination of the energy resolution will be systematically limited by the uncertainty in the width of the Z boson. The uncertainty due to electron removal was conservatively assumed to decrease only by half. Table I also shows an extrapolation of the uncertainty for the ratio method from Run I results by DØ. The systematic uncertainty for this method is smaller than for the transverse mass fit and it may well be the best for high integrated luminosities. We conclude that the W-boson mass will be measured at the end of Run II to a precision of 15 MeV, perhaps even 10 MeV, combining the results from both experiments, using several methods and the $W \to e\nu$ and $W \to \mu\nu$ channels.

TABLE I: Projected uncertainties in MeV of the W-boson mass measurement using the transverse mass fit (left) and the ratio method (right) for $W \to e\nu$ decays.

Run	I	IIA	IIB	IIB*
$\int \mathcal{L}dt$ (fb^{-1})	0.08	2	15	30
statistical uncertainty	96	19	7	5
production/decay model	30	14	13	13
backgrounds	11	2	1	1
detector model	57	13	8	8
total systematic	66	19	16	15
total uncertainty	116	27	17	16

Run	I	IIA	IIB	IIB*
$\int \mathcal{L}dt$ (fb^{-1})	0.08	2	15	30
statistical uncertainty	211	44	16	11
total systematic	50	10	4	3
total uncertainty	217	44	16	12

The improvement in the measurement of M_W at the LHC is due to the large statistics which is expected to result in very small statistical errors and good control of many systematic uncertainties. However, as in Run IIB, theoretical improvements are needed, e.g. for radiative W decays, the modeling of the p_T^W distribution, and for constraining PDFs. In Ref. [6] it has been argued that it should be possible to obtain an uncertainty on the W mass due to PDFs smaller than 10 MeV.

Linear Collider:

As for the M_W measurement at LEP2, the determination of the W mass at the LC at center of mass (CM) energies above the W^+W^- production threshold will be based on direct reconstruction of W-pair events in 4-fermion production processes. For an integrated luminosity of 500 fb^{-1} the uncertainty on M_W is expected to be about 10 MeV[7].

A determination of M_W with the GigaZ option is based on a dedicated threshold scan. About one year of running would be needed to achieve a 5 MeV experimental error on M_W [8]. This also requires that the knowledge of the absolute beam energy can be controlled to better than 2.5 MeV. Although this is of higher precision than currently foreseen for TESLA or NLC, it might be achievable with some additional effort [8].

B. Theoretical Issues

For the envisioned precision measurement of the W-boson mass, M_W, at present and future lepton and hadron colliders it is crucial that the theoretical predictions for the underlying production processes are well under control. The status and prospects of theoretical predictions for weak gauge boson production processes are presented in Ref. [9] and briefly summarized in the following.

Hadron Colliders

QED corrections are known to produce a considerable shift in the W- and Z-boson masses measured at hadron colliders [2, 10, 11, 12]. Given the expected accuracy for M_W in Run II of the Tevatron and at the LHC, calculations including the full $\mathcal{O}(\alpha)$ electroweak corrections to weak-boson production in hadronic collisions are needed. A calculation of the electroweak $\mathcal{O}(\alpha)$ corrections to W production has been carried out in Ref. [13] in the leading-pole approximation, i.e. corrections which are very small at the W pole were ignored. Calculations of the full $\mathcal{O}(\alpha)$ corrections to $p\overset{(-)}{p} \to W \to \ell\nu_\ell$ have recently appeared in Refs. [14] and [15]. While the corrections ignored in Ref. [13] change the differential cross section in the W-pole region by less than 1% [14], they become large at high $\ell\nu_\ell$ invariant masses $m(\ell\nu_\ell)$ due to Sudakov-like logarithms of the form $\ln^2[m(\ell\nu_\ell)/M_W]$. They significantly affect the transverse mass distribution above the W peak, which serves as a tool for a direct measurement of the W width, Γ_W. Taking these corrections into account in future measurements of Γ_W will be important.

The determination of the W-boson mass in a hadron collider environment requires a simultaneous precision measurement of the Z-boson mass, M_Z, and width, Γ_Z. When compared to the value measured at LEP1, the two quantities help to accurately calibrate detector components [2, 10, 11, 12]. It is therefore necessary to also understand electroweak corrections for Z-boson production. A calculation of the full $\mathcal{O}(\alpha)$ QED corrections to

$p\overset{(-)}{p} \to \gamma,\, Z \to \ell^+\ell^-$ was carried out in Ref. [16]. Purely weak corrections were ignored in this first step towards a complete calculation of the $\mathcal{O}(\alpha)$ electroweak corrections to $p\overset{(-)}{p} \to \gamma,\, Z \to \ell^+\ell^-$. However, in order to properly calibrate the Z-boson mass and width using the available LEP data, it is desirable to use exactly the same theoretical input that has been used to extract M_Z and Γ_Z at LEP, i.e. to include the purely weak corrections to $p\overset{(-)}{p} \to \gamma,\, Z \to \ell^+\ell^-$ and the $\mathcal{O}(G_F^2 m_t^2 M_W^2)$ corrections to the effective leptonic weak mixing parameter, $\sin^2\theta_{\rm eff}$, and the W-boson mass [17], in the calculation. Such a calculation has recently been completed [18]. The additional corrections taken into account in Ref. [18] enhance the differential cross section in the Z-peak region by up to 1.2%. Since they are not uniform, they are expected to shift the Z-boson mass extracted from data upward by several MeV. Detailed simulations of this effect, however, have not been carried out yet.

For the analysis of Run IIA data [19] the presently available calculations of W- and Z-boson production will likely be sufficient. However, to measure M_W with a precision of less than 20 MeV in a hadron collider environment as foreseen in Run IIB and at the LHC, it will be necessary to take into account higher-order corrections, in particular multi-photon radiation effects.

