Model Documentation
The Second Generation Model

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Executive Summary

For a full overview of the model see the "SGM Model Overview".

Briefly, the Second Generation Model (SGM) is a computable general equilibrium model designed specifically to analyze issues related to energy, economy, and greenhouse gas emissions. It has 14 global regions, multiple greenhouse gas emissions, vintaged capital stocks, explicit connections between technology and the economy, disaggregated to reflect the relative importance of sectors in determining greenhouse gas emissions, and explicit treatment of energy and land stocks. Model development began in 1991. The first model design paper was published in 1993 (Edmonds, et al., 1993). The SGM was developed to complement the "first generation model," referred to as the MiniCAM. The MiniCAM was also explicitly designed to address long-term, strategic, issues related to energy, economy, and greenhouse gas emissions (Edmonds and Reilly, 1983)[1] and continues to be used for that purpose[2]. In contrast the SGM was designed to address transitional energy-economy-technology-greenhouse-gas-emissions issues.

The detailed documentation below presents the SGM 2003 version. The SGM was developed at the Pacific Northwest National Laboratory (PNNL) and is maintained by the PNNL Joint Global Change Research Institute (JGCRI)[3].

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[1] The MiniCAM evolved to include agriculture, land-use, terrestrial and ocean carbon cycle, radiative forcing, sea level rise, and climate change.

[2] For a brief comparison between the SGM and MiniCAM see Appendix D

[3] The JGCRI is a collaboration between the PNNL and the University of Maryland at College Park. The JGCRI is located on the campus of the University of Maryland in College Park, MD.

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Chapter 1. The Second Generation Model

The Second Generation Model (SGM) is a computable general equilibrium model designed specifically to analyze issues related to energy, economy, and greenhouse gas emissions. It has 14 global regions, multiple greenhouse gas emissions, vintaged capital stocks, explicit connections between technology and the economy, disaggregated to a degree to reflect the relative importance of sectors in determining greenhouse gas emissions. The SGM is maintained at the Joint Global Change Research Institute, operated by Pacific Northwest National Laboratory and the University of Maryland.

The SGM projects economic activity, energy consumption, and greenhouse gas emissions for each region in five-year time steps from 1990 through 2050. It is designed specifically to address issues associated with global change, including (1) projecting baseline carbon-equivalent emissions over time for a country or group of countries; (2) determining the least-cost way to meet any particular emissions constraint; (3) providing a measure of the carbon price, in dollars per metric ton carbon; and (4) providing a measure of the overall cost of meeting an emissions target.

Below we present a quick overview of the chapters that follow. The chapters describe in detail all model processes simulated in SGM 2003.

Chapter two explicates two basic features of the SGM: the basic structure of the SGM, that is, the regional hybrid commodity-by-commodity input-output table and a summary of the SGM's approach to investments and capital stock. The SGM data requirements are such that for timesteps of five years a regional hybrid commodity-by-commodity input-output table can be solved for a set of prices that clear the markets. The table is hybrid in that it expands a traditional economic input-output table to account for an energy balance. Data on the relationship between monetary units and energy units is therefore a prerequisite, as are base year data on capital stock.

As in a typical computable-general-equilibrium (CGE) model, each commodity produced is associated with a production sector represented by a production sector "column" in SGM's input-output table. In Chapter three we describe the production functions employed so that future values in the input-output tables can be projected. All production in the SGM takes place with either a constant-elasticity-of-substitution (CES) production function, or a fixed-coefficient (Leontief) production function. These are constant-returns-to-scale production functions, but they are operated with one fixed input, capital.

Chapter four describes the price calculations. The model must find a set of prices for which demands by producers and consumers for goods, services and primary factors of production are consistent with domestic production and imports/exports. These price calculations need to work not only for base case scenarios but also under large changes in relative prices induced by carbon management strategies. Chapter four also describes how price expectations over the lifetime of plant and equipment are formed and used.

Chapter five describes how technical change is implemented in the SGM. In brief, technical change is imposed on the scale coefficients of the input-output table that relate the supply sectors to the production sectors. In addition, we describe how autonomous efficiency improvements are imposed on energy use in households and the government.

Chapter six contains an explanation of how the production functions relate to profits and demands and how they are operationalized in the SGM, that is, how the operating vintages relate to the technical scale coefficients and elasticities of substitution, including a description of how elasticities of substitution are reduced between *ex ante* investment decision and *ex post* operating decisions.

The model's representation of the competition among different technologies for fuel-specific electricity generation is based on a logit share equation, which is also described in this section, as are different aspects of depletable resources.

Chapter seven describes carbon policy options that can be implemented in the SGM. An important consideration for any climate policy is the time it takes for capital stock to turn over. The five-year time steps and capital vintages in SGM allow simulation of important dynamics with the introduction of a carbon price and the corresponding changes in relative energy prices. Depending on what carbon policies are implemented, investments, households, government and/or trade flows may be affected. SGM regions are operated together when a carbon emission target is set for a group of regions and carbon emission permits are traded between those regions. Carbon emission permits are traded at base year market exchange rates.

Chapter eight describes the final demand sectors: investments, household consumption, government consumption and trade.

In the SGM, capital may be allocated to new production activities in one of two ways. The first allocates capital as a function of the expected profit rate and previous investment. The second allocates capital based on the expected profit rate and previous output. Chapter eight also describes how capital goods are produced by a Leontief production function.

The representative consumer (households) in the SGM supplies labor and land to other sectors, and also acts as the owner of all capital stocks. Income from labor, land, and retained earnings, adjusted for taxes and government transfers, becomes household income. Income is split between savings and expenditure, with a savings function that depends on the interest rate. Demand for each good by households is calculated as a function of expenditure, income elasticity, own-price, and price elasticity (the constant elasticity equation).

Government activity purchases goods and services.

Each region has a trade-balance constraint that must be satisfied within each model time step. In the model base year, the trade-balance constraint is the same as the historical trade balance. The model user has a choice as to whether this trade balance goes to zero over time or is kept at the base year level.

Chapter nine summarizes the major aspects of the economy simulated in the SGM. GNP can be calculated either as the sum of final demand or as payments to primary factors of production.

Chapter ten describes the solution procedure and illustrates how the SGM solves for excess demand – the difference between demand and supply, and the market prices.

Chapter eleven contains a brief description of the calibration procedure that is employed for calibrating SGM's outputs.

Chapter twelve contains a description of how emissions are calculated. Emissions are calculated based on a production sector's emission activity, which is the product of the total demands by a production sector and a conversion factor that converts monetary units to energy units. Carbon-equivalent emissions are then calculated based on these emission activities' emission coefficients and the emissions' global warming potential coefficient.

In Chapter thirteen we provide for some results for the reference case for the USA as region.

Chapter 2. The modeling framework: economic input-output and energy balance tables

The Second Generation Model (SGM) is a set of 14 regional computable-general-equilibrium dynamic recursive (CGE)[4] models, with an emphasis on energy transformation, energy consumption, and greenhouse gas emissions. Regional models may be run independently or as a system with international trade in greenhouse gas (GHG) emissions permits. Table 1 lists the regions.

Table 1 Regions in the SGM

Annex I	Non-Annex I
United States	China
Canada	India
Western Europe[5]	Mexico
Japan	South Korea
Australia[6]	Middle East
Former Soviet Union	Rest of World
Eastern Europe	Brazil (as stand alone region; its trade with other regions is not yet implemented)

Many of the 14 regional models, including Japan, China, India, South Korea, and Brazil, were developed along with local institutions using local data. The models are then available to those institutions for their own analysis. For example, SGM-Japan has been used within Japan in a model comparison exercise of the cost of reducing GHG emissions.

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4 Two types of CGE models exist: inter-temporal optimization and dynamic recursive. The SGM is an example of a dynamic recursive model. The 2 types of CGE models differ primarily in the treatment of savings and gross investment. Both types of models must allocate investment across sectors within a time period, but inter-temporal models also determine an optimal time path of capital accumulation. Inter-temporal optimization creates an additional computational burden, because all time periods are solved simultaneously; that burden usually limits the amount of sectoral detail in a model with many regions. Savings and investment decisions are endogenous in inter-temporal optimization models, making it possible for the trade balance to be endogenous. A dynamic recursive model is in a narrow sense a sequence of static models with rules for determining the amount of savings and therefore the total amount of new capital constructed in each time period. The SGM, is however, not simply static in that its decisions involving capital investment and resource base utilization are explicitly carried forward to subsequent periods. The trade balance must be exogenous in a recursive CGE model. In the SGM base year of 1990, each region is given its historical trade balance for 1990. In subsequent model years, the modeler can either leave the trade balance at its 1990 level or bring the trade balance gradually to zero.

5 Germany can be run as a separate region

6 Australia can be replaced with an Australia /New Zealand composite region in SGM 2003.
PNNL11819: JA Edmonds, SH Kim, CN MacCracken, RD Sands, MA Wise. October 1997.

Figure 1 illustrates in general terms the flow of goods and services in the SGM. Goods are produced in the production sectors, which use the three primary factors of production: land, labor, and capital.
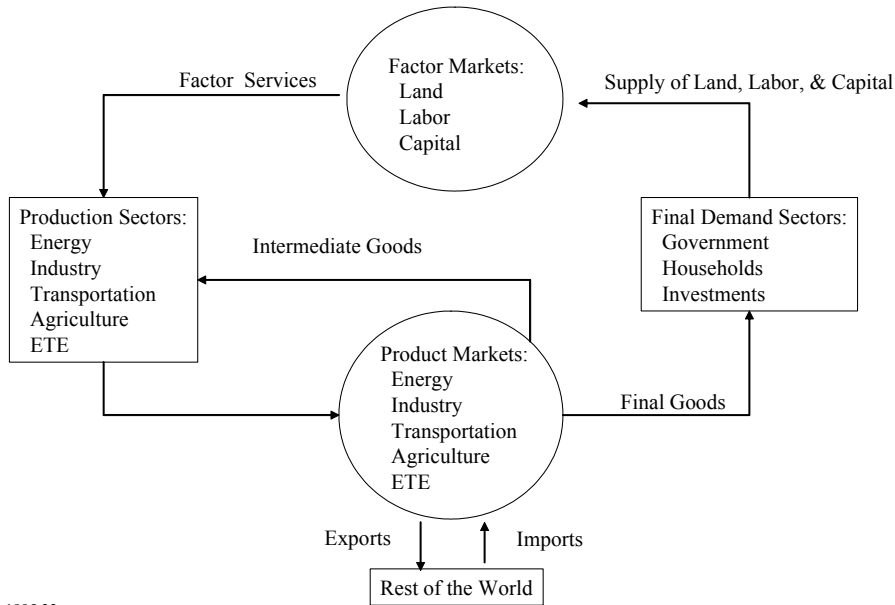
Figure 1 The flow of goods and services in SGM

The flow of goods between industries and consumers in an economy can be described by means of an input-output table in value terms (see Figure 2), which maps directly onto Figure 1. An economic input-output table has three major sections: inter-industry flows, value added, and final demand. An input-output table is organized with inputs as rows and activities (production) as columns. Inputs include direct inputs and intermediate (inter-industry) inputs (those derived from a production process) and primary factors of production. The primary factors of production are rental of land, labor income, other value added, and indirect business taxes.[7] Activities that use inputs are industries, and consumption by the final demand sectors: households, government, investments, and net exports (exports minus imports).

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7 Indirect business taxes, less subsidies, are modeled as a proportional tax on production.
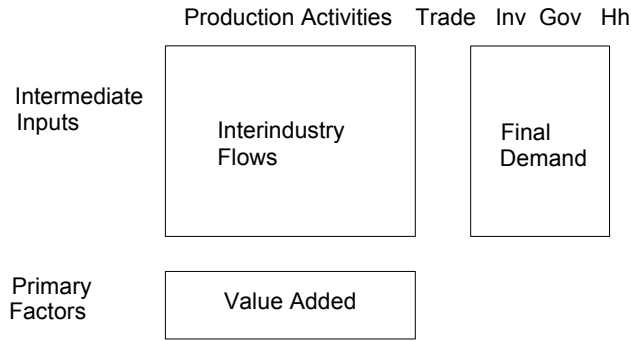
Figure 2 Typical input-output table

In the SGM, regional economic input-output tables are combined with regional energy balance tables to create regional hybrid input-output tables (see also Miller and Blair 1985). Economic input-output tables alone are not sufficient to determine the quantities of oil, gas, coal, electricity, and refined petroleum that are produced and consumed. Supplemental information on energy quantities is required to map monetary units from an input-output table to energy units needed to calculate levels of greenhouse gas emissions.
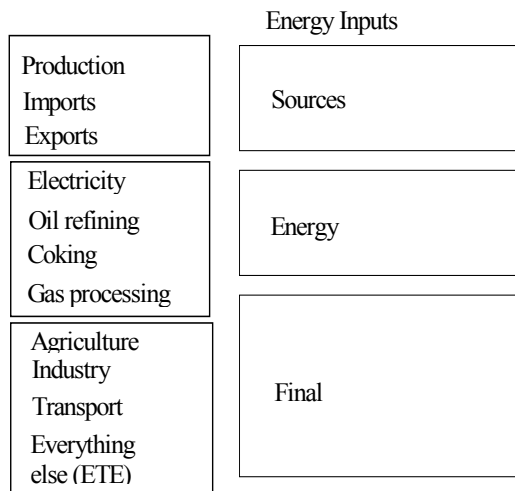
Figure 3 Typical energy balance table

The International Energy Agency (IEA) compiles annual energy balance tables for both OECD and non-OECD countries. These national energy balance tables show the basic supply and demand data for all fuels expressed in common physical units. Moreover, these tables provide the necessary information on the fuel interrelationships (see Figure 3) through conversions of one fuel into another (IEA 1997) and they provide a consistent energy balance table format across countries. Energy balance tables may be summed across countries to form energy balance tables

for multi-country regions. The SGM uses IEA energy balances for all of the multi-country SGM regions, and uses local energy data for all of the following single-country regions, United States, Canada, Japan, China, India, Mexico, South Korea, and Brazil, given that the IEA energy balance tables are not as detailed as may be obtained from these local governments.

Table 2 Production sectors and sub-sectors in SGM 2003

	Sector	SGM 2003	
	No.	Sector / Markets	Subsectors
Everything Else (ETE)	2	EveryThing Else sector	
Energy Production	3	Crude oil production	
	4	Natural gas production	
	5	Coal production	
Energy Transformation	9	Oil refining	
	10	Distributed gas production	
	6	Coke production	
	8	Electricity generation	Oil (8.1)
			Gas (8.2)
			Coal (8.3)
			not active
			Nuclear (8.5)
			Hydro (8.6)
Industry or Manufacturing	11	Paper and pulp	
	12	Chemicals	
	13	Cement	
	14	Primary metals	
	15	Non-ferrous metals	
	16	Other industry and construction	
Transportation	17	Passenger transport	
	18	Freight transport	
Agriculture	1	Other agriculture	
	19	Grains and oil crops	
	20	Animal products	
	21	Forestry	
	22	Food processing	
Carbon	23	Carbon prices and/or GHG emissions	
	24	**Land (Factor market)**	
	25	**Labor (Factor market)**	
	26	**Capital (Factor market) also called "other value added" or OVA**	

Economic input-output tables contain varying amounts of sectoral detail, depending on the country, with anywhere from 30 to over 500 producing sectors. The tables are aggregated to the level of detail that best matches the structure of SGM. Usually, an input-output table contains the same number of producing sectors as there are intermediate inputs. This does not have to be the case however because some industries may produce more than one product and some inputs might not be produced domestically and must be imported. The number of production sectors and markets simulated in the SGM is flexible. In the reference case, production sectors with markets are implemented for the so-called "Everything Else" sector or ETE, three energy production

sectors, four energy transformation sectors, five agriculture sectors, six industrial sectors, a passenger transport sector, a freight transport sector, and a carbon sector (see Table 2).

Since the SGM is an energy model as well as an economic model, attention is paid to maintaining energy balances as the model operates through time. An energy balance table is used for base year calibration of energy production and consumption. Original units might be tons of coal equivalent (China), tons of oil equivalent (International Energy Agency statistics), or calories (Japan). The SGM uses joules for all energy units, expressed as either petajoules (PJ) or exajoules (EJ).

The merging of an economic input-output table with an energy balance table presents a special problem: oil and natural gas are a joint product from a single industry in input-output tables, producing a single composite product. This is one area where energy balance tables provide essential information to maintain a clear distinction between oil and natural gas consumption.

The following steps are used to create a hybrid commodity-by-commodity input-output table from an economic input-output table and energy balance table (Sands 2002).
1. Put the economic input-output table in a format suitable for SGM. This involves aggregation across producing sectors and possible conversion to a 1990 base year.
2. Obtain 1990 energy balance table and convert units to joules.
3. Aggregate energy balance table across fuels to match the SGM format.
4. Rearrange activities (rows) within the energy balance table to match those of the economic input-output table.
5. Transpose the energy balance table so that rows correspond to fuel inputs and columns correspond to energy-consuming activities.
6. Create a hybrid input-output table where the energy rows (inputs) come from the transposed energy balance table and all other rows come from the economic input output-table. This table is no longer in value terms but is now considered to be in quantity terms with units of joules for the energy rows and units of 1990 dollars (or other local currency) for all other rows.
7. Find a set of prices for all intermediate inputs that will rebalance the hybrid input-output table in value terms. By rebalancing, we mean that the value of output in each producing sector is equal to the total value of inputs. A linear equation may be derived for each producing sector, resulting in a system of equations that can be solved to obtain a price for each intermediate input[8]. It is important to note that these prices are derived from the calibration process and are not historical prices. This reflects the SGM modeling philosophy that assumes that technology characteristics, represented by the input-output and energy balance data, should determine relative prices in the model, and not the other way around.
8. Finally, create a new hybrid (economic and energy information) input-output table in value terms by multiplying all quantities by their respective prices (paid). The resulting

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[8] These linear equations are sometimes referred to as zero-profit conditions.

commodity-by-commodity table elements represent quantities expressed in 1990 million dollars.[9]

9. Units for the non-energy inputs in the hybrid input-output table are usually redefined so that prices equal *one* in the base year.
10. Thus, baseline information obtained consists of
    o the regional commodity-by-commodity hybrid input-output tables, including land, and labor prices, information on capital for production sectors and taxes imposed (*e.g.*, Appendix A)
    o information on final demand sector supply and demands in the form of domestic consumption by households, domestic consumption by government, domestic investment, gross exports, and gross imports
    o initial market prices are set to one (see Appendix A)
    o energy price-converters (EJ per million 1990 regional currency; see Appendix A).

The final hybrid input-output tables provide regional representations of the economy that are completely consistent with base year (1990 for SGM 2003) energy balances. Energy production and consumption for each fuel will exactly match the quantities in the base year energy balance table, and emissions can be calculated (Sands 2002).

Production and input-output tables in the SGM

Each region's base year input consists of a commodity-by-commodity hybrid input-output table. Each column in the table represents a production process in the SGM, either represented by a constant-elasticity-of-substitution (CES) or a fixed-coefficient (Leontief) production function.

Through changes over time in population and labor productivity, and through autonomous time trends governing the efficiency of inputs in production processes projections can be made and historical changes in GDP, fossil fuel consumption and electricity generation can be calibrated against.

The hybrid commodity-by-commodity input-output table provides information on the use of produced commodities in the production of other commodities. These interdependencies are the basics in the computation of each production sector's production function technical scale efficiency coefficients (Fisher-Vanden et al. 1993:6), along with value-added information in the form of land and labor prices, and capital and tax information. Calibration of the production function coefficients allows observations of benchmark years to be reproduced. When anticipated technological change is incorporated in these coefficients, the model produces long-term projections.

Appendix A shows an complete example of the input-output elements $X_{i,j,jj,v}$ for all input supplies (rows) and all production sectors (columns), including the electricity production subsectors, the carbon sector and all final demand sectors, where *i* indicates the row number of the input in the input-output table, *j* represents the column number or production sector in the input-output table,

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9 Appendix A provides essential information about transforming energy units (Joules) into monetary values (1990 dollars).

jj represents the production subsector column in the input-output table, and *v* represents the actively operating vintaged capital (technology). The input-output table represents a one-year snapshot of the economy at time *t*; for the base year inputs *v=t*. Units are quantities in monetary values. The summed input values $XS_{i,v}$ (Equation 1) for each of the regions are shown in Table 2 of Appendix A, showing the different regions' input breakdowns. Total production is the sum of the intermediate products that are produced in a production sector *j* of technology vintage *v* ($XS_{j,v}$); this summed production is the gross production of a commodity *j*. When subsectors *jj* are simulated, as in electricity generation, commodity *j* is calculated as the sum over the subsectors (Equation 2).

$$XS_{i,v} = \sum_{j=1}^{22} \left[\sum_{jj=1}^{n} X_{i,j,jj,v} \right] \qquad \text{Eq. 1}$$

$$XS_{j,v} = \sum_{i=1}^{26} \left[\sum_{jj=1}^{n} X_{i,j,jj,v} \right] + X_{i=27,j,jj,v} \qquad \text{Eq. 2}$$

where
  i indicates the row number of the input in the input-output table,
  j represents the column number or production sector in the input-output table,
  jj represents the production subsector column in the input-output table,
  v represents the actively operating vintaged capital (technology),
  n is the number of active production subsectors where $n \leq jj$,
  $X_{i,j,jj,v}$ are the input-output table elements, which for the base year values are called $X_{i,j,jj,v}$ in the documentation and when the model procedures are described are referred to as $EDV_{i,j,jj,v}$ (vintage v, production sector j, or subsector jj, and supply sector i, demands), and
  $X_{i=27,j,jj,v}$ is the production sector or subsector indirect business tax; note that these are added only for the production sectors (j=1:22), and not for the final demand sectors (j=24:27).

When equilibrium conditions exist, supplies (inputs) meet demands, and excess demand – the difference between demand and supply – equals zero for all markets.

Besides input-output tables, key parameters needed are base year capital, elasticities of substitution for the production sectors, and income and price elasticities for the final demand sectors. Individual sector and subsector technologies are characterized by the annualized cost of providing an energy service. Thus, additional data needed to determine the annualized cost include in addition to capital cost, equipment lifetime, annual fuel requirements, the interest rate, and other annual maintenance and operating costs.