Lepton Colliders

At LEP2 and a future Linear Collider (LC), M_W is measured in W-pair production either in a dedicated threshold scan operating the machine at center-of-mass (CM) energies of ≈ 161 GeV, or via direct reconstruction of the W bosons at CM energies above 170 GeV. For state-of-the-art predictions for $e^+e^- \to WW \to 4f$ cross sections including $\mathcal{O}(\alpha)$ corrections, two MC generators are presently available, `RacoonWW` [20, 21, 22] and `YFSWW3` [23, 24, 25]. As it is the case for all present calculations of $\mathcal{O}(\alpha)$ corrections to $e^+e^- \to WW \to 4f$, they rely on a double-pole approximation (DPA): electroweak $\mathcal{O}(\alpha)$ corrections are only considered for the terms that are enhanced by two resonant W bosons. A tuned numerical comparison between `RacoonWW` and `YFSWW3`, supported by a comparison with a semi-analytical calculation [26] and a study of the intrinsic DPA ambiguity with `RacoonWW` [22, 27] and `YFSWW3` [27], shows that the current theoretical uncertainty for the total W-pair production cross section, σ_{WW}, is about 0.5% for CM energies between 170 GeV and 500 GeV [27]. Taking the observed differences [27] between the `RacoonWW` and `YFSWW3` predictions at CM energies of 200 and 500 GeV as a guideline, a theoretical uncertainty of about 1% can be assigned to the distribution of the W production angle and the W invariant-mass distribution in the W resonance region. The theoretical uncertainties of the $e^+e^- \to WW \to 4f$ cross sections translate into uncertainties of the W mass and the triple gauge-boson couplings (TGCs) extracted from data. A recent study [28] based on the MC generators `KoralW` and `YFSWW3` finds a theoretical uncertainty of $\delta M_W = 5$ MeV due to unknown electroweak corrections at LEP2 energies. Using the MC generator `YFSWW3`, the ALEPH collaboration [29] has derived (preliminary) results for the shifts in the extracted values for the TGCs due to the inclusion of electroweak corrections. For M_W measurements well above the W-pair threshold at LEP2 energies and up to about 500 GeV the accuracy of the corresponding predictions is sufficient. More studies, however, are needed to properly estimate the uncertainties due to missing higher-order corrections at LC energies above 500 GeV.

In order to meet the goal of a combined error of $\delta M_W = 7$ MeV [1, 8] in a threshold scan at a LC, the theoretical uncertainty of the extracted value of M_W has to be less than 2–3 MeV. At present there is no study available on how this requirement translates into a constraint on the theoretical precision for σ_{WW} in the threshold region. The DPA is not a valid approximation in the threshold region, as the $e^+e^- \to 4f$ cross section in this region is not dominated by W-pair production. Thus, σ_{WW} in the threshold region is known with an accuracy of only about 1–2% [30, 31], since predictions are based on an improved-Born approximation which neglects non-universal electroweak corrections. Assuming the theoretical predictions do not improve, the uncertainty of the W mass obtained from a threshold scan is completely dominated by the theoretical error, and the precision of the W mass is limited [9] to about $20-30$ MeV. Only the knowledge of the full $\mathcal{O}(\alpha)$ electroweak corrections to $e^+e^- \to 4f$ in the threshold region will allow one to decide whether the theoretical uncertainty of the M_W measurement from a threshold scan can be reduced to the desired level. A full calculation of the $\mathcal{O}(\alpha)$ electroweak corrections to $e^+e^- \to WW \to 4f$ is currently not available, but there is ongoing work in this direction [32, 33]. This calculation is of enormous complexity and the requirement to include finite W-width effects poses severe problems with gauge invariance.

Higher Order Uncertainties:

Concerning the intrinsic uncertainties of M_W from unknown higher orders, recent progress has been made for the prediction of M_W by the inclusion of the full fermionic two-loop corrections [34], superseding the previous expansions in m_t^2/M_W^2. Since this expansion yielded similar values (with the same sign) for the m_t^4/M_W^4 and the m_t^2/M_W^2 terms (casting some doubt on the convergence), the full fermionic two-loop corrections constitute an important step towards a very precise M_W prediction. The difference between the expansion calculation and the full result can reach up to about 4 MeV, depending on M_H. The only missing two-loop corrections to M_W are the pure bosonic contributions. The M_H dependence of the bosonic two-loop contributions to M_W has recently been evaluated [35], indicating corrections of $\mathcal{O}(1$ MeV).

In order to quantify the remaining intrinsic uncertainties of the EWPO, one has to perform estimates of the possible size of uncalculated higher-order corrections. The results of calculations based on different renormalization schemes or on different prescriptions for incorporating non-leading contributions in resummed or expanded form differ from each other by higher-order corrections. One way of estimating the size of unknown higher-order corrections is thus to compare the results for the prediction of the EWPO from different codes in which the same corrections are organized in a somewhat different way. A detailed description of different "options" used in this comparison can be found in Ref. [36] and an update in Ref. [37]. This prescription may lead to an underestimate of the theoretical uncertainty if at an uncalculated order a new source of potentially large corrections (e.g. a certain enhancement factor) sets in. In general, it is not easy to quantify how large the variety of different codes and different "options" should be in order to obtain a reasonable estimate of the theoretical uncertainty.

Other (related) methods to estimate the size of missing higher order corrections are to vary the renormalization scales and schemes. While these methods usually give an order of magnitude estimate and a lower bound on the uncertainty, they can lead to underestimates whenever there are sizeable but scheme- and scale-invariant contributions. For example, the lowest order flavor singlet contribution to Z decay, a separately gauge invariant and finite set of corrections, cannot be estimated by scale variations of the non-singlet contribution or by using different "options" for resumming non-leading contributions in computer codes.

In the following we use a simple minded, but rather robust and, in the past, quite successful method for estimating the uncertainties from unknown higher orders [38]. The idea is to collect all relevant enhancement and suppression factors and setting the remaining coefficient (from the actual loop integrals) to unity. If, in a given order, terms with different group theory factors contribute, one can often choose the largest one as an estimate for the uncertainty. Our results are summarized in Table II [1]. They are in good agreement with the estimates of the current uncertainties of M_W performed in Refs. [35, 39, 40, 41].

TABLE II: Theoretical uncertainties from unknown higher-order corrections to M_W. $\hat{s}$ denotes the $\overline{\rm MS}$ mixing angle, $N = 12$ is the number of fermion doublets in the SM, $C_F = 4/3$ and $C_A = 3$ are QCD factors, and $N_C = 3$ is the number of colors. The corrections in the upper part of the table are assumed to enter the predictions in the same way as $\Delta\alpha$ (only the leading top quark correction of $\mathcal{O}(\alpha\alpha_s^2)$ enters via $\Delta\rho$), while the ones in the lower part are assumed to enter via $\Delta\rho$. The fermionic contributions of $\mathcal{O}(\alpha^2)$ refer to the non-leading terms beyond the next-to-leading term of the expansion in powers of m_t^2/M_W^2. The $\mathcal{O}(\alpha\alpha_s^2)$ corrections, which are completely known both for M_W and $\sin^2\theta_{\rm eff}$, are included in the table for completeness. However, the light fermion corrections are not yet included in all codes currently used for performing electroweak fits (and have not been published yet as an independent explicit formula); our corresponding error estimate for M_W would be ± 1.7 MeV. In order to estimate effects of finite M_H and subleading terms in the lower part of the table, we have taken the average of the individual coefficients of the result in the limit $M_H = 0$ [42] (which in this limit conspire to yield a small answer), resulting in the numerical prefactors there.

order	sector	estimate	size ($\times 10^5$)	M_W[MeV]
α^2	fermionic	$N(\alpha/4\pi\hat{s}^2)^2$	8.7	complete [34]
α^2	bosonic	$(\alpha/\pi\hat{s}^2)^2$	11.6	2.1
$\alpha\alpha_s^2$	top-bottom doublet	$N_C C_F C_A \alpha\alpha_s^2/4\pi^3\hat{s}^2$	4.7	complete [43]
$\alpha\alpha_s^2$	light doublets	$2\ N_C C_F C_A \alpha\alpha_s^2/4\pi^3\hat{s}^2$	9.4	complete [44]
$\alpha^3 m_t^6$	heavy top	$5.3\ N_C^2(\alpha m_t^2/4\pi\hat{s}^2 M_W^2)^3$	7.0	4.1
$\alpha^3 m_t^6$	heavy top	$3.3\ N_C(\alpha m_t^2/4\pi\hat{s}^2 M_W^2)^3$	1.5	0.9
$\alpha^2\alpha_s m_t^4$	heavy top	$3.9\ N_C C_F \alpha^2\alpha_s m_t^4/16\pi^3\hat{s}^4 M_W^4$	7.8	4.5
$\alpha\alpha_s^3 m_t^2$	heavy top	$N_C C_F C_A^2 \alpha\alpha_s^3 m_t^2/4\pi^4\hat{s}^2 M_W^2$	2.3	1.3
	total			7

IV. THE EFFECTIVE LEPTONIC WEAK MIXING ANGLE

A. Experimental Prospects

The current measurement of $\sin^2\theta_{\rm eff}$ is dominated by the left-right asymmetry from SLD[45] and the b-quark forward-backward asymmetry from LEP1[46].

At the Tevatron, the forward-backward asymmetry A_{FB} in the process $u\overline{u} + d\overline{d} \to Z \to \ell^+\ell^-$, measured near the Z pole, gives a value of the weak mixing angle $\sin^2\theta_{\rm eff}$. CDF published $A_{FB} = 0.070 \pm 0.015(\text{stat}) \pm 0.004(\text{syst})$ based on data from Run I[47]. The statistical uncertainty scales to 0.0016 for 10 fb^{-1} and to 0.0009 for 30 fb^{-1}. The most important systematic uncertainty arises from the parton distribution functions. These can be constrained by the charge asymmetry in W decays. A theoretical uncertainty arises from the limited rapidity coverage and the p_T distribution of the Z. Both the rapidity and p_T distribution of the Z will be measured and this uncertainty will be reduced. It is expected that the statistical uncertainty will dominate all systematics[19, 48].

Combining the electron and muon channels from both CDF and DØ and using a linear approximation for the relation between A_{FB} and $\sin^2\theta_{\rm eff}$ leads to a projection for the precision of the $\sin^2\theta_{\rm eff}$ measurement of 0.00028 for 10 fb^{-1}[19, 48], to be compared with the current world-average precision of 0.00017. The precision is degraded somewhat when the full tree level relation between A_{FB} and $\sin^2\theta_{\rm eff}$ is taken into account: 0.00029 for 15 fb^{-1} and 0.00020 for 30 fb^{-1}. This is comparable to the current world average[49], and should clarify the 3.5σ discrepancy between $\sin^2\theta_{\rm eff}$ from $A_{\rm LR}$ measured at SLD[45] and $A_{FB}^{0,b}$, measured at LEP[46].