Investment and capital stocks

The SGM operates in five-year time steps and keeps track of capital stocks in five-year vintages. At the end of each SGM time period, the model converts investment for each producing sector into capital stock, with the capital stock defined to be five-years' worth of investment. Each

capital stock has a specified lifetime typically of four time periods or 20 years, the so-called nameplate lifetime of a technology[10]; the vintage indicator v of a technology keeps track of the operating technology capital stock's age. Four vintages are operating simultaneously at each point in time, representing three operating old vintage technologies and one new technology that do differ in efficiency. Once capital is created it remains with its original sector, subsector and vintage until its planned retirement or when its profits are insufficient in meeting certain criteria. Vintage-specific capital stock is operated across its lifetime without change in technical efficiency.

Thus, the SGM in effect operates as a hybrid commodity-by-commodity input-output table that gets solved by "clearing" its markets after supplies and demands are summed over its vintage-specific determination of supply and demand. In the SGM production of supplies and demands for consumption are determined separately, and the model has therefore has to reach equilibrium through a solution algorithm that adjusts market prices until excess demand ─ the difference between demand and supply, is less than the solution criteria, which is typically a small number (less than one but greater than zero). The solution algorithm needs only to access information on price, supply and demand to operate. SGM 2003's solution algorithm first uses a bisection routine to bring individual markets closer to equilibrium and then uses a Newton-Raphson procedure for final convergence (see also Appendix B). Newton-Raphson relies on partial derivatives of supply and demand with respect to the unknown prices. Once the set of unknown prices is close to its final solution, Newton-Raphson converges very quickly. Prices adjusted by the iterative solution algorithm determine technology costs, demand for inputs, and sector outputs for a point in time.

Old capital has limited ability to respond to changing prices; this is implemented by imposing short-run elasticities of substitution on the operation of old capital. New capital is assumed to be more flexible than old capital in its response to prices. Long-run elasticities of substitution, with values greater than short-run elasticities, allow for this greater flexibility. Long-run elasticities of substitution are imposed when investment decisions are made and when new capital is operated.

Most production sectors use a single production function for each capital vintage. The electricity sector, however, is divided into subsectors that represent alternative processes for generating electricity such as gas-turbine, coal-steam, nuclear, or hydro. Market share within the electricity sector is based on expected the rate of return to new investments. Technologies with the highest expected rate of return receive the greatest market share.

The major behavioral components of the SGM model thus describe the relationship between prices, production, and consumption of goods and services. Prices that are solved for are consistent with demands by producers and consumers for goods, services, and primary factors of production. Energy production and consumption balance, such that emissions can be accounted for. The behavioral components of the system are shown in Figure 4.

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10 The modeler has the option changing any capital stock's lifetime. The SGM has mostly been run with capital stocks lifetimes of 20 years.
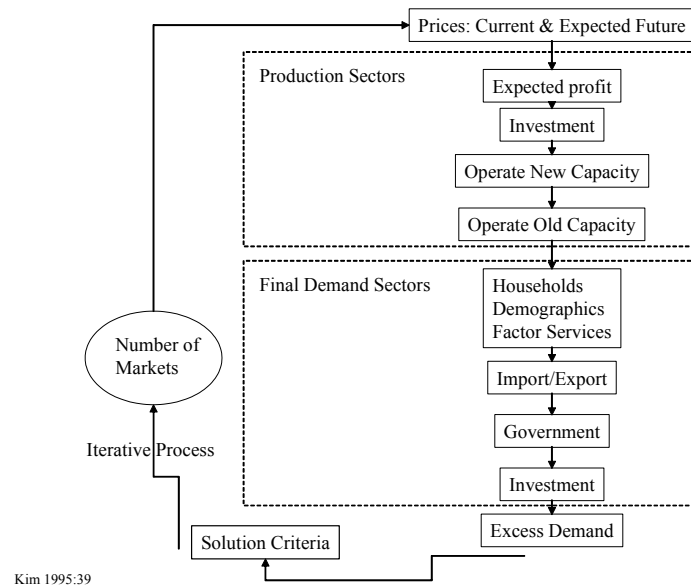
Figure 4 Basic flowchart of the SGM model

Base year calibration of the SGM requires that we determine the quantities of capital needed for all producing sectors by vintage. For a 1990 base year and a capital lifetime of 20 years, we need four capital stocks which equal investment during the time periods 1971-1975, 1976-1980, 1981-1985, and 1986-1990. The following steps are used to create capital stocks for a region.

1. Obtain historical time series of investment data for each producing sector (*e.g.,* Figure 5).
2. Fit an exponential curve to the investment data for each producing sector. This provides for smoothing out the effects of recessions or other temporary deviations from a long-term trend. This also provides a way to extrapolate data backwards in time if the historical time series of investment is not long enough to create all of the needed capital stocks.
3. Convert investment data to real 1990 currency (*e.g.,* 1990 US dollars) using a time series of GDP deflators.
4. Sum investment by sector across each five-year time period to create capital stocks with units of 1990 currency.

Figure 5 provides an example of annual investment data available for papermaking and paper products sector in China. An exponential line is fit to the historical data for China to smooth out the effects of a recession around 1990. The smoothed data are summed over five-year intervals to create capital stock vintages.
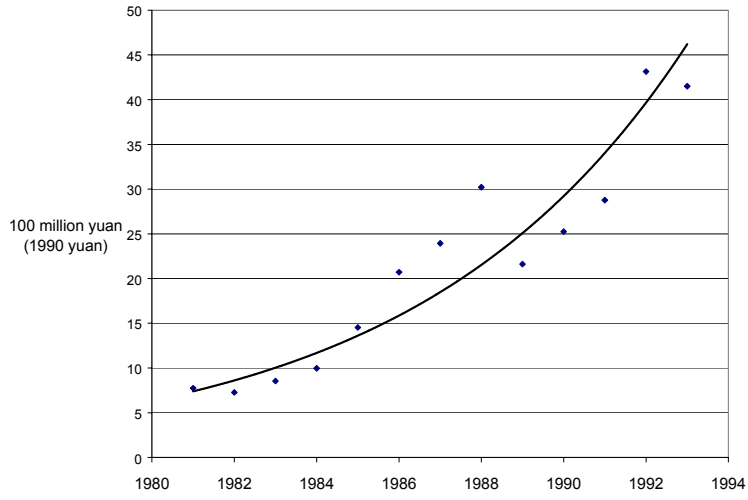
Figure 5 Annual investment: paper making and paper products in China

Chapter 3. Production functions

A basic concept in the economic theory of production is the production function. A production function is a quantitative abstraction of a technology's – or industry's – productive operations. A typical industry transforms inputs (capital, labor, energy, materials, and land) into output. The production function relates the maximum output q_j to any given vector of inputs x_i: that is, $q_j=f(x_i,t)$, where t is time, implying that the maximum output q_j from a given set of inputs x_i can change with time. Technological change can increase the maximum output. Natural resource exhaustion or environmental constraints can decrease the maximum output (Reister et al.).

All goods in the SGM are produced with either a Constant-Elasticity-of-Substitution (CES) production function or a fixed-coefficient (Leontief) production function. These are constant-returns-to-scale production functions[11]; they are operated with one fixed input, capital in SGM 2003. Thus, among the key parameters needed for the SGM are elasticities of substitution for the production sectors.[12]

Oil refining is represented by a fixed-coefficient production function, as is electricity generation from its various energy sources, while most other production sectors are based on the CES production function.

The production functions are described below. We describe in Chapter four "Prices and expected prices" and in Chapter five describe how operating vintages relate to the technical scale coefficients and elasticities. That information is required to understand the way in which the production functions relate to profits and demands as described in Chapter six.

Production functions, vintages, and the input-output matrix

The hybrid commodity-by-commodity input-output table, excluding the final demand sectors, provides representations of the inputs i (rows), to a production process in any sector j or subsector jj (columns). For each input there is an associated price and quantity. Each production process is tracked for its lifetime by vintage v; each vintage is the accumulated capital stock over 5 years of investment. Once capital is created, it must remain within its original sector or subsector and can be operated until its retirement.

The primary advantage of this vintage structure is to better describe technical change over time and provide the option for *putty-semiputty* and *putty-clay* behavior in capital stock (Kim 1995:37) (*putty* for flexible; *clay* for fixed). Because capital stock cannot shift from one sector to another, new technologies affect productivity only at the margin; once a technology is installed, capital

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11 " 'Returns to scale' describes the output response to a proportionate increase of all inputs" (Henderson and Quandt 1971:79).

12 The elasticities of substitution for the reference case for the USA for the production sectors and the income and price elasticities for the final demand sectors can be found in Appendix A.

costs no longer matter. To operate or idle a technology depends solely on the ability of the technology to recover its current operating expenses plus taxes less subsidies.

Vintaging is associated with the SGM's five-year time steps (*Nstep*), that is, each vintage represents 5 years of investment. For the base year 1990, *v* equals zero, implicitly representing investments from 1986 through 1990, while older vintages are represented by negative integers. Vintage *v=-1* represents investments from 1981 through 1985, etc. Table 2 illustrates the relationships between SGM's four vintaged production sectors and subsectors that are operating and the 12 time steps the SGM executes after the base year[13].

Table 3 Relationship between vintages (v) and times of operation (t)

Year	t= 0:12 ↓	**Operating new vintage** (new vintage operates with long-run elasticities of substitution)	**Operating old vintages** (old vintages operate with short-run elasticities of substitution)
1990	0	at t=0 for v=0	at t=0 for v=-3. –2, -1
1995	1	at t=1 for v=1	at t=1 for v=-2, -1, 0
2000	2	at t=2 for v=2	at t=2 for v=1, 0, -1
2005	3	at t=3 for v=3	at t=3 for v=2, 1, 0
2010	4	at t=4 for v=4	at t=4 for v=3, 2, 1
2015	5	at t=5 for v=5	at t=5 for v=4, 3, 2
2020	6	at t=6 for v=6	at t=6 for v=5, 4, 3
2025	7	at t=7 for v=7	at t=7 for v=6, 5, 4
2030	8	at t=8 for v=8	at t=8 for v=7, 6, 5
2035	9	at t=9 for v=9	at t=9 for v=8, 7, 6
2040	10	at t=10 for v=10	at t=10 for v=9, 8, 7
2045	11	at t=11 for v=11	at t=11 for v=10, 9, 8
2050	12	at t=12 for v=12	at t=12 for v=11, 10, 9

CES production by vintage

The Constant-Elasticity-of-Substitution (CES) production function is a well-behaved homogenous, generic function that enables minimum coding for a wide range of analysis (Kim 1995:37). The CES production function has a constant return to scale;[14] that is, output response is proportional to an increase of all inputs. Total factor production is determined by a growth rate applied to the technical scale coefficient ($\alpha_{0,j,jj,v}$ and $\alpha_{i,j,jj,v}$) of the CES production function, affecting all factor demands equi-proportionally.[15]

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13 The model is structured such that nameplate lifetimes of various sizes can be input and additional vintages are simulated.

14 "If output increases by the same proportion, returns to scale are constant for the range of input combinations under consideration" Henderson and Quandt 1971:79.

15 "A production function which belongs to the CES class has 2 major characteristics:
    (1) it is homogeneous of degree one, and
    (2) it has a constant elasticity of substitution." Henderson and Quandt 1971:85

Sands (2002), Kim (1995) and Edmonds et al. (1993) formulate the CES production function[16] in the SGM as follows:

$$q_{j,jj,v} = \alpha_{0,j,jj,v} \left[\sum_{i=1}^{N-1} \alpha_{i,j,jj,v} X_{i,j,jj,v}{}^{\rho} + \alpha_{i=26,j,jj,v} X_{i=26,j,jj,v}{}^{\rho} \right]^{1/\rho} \qquad \text{Eq. 3}$$

where for each production sector j, or subsector jj and each operating vintage v,

   $q_{j,jj,v}$ is the output or gross production of commodity j or jj produced by vintage v capital stock, which is stored in array $PRDV_{j,jj,v}$ in the SGM computer code and referred to as such in the documentation,

   i is an indicator of an input to vintage v's production of commodity j or jj,

   N is the total number[17] of inputs i (26) in SGM 2003; if i equals 26, then i, in our case, refers to the one fixed factor in the model, capital,

   N-1 is the number of variable inputs,

   $\alpha_{0,j,jj,v}$ is a technical coefficient in vintage v's production of commodity j reflecting the overall production efficiencies of the vintages of production sectors j or jj,

   0 indicator denotes that the parameter impacts the production sector as a whole (in this case the technical scale coefficient α),

   $\alpha_{i,j,jj,v}$ is an input-specific (i) technical scale coefficient in the production of commodity j or jj,

   $\alpha_{i=26,j,jj,v}$ is the input-specific (i=26=capital) technical scale coefficient for capital (input N) in the production of commodity j or jj,

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The elasticity of substitution measures the rate at which substitution takes place. It is defined as the proportionate rate of change of the input ratio divided by the proportionate rate of change of technical substitution, where the rate of technical substitution is defined as –dx2/dx1 where x1 and x2 are inputs (Henderson and Quandt 1971:85-86; 62; 59).

16 Equation 3 is analogous to Edmonds et al.'s 1993 formulation:

$$q(x) = \alpha_0 \left[\sum_{i=1}^{N} (\alpha_i X_i)^{\rho} \right]^{1/\rho}$$

where $\rho=(\sigma-1)/\sigma$ and $\sigma=1/(1-\rho)$.

See also Henderson and Quandt (1971:86: eq.3-35) and Chung (1994:111:eq.9.1).

In the equations that follow, both ρ and μ are used as functions of σ:
$\rho = (\sigma-1)/\sigma$ and $\sigma=1/(1-\rho)$
$\mu = (\sigma-1) = \rho/(1-\rho)$ and $\sigma=(\mu+1)$
$-\mu = \rho/(\rho-1)$
$\rho/\mu = 1/\sigma$ and $\mu/\rho = \sigma$
$1/\rho = \sigma/(\sigma-1)$
$-1/\rho = -\sigma/(\sigma-1) = \sigma/(1-\sigma)$
$1/\mu = 1/(\sigma-1)$
$-1/\mu = (\rho-1)/\rho = (-1/\sigma) \bullet (\sigma/\sigma-1) = 1/(1-\sigma)$

17 M is the number of primary factors; $M=3$ in this SGM version: capital (*OVA*, or *KA*; *N=i=26*), which is a fixed factor; and labor (*N-1= i=25*) and land (*N-2=i=24*), which are variable factors; the indirect business tax (*IBT*) associated with the production of commodity j or *jj* is the 27th i input (*N+1=27*).

$X_{i,j,jj,v}$ is demand for input i in vintage v's production of commodity j or jj; this demand is denoted by $X_{i,j,jj,v}$ in the base year in the documentation and stored in array $EDV_{i,j,jj,v}$ in the SGM computer code and denoted as such in the documentation when not referring to the base year,

$X_{i=26,j,jj,v}$ is the demand for the fixed (capital) input in vintage v's production of commodity j or jj, and

ρ is a function of the elasticity of substitution parameter, σ, where $\rho = (\sigma-1)/\sigma$ and $\sigma=1/(1-\rho)$.

Long- and short-run elasticities

The output of a CES production function is dependent on the elasticity of substitution, as shown in Equation 3 (see also Henderson and Quandt 1971: Chapter 3). In the SGM, when capital is operated as a CES production function, new capital operates under long-run elasticity, and all older vintages of capital operate under short-run elasticity. This means that new capital (new vintage) has a higher elasticity of technical substitution among inputs in response to changes in prices than old capital (old vintage). The type of technology simulated in the SGM in that case is so-called *putty-semiputty* technology.[18]

Sands (2002) summarizes how elasticities of substitution, σ, must satisfy the following relationship

$$0 \le \sigma_{j,v=old} \le \sigma_{j,v=new} < 1 \qquad \text{Eq. 4}$$

where
old and new denote vintages, and
j is the sector index

The model structure, with different elasticities of substitution between old and new capital, provides a way to simulate dynamics in marginal abatement curves, *e.g.*, carbon prices must be driven higher in the short-run than in the long-run to meet any given carbon emission target. If the elasticity σ in the CES production function is specified to be less than 0.05, the SGM switches from a CES to a fixed-coefficient production function. The elasticity of substitution between inputs is then zero and the production function turns into a fixed-coefficient (or Leontief) technology. This switch in production functions is so called *putty-clay* technology.

A graphical representation of a putty-semiputty technology, using CES production functions, is shown in Figure 6. Two isoquants are plotted, each with a different elasticity of substitution, sigma. An isoquant shows all combinations of two inputs that will produce one unit of output. An isoquant shows the physical tradeoffs available between these two inputs. For a given set of prices, only one point on the isoquant will minimize the cost of production. At that point, the

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18 *Putty-semiputty* technology provides a lower elasticity of substitution in old capital than in new.
Putty-clay technology assumes that new capital has a certain elasticity of substitution and that old capital is fixed-coefficient where the input-output coefficients do not respond to price.
Putty-putty technology uses the same elasticity of substitution for both new and old capital.

isoquant is tangent to a line showing all combinations of equal expenditure on inputs. The slope of this line is the ratio of prices for the two inputs. A representative line of tangency is also shown.
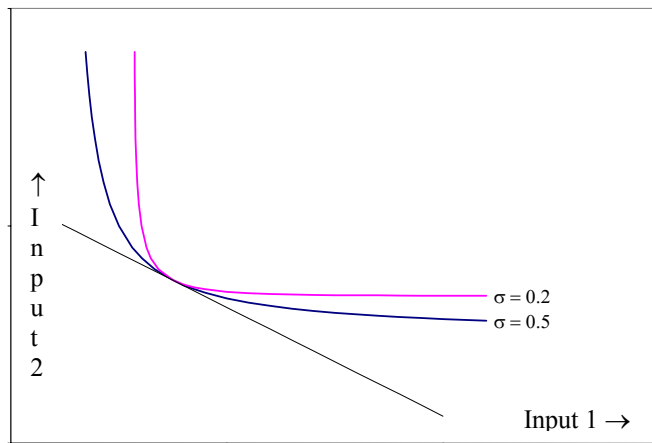
Figure 6 Putty-semiputty isoquants

During each SGM time period, new capital is formed. At the time of capital formation, the isoquant with the greater elasticity of substitution is used. However, at the end of the SGM time period, capital is converted from a higher to a lower elasticity of substitution. Once capital is constructed in SGM, substitution possibilities between inputs are limited.

A *putty-clay* technology assumes that old capital is fixed-coefficient and the input-output coefficients do not respond to price. The input-output coefficients reflect relative prices that were in effect when the capital was new. Therefore, new capital is responsive to prices (putty), but input-output coefficients are locked into place as the capital is converted from new to old (clay). Isoquants for a putty-clay technology are shown in Figure 7.
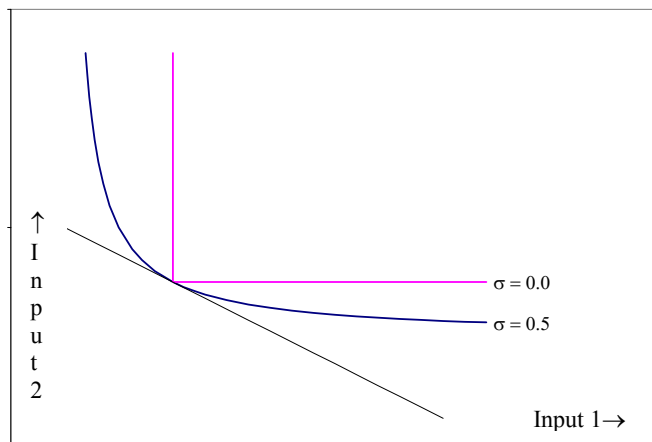
Figure 7 Putty-clay isoquants

26

The Leontief or fixed coefficient production function

Some technologies are considered to be fixed-coefficient[19] and have a Leontief functional form whether new or old, and their mix of input factors of production is not responsive to prices. This technology is referred to as *clay-clay* or simply *clay*. This assumption is sometimes used in the energy transformation sectors, especially oil refining, where the ratio of energy input to energy output is fixed in advance by physical processes and cannot respond to changes in price. All electricity subsectors are fixed coefficient.

When a Leontief production function is implemented, an industry *j* or *jj* chooses the minimal amount of all intermediate and primary inputs *i* to produce output, leaving quantities of the inputs beyond the minimal amount idle (Chung 1994:170; Henderson and Quandt 1971:336).

The Leontief production function can be described as follows:

$$q_{j,jj,v} = \lambda_{0,j,jj,v} \min\left(\frac{X_{1,j,jj,v}}{\lambda_{1,j,jj,v}}, \ldots, \frac{X_{i,j,jj,v}}{\lambda_{i,j,jj,v}} \ldots \frac{X_{N,j,jj,v}}{\lambda_{N,j,jj,v}} \right) \qquad \text{Eq. 5}$$

where for each production sector j, or subsector jj and each operating vintage v,

$q_{j,jj,v}$ is the output or gross production of commodity j or jj produced with vintage v capital stock, later in the documentation denoted by $PRDV_{j,jj,v}$,

i is the input to vintage v's production of commodity j,

N is the number of inputs i (N=26); if i equals 26, then i refers, in our case, to the one fixed factor in the model, capital,

$\lambda_{0,j,jj,v}$ is a technical scale coefficient in vintage v's production of commodity j or jj reflecting the overall production efficiencies of the vintages of production sectors j or jj,

0 indicator denotes that the parameter (in this case the technical scale coefficient λ) impacts the production sector as a whole,

$\lambda_{i,j,jj,v}$ represents the technical scale coefficients, and

$X_{i,j,jj,v}$ is demand for input i in vintage v's production of commodity j or jj; this demand is denoted by $X_{i,j,jj,v}$ in the base year and by $EDV_{i,j,jj,v}$ in the forecasts, where jj denotes a production subsector.

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19 The elasticity of substitution (σ) in CES production functions has a practical lower bound of 0.05 in economic models such as SGM. Elasticities less than 0.05 result in numeric overflows or underflows in double-precision. Fixed-coefficient production functions in SGM have an elasticity of substitution of exactly zero. Therefore, there is a range of substitution elasticities between zero and 0.05 that the SGM can not simulate.

Chapter 4. Prices and expected prices

The model must find a set of prices that is consistent with demands by producers and consumers for goods, services, and primary factors of production. In Chapter ten we detail the market price solution process, but here and in Chapters five through nine we describe the details of the costs of operating capital and the various aspects of demand simulated in the SGM.

Base year prices and initial future prices

In the SGM 2003 model runs, the base year market prices are set to one ($P_{i,t=0}=1$), given that base year input is based on equilibrium and Walras' Law[20] guarantees that (1) if an equilibrium set of prices exists, any positive scalar multiple of each of those prices is also an equilibrium set of prices and (2) any commodity can be chosen as a numeraire,[21] which in the SGM is the commodity of the Everything Else sector. Future prices are determined relative to the price of the Everything Else sector, for which market prices remain one in the projections.

Initial, base year capital interest rates ($i=26$) equal 0.06; land rental prices ($i=24$) do not play a role in the example data set and prices are set to zero. When land plays a role in SGM, the initial market price of land rental, $P_{i=24=ls,t}$, is calculated based on the total land available as supply and land demand ($i=24$; see Equation 6).

$$P_{i=24=ls,t} = \frac{XS_{i=24=ls}}{ED_{i=24=ls,j=27=hh}}$$
Eq. 6

where
  $XS_{i=24=ls}$ is the summed (total) demand for supply of land in the base year and
  $ED_{i=24=ls,j=27=hh}$ is the total demand for supply of land (see "Households").

Similarly, the initial market price for labor, $P_{i=25=lbs,t}$ — the cost of bying a year's worth of one laboror's time, *i.e.*, the average annual wage of labor — could be calculated ($i=25$; see Equation 7) and should deliver the same results as the input parameters listed for the reference case; thus, labor's market rental price is not set to one.

$$P_{i=25=lbs,t} = \frac{XS_{i=25=lbs}}{ED_{i=25=lbs,j=27=hh}}$$
Eq. 7

where
  $XS_{i=25=lbs}$ is the total demand for supply of labor in the base year, and
  $ED_{i=25=lbs,j=27=hh}$ is the total demand for supply of labor (see "Households").

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20 "If all the commodities in an economy are included in a comprehensive market model, the result will be a Walrasian type of general-equilibrium model, in which the excess demand for every commodity is considered a function of the prices of all the commodities in the economy" Chiang (1967:52).

21 Numeraire: " A unit of account, or an expression of a standard of value. Money is a numeraire, by which different commodities can have values compared" (Pearce 1992); Oxford: Numeraire – good used as a standard value for other goods. The price of the numeraire is defined to be 1.

In the base year market prices equal one ($P_{i,t}=1$), with the exception of labor; market prices in the projections result from the market solution processing but can be set exogenously. The prices for crude oil, land rental and the Everything Else sector are set exogenously in the reference case; the gas market price is also set exogenously, assuming that they are set globally and not regionally as in the example carbon policy run to be discussed in Chapter thirteen. When market prices are set exogenously they retain their exogenously set value in the market solution process.

For the base year, prices received for commodity j produced in a production sector are calculated from the difference between the total demand for i inputs minus the indirect business taxes paid ($i=1:27=1:N+1$), divided by the total demand for i inputs. Prices received for a production sector commodity j are identical for all subsectors jj.

$$Pr_{j,t=0} = \frac{\sum_{i=1}^{N+1} X_{i,j,jj,v} - X_{i=N+1,j,jj,v}}{\sum_{i=1}^{N+1} X_{i,j,jj,v}} \qquad \text{Eq. 8}$$

where

$\sum X_{i,j,jj,v}$ is the summed over N+1 demands for inputs i in the production of commodity j or jj (see Equation 2), which includes

$X_{i=N+1,j,jj,v}$, the indirect business tax from the hybrid commodity-by-commodity input-output table, which is a production sector- or subsector-specific input parameter which needs to be substracted.

Determining prices paid for supplies by producers, prices received for produced commodities, and policy potential

For a closed economy, the price paid by producers for supplies at each point in time includes transportation costs, taxes and adjustments.

Prices paid for the variable inputs ($i=1:23$) from the ith supply sector for each of the actively producing vintages v for sectors $j=1:22$ and subsectors $jj=1:6$ at time t are calculated as follows:

$$Pi_{i,j,jj,t} = adj_{i,j,jj,t} \bullet \left(P_{i,t} \bullet Ex\,Im_i \bullet Txpro_{i,j,jj} + Txadd_{i,j,jj} + Cpf_i\right) \qquad \text{Eq. 9}$$

where

$adj_{i,j,jj,t}$ is a time-period-dependent adjustment factor that reflects markups and intra-regional transport costs; these adjustment factors can be input i and sector j or subsector jj specific; $adj_{i,j,jj,t}$ is further described in Equation 11,

$P_{i,t}$ is the market price; in the base year $P_{i,t}$ equals one; in the forecasts $P_{i,t}$ is calculated during the model solution process and is supply and demand dependent,

$ExIm_i$ is a transportation cost multiplier, a rate parameter,

$Txpro_{i,j,jj}$ is a proportional tax rate on the ith product,

$Txadd_{i,j,jj}$ is an additive tax on the ith product, and

Cpf_i, is a carbon permit fee on inputs, i, per dollar of production based on the global warming potential and the emission coefficient of a fuel; thus

$$Cpf_i = P_{i=23,t} \bullet ExchRate_{region} \bullet \sum_{fn} EMC_{fn} \bullet PRconvrt_i \bullet GWP_{fn} \qquad \text{Eq. 10}$$

where

   fn stands for oil, gas, or coal within a production process for which the energy supply carbon fee is calculated,

   $P_{i=23,t}$ is the carbon price, which, dependent on if and which carbon policy is implemented may equal the market price during the model solution process, or may be fixed,

   $ExchRate_{region}$ the monetary exchange rate,

   EMC_{fn} is the fuel-specific carbon content in million tons C per exajoule of the energy source; fn stands for oil, gas, or coal within a production process for which the energy supply carbon fee is calculated,

   GWP_{fn} is the global warming potential; $GWP_{fn}=1$ for carbon,

   $PRconvrt_i$ is the energy conversion factor, converting the relative prices in monetary units to physical energy units.

Note that *PRconvrt* for each supply sector in each region is expressed in energy units produced (EJ) per million 1990 regional currency (that is, per gross production of a commodity *j*, which is the sum of all intermediate products in the base year in 1990 million regional currency). Also note that the final conversion factors are the results of base year calibration.

A time-dependent adjustment factor, $adj_{i,j,jj,t}$, reflects markups and intra-regional transport costs; these adjustment factors can be input *i* and sector *j* or subsector *jj* specific. A (sub)sector-specific price adjustment factor is calculated as a time-dependent interpolation of initial and terminal values when *T* as number of time periods *t* times the time step of five years (*Nstep*) is smaller than *Tadjz_i*:

$$adj_{i,j,jj,t} = Ad_{i,j,jj} \bullet (1 - \frac{T}{Tadjz_{i,j,jj}}) + Adjz_{i,j,jj} \bullet \frac{T}{Tadjz_{i,j,jj}} \qquad \text{Eq. 11}$$

where

   $Ad_{i,j,jj}$, $Tadjz_{i,j,jj}$, and $Adjz_{i,j,jj}$ are input parameters and T is the number of time periods multiplied with the time step (t*Nstep).

In most cases *adj* equals one. However, *adj* is included in the equation calculating the price paid by the producer for inputs (Equation 9) to provide the model with increased flexibility in describing the purchase price.

The proportional tax, *Txpro*, and additive tax, *Txadd*, can be input and (sub)sector specific; they are presently only supply-price-specific, however.

By changing the different rates in the above price-paid equation, researchers can evaluate policy scenarios. For example, carbon prices and energy excess fees can be imposed in the model by

changing *Txadd* or applying a carbon fee *Cpf* on the appropriate products whether they are fossil fuels or energy services.

Technology specific discount rates $Pi_{i=capital,j,jj,t}$ may be adjusted over time as follows:

$$Pi_{i=capital,j,jj,t} = wedge_{i=capital,j,jj,t} + P_{i=26}$$
<div align="right">Eq. 12</div>

The wedge component of the technology specific discount rate can be considered an interest rate wedge or discount rate. They can be calculated in the base year in a spreadsheet model for each of the sectors and subsectors. A target profit rate for each (sub)sector is calculated based on
   o   all active capital (KA), and
   o   an expected profit rate weighed against zero profit by the sector where the expected profit rate calculations are based on the CES production function and long-run elasticity, *σ*, and inputs from the IO table, which are based on base year information

The wedge component of the technology specific discount rate can be modified over time as part of the calibration process (see also Chapter eleven).

Calculation of prices received ($Pr_{j,t}$) for the production sector-specific commodities produced, in relation to the market prices ($P_{i,t}$), are first steps in the model evaluation, since they determine operational and investment decision-making.

Prices received for a production sector commodity are identical for all subsectors. For the projections they are calculated as follows:

$$Pr_{j,t} = \frac{P_{i,t} + Tr_{j,t} \bullet Ex\,Im_j}{1 + TxIBT_j}$$
<div align="right">Eq. 13</div>

where
   $P_{i,t}$ are the market prices which are updated during the model solution processing and are therefore time-period-specific,
   $ExIm_j$ is the transportation cost rate parameter (these input parameters equal one in the reference case),
   $TxIBT_j$ is the production sector indirect business tax; these input parameters are constant over time in the reference case, and
   $Tr_{j,t}$ is a time-period-dependent transportation cost factor.

The region-specific transportation cost factors are calculated as time-dependent interpolations of initial and terminal values as long as *T*, which is the number of time periods multiplied by the time step ($T=t*Nstep$, where *Nstep* equals 5 years), is smaller than Trz_i.

$$Tr_{j,t} = Tri_j \bullet (1 - \frac{T}{Ttrz_j}) + Trz_j \bullet \frac{T}{Ttrz_j}$$
<div align="right">Eq. 14</div>

where
   Tri_j, $Ttrz_j$, and Trz_j are input parameters.

The equations above are cycled through each row i and column j and jj element of the input-output matrix, and each of the vintage-specific $Pi_{i,j,jj,t}$ matrices are filled for each element of the matrix at each point in time for each of the operating vintages. The model is solved after summing the vintage-specific inputs to production and vintage-specific calculated demands. Note that prices for input to production are identical for all operating vintages and will be only differentiated among production sectors by the adjustment factors $adj_{i,j,jj,t}$ that reflect markups and intra-regional transport costs, the proportional tax rates $Txpro_{i,j,jj}$ and the additive taxes $Txadd_{i,j,jj}$.

Expected prices

The simplest rule for projected future prices is to set them equal to current prices. Alternatively, projected future prices can be based on a previous model run. A third option is to determine expected prices as projected future prices discounted to the present. In the SGM expected future prices for all variable inputs ($i=1:25$) are calculated such that investment decisions based on expected profitability can be made. The expected prices paid $Pie_{i,j,jj}$ ($i=1:25$) for the next point in time are calculated based on prices paid by producers for inputs (see Equation 9). These prices are production sector- and production subsector-specific.[22]

$$Pie_{i,j,jj} = Pi_{i,j,jj,t} \bullet \left(\frac{1}{\left(1 + Pi_{i=26,j,jj,t}\right)} \right)^{0.5} \bullet \left(1 + \frac{1}{Pi_{i=26,j,jj,t}} \right) \bullet \left(1 - \left(\frac{1}{\left(1 + Pi_{i=26,j,jj,t}\right)} \right)^{T_{exp}} \right) \quad \text{Eq. 15}$$

Expected prices received for the commodities produced by the production sectors and subsectors are based on the prices paid for inputs (see Equation 9) and the prices received for commodities produced (see Equation 13).

$$Pe_{i,j,jj} = Pr_{j,t} \bullet \left(\frac{1}{\left(1 + Pi_{i=26,j,jj,t}\right)} \right)^{0.5} \bullet \left(1 + \frac{1}{Pi_{i=26,j,jj,t}} \right) \bullet \left(1 - \left(\frac{1}{\left(1 + Pi_{i=26,j,jj,t}\right)} \right)^{T_{exp}} \right) \quad \text{Eq. 16}$$

where
   $Pi_{i,j,jj,t}$ is the prices paid by the producer for supplies (see Equation 9),
   $Pr_{j,t}$ is the price received for a commodity produced (see Equation 13),
   $Pi_{i=26,j,jj,t}$ is the interest rate, which is production sector- and subsector-specific (Equation 12),
   T_{exp} is the nominal life of the investment (years) and a parameter of technology, and

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[22] Note that the equations as described here are easily solved analytical equations and replace the original equation that had to be solved iteratively (eq. 15 in Edmonds et al. 1993). The original equations were described as follows: Pe_j, for time t is calculated as discounted price for product j (either an input or an output) by

$$Pe_{j,t}(T) = \sum_{t=1}^{T_{exp}} P_{j,t}(t_0) \bullet \left(\frac{1 + r_j}{(1 + dis)} \right)^t$$

where $Texp$ is the nominal life of the investment; r_j is the rate of expected price change; and dis is the discount rate. The nominal life of the investment is a parameter of the technology. This parameter value may be different from the maximum potential life of the technology. Both the rate of expected price change r_j and the discount rate dis are behavioral variables.

Texp equals t•Nstep, where t is the number of time-step periods the investment is operational; thus Texp=20 when t=4, given that SGM's timestep (Nstep) equals five years.

The difference between the expected prices for input supplies $Pie_{i,j,jj}$ and the expected prices for the output products $Pe_{j,jj}$ is thus based on the difference between the price paid for input $Pi_{i,j,jj,t}$ and the price received for a commodity j produced $Pr_{j,t}$ at time t.

Chapter 5. Technical change

Technology is a central feature shaping anthropogenic greenhouse emissions. Given that technology is "the broad set of processes covering know-how, experience and equipment, used by humans to produce services and transform resources" (Edmonds and Moreira, 2004), technology is a central issue in understanding climate change. Technology unifies the elements of the system. Technology shapes many elements of SGM design. This chapter describes how technical change is implemented in the SGM. In brief, technical change is imposed on the scale coefficients of the input-output table that relate the supply sectors to the production sectors. In addition, we describe how autonomous efficiency improvements are imposed on energy use in households and the government and the changes over time.

Input-output coefficients and their relationship to the technical scale coefficients

The SGM keeps track of capital stocks in five-year vintages. Vintage-specific capital stock is operated across its lifetime without change in technical efficiency. The four vintages operating simultaneously at each point in time represent three old vintage technologies and one new technology. Old and new vintages may be characterized by different elasticities (denoted by "σ") of substitution given that new capital can be expected to be more flexible than old capital and its elasticity of substitution will therefore be greater than or equal to the corresponding elasticity of substitution for old capital.

The SGM's framework is a hybrid commodity-by-commodity input-output table in monetary values where for energy the input-output dollar values remain related to the per-joule cost of energy.

The relationship between input-output coefficients (denoted by "$\mathbf{a}$") and the technical scale coefficients (denoted with "α" for a CES production function and with "λ" for a Leontief or fixed-coefficient production function) that are implicit in an input-output table was described by Sands (2000) and can be summarized as follows. The physical input-output coefficients can be derived from a hybrid input-output table for each combination of input and production processes. An input-output coefficient is the quantity of input divided by the quantity of output. The quantity of output is found by dividing the value of output by its price. Similarly, the quantity of input is found by dividing the value of input by its price.

In the special case of fixed-coefficient production, the technical coefficients of the production function are related to input-output coefficients in a very simple way which is independent of price:

$$a_{i,j} = \frac{\lambda_{i,j}}{\lambda_{0,j}}$$
Eq. 17

where

$\lambda_{0,j}$ are production sector-specific technical coefficients,

$\lambda_{i,j}$ are technical coefficients that can incorporate technology-specific (*i.e.,* non-neutral) technical change parameters.

In general though, input-output coefficients are related to the technical coefficients of production functions through prices. The production process should substitute away from inputs that become relatively more expensive. The rate at which one input is substituted for another, relative to the rate of change in prices of those inputs, is accomplished through the implementation of the elasticity of substitution among inputs in production. In a constant-elasticity-of-substitution (CES) production function, this elasticity of substitution remains constant across changes in price. Equation 18 shows the dependence of input-output coefficients on CES technical coefficients, relative prices, and the assumed elasticity of substitution. In the SGM we invert this relationship and solve for technical coefficients as a function of input-output coefficients, prices, and the elasticity of substitution.

$$a_{i,j}(p) = \left[\alpha_{0,j} \bullet \alpha_{i,j}\right]^{\rho/(1-\rho)} \bullet \left[\frac{p_j}{p_i}\right]^{1/(\rho-1)} \qquad \text{Eq. 18}$$

where
   $a_{i,j}$ is the amount of input i required per unit of output j; $a_{i,j}$ is called the input-output coefficient when dealing with input-output matrices,
   $\alpha_{0,j}$ is a production sector-specific technical coefficient that can incorporate a Hick's neutral technical change parameter[23]
   $\alpha_{i,j}$ is a technology-specific technical coefficient that can incorporate a technology-specific (*i.e.*, non-neutral) technical change parameter,
   ρ is a function of the substitution elasticity; $\rho=(\sigma-1)/\sigma$, where σ is a production sector-specific elasticity of substitution,
   p_j is the price received for the commodity produced, and
   p_i is the price paid by the producer for input.

Note that the above equation uses subscripts for inputs and outputs, except for the exponent. This exponent actually does vary by producing sector as described above.

As the SGM searches for equilibrium prices, technical coefficients remain invariant with respect to price. However, input-output coefficients will respond to changes in price. This response, governed by the elasticity of substitution is important for climate policy since changes in fossil fuel prices are an important component of the change in emissions.

Note that the input-output table tells us nothing about the elasticity of substitution among inputs in production. That parameter must be set exogenously using published results from econometric studies, simulations of bottom-up technology models, or expert judgment.

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23 Hick's neutral technical change parameter reflects the overall production efficiency of a vintage (Fisher-Vanden 1993:12).

The SGM contains a large set of parameters that can be used to simulate technical change over time for any given production sector. Separate parameters are available for each input to each production process (and each vintage)[24]. The rates of change in input efficiency can be varied at each five-year time step in the SGM. For example, we could assume that energy efficiency in coal-fired electric generating plants is improving at a rate of 0.5% per year for 10 years, with no change after that. These technical change parameters are very powerful in determining energy consumption and emissions in a reference case, the impact of which is similar to the Autonomous Energy Efficiency Improvement (AEEI) that is used in some other energy models. In the SGM, exogenous rates of technical change can be specified for all inputs to production, not just energy.

In the case of Hick's neutral technological change, all of the α or technical scale coefficients (the parameters that relate the inputs to the production processes) within a production function are assumed to change at the same rate. Isoquants for the case of two inputs with neutral technical change are shown in Figure 8. Note that as technology improves from T_0 to T_2, the isoquant moves inward toward the origin, showing that less of each input is required to produce one unit of output.
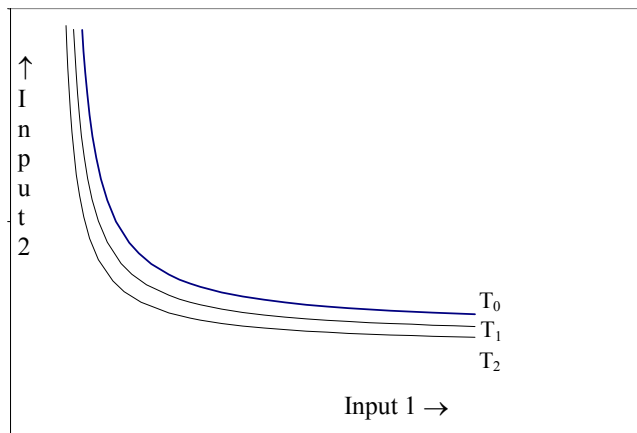
Figure 8 Neutral technical change when the elasticity of substitution coefficient (σ) equals 0.5

Instead of adjusting α_0 to represent technical change, each of the $\alpha_{t,j}$ parameters can be adjusted with its own technical change parameters. If the rate of change in efficiency varies across inputs, then technical change is said to be non-neutral or biased.

Figure 9 provides an example of biased technical change, with the isoquant shifting in towards the origin, but at different rates for different inputs. For example, we could have a technology for which labor productivity is improving at a faster rate than energy efficiency. In this case the amount of labor needed per unit of output is falling faster than the amount of energy needed per unit of output.

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24 The technical scale coefficients remain constant over the lifetime of the vintage, while they differ among vintages.
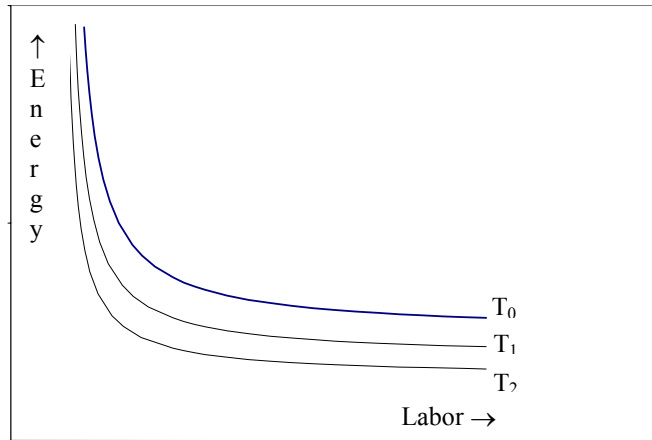
Figure 9 Biased technical change when the elasticity of substitution coefficient (σ) equals 0.5

If the production process is fixed-coefficient, then these technical change parameters λ can be used to precisely control input efficiency. However, most production processes are not fixed-coefficient, at least in the long run as old capital is replaced by new capital. Most production processes respond to changes in prices of inputs by substituting away from those inputs that are relatively expensive. In this case, input efficiency is affected by both the exogenous technical change parameters described in this section and by changes in relative prices. Changes in input efficiency are a combination of exogenous technical change and price-induced technical change.[25] Exogenous technical change is represented by a shifting isoquant; price-induced technical change is represented by shifts along an isoquant.

Besides production efficiency, there are parameters that determine land, labor, and capital productivity. Labor productivity parameters are the primary determinants of economic growth in SGM and are used to influence the time path of GDP in a reference case. Both energy efficiency and labor productivity assist in constructing reference scenarios. The labor productivity parameters are used to determine a time path for GDP and the energy efficiency parameters are used to determine consumption efficiency of energy by fuel (see also Chapter eleven).

How technical change is operationalized in the SGM

For each region technical change parameters for each sector j, each subsector jj, and each time period t determine the efficiency improvements in energy production, energy transformation (*e.g.,* oil refining, gas processing, coal and electricity generation), industry, transportation and agriculture, and the efficiency of labor productivity, land productivity, and capital.