In pp collisions at LHC, a forward-backward lepton asymmetry can also be measured, where the quark direction in the initial state has to be extracted from the boost direction of the $\ell^+\ell^-$ system with respect to the beam axis. In order to improve the experimental uncertainty of $\sin^2\theta_{\rm eff}$ at the LHC, it will be necessary to detect one of the leptons originating from $Z \to \ell^+\ell^-$ over the entire pseudorapidity range of $|\eta| < 5$ [6]. This requires an electron jet rejection factor of < 0.01 in the forward region ($2.5 < |\eta| < 5$) of the electromagnetic calorimeter. The relevance of a more precise determination of PDFs in this respect remains to be investigated.

The determination of the effective leptonic mixing angle at the LC could in principle be performed by a fixed-target Møller scattering experiment [50, 51] (but see also Ref. [52].) However, this would yield $\sin^2\theta_{\rm eff}$ at a scale of $\mathcal{O}(0.5$ GeV$)$, making the extrapolation to M_Z difficult. We therefore do not consider this option here in more detail (although sensitivity to physics beyond the SM might be possible.)

At GigaZ one hopes to improve the current precision of $\sin^2\theta_{\rm eff}$ by more than an order of magnitude. This is envisaged by a precise measurement of $A_{\rm LR}$ [53, 54] using the Blondel scheme [55]. $A_{\rm LR}$ is then given as a function of polarized cross sections, where both beams have possibly different combinations of polarizations. Due to the anticipated drastic improvement in the accuracy, a reanalysis of the effect of primordial uncertainties in the determination of $\sin^2\theta_{\rm eff}$ might become necessary. This determination of $A_{\rm LR}$ requires that both beams can be polarized independently and that the polarizations of the colliding e^+ and e^- bunches with opposite helicity states are equal (or that their difference is precisely determined; see Ref. [53] for details). A precision of $\delta A_{\rm LR} \approx 8 \times 10^{-5}$ seems to be feasible [53, 56], resulting in $\delta\sin^2\theta_{\rm eff} \approx 10^{-5}$.

B. Theoretical Issues

Concerning the intrinsic uncertainties of $\sin^2\theta_{\rm eff}$ from unknown higher orders, the situation is slightly worse than for M_W, since a result for the full fermionic two-loop corrections is not yet available, and one has to rely on the expansion in powers of m_t^2/M_W^2 [57]. Beyond two-loop order, the results for the pure fermion-loop contributions (incorporating in particular the leading terms in $\Delta\alpha$ and $\Delta\rho$) are known up to the four-loop order [58]. Furthermore, the QCD corrections of $\mathcal{O}(\alpha\alpha_s^2)$ are known [43, 44]. More recently, also the leading three-loop terms of $\mathcal{O}(G_F^3 m_t^6)$ and $\mathcal{O}(G_F^2\alpha_s m_t^4)$, which enter via the quantity $\Delta\rho$, have been calculated in the limit of vanishing Higgs-boson mass. The results have been found to be quite small, which is familiar from the $M_H = 0$ limit of the $\mathcal{O}(G_F^2 m_t^4)$ result [59]. In the latter case, the extension to finite values of M_H and the inclusion of subleading terms led to an increase in the numerical result by a factor of up to 20.

A shift in the prediction for M_W, on the other hand, induces a shift in $\sin^2\theta_{\rm eff}$ according to

$$\sin^2\theta_{\rm eff} = \left(1 - \frac{M_W^2}{M_Z^2}\right)\kappa(M_W^2), \tag{1}$$

where κ is a calculable function in the SM. While the shift in M_W induced by going from the result of the expansion in powers of m_t^2/M_W^2 to the result of the full fermionic two-loop corrections is known, the corresponding result for $\kappa(M_W^2)$ is still missing. The effect of inserting the new result for M_W in Eq. (1), which amounts to an upward shift of about 8×10^{-5} in $\sin^2\theta_{\rm eff}$ (for $M_H \approx 115$ GeV), has been (conservatively) treated as a theoretical uncertainty in the "Blue Band" of Fig. 1 (see Sec. VII).

The same method described earlier for M_W is used to quantify the remaining intrinsic uncertainties of $\sin^2\theta_{\rm eff}$ [38]. Our results are summarized in Table III, with a total contribution to the uncertainty of 7×10^{-5}. They are in good agreement with the estimates of the current uncertainties of $\sin^2\theta_{\rm eff}$ performed in Refs. [35, 39, 40, 41].

TABLE III: Theoretical uncertainties from unknown higher-order corrections to $\sin^2\theta_{\rm eff}$. See Table II for a key to the notation and assumptions. The uncertainty in $\sin^2\theta_{\rm eff}$ has been estimated from the known correction to M_W using Eq. (1) (see text). The light fermion corrections are not yet included in all codes currently used for performing electroweak fits (and have not been published yet as an independent explicit formula); our corresponding error estimate for $\sin^2\theta_{\rm eff}$ would be $\pm 3.3\times 10^{-5}$.

order	sector	estimate	size ($\times 10^5$)	$\sin^2\theta_{\rm eff}$ ($\times 10^5$)
α^2	fermionic	$N(\alpha/4\pi\hat{s}^2)^2$	8.7	4.1
α^2	bosonic	$(\alpha/\pi\hat{s}^2)^2$	11.6	4.1
$\alpha\alpha_s^2$	top-bottom doublet	$N_C C_F C_A \alpha\alpha_s^2/4\pi^3\hat{s}^2$	4.7	complete [43]
$\alpha\alpha_s^2$	light doublets	$2\ N_C C_F C_A \alpha\alpha_s^2/4\pi^3\hat{s}^2$	9.4	complete [44]
$\alpha^3 m_t^6$	heavy top	$5.3\ N_C^2(\alpha m_t^2/4\pi\hat{s}^2 M_W^2)^3$	7.0	2.3
$\alpha^3 m_t^6$	heavy top	$3.3\ N_C(\alpha m_t^2/4\pi\hat{s}^2 M_W^2)^3$	1.5	0.5
$\alpha^2\alpha_s m_t^4$	heavy top	$3.9\ N_C C_F \alpha^2\alpha_s m_t^4/16\pi^3\hat{s}^4 M_W^4$	7.8	2.5
$\alpha\alpha_s^3 m_t^2$	heavy top	$N_C C_F C_A^2 \alpha\alpha_s^3 m_t^2/4\pi^4\hat{s}^2 M_W^2$	2.3	0.8
	total			7

V. TOP QUARK PROPERTIES

For the precision EW fits, the most important top quark property is its mass, m_t. Although from an experimental standpoint, m_t is on the same footing as M_W and $\sin^2\theta_{\rm eff}$, theoretically it is handled a bit differently and affects the theoretical predictions for M_W and $\sin^2\theta_{\rm eff}$.

In Tevatron Run I, the top quark mass was measured to ≈ 5 GeV[60, 61]. For Run II, $t\bar{t}$ data samples will be large enough to allow a double b-tag. For 15 fb^{-1} per experiment, 3200 double-tagged single-lepton and 1200 untagged dilepton events are expected.

The main systematic uncertainty for the top quark mass measurement is the jet energy scale. Using Run I methods, this uncertainy cannot be reduced below a couple of GeV. However, both experiments plan to use $p\bar{p}\to Z\to b\bar{b}$ events, which will help set the energy scale to a precision of about 0.5 GeV[62]. In addition, the hadronically decaying W in single-lepton $t\bar{t}$ events provides an independent calibration point[63].

The next most important systematic uncertainty is modeling of initial and final state gluon radiation in $t\bar{t}$ events. In Run I, this uncertainty was estimated mainly by comparing predictions of different event generators. In Run II, the modeling of jet activity in top quark events can be constrained better by comparing double-tagged events with simulations. We estimate this uncertainty to be about 1 GeV, and expect it to decrease only slightly with increasing integrated luminosity. Other systematics will scale inversely with the square root of the integrated luminosity.

We extrapolate the uncertainty on the top mass based on Run I DØ results in the single-lepton channel[61] in Table IV. We take the higher cross section at $\sqrt{s}=2$ TeV in account and we assume double b-tagging with an efficiency per b-jet of 65%. Double b-tagging will essentially eliminate the uncertainty due to the W+jets background. The uncertainty in the dilepton channel[64] is also extrapolated in Table IV. No b-tagging is assumed here. For each channel and each experiment, a precision of abut 1.2 GeV is projected. By combining both channels and experiments an overall precision close to 1 GeV should be achievable.

At the LC, the top quark mass can either be extracted from a $t\bar{t}$ threshold scan that would determine a suitably defined threshold mass [65, 66], or in the continuum by direct kinematical reconstruction of $e^+e^-\to t\bar{t}\to W^+W^-b\bar{b}\to \ell^+\nu\ell^-\bar{\nu}b\bar{b}$ events [67] which determines the pole mass. The remaining theoretical uncertainties are sufficiently small to allow a measurement of the threshold mass with a precision of ~ 100 MeV [65, 66, 68]. (For the conversion of the $\overline{\rm MS}$ top quark mass to other top quark mass definitions see e.g. Ref. [69, 70] and references therein.) The measurement of the pole mass at higher energies with an accuracy of 200 MeV or better may be possible [67], but is limited in precision by QCD renormalon effects which are of $\mathcal{O}(\Lambda_{QCD})$.

The precise measurement of m_t at the LC will eliminate the main source of parametric uncertainties of the EWPO. The uncertainties induced in M_W and $\sin^2\theta_{\rm eff}$ by the experimental error of m_t will be reduced by the LC measurement to the level of 1 MeV and 0.5×10^{-5}, respectively, i.e. far below the uncertainty corresponding to the present error of $\delta\Delta\alpha_{\rm had}$.

The $t\bar{t}$ threshold analysis at the LC will result in correlated measurements of α_s and m_t. Since an independent and more precise determination of $\alpha_s(M_Z)$ would be possible with GigaZ (to ± 0.0010, from other GigaZ

TABLE IV: Projected uncertainties in GeV of the top quark mass measurement in the single-lepton channel (left) and dilepton channel (right).

Run	I	IIA	IIB	IIB*
$\int \mathcal{L}dt$ (fb^{-1})	0.1	2	15	30
statistical uncertainty	5.6	1.7	0.63	0.44
jet scale ($W \to q\bar{q}$)	4.2	1.8	0.64	0.45
jet scale ($Z \to b\bar{b}$)	—	0.53	0.19	0.14
MC model (gluon radiation)	1.9	1.1	0.97	0.96
event pile-up	1.6	0.49	0.18	0.13
W+jets background	2.5	0	0	0
b-tag	0.4	0	0	0
total systematic	5.5	2.1	1.2	1.1
total uncertainty	7.8	2.7	1.3	1.2

Run	I	IIA	IIB	IIB*
$\int \mathcal{L}dt$ (fb^{-1})	0.1	2	15	30
statistical uncertainty	12.3	2.4	0.87	0.62
jet scale	2.0	0.88	0.32	0.23
MC model	2.3	1.0	0.96	0.96
event pile-up	1.4	0.27	0.10	0.07
background	0.9	0.17	0.06	0.05
method	0.9	0.17	0.06	0.05
total systematic	3.6	1.4	1.0	1.0
total uncertainty	12.8	2.8	1.3	1.2

observables: the Z width with 1 MeV uncertainty, and R_l with 0.05% uncertainty [71, 72]), an improved value for m_t can be expected as well.