The *TECHNN* parameters or γ parameters are set to identical values for all industries, while coal production, gas production and distribution, oil refining and electricity production have their

25 Note that the SGM does not attempt to simulate induced technical change. In that case, an isoquant would shift inward in response to a change in prices or some other policy signal, perhaps with a time lag.

unique efficiency parameters (see Appendix A for example values). Technical change over time is calculated as a multiplier called *Factor* of the technical scale coefficients α for the CES formulation and λ for the Leontief formulation.

$$Factor_{i,j,jj,t} = Factor_{i,j,jj,t-1} \bullet (1 + TECHNN)^{Nstep}$$
(unitless)   Eq. 19

where
   Nstep is the time step of 5 years,
   t denotes the number of 5 year time steps since the base year; in the base year t=0,
   Parmname (TECHNN or γ) indicates the sector-, or subsector-dependent regional technology-dependent technological rate change (unitless).

Technical change parameters are also implemented for household and government fuel use such that improved energy-use efficiency can be simulated.

Household (*j=hh*) fuel-use changes over time are calculated by means of the following factor, the implementation of which is discussed under "Household Demands":

$$Factor_{i,hh,t} = (1 + HHAEEI_{i,hh})^{Nstep}$$
(unitless)   Eq. 20

where
   $HHAEEI_{i,hh}$ is a technical change parameter for household fuel use.

Government (*j=gv*) fuel-use changes over time are calculated by means of a similar calculation:

$$Factor_{i,gv,t} = (1 + GVAEEI_{i,gv})^{Nstep}$$
(unitless)   Eq. 21

where
   $GVAEEI_{i,gv}$ is a technical change parameter for government fuel use.

Implementation of the government fuel use factor is discussed under "Government Demands."

Technical change as described above is implemented in exactly the same way for all production functions in the SGM. Thus, *Factor* is used as a multiplier of a technical coefficient such that a change over time in technology is simulated.

The technical coefficients ($\alpha_{i,j}$) in the CES production function

The technical scale coefficients ($\alpha_{i,j}$) are extracted from CES production functions for the base year as follows:

$$\alpha_{i,j,jj,v} = \left(\frac{X_{i,j,jj,v} / P_{i,t}}{X_{i=ETE,j,jj,v} / P_{i=ETE,t}} \right)^{\frac{\rho}{\mu}} \bullet \frac{P_{i,t}}{P_{i=ETE,t}}$$
Eq. 22

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where

v equals zero for the base year,

$\alpha_{i,j,jj,v}$ is the technical coefficient of the ith variable input in the production of commodity j or jj, i denotes the variable input (i=1:22),

j or jj denotes the production (sub)sector commodity output,

$X_{i,j,jj,v}$ is demand for the ith input in the production of commodity j or jj (each $X_{i,j,jj}$ is a value of the input-output table in the base year, which comprises the input-output values within the regional hybrid commodity-by-commodity input-output table in value terms (in monetary units) by row i, column j, subsector (column jj) for electricity, and region L,

$P_{i,t}$ is the market price, which equals one in the base year,

$X_{i=ETE,j,jj,v}$ is the demand for the Everything Else sector input in the production of product j or jj in the base year,

$P_{i=ETE,t}$ for the base year is the market price of the Everything Else sector and equals one,

ρ is a function of the elasticity of substitution parameter σ, a unitless input parameter that is a base year long-run elasticity of substitution for each production sector, where $\rho = (\sigma-1)/\sigma$ and μ if expressed in terms of σ then $\mu = \sigma-1$; thus, $\rho/\mu = 1/\sigma$.

Note that the alphas are extracted without consideration of vintage. The vintage indicator is set to zero (*v=t=0*). It is assumed that total output from production sectors is produced from one vintage (*i.e.,* total production can be described by one CES production function) rather than from a summation of multiple vintage-specific CES production functions in the base year (Fisher-Vanden 1993:14). Formulation of that assumption is a normalization of the technical scale coefficients after the the fixed factor capital stock technical scale coefficient is calculated based on data on prior active vintage stock.

The production sector-specific technical scale parameters ($\alpha_{0,j}$)

The production sector- and subsector-specific efficiency coefficients ($\alpha_{0,j,jj,v}$) are calculated for the base year as follows:

$$\alpha_{0,j,jj,v} = \left(\frac{P_{0,j,jj}}{P_{i=ETE,t}} \right)^{-1/\rho} \bullet \left(\sum_{i=1}^{N+1} X_{i,j,jj,v} \right)^{-1/\mu} \bullet \left(\frac{X_{i=ETE,j,jj,v}}{P_{i=ETE,t}} \right)^{1/\mu} \qquad \text{Eq. 23}$$

where

v=0 for the base year,

$P_{0,j,jj}/P_{i=ETE,t}$ is the normalized price index for commodity j or jj,

$P_{i=ETE,t}$ is the market price of the numeraire sector ETE, which equals one,

$\sum X_{i,j,jj,v}$ is the summed over N+1 base year demand (variable plus fixed factors + the indirect business taxes) for the inputs i in the production of commodity j or jj (see Equation 2),

$X_{i=ETE,j,jj,v}$ is demand for the ETE input in the production of commodity j or jj,

$P_{0,j,jj}$ is the price received for commodity j produced in a production sector. These sector prices are calculated from the difference between the total demand for inputs i minus the indirect business taxes paid (i=1:27=1:N+1), divided by the total demand for inputs i for the base year:

$$P_{0,j,jj} = \frac{\sum_{i=1}^{N+1} X_{i,j,jj,v} - X_{i=N+1,j,jj,v}}{\sum_{i=1}^{N+1} X_{i,j,jj,v}}$$
Eq. 24

where

v=0 for the base year,

$\sum X_{i,j,jj,v}$ is the summed over N+1 demands for the inputs i in the production of commodity j or jj (see Equation 2) in the base year, which includes

$X_{i=N+1,j,jj,v}$, the indirect business tax from the hybrid commodity-by-commodity input-output table, which is a production sector- or subsector-specific input parameter which needs to be substracted.

The fixed-factor-specific scale coefficients for capital stock ($\alpha_{i=N=capital\,stock,j}$)

The fixed-factor-specific scale coefficients for capital stock ($\alpha_{i=26=capital,j,jj,v}$) for each production sector *j* and subsector *jj* for the base year are calculated in steps and are a direct result of SGM's profit-making assumption:

$$\alpha_{i=26,j,jj,v} = \left(\frac{\sum_{i=1}^{N+1} X_{i,j,jj,v}}{\alpha_{0,j,jj,v}} \bullet \frac{1}{\sum_{v=-3}^{0} KAprior_{j,jj,v}} \right)^{\rho} \bullet Z_{j,jj,v}$$
Eq. 25

where

$\sum X_{i,j,jj,v}$ is the summed demand for the inputs i in the production of commodity j or jj in the base year and v equals zero for the base year,

$KAprior_{j,jj,v}$ is the vintage capital stocks that is active prior to the base year 1990, and

$Z_{j,jj,v}$ is calculated as follows for the base year when no or zero profit expectations are incorporated:

$$Z_{j,jj,v} = 1 - (\alpha_{0,j,jj,v} P_{0,j,jj})^{\mu} \bullet \sum_{i=1}^{N-1} \left(P_{i,t} \bullet \frac{\alpha_{i,j,jj,v}}{\alpha_{i=ETE,j,jj,v}} \right)$$
Eq. 26

where

v equals zero in the base year,

$P_{0,j,jj}$ is the price received for commodity j or jj in the base year (see Equation 24),

$\alpha_{0,j,jj,v}$ reflects the overall production efficiency of the vintage,

$P_{i,t}$ is the market price, which equals one in the base year,

$\alpha_{i,j,jj,v}$ is a technical scale coefficient of the ith input (supply) in(to) the production of commodity j or jj,

$\alpha_{i=ETE,j,jj,v}$ is the technical scale coefficient of the numeraire sector.

Normalizing the alphas ($\alpha_{i,j}$ where i=0:26 and j=1:22)

For normalization of the alphas, all alphas (*i=0:26*; *N=26*) are summed, including $\alpha_{0,j,jj,v}$ and $\alpha_{i=26=capital,j,jj,v}$ and each of the alphas is recalculated by dividing by this sum:

$$\alpha_{i,j,jj,v} = \frac{\alpha_{i,j,jj,v}}{\sum\limits_{i=0}^{N=26}\alpha_{i,j,jj,v}} \qquad \text{for i=1:26 and j=1:22}$$

Eq. 27

The production sector-specific efficiency parameters, $\alpha_{0,j,jj,v}$, are re-calculated as follows:

$$\alpha_{0,j,jj,v} = \left(\frac{\alpha_{0,j,jj,v}}{\sum\limits_{i=0}^{N=26}\alpha_{i,j,jj,v}}\right)^{1/\rho}$$

Eq. 28

The vintage alphas and incorporation of technical change

The vintage alphas are calculated in three steps:

(1) First, assumed future technical change is incorporated. All alphas (*i=0:26*; *j=1:22*; *jj=1:6*), from the oldest vintage (*v=-3*) through the time zero vintage (*v=t=0*), $\alpha_{i,j,jj,v=-3:0}$, are set equal to $\alpha_{i,j,jj,v=0}$, which are calculated as shown above (Equations 22 through 27) and all alphas for the variable factors and for capital stock are projected into the future (*t=1:12*) as follows:

$$\alpha_{i,j,jj,v} = \alpha_{i,j,jj,v-1} \bullet \text{Factor}_{i,j,jj,t}{}^{\rho}$$

Eq. 29

where

   v equals t-3:t,

   $\alpha_{i,j,jj,v-1}$, the previous vintage-active alpha, is updated for each time period by implementing the technological change assumptions:

   $$\text{Factor}_{i,j,jj,t} = \text{Factor}_{i,j,jj,t-1} \bullet (1 + \text{Tech}_{i,j,jj})^{\text{Nstep}} \qquad \text{(see Equation 19)}$$

   where

      Nstep is the time step of 5 years, and

      Tech i,j,jj is the production sector-dependent regional technology-dependent technological rate change, which for αi,j,jj,v is referred to as Parmname, and

      ρ is a function of the production sector-specific long-run elasticity σ.

Note that *Factor*$_{i,j,jj,t}$*=1* in the base year when *t=0*.

(2) Second, the overall production efficiency, $\alpha_{0,j,jj,v}$, is incorporated into the alphas. These transformed alphas – by region, sector, subsector, input, and vintage, including the capital stock (*i=26*) – are calculated as follows:

$$\alpha_{i,j,jj,v} = \alpha_{0,j,jj,v}{}^{\rho} \bullet \alpha_{i,j,jj,v}$$

Eq. 30

$$\alpha\text{cst}_{i,j,jj,v} = \left(\alpha_{i,j,jj,v}\right)^{1/(1-\rho)}$$

Eq. 31

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where
ρ is a function of the long-run elasticity of substitution and equals (σ-1)/σ.

(3) Third, the alpha$_0$s ($\alpha_{0,j,jj,v}$) for all production (sub)sectors (*j* and *jj*), all vintages (*v*) and time periods are set to one after being "used" in the equation above. This allows alpha$_0$ to be used for energy supply calibration.

$$\alpha_{0,j,jj,v} = 1 \qquad\qquad \text{Eq. 32}$$

and

$$\alpha cst_{0,j,jj,v} = \alpha_{0,j,jj,v}{}^{\sigma \bullet \rho} = 1 \qquad\qquad \text{Eq. 33}$$

Imposing changes in elasticities of substitution

When capital becomes "old vintage" the technical coefficients' long-run elasticity *σ1* of substitution is converted to short-run elasticity *σ2* and profit rate is included:

$$\alpha_{i=26,j,jj,v} = \alpha_{i=26,j,jj,v}^{(1-\rho 2)/(1-\rho 1)} \bullet \left(\frac{Pr_{j,t}}{\pi_{j,jj,v}/KA_{j,jj,v}} \right)^{(\rho 1 - \rho 2)/(1-\rho 1)} \qquad\qquad \text{Eq. 34}$$

where
v=t-3:t,
$\alpha_{i=26=capital,j,jj,v}$ was normalized (in Equation 27 based on Equation 25),
ρ1 is a function of the long-run substitution elasticity [(σ1-1)/ σ1], and
ρ2 is a function of the short-run substitution elasticity [(σ2-1)/ σ2],
$Pr_{j,t}$ is the price received for commodity j (see Equation 13 in the "Price and expected price" section) at time t,
$\pi_{j,jj,v}/ KA_{j,jj,v}$ is the vintage-specific production (sub)sector profit rate — that is the profit relative to capital cost, or annual return per dollar of invested capital — for which the calculations are described in Chapter six, and
$KA_{j,jj,v}$ is the vintage-specific production sector or subsector capital stock; for the base year $KA_{j,jj,v}$=KAprior$_{j,jj,v}$ and in the projections $KA_{j,jj,v}$ is set equal to the linearly interpolated annual capital investment flow over a five-year period (from the current point in time back to the previous model solution-point in time [see Equation 118]).

$$KA_{j,jj,v} = KAprior_{j,jj,v} \qquad\qquad \text{Eq. 35}$$

The vintaged-specific production sector or subsector-specific technical scale coefficients (for *i=1:25* and *j=1:22*; for *jj=1:6* if active) for old vintages are calculated as follows:

$$\alpha_{i,j,jj,v} = \alpha_{i,j,jj,v}^{(1-\rho 2)/(1-\rho 1)} \bullet \left(\frac{Pr_{j,t}}{Pi_{i,j,jj,t}} \right)^{(\rho 1 - \rho 2)/(1-\rho 1)} \qquad\qquad \text{Eq. 36}$$

where
v equals t-3:t,

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$\alpha_{i,j,jj,v}$, is based on the calculation in Equation 28,

$Pi_{i,j,jj,t}$ are the prices paid by the producer for supplies (see Equation 9),

$Pr_{j,t}$ is the price received for a commodity produced (see Equation 13).

All alphas, including alpha for capital interest ($i=26$) are then once more transformed before being used in the calculations when operating old vintages (see "Profits, demands, expected profit rates, and the operation of capital").

$$\alpha cst_{i,j,jj,v} = \left(\alpha_{i,j,jj,v}\right)^{1/(1-\rho 2)} \qquad \text{Eq. 37}$$

where

v equals t-3:t,

$\alpha_{0,j,jj,v}$ equals one,

$\alpha_{i,j,jj,v}$ is based on its last transformation in Equations 34 for capital costs and in Equation 36 for the variable factors, and

$\sigma 2$ is the short-run elasticity of substitution, which can also be expressed as $1/(1-\rho 2)$

and

$$\alpha cst_{0,j,v} = \alpha_{0,j,v}^{\sigma 2 \bullet \rho 2} = 1 \qquad \text{Eq. 38}$$

where

$\rho 2$ is a function of the short-run substitution elasticity [$\rho 2 = (\sigma 2 - 1)/\sigma 2$].

Note that after the transformation processes the adjusted $\alpha_{0,j,v}$ for each production sector is set to one.

The technical coefficients ($\lambda_{i,j}$) in the Leontief production function

For potential substitution of each of the production functions' technical coefficients based on the CES production function ($\alpha_{0,j,jj,v}$; $\alpha_{i,j,jj,v}$; $\alpha_{i=26=capital,j,jj,v}$) by technical coefficients based on the fixed coefficient or Leontief production function ($\lambda_{0,j,jj,v}$; $\lambda_{i,j,jj,v}$; $\lambda_{i=26=capital,j,jj,v}$), the Leontief technical coefficients are calculated. These Leontiefs are, as are the alphas, calculated from the hybrid commodity-by-commodity input-output table values and prices. Base year computations are given in Equations 39-32, projections in Equation 43.

For the base year the technical coefficients for the Leontief production function ($\lambda_{i,j,jj,v}$ for $i=1:25$ and $j=1:22$ and $jj=1:6$ when active) are computed as follows:

$$\lambda_{i,j,jj,v} = \frac{X_{i,j,jj,v} / Pi_{i,j,jj,t=0}}{\sum_{i=1}^{N} X_{i,j,jj,v} / Pr_{j,t=0}} \qquad \text{Eq. 39}$$

where

v=0 for the base year,

N equals the variable plus fixed factors,

$X_{i,j,jj}$ is base year demand for the ith input in the production of commodity j or jj (each $X_{i,j,jj}$ is a value within the regional hybrid commodity-by-commodity input-output table),
$\sum X_{i,j,jj,v}$ is the summed base year demand for the inputs i in the production of commodity j or jj,
$Pi_{i,j,jj,t}$ are the prices paid by the producer for supplies (see Equation 9), and
$Pr_{j,t}$ is the price received for a commodity produced (see Equation 13).

For the base year the production sector-specific efficiency coefficients for the Leontief production function $\lambda_{0,j,jj,v}$ are set to one.

$$\lambda_{0,j,jj,v} = 1 \qquad\qquad \text{Eq. 40}$$

The capital cost technical scale coefficients (*i=26*) for the base year are calculated as follows:

$$\lambda_{i=26=capital,j,jj,v} = \frac{\displaystyle\sum_{v=-3}^{0} KAprior_{j,jj,v}}{\displaystyle\sum_{i=1}^{N} X_{i,j,jj,v} / Pr_{j,t}} \qquad\qquad \text{Eq. 41}$$

where
    $KAprior_{j,jj,v}$ is the capital stock vintage that is active for a production sector in the base year,
    $\sum X_{i,j,jj,v}$ is the summed base year demand for the inputs in the production of commodity j in the base year,
    $Pr_{j,t}$ is the price received for commodity j at time t.

Note that the vintaged and projected technical coefficients λ are only calculated when the elasticity of substitution for the production sector equals or is smaller than 0.05. Note that these elasticities are inputs for each production sector (for an example, see Appendix A). Vintaged production sector or subsector-specific efficiency coefficients $\lambda_{0,j,jj,v}$ for a Leontief production function are always set to one in the reference case.

$$\lambda_{0,j,jj,v} = 1 \qquad\qquad \text{Eq. 42}$$

Implementation of technical change is formulated as follows:

$$\lambda_{i,j,jj,t} = \frac{\lambda_{i,j,jj,t-1}}{(1+\gamma_i)^{Nstep}} \qquad\qquad \text{Eq. 43}$$

thus, with large values of γ_i, λ_{ij} gets smaller and demand decreases, or differently put, supplies are used more efficiently. For the technical coefficient of capital any value besides zero, of γ_i will result in more costly capital over time.

In the case of *putty-clay* behavior a sector's vintaged and projected technical coefficients for the Leontief production function ($\lambda_{i,j,jj,v}$ for *i=1:25* and *j=1:22*) are based on the vintage technical coefficients calculated for the CES production function.

$$\lambda_{i,j,jj,v} = \alpha_{0,j,jj,v}^{\rho 1/(1-\rho 1)} \bullet \left(\alpha_{i,j,jj,v} \bullet \frac{Pr_{j,t}}{Pi_{i,j,jj,t}} \right)^{1/(1-\rho)} \qquad \text{Eq. 44}$$

where

    $\alpha_{0,j,v}$ equals one,

    $\rho 1$ is a function of the long-run elasticity $\sigma 1$ which equals less or equal 0.05,

    $\alpha_{i,j,jj,v}$ is based on long-run elasticity and has technological change incorporated,

    $Pi_{i,j,jj,t}$ are the prices paid by the producer for supplies (see Equation 9), and

    $Pr_{j,t}$ is the price received for a commodity produced (see Equation 13).

And the Leontief vintaged and projected technical coefficients for capital are then also based on the vintage technical coefficients calculated for the CES production and on the profit rate:

$$\lambda_{i=26=capital,j,jj,v} = \alpha_{0,j,jj,v}^{\rho 1/(1-\rho 1)} \bullet \left(\alpha_{i=26,j,jj,v} \bullet \frac{Pr_{j,t}}{\pi_{j,jj,v} / Ka_{j,jj,v}} \right)^{1/(1-\rho 1)} \qquad \text{Eq. 45}$$

where

    $\alpha_{0,j,jj,v}$ equals one,

    $\alpha_{i=26=capital,j,jj,v}$ will be based on long-run elasticity and has technical change incorporated,

    $Pr_{j,t}$ is the price received for the commodity j produced at time t,

    $Ka_{j,jj,v}$ is the capital stock vintage that is active,

    $\pi_{j,jj,v}$ are profits for which the calculations are described in "Profits, demands, expected profit rates, and the operation of capital."

Chapter 6. Profits, demands, expected profit rates, and the operation of capital

Profits equal the residual payments to owners of capital; alternatively defined, profit is the difference between total revenue and the summation of all variable costs. Profit rates equal the annual return per dollar of invested capital and profit maximization implies that the value of the additional output (marginal revenue) from increasing input i by one unit should be equal to the price of that input.

Capital is the only fixed input in SGM. With one or more fixed factors of production, a profit function can be derived.

The relationship between profits and demands is addressed in the next two sections, followed by a description of expected profit rate calculations. Next, operation of old and new vintages in relation to production of supplies, demand, profit and cost as simulated in the SGM is described.

Profits and the production functions

Payments to owners of capital at a point in time, *i.e.*, profits are the difference between total revenue and the summation of all variable costs.

$$\pi_{j,jj,v} = Pr_{j,t} \bullet q_{j,jj,v} - \sum_{i=1}^{N-1} Pi_{i,j,jj,t} \bullet X_{i,j,jj,v} \qquad \text{Eq. 46}$$

where
$\quad$ $_{j,jj}$ refers to the total profit in the production of sector commodity j or subsector jj,
$\quad$ $Pi_{i,j,jj,t}$ are the prices paid by the producer for supplies (see Equation 9),
$\quad$ $Pr_{j,t}$ is the price received for a commodity produced (see Equation 13),
$\quad$ $q_{j,jj,v}$ is the gross output of the production process in production sector j or jj by operating vintage v at a point in time (see Equations 3 and 5), later referred to as $PRDV_{j,jj,v}$,
$\quad$ $X_{i,j,jj,v}$ is the amount of input i demanded in the production of commodity j or jj at a point in time, later referred to as $EDV_{i,j,jj,v}$,
$\quad$ N is the number of variable plus fixed inputs (N=26) where capital is the only fixed input in SGM.

Profit maximization implies that the value of the additional output (marginal revenue) from increasing input i by one unit should be equal to the price of that input (*HP pers*) in a perfectly competitive market; thus,

$$Pi_{i,j,jj,t} = Pr_{j,t} \bullet \frac{\partial q_{j,jj,v}}{\partial X_{i,j,jj,v}}, \ i=1, \ldots N\text{-}1 \qquad \text{Eq. 47}$$

where the variables are defined above.