VI. RARE B DECAYS

Rare B meson decay results are not among the inputs to the global fits for M_H, but they provide important constraints on the EW symmetry breaking sector, so they were included in this study. Rare B meson decays that first appear at one loop in the Standard Model (SM) are especially sensitive to new physics that also enters at one loop. We considered the processes $b \to s\gamma$, $B \to \ell^+\ell^-$, and $B \to X_s\ell^+\ell^-$.

The decay $\bar{B} \to X_s\gamma$ has been measured [73] and is in agreement with the SM prediction. This allows constraints to be placed on new physics, most notably the two Higgs doublet model (2HDM) [74] and the Minimal Supersymmetric Standard Model (MSSM) [75]. The constraints on new physics depend sensitively on the SM result; recent inclusion of quark mass effects [76] in the SM calculation shifted the SM prediction by $\sim 1\sigma$. The current SM theoretical uncertainty in BR($\bar{B} \to X_s\gamma$) is about the same size as the experimental uncertainty and is limited by the charm quark mass dependence [76]. The BaBar and BELLE experiments expect to reduce the current 12% statistical uncertainty in BR($\bar{B} \to X_s\gamma$) to about 5%, which is limited by the amount of running off the $\Upsilon(4s)$ resonance to measure backgrounds.

The decay $B \to \ell^+\ell^-$ is highly suppressed in the SM and has not been observed. The best current bound is BR($B_s \to \mu^+\mu^-$) $< 2.6 \times 10^{-6}$ [77] while the corresponding SM prediction is $\sim 4 \times 10^{-9}$ [78, 79]. New physics contributions to this decay from the 2HDM can enhance or suppress the branching ratio by a factor of two [78], while in the MSSM a much more dramatic enhancement is possible, even up to the current experimental bound at large $\tan\beta$ values [80]. Both CDF and DØ expect to observe a few events at the SM rate with 15 fb^{-1} at Run II of the Tevatron; severe constraints or a discovery are possible in the MSSM.

The decay $B \to X_s\ell^+\ell^-$ is not as highly suppressed and inclusive branching ratios of several $\times 10^{-6}$ are expected in the SM; experimental bounds from BaBar are BR($B \to K\ell^+\ell^-$) $< 0.6 \times 10^{-6}$ and BR($B \to K^*\ell^+\ell^-$) $< 2.5 \times 10^{-6}$ [81], close to the SM predictions. BELLE has recently reported observation of $B \to K\mu^+\mu^-$ [82]. The spectrum and asymmetries in this decay allow separate measurements of the signs and magnitudes of three Wilson coefficients [83] and are sensitive to MSSM contributions. More data is expected soon at BaBar and BELLE. CDF and DØ expect to be able to measure the BR, spectrum, and energy-dependent asymmetry for $B \to K^*\mu^+\mu^-$ at Run II of the Tevatron. Further refinements in the asymmetry and spectrum can be expected at BTev.

VII. ELECTROWEAK GLOBAL FITS

The LEP Electroweak Working Group's most recent fit to the current precision electroweak data is shown in Fig. 1 [84]. The figure shows $\Delta\chi^2 \equiv \chi^2 - \chi^2_{\min}$ as an approximately quadratic function of $\log M_H$. Therefore, the 95% CL upper limit can be approximated by $\Delta\chi^2 = 2.71$, corresponding to a 95% CL upper bound of $M_H < 196$ GeV at present.

The precision of the fit results depends on the experimental uncertainties of the measured values of the EWPO and the theoretical uncertainties of their predictions. The SM predictions for the EWPO are calculated in terms

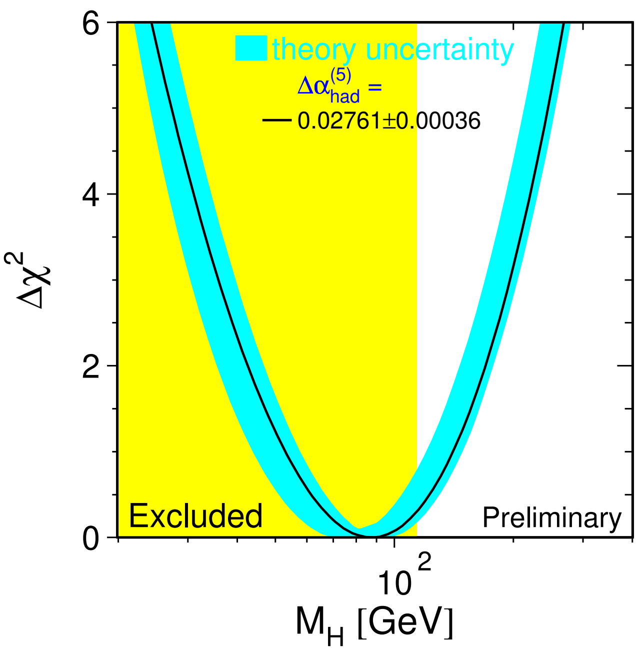

FIG. 1: $\Delta\chi^2 = \chi^2 - \chi^2_{\rm min}$ from a global fit to all available data [84] as a function of the SM Higgs-boson mass, M_H. The width of the "Blue Band" indicates the effect of intrinsic uncertainties from unknown higher order corrections. The yellow region is excluded by direct Higgs searches at LEP2 [85].

of a small set of input parameters: $M_Z, G_\mu, \alpha(M_Z), m_\ell, m_q, m_t, M_H$, and $\alpha_s(M_Z)$. The fine structure constant, $\alpha(0)$, the Z-boson mass, M_Z, the lepton masses, m_ℓ, and the Fermi constant, G_F, are currently the most precisely measured input parameters [49], and their errors have negligible effects on the fit results [46, 86, 87]. The dominant uncertainties presently arise from the experimental error on the top quark mass, $m_t = 174.3 \pm 5.1$ GeV [49], the hadronic contribution to the fine structure constant at the Z-boson mass, $\Delta\alpha_{\rm had}$ [88, 89] (the value used in Fig. 1 is from Ref. [90]), as well as M_H. $\alpha_s(M_Z)$ is constrained mainly by Γ_Z, R_l, and $\sigma^0_{\rm had}$, with little theoretical uncertainty as long as one ignores the possibility of large new physics effects.