Profits and the CES production function

Through algebraic manipulations of Equations 46 and 47 and the CES production equation (Equation 3), profits can be expressed as a function of price and the quantities of M non-variable or fixed factors of production (M=1 in the SGM) captured in $Y_{j,jj,v}$, and the quantities of variable factors, captured in $Z_{j,jj,v}$.

$$\pi_{j,jj,v} = \alpha_{0,j,jj,v} \bullet Pr_{j,t} \bullet Z_{j,jj,v}^{-1/\mu} \bullet Y_{j,jj,v}^{1/\rho} \qquad \text{Eq. 48}$$

where for each operating vintage v

   $\alpha_{0,j,jj,v}$ equals one (see "Technical change"),
   $Pr_{j,t}$ is the price received for the commodity produced in sector j at a point in time,
   μ and ρ are functions of the elasticity of substitution σ where $1/\mu=1/(\sigma-1)$ and $1/\rho=\sigma/(\sigma-1)$,
   $Z_{j,jj,v}$ denotes the participation of the variable factors at a point in time and is calculated as follows:

$$Z_{j,jj,v} = 1 - (\alpha_{0,j,jj,v} \bullet Pr_{j,t})^{\mu} \bullet \left(\sum_{i=1}^{N-M} \alpha cst_{i,j,jj,v} \bullet Pi_{i,j,jj,t}^{-\mu} \right) \qquad \text{Eq. 49}$$

   where
   M=1, since the SGM has one fixed factor supply, capital,[26]

   $Pi_{i,j,jj,t}$ are the prices paid by the producer for supplies (see Equation 9), and
   $\alpha cst_{i,j,jj,v}$ is the transformed and vintaged technical scale coefficient between a supply and production sector; note that this coefficient incorporates the technical change $\alpha_{0,j,jj,v}$ coefficient; short-run elasticities may be substituted for the long-run elasticities when old vintage is operational (see Equations 22-38), and
   $\alpha_{0,j,jj,v}$ is a potential energy supply calibration parameter, presently set to one.

Note that $Z_{j,jj,v}$ is for each point in time t for each production sector j, production subsector jj, vintage v-specific; v and t may be suppressed for simplicity.

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[26] Note that if only one fixed factor, capital, is simulated Equations 47-49 are as follows:

$$\pi_{j,jj,v} = \alpha_{0,j,jj} \, Pr_{j,t} \, Z_{j,jj,v}^{-1/\mu} Y_{j,jj,v}^{1/\rho}$$

$$Z_{j,jj,v} = 1 - (\alpha_{0,j,jj,v} \, Pr_{j,t})^{\mu} \left(\sum_{i=1}^{N-1} \alpha cst_{i,j,jj,v} \, Pi_{i,j,jj,t}^{-\mu} \right)$$

$$Y_{j,jj,v} = \alpha cst_{i=26,j,jj,v} \, X_{i=26,j,jj,v}^{\rho}$$

for acst equations see Equations 20-36 Section 2.5.2:

$$\alpha cst_{i,j,jj,v} = \left(\alpha_{0,j,jj,v}^{\rho} \alpha_{i,j,jj,v} \right)^{1/(1-\rho)} \quad \text{or}$$

$$\alpha_{i,j,jj,v} = \alpha_{i,j,jj,v}^{(1-\rho2)/(1-\rho1)} \left(\frac{Pr_{j,t}}{Pi_{i,j,jj,t}} \right)^{(\rho1-\rho2)/(1-\rho1)} \quad \text{and} \quad \alpha cst_{i,j,jj,v} = \left(\alpha_{0,j,jj,v}^{\rho} \alpha_{i,j,jj,v} \right)^{1/(1-\rho2)}$$

The last part of the profit equation $Y_{j,jj,v}$ denotes the participation of capital,

$$Y_{j,jj,v} = \sum_{i=N-M+1}^{N} \left(acst_{i=26,j,jj,v} \bullet KA^{\rho}_{j,jj,j} \right)$$

Eq. 50

where

$Ka_{i,j,jj,v}$ is the demand of the fixed factor input (capital) in the production of commodity j or jj of vintage v at time t (denoted by $X_{i=26,j,jj,v}$ in the base year) and

$\alpha cst_{i,j,jj,v}$ is the fixed factor technical scale coefficient (see Equations 22 through 38).

Profits and the Leontief production function

Through algebraic manipulations of Equations 46 and 47 and the Leontief production equation (Equation 5), profits can be expressed as a function of price and the quantities of *M* non-variable or fixed factors of production (*M=1*). Profits are not dependent on the quantities of variable factors, which are in the CES production function captured in $Z_{j,jj,v}$. Thus, the Leontief profit function is simply revenue[27] less variable costs. Profit based on prices received and prices paid and on the fixed factor of production (capital) is calculated as follows:

$$\pi_{j,jj,v} = \left[\frac{\lambda_{0,j,jj,v}}{\lambda_{i=capital,j,jj,v}} \bullet Pr_{j,t} - \sum_{i}^{N-1} \left(\frac{\lambda_{i,j,jj,v}}{\lambda_{i=capital,j,jj,v}} \bullet Pi_{i,j,jj,t} \right) \right] \bullet KA_{j,jj,v}$$

Eq. 51

where for each operating vintage v at each point in time

$KA_{j,jj,v}$ is the vintage-specific capital stock,

$Pi_{i,j,jj,t}$ are the prices paid by the producer for supplies (see Equation 9),

$Pr_{j,t}$ is the price received for a commodity produced (see Equation 13), and

$\lambda_{0,j,jj,v}$, $\lambda_{i,j,jj,v}$ and $\lambda_{i=26=capital,j,jj,v}$ are the Leontief technical coefficients.

The relationship between profits and demands

Hotelling's Lemma states that the partial derivative of the profit function with respect to the price of the ith input results in negative profit-maximizing total factor demand ($X_{i,j,jj,v}$) for that input:

$$\frac{\partial \pi_{j,jj,v}}{\partial Pi_{i,j,jj,t}} = -X_{i,j,jj,v}$$

Eq. 52

where

v indicates the active vintage,

$\pi_{j,jj,v}$ is the vintage-specific profit in the production of commodity j or jj,

$Pi_{i,j,jj,t}$ is the price paid by the producer for input at time t, and

$X_{i,j,jj,v}$ is the demand of the ith input in the production of commodity j or jj.

Demands are thus partial derivatives of the profit function with respect to prices.

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27 Revenue equals the product of output (*PRDV*) and the price received (*PR_j*). Variable costs equal the product of demand (*EDV*) and the price paid for supplies (*Pi*).

Demands and the CES production function

The demand for the ith factor input to the production process[28] in relation to the technical scale coefficients (the individual supply-to-production sector technical scale coefficients $\alpha_{i,j,jj,v}$ and the overall production technical sector scale coefficient $\alpha_{0,j,jj,v}$) can therefore be calculated directly if the production function is a CES production function as:

$$X_{i,j,jj,v} = \left(\alpha_{0,j,jj,v} \bullet Pr_{j,t} \bullet \frac{\alpha_{i,j,jj,v}}{Pi_{i,j,jj,t}} \right)^{\mu/\rho} \bullet \left(\frac{Y_{j,jj,v}}{Z_{j,jj,v}} \right)^{1/\rho} \qquad \text{Eq. 53}$$

where for each vintage v at each point in time

$\alpha_{0,j,jj,v}$ is a potential energy supply calibration parameter, presently set to one,

ρ and μ are functions of the elasticity of substitution parameter σ, where $\rho = (\sigma-1)/\sigma$ and $\mu=\sigma-1$,

$Z_{j,jj,v}$ denotes the participation of the variable factors (see Equation 49), and

$Y_{j,jj,v}$ stands for the (single) fixed factor part (capital) of the profit calculation,

$$Y_{j,jj,v} = \alpha cst_{i=26,j,jj,v} \bullet X_{i=26,j,jj,v}{}^{\rho} \qquad \text{(see Equation 50)}$$

where

$\alpha cst_{i=26=capital,j,jj,v}$ is the fixed factor capital scale coefficient, and

$X_{i=26=capital,j,jj,v}$ is the demand of the fixed factor inputs (in SGM, capital).

Demands and the Leontief production function

When the production function represents a Leontief technology, demand is independent of the variable factors and the expression for profit is linear. Demand for the ith input in the production process j or jj by operating vintage v at time t is described as follows:

$$X_{i,j,jj,v} = \frac{\lambda_{i,j,jj,v}}{\lambda_{0,j,jj,v}} \bullet \frac{\lambda_{0,j,jj,v}}{\lambda_{i=26,j,jj,v}} \bullet KA_{j,jj,v} = \frac{\lambda_{i,j,jj,v}}{\lambda_{i=26,j,jj,v}} \bullet KA_{j,jj,v} \qquad \text{Eq. 54}$$

where

$KA_{j,jj,v}$ is the vintage-specific investment capital, and

$\lambda_{0,j,jj,v}$, $\lambda_{i,j,jj,v}$ and $\lambda_{i=26=capital,j,jj,v}$ are the Leontief technical scale coefficients.

Expected profit rates

One way in which economic growth is achieved from one model period to the next is through investments in new production capacity. Demand for investment in new technology is determined

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$$EDV_{i,j,jj,v} = \left(\alpha_{0,j,jj,v} \bullet Pr_j \bullet \frac{\alpha_{i,j,jj,v}}{Pi_{i,j,jj,t}} \right)^{1/(1-\rho)} \bullet Z_{j,jj,v}{}^{-1/\rho} \bullet \left(\alpha_{i=26,j,jj,v} \bullet KA_{j,jj,v}{}^{\rho} \right)^{1/\rho}$$

[28] see Equations 61 & 76

by the expected profit rate. Once expected prices of supplies and expected prices to be paid for commodities produced are calculated, expected profit rates for new capacity can be calculated by substituting expected future prices into the profit rate function. The results are sensitive to the interest rate as well as other model prices. Decision-making on investments is part of the iterative process of (1) determining supply prices and prices received for commodities produced based on the operation of the active capital stocks, and (2) of determining expected prices and expected profit rates.

Expected profit rates and the CES production function

The expected profit rate is calculated based on expected prices and current technology. The expected profit rate $E\pi r_{j,jj,t}$ relative to capital cost $KA_{j,jj,v=t}$ is based on long-run elasticities of substitution. The vintage subscript for the expected profit rate is denoted not by v but by t, given that the operating vintage will have a vintage $v=t$ for the time period at hand.

$$\mathrm{E}\pi r_{j,jj,t} = \alpha_{0,j,jj,v} \bullet \frac{\mathrm{Pe}_{j,jj}}{\mathrm{Pie}_{i=ETE,j,jj,t}} \bullet Z_{j,jj,v}^{\;(\rho-1)/\rho} \bullet \alpha_{i=26,j,jj,v}^{1/\rho} \qquad \text{Eq. 55}$$

where

$\alpha_{0,j,jj,v}$ equals one and remains one over time,

$\mathrm{Pe}_{j,jj}$ is the expected price to be received for a commodity (see Equation 16), which changes over time,

$\mathrm{Pie}_{i=ETE,j,jj,t}$ is the price of the numeraire Everything Else sector, which remains one over time,

$\alpha_{i=26=capital,j,jj,v}$ is the technical scale parameter for capital costs; this is the transformed capital stock scale coefficient based on long-run elasticities of substitution (see Equations 25, 27 and 34),

ρ and μ are functions of the long-run elasticity of substitution, and

$Z_{j,jj,v}$ denotes the participation of the variable factors as described above (Equation 49) and is calculated for each production sector j, based on active technology and expected prices as follows:

$$Z_{j,jj,v=t} = 1 - (\alpha_{0,j,jj,v} \bullet \mathrm{Pe}_{j,jj})^{\rho/(1-\rho)} \bullet \sum_{i=1}^{N-1} \left(\alpha cst_{i,j,jj,v} \bullet \mathrm{Pie}_{i,j,jj}^{\;\rho/(\rho-1)} \right) \qquad \text{Eq. 56}$$

where the variables are defined above, and

$\alpha cst_{i,j,jj,v}$ is the technical scale coefficient described in Equation 31 and

$\mathrm{Pie}_{i,j,jj}$ is the expected price paid (Equation 15).

Expected profit rates and the Leontief production function

Based on the Leontief production function the expected profit rates based on expected prices and the technical scale coefficients of capital.

$$\mathrm{E}\pi r_{j,jj,t} = \frac{\lambda_{0,j,jj,v}}{\lambda_{i=26=capital,j,jj,v}} \bullet \mathrm{Pe}_{j,jj} - \sum_{i}^{N-1} \left(\frac{\lambda_{i,j,jj,v}}{\lambda_{i=26=capital,j,jj,v}} \bullet \mathrm{Pie}_{j,jj} \right) \qquad \text{Eq. 57}$$

where

v equals t,

$\lambda_{0,j,jj,v}$, $\lambda_{i,j,jj,v}$, and $\lambda_{i=26=capital,j,jj,v}$ are the Leontief technical coefficients (Equations 39-45),

$\mathrm{Pe}_{j,jj}$ is the expected price received for commodity j or jj (Equation 16), and

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Pie$_{i,j,jj}$ is the expected price to be paid by the producer j or jj for input i.

Note that the elasticity of substitution σ is not less than 0.05 for any of the production sectors or subsectors (see Appendix A for the list of the sigmas for the production sectors and subsectors) with the result that expected profit rates are always based on the CES production function in the reference case. Investment is based on expected profit rates and one may assume some price elasticity with regard to investment choices.

Operation of capital stock in the SGM: determining production and interindustry demand

As was stated above, profits π are the difference between total revenue and the summation of all costs; demand is the amount of input i demanded in the production of commodity j or jj by operating vintage v (denoted by $EDV_{i,j,jj,v}$ and, as previously discussed, the same as the $X_{i,j,jj}$ elements of the hybrid commodity input-output table in the base year); supply is the gross output of the production process in production (sub)sector j or jj by operating vintage v ($PRDV_{j,jj,v}$ or as previously denoted by $q_{j,v}$).

Excess demand is the difference between demand and supply. For each of the market commodities excess demand is calculated when operating either old or new capital. An equilibrium solution requires the absence of excess demand for each and every commodity included in the model. When the model solution is reached there will be a set of prices and quantities of the n-commodity market such that all n equations in the equilibrium condition will be simultaneously satisfied.[29]

When operating old vintages in the base year, vintages v=–3 through v=–1 are operated. When operating old vintages in the years following the base year, three vintages v=t-3 through v=t-1 are operated given that in the reference case the expected lifetime of a vintage is 20 years, the time step is five years and a vintage in its first time period is operated as new vintage, v=t. The time step indicator t is therefore, at each time step, three steps ahead of the oldest vintage and one step ahead of the latest vintage and with new vintages the time step t and the vintage v are identical.

All production activities, regardless of vintage, have a fixed input. First, profits and profit rates are calculated. Second, input demands are calculated. Third, output is calculated by substituting input demands into the production function. We first describe operation of old vintages, where the capital stock is known in advance.

Old vintages

Operating old vintage and the CES production function

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[29] "If all the commodities in an economy are included in a comprehensive market model, the result will be a Walrasian type of general-equilibrium model, in which the excess demand for every commodity is considered a function of the prices of all the commodities in the economy" Chiang (1967:52).

Before profits are calculated from the operation of old capital, its lifetime is checked to determine if it should operate at all (see Appendix A for examples of nameplate life of capital). If the vintage-specific capital stock ($KA_{j,jj,v}$) exceeds its nameplate life, that capital will be scrapped. If capital is not scrapped, the profit rate for each operating old vintage is calculated. When a profit rate is negative, capital stock is retired and production for that vintage is set to zero.

The fixed input for old vintage profit is the industry- and vintage-specific capital stock carried over from the previous time step. In the case of a CES production function, profit is calculated as follows:

$$\pi O_{j,jj,v=t-3:t-1} = \alpha_{0,j,jj,v} \bullet Pr_j \bullet \left(Z_{j,jj,v}^{(-1/\rho)} \right)^{(1-\rho)} \bullet \left(\alpha_{i=26,j,jj,v} \bullet KA_{j,jj,v}^{\rho} \right)^{1/\rho} \qquad \text{Eq. 58}$$

where
  v equals t-3 through t-1,
  $\alpha_{0,j,jj,v}$ equals one,
  $Pr_{j,t}$ is the price received for the commodity produced in sector j,
  $Z_{j,jj,v}$ is described in Equation 49 and implemented in Equation 58 with long-run elasticities for old vintages older than the base year of 1990; in the base year and in the projections, short-run elasticities are implemented when vintages are old capital,
  ρ is represented by long-run elasticity for vintages older than the base year and ρ is represented by short-run elasticity for old vintages for the base year and beyond (then $\rho=\rho 2$),
  $KA_{j,jj,v}$ is the vintage-specific capital stock,
  $\alpha_{i,=26=capital,j,jj,v}$ is the vintage-specific capital stock technical scale coefficient in which technical change is incorporated; short-run elasticities are implemented when calculations are made for old vintages (Equation 34); however, when vintages are older than the base year of 1990, long-run elasticities are implemented (see Equations 25 and 27).

Operating old vintage and the Leontief fixed factor production

For old operating capital, profit based on the Leontief production function is calculated as follows:

$$\pi O_{j,jj,v=t-3:t-1} = \left[\frac{\lambda_{0,j,jj,v}}{\lambda_{i=26,j,jj,v}} \bullet Pr_{j,t} - \sum_{i}^{N-1} \left(\frac{\lambda_{i,j,jj,v}}{\lambda_{i=26,j,jj,v}} \bullet Pi_{i,j,jj,t} \right) \right] \bullet KA_{j,jj,v} \qquad \text{Eq. 59}$$

where
  v equals t-3 through t-1,
  $\lambda_{0,j,jj,v}=1$,
  $KA_{j,jj,v}$ is the vintage-specific capital
  $\lambda_{0,j,jj,v}$, $\lambda_{i,j,jj,v}$, and $\lambda_{i=26=capital,j,jj,v}$ are the Leontief technical coefficients (Equations 39-45),
  $Pi_{i,j,jj,t}$ are the prices paid by the producer for supplies (see Equation 9), and
  $Pr_{j,t}$ is the price received for a commodity produced (see Equation 13).

Note that only in the cases of operating old capital for electricity generation and for refining oil, the Leontief production functions are actually implemented in this SGM's documented version.

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Profit rates ─ that is profits relative to capital cost, or annual return per dollar of invested capital ─ are calculated by production sector or if subsector production takes place, as follows:

$$\pi Or_{j,jj,v} = \frac{\pi O_{j,jj,v}}{KA_{j,jj,v}}$$

Eq. 60

When a profit rate is negative, that is when payments to owners of capital can not be met, capital stock is retired and production for that vintage is set to zero.

Old vintages, demand, and gross production

Demands by production sectors *j* or subsectors *jj*, when operating old vintages, are calculated as in Equation 61 when the production function is a CES production function. A multiplicative factor reduces output smoothly to zero as profit rate goes to zero. This is *fac* in the equation for input demands ($EDV_{i,j,jj,v}$). The same factor is applied when calculating production or output ($PRDV_{j,jj,v}$) in the operation of old vintages. This term *fac* is used to ensure that input demands are continuous as relative prices change.

$$EDV_{i,j,jj,v=t-3:t-1} = fac \bullet \left(\alpha_{0,j,jj,v} \bullet Pr_{j,t} \bullet \frac{\alpha_{i,j,jj,v}}{Pi_{i,j,jj,t}} \right)^{1/(1-\rho)} \bullet Z_{j,jj,v}^{-1/\rho} \bullet \left(\alpha_{i=26,j,jj,v} \bullet KA_{j,jj,v}^{\rho} \right)^{1/\rho}$$

Eq. 61

where
   i is the supply sector indicator (i=1:25), j the production sector indicator (j=1:22), jj the production subsector indicator (j ≤ 6), and v the vintage indicator,
   v equals t-3:t-1,
   fac is a scaling factor described below,
   $Z_{j,jj,v}$ is described under Equation 49[30] and used in Equation 61 with long-run elasticity of substitution for the base year's old vintages and short-run elasticity of substitution for 1990 and beyond; it is similarly calculated for subsector production,

$$Z_{j,jj} = 1 - (\alpha_{0,j,jj,v} \bullet Pr_{j,t})^{\mu} \bullet \left(\sum_{i=1}^{N-1} \alpha cst_{i,j,jj,v} \bullet Pi_{i,j,jj,t}^{-\mu} \right)$$

$[\alpha_{i,j,jj,v}^{1/(1-\rho)}]$ is described as $\alpha cst_{i,j,jj,v}$ in Equation 31 with long-run elasticities and in Equation 37 for old vintages with short-run elasticities,
   ρ is a function of the long-run elasticity of substitution for the base year's old vintages and short-run elasticity of substitution for 1990 and beyond,
   $Pi_{i,j,jj,t}$ are the prices paid by the producer for supplies (see Equation 9),
   $Pr_{j,t}$ is the price received for a commodity produced (see Equation 13), and
   fac represents a scaler also to be applied to the production sector's productivity (Equation 61 and 63). This scaler may be applied when the profit rate is less than a threshhold profit rate of 0.01 (see Equation 62 for the calculation of the scaler in that case). Alternatively, the threshold can be set such that for depletable resources one makes sure that production does not exceed available optimal production levels from resources through the calculation of the total amount

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[30] $$Z_{j,jj,v} = 1 - (\alpha_{0,j,jj,v} Pr_{j})^{\mu} \left(\sum_{i=1}^{N-1} \alpha cst_{i,j,jj,v} Pi_{i,j,jj,t}^{-\mu} \right)$$

(integrated over the five-year time period) of resources needed to meet expected demand. Under one more alternative, production sectors may be idled based on emission limitations.

$$\text{fac} = 3 \bullet \left(\frac{\pi Or_{j,jj,v}}{0.01} \right)^2 - 2 \bullet \left(\frac{\pi Or_{j,jj,v}}{0.01} \right)^3 \qquad \text{Eq. 62}$$

The annual vintage-specific gross production by the production sectors and subsectors through the CES production function is calculated as follows**:**

$$PRDV_{j,jj,v=t-3:t-1} = \text{fac} \bullet \alpha_{0,j,jj,v} \bullet Z_{j,jj,v}^{-1/\rho} \bullet \left(\alpha_{i=26,j,jj,v} \bullet KA_{j,jj,v}^{\ \rho} \right)^{1/\rho} \qquad \text{Eq. 63}$$

where
fac is described above,
$\alpha_{0,j,jj,v}$ equals one,
$Z_{j,jj,v}$ is described under Equation 61,
$\alpha_{i,=26,j,jj,v}$ is described under Equation 61,
$KA_{j,jj,v}$ is the vintage-specific capital, and
ρ is a function of the elasticity of substitution.

When the production function represents a Leontief technology – that is, when the elasticity of substitution $\sigma2$ is smaller than 0.05 – demand is calculated as follows:

$$EDV_{i,j,jj,v=t-3:t-1} = PRDV_{j,jj,v} \bullet \frac{\lambda_{i,j,jj,v}}{\lambda_{0,j,jj,v}} \qquad \text{Eq. 64}$$

where
$\lambda_{0,j,jj,v}$ and $\lambda_{i,j,jj,v}$; are the Leontief technical coefficients, and gross production by the production sectors or subsectors is calculated as follows:

$$PRDV_{j,jj,v=t-3:t-1} = \text{fac} \bullet \frac{\lambda_{0,j,jj,v}}{\lambda_{i=26,j,jj,v}} \bullet KA_{j,jj,v} \qquad \text{Eq. 65}$$

where
fac is described above, and
$KA_{j,jj,v}$ is the vintage-specific capital.

Checks on depletable resources after calculations regarding operating old vintages
Oil, gas and coal production sectors involve depletable resources. Therefore, available resources have to be checked against desired investments.

The sum of linearly interpolated annual energy consumption *PRDV*$_{j,jj,v}$ for the five-year period starting with the current model period *T* (which is *t*Nstep*) and going back over the previous four years can be shown to equal two times the annual flow at time *T-5* plus three times the flow at time *T*. The anticipated consumption or depletion potential of an energy source over a five-year time period based on anticipated energy production is calculated as follows:

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$$\text{Deplete}_j = ((2 \bullet \text{PRDV}_{j,jj,v}) + (3 \bullet \text{PRDV}_{j,jj,v})) \bullet \text{PRconvrt}_j \qquad \text{Eq. 66}$$

where

   v equals t-3 through t-1,

   $\text{PRDV}_{j,jj,v=t\text{-}3:t\text{-}1}$ is the annual anticipated gross production of the oil, gas, and coal production sectors, and

   PRconvrt_j is the energy conversion factor, converting the relative prices in monetary units to physical energy units.

If, over the next five-year investments, the required energy production exceeds the oil, gas, or coal supply available, demand for production is decreased (Equation 68), as are the related anticipated profits (Equation 69). Thus,

$$\text{PctDeplete} = \frac{\text{DRSVtmp}_{j,jj,t}}{\text{Deplete}_j} \qquad \text{Eq. 67}$$

$$\text{PRDV}_{j,jj,v=t-3:t-1} = \text{PctDeplete} \bullet \text{PRDV}_{j,jj,v} \qquad \text{Eq. 68}$$

where

   $\text{DrsVtmp}_{j,jj,v}$ is the energy resource available at a point in time initiated with the energy resource data information (see Appendix A) and calculated through resource investment calculations in the projections (Equations 119-132) and updated through Equation 70.

Profits would be reduced accordingly:

$$\pi O_{j,jj,v=t-3:t-1} = \text{PctDeplete} \bullet \pi_{j,jj,v} \qquad \text{Eq. 69}$$

The energy sources are reduced by the amount of energy consumed as follows:

$$\text{DrsVtmp}_{j,jj,t} = \text{DrsVtmp}_{j,jj,t} - \text{Deplete}_j \qquad \text{Eq. 70}$$

Analogously, capital investment demands into fossil fuel production would be reduced (Equation 71). This demand $Kdem_{j,jj}$ will be described under "Investments" in the "Final Demand" section (see Equation 133 where capital demand, $Kdem$, is set equal to the flow of capital $Kaflow$ when investment into depletable resources is described).

Note that $Kaflow_{j,jj,t}$ refers to the yearly capital flow of investments into vintage production sectors or subsectors while $KA_{j,jj,v=t}$ refers to the total capital investments or capital stock over the five-year time period t simulated (see Equation 118).[31]

$$\text{Kdem}_{j,jj} = \text{PctDeplete} \bullet \text{Kdem}_{j,jj} \qquad \text{Eq. 71}$$

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31 Given that $Kaflow_{i,jj,v=t}$ refers to an investment flow at a certain point in time when new capital investment is calculated, the vintage indicator v equals t. $Kaflow$ has, in effect, in the SGM code, at times a v subscript and at other times a t subscript.

New vintages

New vintages differ from old vintages in two important ways. First, capital stock is not known in advance and must be provided by an investment mechanism (see "Investments"). Second, new capital has an elasticity of substitution that is equal to or greater than old capital in that sector or subsector. Therefore, new capital has more flexibility than old capital to respond to changes in relative prices of inputs. Equations for input demands and output for new vintages are identical to equations for old vintages.

Operating new vintages and calculating profit rates
When new capital with a vintage notation of *v=t* is operating at the first iteration of finding a model solution, the profit rate or profit per unit of capital is calculated as follows when calculated by means of the CES production funcion with long-run elasticities:

$$\pi Nr_{j,jj,v=t} = \alpha_{0,j,jj,v} \bullet Pr_{j,t} \bullet Z_{j,jj,v}^{-1/\mu} \bullet \alpha_{i=26,j,jj,v}^{1/\rho} \qquad \text{Eq. 72}$$

where
    v=t denoting new vintage investment,
    $\alpha_{0,j,jj,v} = 1$,
    $Pr_{j,t}$ is the price received for the commodity produced in sector j at a point in time,
    $Z_{j,jj,v}$ is described in Equation 49, and is implemented with long-run elasticities in Equation 72,
    μ is a function of the long-run elasticity $\sigma 1$,
    $\alpha_{i,=26=capital,j,jj,v}$ is the vintaged capital stock scale coefficient in which technical change is incorporated (see Equation 27, which provides for a normalized value of Equation 25; see Appenxix A for an example of capital alpha transformations for j=1), and
    ρ is a function of the long-run elasticity $\sigma 1$.

Similarly, at the first iteration of finding a model solution, the profit rate or profit per unit of capital if calculated by means of the Leontief production funcion is as follows:

$$\pi Nr_{j,jj,v=t} = \frac{\lambda_{0,j,jj,v}}{\lambda_{i=26,j,jj,v}} \bullet Pr_{j,t} - \sum_{i}^{N-1}\left(\frac{\lambda_{i,j,jj,v}}{\lambda_{i=26,j,jj,v}} \bullet Pi_{i,j,jj,t} \right) \qquad \text{Eq. 73}$$

where
    v=t,
    $\lambda_{0,j,jj,v}$, $\lambda_{i,j,jj,v}$, and $\lambda_{i=26=capital,j,jj,v}$ are the Leontief technical coefficients (see Equations 39-45),
    $Pi_{i,j,jj,t}$ are the prices paid by the producer for supplies (see Equation 9), and
    $Pr_{j,t}$ is the price received for a commodity produced (see Equation 13).

Operating new vintage and profit
Profits, after the first iteration of finding a model solution when operating new capital, are calculated based on the CES production function with long-run elasticities:

$$\pi N_{j,jj,v=t} = \alpha_{0,j,jj,v} \bullet Pr_{j,t} \bullet \left(Z_{j,jj,v}^{(-1/\rho)} \right)^{(1-\rho)} \bullet \left(\alpha_{i=26,j,jj,v} \bullet KA_{j,jj,v}^{\rho} \right)^{1/\rho} \qquad \text{Eq. 74}$$

where

v=t,

ρ is a function of the long-run elasticity σ1,

$Z_{j,jj,v}$ is described in Equation 48, and is implemented with long-run elasticities in Equation 74,

$\alpha_{i,=26,j,jj,v}$ is the vintaged capital stock scale coefficient in which technical change is incorporated (see Equation 28 in "Technical change"), and

$KA_{j,jj,v}$ is the vintage-specific capital stock.

If the long-run elasticity of substitution is smaller than 0.05 and new capital is operating, profit is calculated based on the Leontief production function:

$$\pi N_{j,jj,v=t} = \left[\frac{\lambda_{0,j,jj,v}}{\lambda_{i=26,j,jj,v}} \bullet Pr_{j,t} - \sum_{i}^{N-1} \left(\frac{\lambda_{i,j,jj,v}}{\lambda_{i=26,j,jj,v}} \bullet Pi_{i,j,jj,t} \right) \right] \bullet KA_{j,jj,v} \qquad \text{Eq. 75}$$

where

v=t,

$KA_{j,jj,v}$ is the vintage-specific capital,

$\lambda_{0,j,jj,v}$, $\lambda_{i,j,jj,v}$, and $\lambda_{i=26=capital,j,jj,v}$ are the Leontief technical coefficients (see Equations 39-45),

$Pi_{i,j,jj,t}$ are the prices paid by the producer for supplies (see Equation 9), and

$Pr_{j,t}$ is the price received for a commodity produced (see Equation 13).

New vintages, demand,and gross productivity

When new vintages are operating and the production function is a CES production function, demands by production sectors and subsectors are calculated as follows:

$$EDV_{i,j,jj,v=t} = \left(Pr_{j,t} \bullet \alpha_{0,j,jj,v} \bullet \frac{\alpha_{i,j,jj,v}}{Pi_{i,j,jj,t}} \right)^{1/(1-\rho)} \bullet Z_{j,jj,v}^{-1/\rho} \bullet \left(\alpha_{i=26,j,jj,v} \bullet KA_{j,jj,v}^{\rho} \right)^{1/\rho} \qquad \text{Eq. 76}$$

where

i is the supply sector indicator (i=1:25), j the production sector indicator (j=1:22), jj the production subsector indicator (j ≤ 6), and v the vintage indicator,

v=t,

$Z_{j,jj,v}$ is described in Equation 49 and implemented with long-run elasticities in Equation 76,

$$Z_{j,jj,v} = 1 - (\alpha_{0,j,jj,v} \, Pr_j)^{\mu} \left(\sum_{i=1}^{N-1} \alpha cst_{i,j,jj,v} Pi_{i,j,jj,t}^{-\mu} \right)$$

$[\alpha_{i,j,jj,v}^{1/(1-\rho)}]$ is described as αcst in Equation 31 with long-run elasticities,

$Pi_{i,j,jj,t}$ are the prices paid by the producer for supplies (see Equation 9), and

$Pr_{j,t}$ is the price received for a commodity produced (see Equation 13).

The vintage-specific gross production through the CES production function is calculated as follows when new vintage is operated (*v=t*):

$$PRDV_{j,jj,v=t} = \alpha_{0,j,jj,v} \bullet Z_{j,jj,v}^{-1/\rho} \bullet \left(\alpha_{i=26,j,jj,v} \bullet KA_{j,jj,v}^{\rho} \right)^{1/\rho} \qquad \text{Eq. 77}$$

where

$\alpha_{0,j,jj,v} = 1$,

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$Z_{j,jj,v}$ is described in Equation 49 and implemented with long-run elasticities in Equation 77,

$\alpha_{i,=26,j,jj,v}$ is the vintaged capital stock scale coefficient in which technical change is incorporated,

$KA_{j,jj,v}$ is the vintage-specific capital,

ρ is a function of the long-run elasticity of substitution.

When the production function represents a Leontief technology demand is calculated as follows:

$$EDV_{i,j,jj,v=t} = PRDV_{j,jj,v} \bullet \frac{\lambda_{i,j,jj,v}}{\lambda_{0,j,jj,v}}$$

Eq. 78

where

$\lambda_{0,j,jj,v}$ and $\lambda_{i,j,jj,v}$ are the Leontief technical coefficients,

and gross production is calculated as follows:

$$PRDV_{j,jj,=t} = \frac{\lambda_{0,j,jj,v}}{\lambda_{i=26,j,jj,v}} \bullet KA_{j,jj,v}$$

Eq. 79

Checks on depletable resources after calculations regarding operating new vintages

New vintage capital in the oil, gas and coal production sectors, like old vintage capital in these sectors, use depletable resources when operating. Resource consumption and potential depletion is again calculated by linear interpolation of annual flows for the five-year period starting with the current model period *T* and going back over the previous four years. If the consumption of resources is larger than or equals the resources available, the resources are exhausted and set to zero, and consumption is set equal to the available resources (Equations 66-71).

Demands by production sectors and cost calculations

To obtain production sector demands, the vintage and subsector demands are summed over the operating vintages and subsectors if active:

$$ED_{i=1:25,j=1:22} = \sum_{v=t-3}^{t} \left[\sum_{jj=1}^{n} EDV_{i,j,jj,v} \right]$$

Eq. 80

where

$EDV_{i,j,jj,v}$ denotes demand by a vintaged production sector or subsector,

n is the number of subsectors where $n \le jj$, and

v denotes the operating vintages.

When production is simulated, costs for each vintage-specific production sector and subsector are calculated based on the vintage sector or subsector demands and prices paid at time *t* of the model solution; costs are here expressed as the product of quantity produced in units of 1990 millions of dollars and the relative purchase price paid accounting for transportation costs, taxes and adjustments.

58

$$\text{Cost}_{i,j,jj,v=t-3:t-1} = \text{EDV}_{i,j,jj,v} \bullet \text{Pi}_{i,j,jj,t}$$

Eq. 81

where

EDV$_{i,j,jj,v}$ denotes demand by a vintaged production sector or subsector, and

Pi$_{i,j,jj,t}$ are the prices paid by the producer for supplies (see Equation 9).

Note that these cost calculations are performed for model output only.

Additional costs when operating any of the active vintages

Adjustment factors *WHSL$_i$*, representing distribution and markup factors for the supply sectors ─ that is, for the variable factor inputs ─ are calculated next. The same adjustment factors are calculated for household distribution and markup costs (see Equation 193). The summations of both costs are used when calculating Everything Else sector demands[32] (Equation 137). All variables in this equation are described above. The adjustment factors *WHSL$_i$* will be factors with values that equal zero, given that in our reference case *adj$_{i,j,jj,t}$=1* for all supply sectors (see Equation 11). For all supply sectors (*i=1:25*), for each production sector *j* and subsector *jj*, the adjustment factor *WHSL$_i$* is calculated as follows:

$$\text{WHSL}_i = \sum_{j=1}^{22} \left[\sum_{v=t-3}^{t} \left[\sum_{jj=1}^{n} \left[\frac{\left(\text{adj}_{i,j,jj,t} - 1\right) \bullet \left[\left(P_{i,t} + \text{Tr}_{i,t}\right) \bullet \text{Ex Im}_i \bullet \text{Txpro}_{i,j,jj} + \text{Txadd}_{i,j,jj}\right] \bullet}{\text{EDV}_{i,j,jj,v=t-3:t}} \Big/ \text{Pi}_{i=\text{ETE},j,jj,t} \right] \right] \right]$$

Eq. 82

where

v refers to all actively operating vintages (v=t-3:t-1 for old vintages and v=t for new vintages),

n is the number of subsectors with n ≤ jj,

j is the number of production sectors,

i refers to input sectors,

adj$_{i,j,jj,t}$, is a time-period-dependent adjustment factor that reflects markups and intra-regional transport costs; these adjustment factors can be input- (i) and sector- (j) or subsector- (jj) specific,

P$_{i,t}$ is the market price; in the base year P$_i$ equals one; in the forecasts P$_i$ is calculated during the model solution process and is supply and demand dependent,

Tr$_{i,t}$ is a time-period-dependent supply sector transportation cost factor,

ExIm$_i$ is the transportation cost rate parameter,

Txpro$_{i,j,jj}$ is a proportional tax rate on the ith product,

Txadd$_{i,j,jj}$ is an additive tax on the ith product,

EDV$_{i,j,jj,v}$ denotes demand by a vintaged production sector or subsector (see Equations 61 and 64 and 76 and 78), and

Pi$_{i=\text{ETE},j,jj,t}$ is the price paid by the producer for Everything Else input.

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32 Equations described in this section are calculated separately when operating old vintages (*v=t-3:t-1*) and new vintages (*v=t*). Given that these equation in both subroutines in the SGM code are the same we describe them here once for *v=t-3:t*.

Transportation costs (*TRNP_i*) for markets with imports and exports for all variable factor inputs are calculated in Equation 83. They are calculated in a similar fashion based on demands by households (Equation 196). The summation of both costs are used when calculating Everything Else sector demands (Equation 137).

$$TRNP_i = \sum_{j=1}^{22}\left[\sum_{v=t-3}^{t}\left[\sum_{jj=1}^{n}\left(Tr_{i,t} \bullet Ex\,Im_i \bullet EDV_{i,j,jj,v=t-3:t}\Big/Pi_{i=ETE,j,jj,t}\right)\right]\right]$$ Eq. 83

Taxes on production (*Tax_{1,j}* for *i=1:22*) are summed over all supply related transfers and for electricity generation over the subsectors:

$$Tax_{1,j} = \sum_{i=1}^{22}\left[\sum_{v=t-3}^{t}\left[\sum_{jj=1}^{n}\left(P_{i,t} + Tr_{i,t} \bullet Ex\,Im_i \bullet EDV_{i,j,jj,v=t-3:t}\right) \bullet \left(Txpro_{i,j,jj} - 1\right)\right]\right]$$ Eq. 84

Taxes on labor (*Tax_{2,j}* for *i=25*) for each production sector *j* are kept separate in *Tax_{2,j}*. These taxes are for the *v=t*-specific production sectors and summed for electricity generation over the subsectors:

$$Tax_{2,j} = \sum_{v=t-3}^{t}\left[\sum_{jj=1}^{n}\left(P_{i=25,t} + Tr_{i=25,t} \bullet Ex\,Im_{i=25} \bullet EDV_{i=25,j,jj,v=t-3:t}\right) \bullet \left(Txpro_{i=25,j,jj} - 1\right)\right]$$ Eq. 85

where
$P_{i=25,t}$ is the market price for labor rental during the model solution process.

Supply subsidies *Subsid_i* equals the production taxes *Tax_{1,j}* for *i=1:25*:

$$Subsid_i = \sum_{j=1}^{22}\left[-1 \bullet \sum_{v=t-3}^{t}\left[\sum_{jj=1}^{n}\left(P_{i,t} + Tr_{i,t} \bullet Ex\,Im_i \bullet EDV_{i,j,jj,v=t-3:t}\right) \bullet \left(Txpro_{i,j,jj} - 1\right)\right]\right]$$ Eq. 86

Taxes on production per unit production and subsidies to supplies per unit supply *TaxpU_{1,j}* are only calculated when the additive taxes are greater than zero,

$$TaxpU_{1,j} = \sum_{i=1}^{25}\left[\sum_{v=t-3}^{t}\left[\sum_{jj=1}^{n}\left(Txadd_{i,j,jj} \bullet EDV_{i,j,jj,v=t-3:t}\right)\right]\right]$$ Eq. 87

Taxes on labor *TaxpU_{2,j}* (for *i=25*) for each production sector *j* per unit labor supply are kept separate in *TaxU_{2,j}*:

$$TaxpU_{2,j} = \sum_{v=t-3}^{t}\left[\sum_{jj=1}^{n}\left(Txadd_{i=25,j,jj} \bullet EDV_{i=25,j,jj,v=t-3:t}\right)\right]$$ Eq. 88

Subsidies per unit supply *SubsidpU$_1$* (for *i=1,25*) equal the taxes on the production (sub)sectors.

$$\text{SubsidpU}_i = \sum_{j=1}^{22}\left[-1 \bullet \sum_{v=t-3}^{t}\left[\sum_{jj=1}^{n}\left(\text{Txadd}_{i,j,jj} \bullet \text{EDV}_{i,j,jj,v=t-3:t}\right)\right]\right] \qquad \text{Eq. 89}$$

Final carbon permit fees *CPermF$_{5,j}$* are a function of the energy demands by the production sectors and the carbon fees per dollar of energy production, which are a function of the global warming potential and the emission coefficient of the fuel combusted. Thus,

$$\text{CPermF}_{5,j=1:22} = \sum_{i=1}^{25}\left[\sum_{v=t-3}^{t}\left[\sum_{jj=1}^{n}\left(\text{Cpf}_i \bullet \text{EDV}_{i,j,jj,v=t-3:t}\right)\right]\right] \qquad \text{Eq. 90}$$

where

$$\text{Cpf}_i = P_{i=23,t} \bullet \text{ExchRate}_{\text{region}} \bullet \sum_{fn}\text{EMC}_{fn} \bullet \text{PRconvrt}_i \bullet \text{GWP}_{fn} \text{ (see Equation 10)},$$

where

fn stands for oil, gas, or coal within a production process for which the energy supply carbon fee is calculated,

$P_{i=23,t}$ is the carbon price, which equals the market price during the model solution process,

ExchRate$_{\text{region}}$ the monetary exchange rate,

EMC$_{fn}$ is the fuel-specific carbon content in million tons C per exajoule energy of the energy source,

GWP$_{fn}$ is the global warming potential, which equals one for carbon,

PRconvrt$_i$ is the energy conversion factor, converting the relative prices in monetary units to physical energy units.

Chapter 7. Carbon policies

The SGM may impose carbon fees to provide an economic incentive for the economy to substitute away from carbon. Revenues obtained from the carbon fee can be large, and how the revenues are recycled or redistributed to the economy makes a difference in the economic costs. The GHG market operates using the same mechanism as each of the other markets in the SGM. Implementation of a carbon market can be achieved by (1) imposing a global or regional carbon price and have the model solve for additions in prices paid with the objective of emission reductions, or (2) imposing an emission limit or target and have the model solve for the necessary carbon price to reach that target. We describe the different options in the next section.

For the cases in which emissions permits are traded between regions, each SGM region is allocated an initial number of carbon emission permits based on the stated mitigation policy (*e.g.*, 1990 emissions levels). Carbon permits can then be traded between countries at a price that clears the global market in these permits.

Carbon prices and revenue cycling

Carbon policies in the SGM can be expressed in a number of different forms. Sets of switches (*CarbFeeP* and *PolType*) activate the no-policy or policy-specific options. Appendix A lists the first set of switches. To implement any carbon policy the *CarbFeeP₁* switch has to be set to one.