In practice, both EWPO and input parameters are used as constraints in the fits subject to their experimental uncertainties (which, as explained above, contain the primordial theoretical uncertainties related to extraction of the EWPO). The only distinction is that the input parameters are treated as fit parameters, and the EWPO are computed in terms of these. For example, m_t which appears only in loops is chosen as input. Moreover, one usually prefers to compute less precise quantities in terms of more precise ones. The fit results are insensitive to these choices.

Table V summarizes the current status of the experimental uncertainties and the precision one expects to achieve at future colliders for the most relevant EWPO, M_W and $\sin^2\theta_{\rm eff}$, and the top quark mass, together with the expected experimental error on M_H[91], assuming the SM Higgs boson has been discovered with $M_H \approx 115$ GeV. The entries in the table attempt to represent the combined results of all detectors and channels at a given collider, taking into account correlated systematic uncertainties.

The dominant parametric uncertainty of the EWPO presently arises from the experimental error of the top quark mass, $\delta m_t = 5.1$ GeV. This error induces a parametric uncertainty of 32 MeV and 16×10^{-5} in the W mass and the leptonic effective mixing angle, respectively. The value of $\Delta\alpha_{\rm had}$ used in the fit is calculated according to [89], with the unsubtracted dispersion relation (UDR) approach, and the corresponding errors from its uncertainty, $\delta\Delta\alpha_{\rm had} = 0.0002$, are 3.7 MeV and 7×10^{-5}. Furthermore, the imperfect knowledge of the strong coupling constant, $\delta\alpha_s(M_Z) = 0.0028$, introduces uncertainties of 2 MeV and 3.5×10^{-5} and also an uncertainty in $\Delta\alpha_{\rm had}$ of about $\delta\Delta\alpha_{\rm had} = 0.0001$. While the uncertainty induced by the top quark mass is about as large as the present experimental error of M_W and $\sin^2\theta_{\rm eff}$, the parametric uncertainties induced by the errors of $\Delta\alpha_{\rm had}$ and $\alpha_s(M_Z)$ are already smaller than the prospective experimental errors on M_W and $\sin^2\theta_{\rm eff}$ at the Tevatron and the LHC (see Table V). On the other hand, the accuracies reachable at GigaZ will clearly require a significantly improved experimental precision not only of m_t (see Table V), but also of $\Delta\alpha_{\rm had}$

TABLE V: The expected experimental uncertainties (including theory errors for the experimental extraction) at various colliders are summarized for $\sin^2\theta_{\rm eff}$, M_W, m_t, and M_H (the latter assuming $M_H = 115$ GeV[92]). Each column represents the combined results of all detectors and channels at a given collider, taking into account correlated systematic uncertainties. (The entry in parentheses assumes a fixed target polarized Møller scattering experiment using the e^- beam [50, 51], thus corresponding to an effective mixing angle at a scale of $\mathcal{O}(0.5$ GeV). It is not used in the fits.)

	now	Tev. Run IIA	Run IIB	Run IIB*	LHC	LC	GigaZ
$\delta\sin^2\theta_{\rm eff}(\times 10^5)$	17	78	29	20	14–20	(6)	1.3
δM_W [MeV]	33	27	16	12	15	10	7
δm_t [GeV]	5.1	2.7	1.4	1.3	1.0	0.2	0.13
δM_H [MeV]	—	—	$\mathcal{O}(2000)$		100	50	50

and $\alpha_s(M_Z)$. An improved determination of $\alpha_s(M_Z)$ with little theoretical uncertainty is, in fact, expected from GigaZ itself [71, 72].

The "Blue Band" shown in Fig. 1 is obtained by comparing the predictions of the EWPO using the codes **ZFITTER** [93] and **TOPAZ0** [94]. At present, the theoretical uncertainty represented by the width of the "Blue Band" mainly arises from the intrinsic uncertainties in the prediction for $\sin^2\theta_{\rm eff}$, while the intrinsic uncertainty in the prediction for M_W, being significantly smaller than the experimental error, is less important.

We have performed a global fit to all data in the Standard Model based on the present experimental and parametric uncertainties and using the estimates of Tables II and III for the intrinsic theoretical uncertainties from unknown higher-order corrections. For the theoretical predictions the program **GAPP** [38] has been used. In contrast to the fit in Fig. 1, where the theory uncertainties are represented by the width of the blue band, we have added theoretical and experimental errors in quadrature. As a result we find

$$M_H = 97^{+53}_{-36} \text{ GeV}, \tag{2}$$

and a 95% CL upper bound of $M_H < 194$ GeV. These numbers are very close to the result of the fit in Fig. 1 [84].

Concerning the interpretation of the fit result, it should be kept in mind that it is based on the assumption that the Standard Model provides the correct description of the experimental measurements. This means, in particular, that the resulting bound on M_H does not reflect the quality of the fit, i.e. it does not contain information about how well the SM actually describes the data.

VIII. FUTURE INDIRECT DETERMINATIONS OF M_H

For the analysis in this section, we anticipate that in the future the currently missing corrections indicated in the upper part of Tables II and III will become available, and that the uncertainties listed in the lower part will be reduced by a factor of two.