Appendix A also lists the second set of switches (*PolType*) and the different sets of input data (*CarbVar*) that are required under the different policy options that can be implemented in the SGM.

A. Under Policy type zero (*PolType=0*) carbon prices are fixed; *CarbFeeP₂* is set to zero which implies that there are no carbon emission limits (*CarbonLim = 0.0*); the fixed price switch is set for carbon (*IPfix_{i=23} = True*) and carbon prices are input (*CarbVar_t*) such that

$$P_{i=23,t} = \text{CarbVar}_t \qquad\qquad\qquad \text{Eq. 91}$$

If, in addition to *CarbFeeP₁=1*, *CarbFeeP₆* has a value larger than the base year (where *t=0*), then the time period of implementing the carbon price is determined by the value *t* of *CarbFeeP₆*.

Alternatively, under other types of policies, a set of switches can be set to solve for a market price for carbon in response to emission limits. The carbon price is then initially set to a small number: *e.g.*, $P_{i=23,l,m}=0.1$ and the model is allowed to solve for the carbon price; the *CarbFeeP₁* and *CarbFeeP₂* switches have to be set to one.

$$P_{i=23,t} = 0.1 \qquad\qquad\qquad \text{Eq. 92}$$

B. Under Policy type one (*PolType=1*) external carbon emission limits are set; *CarbFeeP₂* is set to one; carbon emission limits are then equivalent to *CarbVar_t*:

$$CarbonLim_t = CarbVar_t \qquad \text{Eq. 93}$$

where
 $CarbVar_t$ values are externally set emission level limits illustrated in Appendix A (*e.g.*, 1429 mt C/yr in year 2000; 1345 mt C/yr in 2005, etc.) and
 $P_{i=23=carbon}$ is set initially to 0.1 or one (Equation 91) and when solving the model becomes the market price.

C. Under Policy type "larger than 1990" an internal carbon emission target is determined; both *CarbFeeP$_2$* and *CarbFeeP$_4$* are set to one and *CarbFeeP$_5$* is set to a number of time steps (*t=(year-1990)/5*) into the future to function as the initial year for an internal carbon emission target to be reached. Thus,

$$CarbonLim_t = EmTot_t \bullet CarbVar_t \qquad \text{Eq. 94}$$

where
 $EmTot_t$ is total emissions (Equation 247),
 $CarbVar_t$ values are endogenously set emission level limits (*e.g.*, 1.1 times the year 2005 level in 2010; 1.2 times the 2005 level in 2015, etc.), and
 $P_{i=23=carbon,t}$ is set initially to 0.1 or one (Equation 91) and when solving the model becomes the market price.

Ensuring that the model runs contain relevant data is dependent on the relationships between the switches and values listed above. These relationships are summarized in Appendix A.

When the switch *CarbFeeP$_1$* is set to one − that is, when a carbon policy is to be implemented − all demands (*Ed$_{i=23=carbon,j=1:22}$=0*) for carbon are set initially to zero. Next, if there is an active carbon market, the "carbon demand" (*i=23* and *j=23*) is set equal to the negative value of the carbon emission limit (Equation 95) and production of carbon emissions (Equation 96) is set to the positive value of the carbon emission limit (see also Equation 238 where *SalesV$_{i=1:23}$=PRD$_{j=1:23}$*). Carbon prices are then determined as for any market (see "The solution procedure").

$$ED_{i=23,j=23} = -CarbLim_t \qquad \text{Eq. 95}$$

$$PRD_{j=23} = CarbLim_t \qquad \text{Eq. 96}$$

If carbon fees are fixed, "carbon demand" is determined by the related emissions (see Equations 244 for oil, 245 for gas, and 246 or coal).

$$ED_{i=23,j=3,4.5} = EmissionsC_{j=3,4,5} \qquad \text{Eq. 97}$$
 for carbon emissions from oil, gas and coal combustion (*i=3, 4*, and *5*)

Similarly, if carbon fees are fixed and carbon-equivalent emissions are calculated (see Equation 98 and 249), "carbon-demand" is tracked as equivalent to the emissions by a production sector based on their total demands (*TotDem*) by pairing the correct global warming potential (*GWP*)

values of the different gases with the type of gases emitted (as related to their emission coefficients, *EMC*) and their conversion factors of monetary units to energy units (*PRconvrt*)

$$ED_{i=23, j=1:22} = Emissions_j = \sum_i TotDem_i \bullet PRconvrt_i \bullet GWP_i \bullet EMC_i \qquad \text{Eq. 98}$$

Carbon permit trade

If carbon permits are traded, the value of *CrbTrade$_t$* can be set exogenously in the base year (variable *CrbTrExo$_t$*).

Alternatively, under each of the carbon policy implementations listed in the previous section, *CrbTrade$_t$* may be based on a fixed permit price ($P_{i=23,t}$) or a market price for carbon and on a carbon emission limit set exogenously or endogenously. Carbon permit trading only takes place when regions are linked through trade markets.

$$CrbTrade_t = P_{i=23,t} \bullet ExchRate_{region} \bullet (CarbLim_t - EmTot_t) \qquad \text{Eq. 99}$$

where
   $P_{i=23,t}$ is the carbon price at time t
   ExchRate$_t$ is the exchange rate,
   CarbLim$_t$ is the emission limit, and
   EmTot$_t$ is total carbon emissions, which are calculated in Equations 244-247.

Carbon policy impacts

Impacts of carbon policies can include the following: (a) energy prices are impacted by adding the carbon fee to the energy price (see Equation 10); and (b) household, government and investment demands are impacted by accounting for a carbon fee per dollar of energy demand (see Equations 205, 206, and 140).

Carbon fees collected can be recycled as revenue in various ways, described below.

If carbon prices are market-based then the household, government, and investment carbon permit fees are part of a carbon market. If carbon trade is implemented and the market demand and supply are determined by emissions (see the previous section) the domestic carbon price receipts have to be taken out of the carbon trade estimate. The costs of carbon prices are recycled as revenue when carbon prices are market-based and when they are fixed. If no carbon trade is implemented, the *CrbTrade$_t$* variable will equal zero (Equation 100). The carbon fees imposed on investment, households, and government are based on carbon prices (see Equation 10 for determining the *Cpf$_i$* values). The carbon fees to be recycled as carbon revenue within a region are the carbon fees traded and corrected for the carbon fees levied on households, government and investments. Appendix A summarizes the carbon policy option impacts through the recycling of the revenues obtained from the carbon fees and, if active, from carbon permit trade. The details of the revenue recycling are described in the investment, household and government sections in this document.

$$CrbTrade_t = CrbTrade_t - CPermF_{5, j=25=imex} - CPermF_{5, j=26=gv} - CPermF_{5, j=27=hh} \qquad \text{Eq. 100}$$

where

Carbon fees on fuel combusted and demanded by households ($CPermF_{5,j=27}$) is calculated in

Equation 199: $\quad CPermF_{5,j=27=hh} = \sum_{i=1}^{24}\left(Cpf_i \bullet ED_{i,j=27=hh}\right)$ where Cpf_i is a carbon fee per dollar

of energy demand.

Carbon fees on fuel combusted and demanded by government ($CPermF_{5,j=26}$) is calculated in

Equation 205: $\quad CPermF_{5,j=26=gv} = \sum_{i=1}^{25}\left(Cpf_i \bullet ED_{i,j=26=gv}\right)$

Carbon fees on fuel combusted and demanded by investment ($CPermF_{5,j=25}$) is calculated in

Equation 140: $\quad CPermF_{5,j=25=imex} = \sum_{i=1}^{22}\left(Cpf_i \bullet ED_{i,j=25=inv}\right)$

where

$\quad Cpf_i = P_{i=23,t} \bullet ExchRate_{region} \bullet \sum_{fn} EMC_{fn} \bullet PRconvrt_i \bullet GWP_{fn}$ (see Equation 10)

$$CrbFeeTot_t = CrbFeeTot_t + CrbTrade_t \qquad\qquad Eq.\ 101$$

where

$\quad CrbFeeTot_t$ are the summed carbon fees: $\quad CrbFeeTot_t = \sum_{j=1}^{27} CPermF_{5,j}$

Chapter 8. The final demand sectors

The four final demand sectors that demand and produce goods and services are
- investment
- households
- government
- net exports.

The final demand sectors are integrated with the rest of the system (see Figure 1) through their demand for final goods from the production sectors and their supply of land, labor and capital[33].

Below we describe investments followed by sections on government and household supply and demand and finally by a section on imports and exports. The export/import aspects of the final demand are integrated with the product markets by including the changes in inventory as net exports to eliminate the need for modeling inventory fluctuations.

Investment

The SGM requires a fixed factor, that is, the amount of capital to be known before production can take place. To close the model, an investment mechanism is required to determine how much capital is to be allocated to each of the sectors and subsectors. New capital requirements, in turn, determine the inputs necessary to produce new capital. Capital required for new production can be determined in two ways. New investment can be determined as a function of expected profit rate and investment in the previous period. Alternatively, new investments can be based on expected profit rates and the gap between expected production levels and the potential output by existing capital stocks.

Whenever a distinct product is being produced, it is represented by means of a production sector. Subsectors represent different ways of making that product. Subsectors are implemented in the SGM for electricity generation. The competition among the technologies based on expected profit rates determines the relative contributions (shares) the technologies make towards electricity generation investments and this is formulated by means of the logit share equation after McFadden (1981). In that formulation the shares of the competing fuels add to one, and are

33 Demand sector indicators

Final demand sectors (j)				
j	24	25	26	27
	Investment	Net Exports	General Government Consumption	Household Consumption
	inv	imex	Gv	Hh

Primary factor supply indicators

i	Primary factor supply sectors	
24	Land	ls
25	Labor	lbs
26	Capital or OVA	capital or capital cost
27	Indirect business taxes or IBT	ibt

dependent on an assumed base share value for each of the competing fuels and on the elasticities of their prices. Thus, more than one fuel source and its accompanying technology (oil-fired, gas-fired, coal-fired, nuclear, and hydro in SGM 2003[34]) can meet the electricity generation demand.

Delays with regard to investments that take more than one period before coming on line are not simulated in SGM 2003[35].

New investments based on expected profit rates and investments in the previous period

Investment as a function of expected profit rate $E\pi r_{j,jj,t}$ and investment in the previous period $KAflow_{t-1}$ is described first.

Initial annual investments *KAflow* for the base year ($v=t=0$) and old vintages ($v=-1$) are input parameters, as are initial expected profit rates. Thus,

$$K_{j,t} = \sum_{jj=1}^{n} KAflow_{j,jj,t} \qquad \text{for the base year} \qquad \text{Eq. 102}$$

$$E\pi_{j,jj,t=0} = EXPPROF_{j,jj,t=0} \qquad \text{for the base year} \qquad \text{Eq. 103}$$

When based on expected profit rate and investment in the previous period, sector-level investment in the projections for all production sectors is a function of a base rate of growth, the growth in working-age population, and the expected return to capital. This investment is calculated in relation to what the marginal dollar should earn under perfect competition (*margvalue=1*); the production sector-specific variable *Temp$_j$* determines the earnings and $K_{j,t}$ denotes the potential investment into a production sector *j* at time *t*.[36] The investment calculation is then

$$K_{j,t} = Temp_j \bullet \left(\frac{\sum_{jj=1}^{n} E\pi_{j,jj,t}^{rhoinv+1}}{\sum_{jj=1}^{n} E\pi_{j,jj,t}^{rhoinv}} \right)^{rinv} \qquad \text{Eq. 104}$$

where
  $E\pi r_{j,jj,t}$ is a sector or subsector's expected profit rate at time t (see Equations 55 and 57),
  n denotes the number of active subsectors where $n \leq jj$; the summation over jj takes place only when subsector production takes place,
  rhoinv is the expected profit rate exponential, which equals one in the reference case, and

---

34 Presently (October 2004) renewables (biomass, waste, solar, geo, wind-on shore, wind-off shore) and new technologies (coal IGCC with and without carbon sequestration and disposal, gas NGCC with and without carbon sequestration and disposal, pulverized coal with carbon sequestration and disposal are implemented in various nesting structures to achieve optional knife-edge investment behavior in the model).

35 Presently (October 2004) delayed investment is a viable option in the model and implemented for new technologies to come on line.

36 calculations concern generating a new vintage *v* with an indicator value of *v=t*

rinv is the expected profit rate function exponential, which equals one in the reference case, and

Temp$_j$ is the variable denoting investment dependency on the change in demography, the marginal value of the dollar, and some basic assumption with regard to annual investments for all production sectors:

$$\text{Temp}_j = \text{basekap}_j \bullet \text{scaler} \bullet \left[\frac{\text{working\_age\_pop}_t}{\text{working\_age\_pop}_{t-1}} \right]^{\text{accinv}} \bullet \text{margvalue}^{-\text{rinv}} \qquad \text{Eq. 105}$$

where

basekap$_j$ is a production sector-specific annual investment which may be based on various assumptions:

a. basekap may be based on the production-sector-specific annual investment (KAflow) from the previous time-period, thus,

$$\text{basekap}_j = \text{KAflow}_{j,jj,t-1}, \text{ or}$$

b. when the annual investment is very small, basekap is 1% of the total available annual investment into all production sectors, thus $\text{basekap}_j = 0.01 \bullet \sum_{jj} \text{KAflow}_{j,jj,t-1}$, or

c. when subsectors are involved (electricity generating subsectors), basekap$_j$ is the summed annual investments over the subsector previous time-period's investments:

$$\text{basekap}_j = \sum_{jj} \text{KAflow}_{j,jj,t-1}$$

scaler — an investment accelerator — is a scale coefficient and has a value of 1.2,

working_age_population$_t$ at a point in time is the number of people in the working age bracket,

accinv is the working age population ratio exponential, which equals one in the reference case, and

margvalue is the marginal one dollar.

New investments based on anticipated output

As an alternative, the model can determine demand for capital based on the expected profit rate $E\pi r_{j,jj,t}$ and the gap between gross output PRD_j and the potential output by existing capital stocks.

When old and new vintages are operated, vintage-specific gross production $PRDV_{j,jj,v}$ is calculated for each operating vintage for each production sector j and subsector jj. Total gross production, PRD_j[37] is the summed vintage-specific production.

$$\text{PRD}_j = \sum_{v=t-3}^{t} \left[\sum_{jj=1}^{n} \text{PRDV}_{j,jj,v} \right] \qquad \text{Eq. 106}$$

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37 In the SGM code *SalesV*$_j$ is set equal to *PRD*$_j$ at this stage, and the *ED* matrix at a certain stage has diagonal elements calculated as *Ed*$_{i,j}$-*PRD*$_j$; in this documentation the *SalesV* vector is only initialized when describing the solution algorithms (see Equation 239) and the *Ed*$_{i,j}$ matrix always refers to the demand matrix. Appendix A shows complete *Ed*$_{i,j}$-*PRD*$_j$ matrices which solve to (close to) zero for the base year (*t=0*) and for year 1995 (*t=1*).

$PRDV_{j,jj,v}$ calculations are either based on CES or on Leontief production functions, dependent on the elasticity of substitution parameter of the production sector or subsector.

Anticipated increases in investments for all production sectors are in this formulation determined by an exogenous parameter in SGM 2003 ($Qproj_t$). This parameter functions as a multiplier to the production sector's gross production of the previous point in time. Output needed from the to-be-invested-in new vintage $Qnew_j$ is calculated as the difference between the summed active vintage-specific production sector output and the anticipated needs calculated as the summed previous vintage-specific operating production sector output multiplied by the desired increase in production. This is expressed in the following equation:

$$ Qnew_j = Qproj_t \bullet \sum_{v=t-4}^{t-1} \left[\sum_{jj=1}^{n} PRDV_{j,jj,v} \right] - \sum_{v=t-3}^{t} \left[\sum_{jj=1}^{n} PRDV_{j,jj,v} \right] \qquad \text{Eq. 107} $$

A testable demand for capital $TestK_j$ to produce output needed from new vintages is calculated as the product of the desired investment $Qnew_j$ and the calculated capital-to-output ratio $Capq$ if no subsector production takes place:

$$ TestK_j = Capq_{j,jj} \bullet Qnew_j \qquad \text{for jj=1} \qquad \text{Eq. 108} $$

where
$Capq_{j,jj}$ is the capital-to-output ratio.

The capital-to-output ratio is calculated based on the CES production function given that one may assume some price elasticity with regard to investment choices. Therefore, investment decisions in the SGM are based on the CES production function assumptions and not on the Leontief production function assumptions where price elasticities are zero[38].

$$ Capq_{j,jj} = \alpha_{0,j,jj,v}^{\rho/(1-\rho)} \bullet \alpha_{i=26,j,jj,v}^{1/(1-\rho)} \bullet \left(\frac{Pr_{j,t}}{\pi Nr_{j,jj,v=t}} \right)^{1/(1-\rho)} \qquad \text{Eq. 109} $$

where

38 When the elasticity of substitution ($\sigma 1$) is less than 0.05 this intermediate step could be calculated based on the Leontief production funcion coefficients:

$$ Capq_{j,jj} = \lambda_{0,j,jj,v} \bullet \lambda_{i=26,j,jj,v} $$

This is not implemented, however. This is for theoretical reasons: one may assume some price elasticity with regard to investment choices. Therefore investment decisions in the SGM are based on the CES production function assumptions and not on the Leontief production function assumptions where price elasticities are zero.

$\pi \mathrm{Nr}_{j,jj,v=t}$ is the profit rate calculated based on active prices and long-run elasticities, which in the CES formulation is as follows (see Equation 72)

$$\pi \mathrm{Nr}_{j,jj,v=t} = \alpha_{0,j,jj,v} \bullet \mathrm{Pr}_{j,t} \bullet Z_{j,jj,v}^{-1/\mu} \bullet \alpha_{i=26,j,jj,v}^{1/\rho}$$

and for the Leontief formulation as (see Equation 73):

$$\pi \mathrm{Nr}_{j,jj,v=t} = \frac{\lambda_{0,j,jj,v}}{\lambda_{i=26,j,jj,v}} \bullet \mathrm{Pr}_{j,t} - \sum_{i}^{N-1}\left(\frac{\lambda_{i,j,jj,v}}{\lambda_{i=26,j,jj,v}} \bullet \mathrm{Pi}_{i,j,jj,t} \right)$$

These calculations are more complex, however, when subsector production is simulated. Subsector investment is simulated as a shared investment. This logit sharing is only relevant to electricity production given that this sector is the only production sectors with subsectors in the reference case. Before 2040 hydropower and nuclear power-generated electricity are determined by exogenously determined investments[39]. Hydropower and nuclear energy produced electricity take part in the electricity generation share equation after 2035.

To calculate the investment in each of the electricity technology subsectors, the expected profit rates for the fixed (hydro and nuclear) electricity subsectors are set to zero. Then, investment is allocated at the remaining subsectors through the logit function. The logit function calculates the sectoral investment shares based on the ratio of the relative expected profitability of each of the subsectors. As a consequence, subsectors with the highest expected profit rates $E\pi r_{j,jj,v=t}$ receive the largest share of that sector's investment. The investment subsector shares sum to one for each production sector. The logit share equation at time t is formulated as follows:

$$\mathrm{Share}_{j,jj} = \frac{E\pi r_{j,jj,t}^{\mathrm{rhoinv}}}{\sum_{jj=1}^{n=3}\left(E\pi r_{j,jj,t}^{\mathrm{rhoinv}}\right)} \qquad \text{for j=8 and jj=1:6} \qquad \text{Eq. 110}$$

where

  $E\pi r_{j,jj,t}$ is the expected profit rate at time t (Equation 55),
  rhoinv is the logit share exponential, or elasticity, a parameter that controls the rate that investment shares change in response to changes in expected profit rates, and
  n denotes the investment demands for oil, gas and coal generated electricity.

Next, the inverse of capital to output ratio *QCap* is calculated:

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39 The history of hydroelectric and nuclear generation installations over the last two decades indicates that factors other than the cost and performance of these technologies and the demand for electricity are the main drivers determining electricity capacity installed. The penetration of these technologies is therefore modeled exogenously.

$$QCap_{j=8} = \sum_{jj=1}^{n=3} \left(\frac{Share_{j,jj}}{Capq_{j,jj}} \right)$$
Eq. 111

And *TestK*$_{j=8}$ is corrected for the revised capital demands:

$$TestK_j = Qnew_j - \sum_{jj=1}^{n} \left[\frac{\dfrac{2 \bullet Kaflow_{j,jj,t-1}}{Capq_{j,jj}} + \dfrac{3 \bullet ExoElec_{jj}}{Capq_{j,jj}}}{QCap_j} \right] +$$
for j=8   Eq. 112

$$2 \bullet \sum Kaflow_{j,jj,t-1} + 3 \bullet \left(ExoElec_{jj=5} + ExoElec_{jj=6} \right)$$

where

n is the number of active subsectors and $n \leq jj$,

KAflow$_{j,jj,t-1}$ is the previous time period's annual investment (see Equation 117 for the present period's investment),

Capq$_{j,jj}$ is the capital to output ratio of a production sector or subsector calculated in Equation 109 (CES production function-based),

QCap$_j$ is the share dependent (based on expected profit rates) inverse of the capital to output ratio,

Qnew$_j$ is the production sector related assumed improvement factor in gross production, and

ExoElec$_{jj}$ are the exogenously set electricity-generating annual investments (hydro and nuclear or when more subsector investments are set exogenously $\sum_{jj}^{n} ExoElec_{jj}$).

Finally, Equation 113 can be executed, analogous to Equation 104, and annual investment $K_{j,t}$ is determined by either of the two methods.

$$K_{j,t} = TestK_j \bullet \left(\frac{\displaystyle\sum_{jj=1}^{n} E\pi_{j,jj,t}^{rhoinv+1}}{\displaystyle\sum_{jj=1}^{n} E\pi_{j,jj,t}^{rhoinv}} \right)^{rinv}$$
Eq. 113

Some sectors may have investment set exogenously (but when *ExoInvst*$_j$ equals one or less, no exogenous investment takes place).

$$K_{j,t} = ExoInvst_j$$
Eq. 114

Since investment is allocated at the subsector *jj* level through a logit function, the above described logit formulation has to be executed once more to correct for the share of investment into hydro-

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electric and nuclear generated electric production and the subsector annual investments that are set exogenously. Thus, the annual subsector investments, *KAflow_j* for the electricty sector for hydro and nuclear and the annual sector investment $K_{j=8,,t}$ denoting the potential investment into the electricity sector have to be set (Equation 115), and the recalculated shares determine the remaining subsector investments, given that those shares equal one when no subsectors are active.

$$\text{KAflow}_{j=8,jj,t} = \text{ExoElec}_{jj} \qquad \text{for j=8 and jj=5 or 6} \qquad \text{Eq. 115}$$

The investment into electricity is updated with the exogenously set investments.

$$\text{K}_{j=8,t} = \text{K}_{j=8,t} + \sum_{jj=1}^{n} \text{ExoElec}_{jj} \qquad \text{Eq. 116}$$

where
  $n \leq jj$ and denotes the exogenously set investments, *e.g.*, nuclear and hydro power generating electricity.