In the following we will discuss the anticipated future experimental precisions of the EWPO reachable at the next generation of colliders as given in Table V in view of necessary improvements of the primordial theoretical uncertainties. In each case we also investigate whether the prospective parametric and intrinsic theoretical uncertainties of the EWPO will be sufficiently under control in order to match the projected experimental precision.

The improvement in the experimental determination of m_t at Run IIA will reduce the parametric theoretical uncertainties of M_W and $\sin^2\theta_{\rm eff}$ to values below the experimental errors of these observables. Similarly, the present values of the intrinsic theoretical uncertainties of M_W and $\sin^2\theta_{\rm eff}$ (see Tables II and III) are smaller than the envisaged experimental errors (see Table V). On the other hand, an improvement of the theoretical prediction of $\sin^2\theta_{\rm eff}$, in particular the inclusion of the missing corrections of $\mathcal{O}(\alpha^2)$, would lead to a significant reduction of the width of the "Blue Band" shown in Fig. 1. In the near future, the full two-loop results for M_W and $\sin^2\theta_{\rm eff}$ constitute the most important contributions in the MSM theory calculations, and are needed on the time scale of Run II, which is taking place over the next ~ 5 years.

In view of the increased precision of $\sin^2\theta_{\rm eff}$ at GigaZ and the largely reduced error of m_t at the LC, it will be very important to reduce the uncertainty of $\delta\Delta\alpha_{\rm had}$ at least to the level of $\delta\Delta\alpha(M_Z) = 7\times 10^{-5}$ [88], corresponding to parametric uncertainties of M_W and $\sin^2\theta_{\rm eff}$ of 1.5 MeV and 2.5×10^{-5}, respectively. This will require improved measurements of $R \equiv \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ below about $\sqrt{s} \le 5$ GeV. In case the uncertainty of $\Delta\alpha(M_Z)$ could even be improved by another factor of two (and taking also into account the expected improvement in the $\alpha_s(M_Z)$ determination at GigaZ), the limiting factor in the parametric uncertainty of $\sin^2\theta_{\rm eff}$ would arise from the experimental error of M_Z ($\delta M_Z = 2.1$ MeV induces an uncertainty of 1.4×10^{-5} in $\sin^2\theta_{\rm eff}$), which is not expected to improve in the foreseeable future.

With the prospective future improvements of higher order corrections to the EWPO discussed above (i.e. complete electroweak two-loop results and a reduction of the uncertainties in the lower parts of Tables II and III by a factor of two), the intrinsic theoretical uncertainties of the EWPO will be comparable to or smaller than the parametric uncertainties and the experimental errors at GigaZ (see also Ref. [95].)

In summary, the projected experimental accuracies at GigaZ require on the theory side a considerable effort to reduce primordial theoretical uncertainties. In addition, improvements of the intrinsic and parametric uncertainties of the EWPO are needed. These tasks appear challenging, but, in view of the time scale of at least a decade, not unrealistic.

Based on the uncertainties expected at the next generation of colliders and our estimates of present and future theoretical uncertainties, we list in Table VI the (cumulative) precision of M_H at different colliders which one hopes to achieve from EWPOs. Results are given for $\delta M_H/M_H$ obtained from M_W alone, from $\sin^2\theta_{\rm eff}$ alone, and from all precision data, taking into account the intrinsic and the parametric theoretical uncertainties and their correlated effects. These future estimates range from 25% at the end of Tevatron Run IIB to 18% after LHC to 8% after LC/GigaZ.

TABLE VI: The expected *cumulative* precision, $\delta M_H/M_H$, from future collider data, given the error projections in Tables V, II, and III. Intrinsic theoretical and parametric uncertainties and their correlated effects on M_W and $\sin^2\theta_{\rm eff}$ are taken into account. In the first row, our estimate for the current intrinsic uncertainties in M_W and $\sin^2\theta_{\rm eff}$ from unknown higher order corrections as given in Tables II and III is used. In the other rows we assume that complete two-loop results for the most relevant EWPO are available, and that the uncertainties in the lower parts of Tables II and III have been reduced by a factor of two. This corresponds to future intrinsic theoretical uncertainties in M_W and $\sin^2\theta_{\rm eff}$ of 3 MeV and 1.7×10^{-5}, respectively[a]. As in Eq. (2) we have added the theoretical and experimental errors in quadrature. We also assume $\delta\Delta\alpha(M_Z) = 7 \times 10^{-5}$ [88]. (Using the very optimistic value of 5×10^{-5} would improve the δM_H uncertainty at GigaZ to 7%.) The last row also assumes a determination of $\alpha_s(M_Z)$ with an uncertainty of ± 0.0010 from other GigaZ observables.

$\delta M_H/M_H$ *from:*	M_W	$\sin^2\theta_{\rm eff}$	all
now	106 %	60 %	58 %
Tevatron Run IIA	72 %	39 %	35 %
Tevatron Run IIB	37 %	33 %	25 %
Tevatron Run IIB*	30 %	29 %	23 %
LHC	22 %	25 %	18 %
LC	15 %	24 %	14 %
GigaZ	12 %	8 %	8 %

If the SM is the correct low energy theory, the Higgs boson will be discovered in Run II at the Tevatron or at the LHC. In this case, the indirect determination of M_H from precision electroweak measurements will constitute an important internal consistency check of the SM. Possible new scales beyond the SM could manifest themselves in a disagreement of the directly and indirectly determined M_H value [71, 96].

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[a] Technically within GAPP this is realized by assuming an uncertainty in the T parameter, $T = 0 \pm 0.007$.

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