Capital investment for all production sectors and subsectors besides the exogenously set investments is then re-calculated as:

$$\text{KAflow}_{j,jj,t} = \text{Share}_{j,jj,t} \bullet \text{K}_{j,t} \qquad \text{Eq. 117}$$

where
  $\text{Share}_{j,jj,t}$ are the calculated shares which equal one when there are no subsector production sectors, and
  $\text{K}_{j,t}$ is the potential investment into production sector j.

Finally, the actual annual investment in the form of capital flow is retained as total capital stock over the five-year time period as *KA_{j,jj,t}* for each production sector and subsector through linearly interpolated annual flows starting with the current period and going back to the previous model solution-period:

$$\text{KA}_{j,jj,t} = 2 \bullet \text{KAflow}_{j,jj,t-1} + 3 \bullet \text{KAflow}_{j,jj,t} \qquad \text{Eq. 118}$$

Investment in energy production

The investment in energy resources is more complicated. A set of triggers function as indicators for the oil, gas, and coal production sector implementation of resource-related investment functions.

At the start of the model run, fossil fuel resource and reserve[40] variables for a region are initialized:

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40 http://www.eere.energy.gov/: "Known resources can be classified from two standpoints: 1) purely geologic or physical/chemical characteristics – such as grade, tonnage, thickness, and depth – of material in place, and 2) profitability analysis based on costs of extracting and marketing the material in a given economy at a given time. The former constitutes important objective information of the resource and a relatively unchanging foundation upon which the latter economic delineation can be based". " For mineral resources, the reserve chosen for this index is the reserve

$$\text{DrsCtmp}_{j,jj} = \text{Drsce}_{j,jj} \qquad \text{Eq. 119}$$

$$\text{DrsVtmp}_{j,jj,t} = \text{Drsve}_{j,jj,t} \qquad \text{Eq. 120}$$

Over time, oil, gas and coal investments are turned into into available reserves (*Reserve_j* in energy units) that can be consumed; these are calculated as follows based on investments in monetary values (*Eqdep_j*):

$$\text{Re serve}_j = \text{PRconvrt}_j \bullet \text{Eqdep}_j \qquad \text{Eq. 121}$$

where

    v=t,

    jj equals one, given that no subector production is active for fossil fuel production,

    PRconvrt_j is the energy conversion factor, converting the investment costs in monetary units to physical energy units, and

    Eqdep_j is the investment in fossil fuel production, where

$$\text{Eqdep}_j = \text{xnp}_j \bullet \alpha_{0,j,jj,v} \bullet Z_{j,jj,v} \bullet \alpha_{i=26,j,jj,v}^{1/(1-\rho)} \bullet \text{KA}_{j,jj,t} \qquad \text{Eq. 122}$$

where

    xnp_j equals the product of the nameplate lifetime of the technology (4 in the reference case for oil, gas and coal production) and the timestep (five years for SGM 2003),

    $\alpha_{0,j,jj,v}$ equals one,

    $\alpha_{i,=26,j,jj,v}$ is the vintaged capital stock scale coefficient in which technical change is incorporated (see normalizalion in Equation 27 of Equation 25 in "The vintage alphas and incorporating technical change").

    $\text{KA}_{j,jj,v}$ is the active vintage-specific production sector or subsector capital, and

    $Z_{j,jj,v}$ incorporates the participation of the variable factors, where

$$Z_{j,jj,v} = 1 - (\alpha_{0,j,jj,v} \bullet \text{Pe}_{j,jj})^{\mu} \bullet \sum_{i=1}^{N-1} \left(\alpha \text{cst}_{i,j,jj,v} \bullet \text{Pie}_{i,j,jj}^{-\mu} \right)$$ which is calculated with

expected prices in the expected profit equation and implemented with long-run elasticities (see Equation 56),

    $\text{Pe}_{j,jj}$ is the expected commodity price (see Equation 16),

    $\text{Pie}_{i,j,jj}$ is the expected price paid (see Equation 15),

    ρ and μ are functions of the long-run elasticity $\sigma 1$,

    $\alpha \text{cst}_{i,j,jj,v}$ equals $(\alpha_{0,j,jj,v}{}^{\rho} \bullet \alpha_{i,j,jj,v})^{1/(1-\rho)}$, (see Equations 30-31 in the section "The vintage alphas and incorporating technical change").

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base as defined by the U.S. Bureau of Mines: 'part of the identified resource that meets specified minimum physical and chemical criteria related to current mining and production practices, including those for grade, quality thickness, and depth. The reserve base encompasses those parts of the resources that have a reasonable potential for becoming economically available within planning horizons beyond those that assume proven technology and current economics. It includes those resources that are currently economic (reserve), marginally economic (marginal reserves) and currently sub-economic". For fossil fuels (including uranium) the reserve chosen can be based on information supplied by the World Energy Council (WEC).

Note that the investments into available energy reserves are calculated with the CES production function with long-run elasticities σl. The values of $\alpha_{i,=26,j,jj,v}$ used in calculating the reserves cab be found in Appendix A. Also note that jj equals one in the reference case.

The quantities of uninvested depletable resources $Drsce_{j,jj}$ change over time, because of a combination of resource growth and investment into reserves, that is, into energy for consumption. Thus,

$$Drsce_{j,jj} = DrsCtmp_{j,jj} + Drsce_{j,jj} \bullet ((1 + Resgro_{j,jj}^{nt*Nstep}) - (1 + Resgro_{j,jj}^{(nt-1)*Nstep}))$$

Eq. 123

where

    $DrsCtmp_{j,jj}$ is an exogenous value in the base year and updated after reserves are calculated (see Equation 126),

    $Drsce_{j,jj}$ is the uninvested depletable resource for which the initial value is an input parameter value (Equation 119 for the initial value),

    $Resgro_{j,jj}$ is the calculated growth in the resource base for depletables (oil, gas, coal) as a time-dependent interpolation of initial and terminal values, as long as T, which is 5 times the number of time periods (Nstep*t), is smaller than $Tresgrz_i$.

$$Resgro_{j,jj} = Resgro_{j,jj} \bullet (1 - \frac{T}{Tresgrz_{j,jj}}) + Resgrz_{j,jj} \bullet \frac{T}{Tresgrz_{j,jj}}$$

Eq. 124

    where

        $Resgro_{j,jj}$, $Tresgrz_{j,jj}$, and $Resgrz_{j,jj}$ are input parameters.

Calculated reserves associated with desired investment are checked to see if they exceed available reserves.

$$DrsVtmp_{j,jj,t} = Reserve_j$$

Eq. 125

$$DrsCtmp_{j,jj} = DrsCtmp_{j,jj} - Reserve_j$$

Eq. 126

where

    $DrsCtmp_{j,jj}$ are the available energy resource at a point in time, which are updated during the iterations finding the model solution,

    $DrsVtmp_{j,jj,t}$ is the energy resource available at a point in time initiated with the energy resource data information (Equation 120 for the base year value) and calculated through resource investment calculations in the projections and updated through Equation 70.

Resources are decreased as long as the amount of production $Reserve_j$ is less than the resources available.

Note that the iterative model results for one region for each of the 12 time steps (1990 through 2050 in five-year time steps) are shown in "The solution procedure" for the resource reserves invested in to be consumed, as well as for the depletable oil, gas, and coal resources.

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The ratio of available reserves and desired reserves is calculated as

$$\mathrm{Pct\,Re\,serves} = \frac{\mathrm{DrsCtmp}_{j,jj}}{\mathrm{Re\,serve}_j} \qquad\qquad \text{Eq. 127}$$

Energy sources available for consumption are set equal to the calculated available reserves, which in turn are checked for depletion when vintages are operated. (See "Checks on depletable resources" after the calculations regarding operating old and new vintages.)

$$\mathrm{DrsVtmp}_{j,jj,t} = \mathrm{Re\,serve}_j \qquad\qquad \text{Eq. 128}$$

If the ratio of available reserves and desired reserves is smaller than one, investment is scaled down:

$$\mathrm{Eqdep}_{j,jj} = \mathrm{Pct\,Re\,serves} \bullet \mathrm{Eqdep}_{j.jj} \qquad\qquad \text{Eq. 129}$$

$$\mathrm{KAflow}_{j,jj,t} = \mathrm{Pct\,Re\,serves} \bullet \mathrm{KAflow}_{j.jj,t} \qquad\qquad \text{Eq. 130}$$

and the previously calculated investment capital over the last five years into the production sectors for oil, gas and oil is updated based of resource availability as follows:

$$\mathrm{KA}_{j,jj,v=t} = \mathrm{Pct\,Re\,serves} \bullet \mathrm{KA}_{j.jj,v=t} \qquad\qquad \text{Eq. 131}$$

and the investment into reserves is set to the maximum available:

$$\mathrm{Re\,serve}_j = \mathrm{DrsCtmp}_{j.jj} \qquad\qquad \text{Eq. 132}$$

Figure 10 illustrates investment in production sectors over time in the USA. The highest level of investment is in the Everything Else sector; coke production shows the lowest level of investment. Note that the vertical axis is logarithmic-transformed. The horizontal axis shows the time line; for 1990 $t=0$ and for 2045 $t=11$.

Investments into Production Sectors other than Energy in the Reference Case
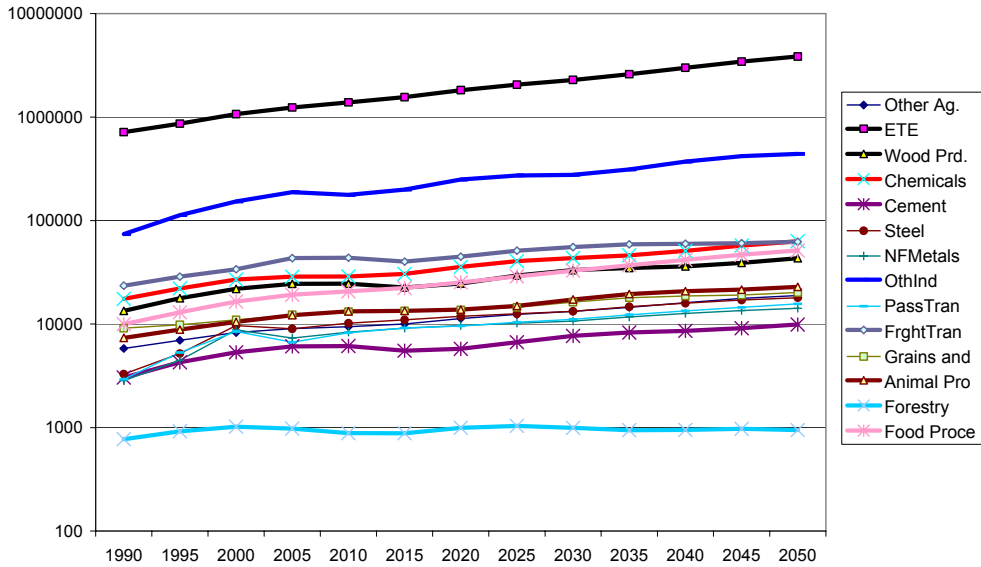
Figure 10 Investment into production sectors over time

Investments in fossil fuel production and electricity are depicted in more detail in Figures 11 and 12. The vertical axes are not log-transformed in these figures. Results are specific with regard to the input values used (carbon fees, emission limits, technical change) and assumptions made with regard to changes in population and labor productivity.

Investments in Fossil Fuel Related Production in the Reference Case
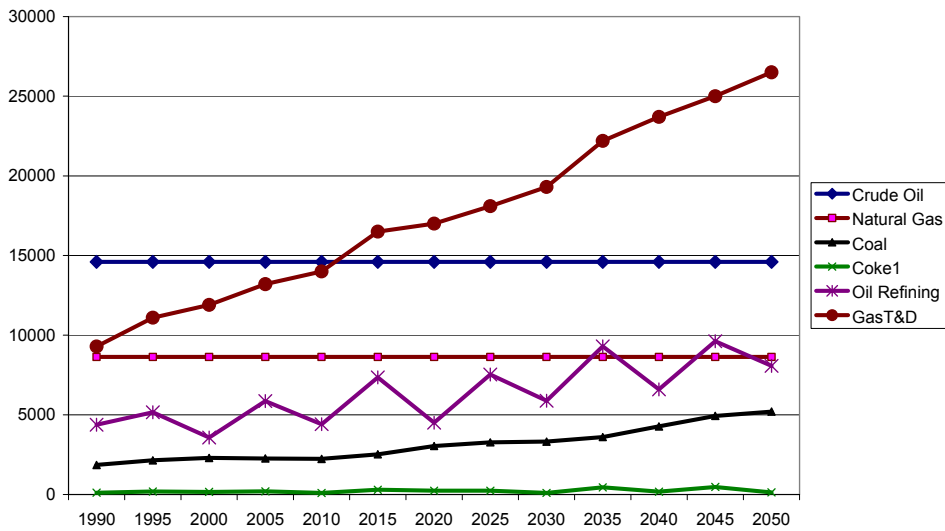
Figure 11 Investments in fossil fuel related production over time

Investments in Electricity Production in the Reference Case
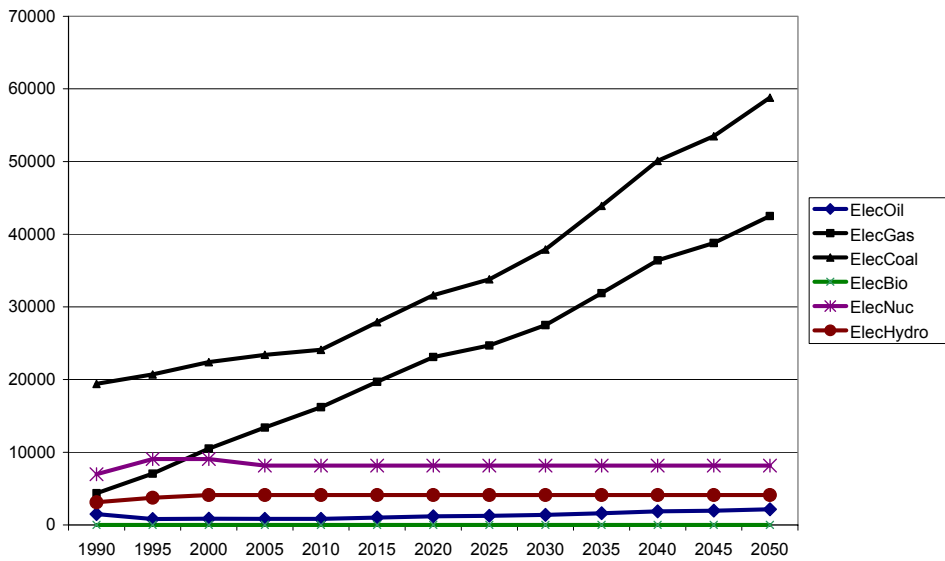
Figure 12 Investment in electricity production over time

Demand for capital

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As part of the iterative model solution, where annual capital flow is the result of the investment equations described above, demand for annual capital $Kdem_{j,jj}$ by each of the production sectors and subsectors is set equal to the annual capital flow $KAflow_{j,jj,t}$:

$$\text{Kdem}_{j,jj} = \text{KAflow}_{j,jj,t} \qquad\qquad \text{Eq. 133}$$

Intermediate capital goods ($i=1{:}22$) are produced as $ED_{i,j=inv}$ using a fixed coefficient or Leontief production function, whose coefficients are the $CapMat_{i,j}$ matrix whose row values i are the *ShareInv* coefficients which are inputs (see Appendix A).

$$\text{ED}_{i,j=25=inv} = \sum_{jj=1}^{n}\left[\sum_{j=1}^{22}\text{Kdem}_{j,jj} \bullet \text{CapMat}_{i,j}\right] \qquad\qquad \text{Eq. 134}$$

Given that $Kdem_{j,jj}$ is the capital demanded by a production sector j or subsector jj and $CapMat_{i,j}$, are the fixed coefficients of a fixed coeficient production function, the demand equation for capital (Equation 134) can be mapped onto the Leontief demand equation (see Equation 54) where demands are described as $\text{ED}_{i,j} = \sum_{jj=1}^{n}\dfrac{\lambda_{i,j,jj}}{\lambda_{i=26,j,jj}} \bullet \text{KA}_{j,jj}$. The quotient of the scale factors of the inputs to production and capital $\lambda_{i,j,jj}/\lambda_{i=26,j,jj}$ is equivalent to the elements of the $CapMat_{i,j}$ vector for each of the inputs, and $KA_{j,jj}$ is equivalent to the (demanded) investment capital $Kdem_{j,jj}$.

Total (summed over all production sectors) capital investment ($i=26$) demand is the summed demand for all capital inputs multiplied by the relevant supply prices paid:

$$\text{ED}_{i=26,j=25=inv} = \sum_{i=1}^{22}\left[\sum_{jj=1}^{n}\left[\sum_{j=1}^{22}\text{Kdem}_{j,jj} \bullet \text{CapMat}_{i=26,j} \bullet \text{Pi}_{i,j,jj,t}\right]\right] \qquad\qquad \text{Eq. 135}$$

Total investment demand needed for GNP accounting is:

$$\text{Ktot}_{t} = \sum_{i=1}^{22}\left[\sum_{jj=1}^{n}\left[\sum_{j=1}^{22}\text{Kdem}_{j,jj} \bullet \text{CapMat}_{i,j}\right]\right] \qquad\qquad \text{Eq. 136}$$

Additional costs towards demands

Transportation demands and distribution costs, $WHSL_i$ and $TRNP_i$, are added to the final Everthing Else demands:

$$\text{ED}_{i=ETE,j} = \text{ED}_{i=ETE,j} + \sum_{i=1}^{22}\text{WHSL}_{i} + \sum_{i=1}^{22}\text{TRNP}_{i} \qquad\qquad \text{Eq. 137}$$

where

WHSL$_i$ is the summed over all adjustment factors to the supply sectors calculated when operating production sector vintages (see Equation 82) and when household consumption is calculated for j=27 (Equation 193 for j=27, v=t, and n=1):

$$\text{WHSL}_i = \sum_{j=1}^{22} \left[\sum_{v=t-3}^{t} \left[\sum_{jj=1}^{n} \left[\frac{\left(\text{adj}_{i,j,jj,t} - 1\right) \bullet \left[\left(P_{i,t} + Tr_{i,t}\right) \bullet \text{Ex Im}_i \bullet \text{Txpro}_{i,j,jj} + \text{Txadd}_{i,j,jj}\right] \bullet}{\text{EDV}_{i,j,jj,v}} \Big/ Pi_{i=\text{ETE},j,jj,t} \right] \right] \right] +$$

$$\left[\frac{\left(\text{adj}_{i,j=27=hh,jj=1,t} - 1\right) \bullet \left[\left(P_{i,t} + Tr_{i,t}\right) \bullet \text{Ex Im}_i \bullet \text{Txpro}_{i,j=27=hh,jj=1} + \text{Txadd}_{i,j=27=hh,jj=1}\right] \bullet}{\text{ED}_{i,j=27=hh}} \Big/ Pi_{i=\text{ETE},j=27=hh,jj=1,t} \right]$$

Eq. 138

where

    v refers to all actively operating vintages (v=t-3:t-1 for old vintages and v=t for new vintages),
    n is the number of subsectors with n ≤ jj,
    j is the number of production sectors,
    i refers to input sectors,
    adj$_{i,j,jj,t}$, is a time-period-dependent adjustment factor that reflects markups and intra-regional transport costs,
    $P_{i,t}$ is the market price,
    $Tr_{i,t}$ is a time-period-dependent supply sector transportation cost factor,
    ExIm$_i$ is the transportation cost rate parameter,
    Txpro$_{i,j,jj}$ is a proportional tax rate on the ith product,
    Txadd$_{i,j,jj}$ is an additive tax on the ith product,
    EDV$_{i,j,jj,v}$ denotes demand by a vintaged production sector or subsector, and
    $Pi_{i=\text{ETE},j,jj,t}$ is the price paid by the producer for Everything Else input.

and

TRNP$_i$ is the summed over all supply sector transportation costs calculated when old vintages are operating (see Equation 83) and when household consumption is calculated for *j=27* (Equation 194 for *j=27*, *v=t*, and *n=1*):

$$\text{TRNP}_i = \sum_{j=1}^{22} \left[\sum_{v=t-3}^{t} \left[\sum_{jj=1}^{n} \left(Tr_{i,t} \bullet \text{Ex Im}_i \bullet \frac{\text{EDV}_{i,j,jj,v}}{Pi_{i=\text{ETE},j,jj,t}} \right) \right] \right] +$$

$$\left(Tr_{i,t} \bullet \text{Ex Im}_i \bullet \frac{\text{ED}_{i,j=27=hh}}{Pi_{i=\text{ETE},j=27=hh,jj,t}} \right)$$

Eq. 139

Final carbon permit fees, *CPermF$_{5,j}$,* are a function of the energy demands and the carbon fees per dollar of energy use, which are a function of the global warming potential and the emission coefficient of the fuel combusted and demanded by investments (see Equation 10 for the calculation of *Cpf$_i$*). Thus,

$$\text{CPermF}_{5,\,j=25=\text{inv}} = \sum_{i=1}^{22} \left(\text{Cpf}_i \bullet \text{ED}_{i,\,j=25=\text{inv}} \right)$$

<div align="right">Eq. 140</div>

Households

A great many operations are carried out in the model's household sector. The household sector concerns
- Labor supply
- Land supply
- Household savings
- Household demand for final products.

Demographic characteristics are the basis for household behavior as a representative consumer with regard to labor supply, land supply, savings, and demand for final products. Household income is derived after adjusting for taxes and government transfers. Income is then split between savings and expenditure, with a savings function that depends on the interest rate. Labor supply, land supply, household income and savings, and household demand for final products are described below. In the subsequent discussion, personal income in the household sector is the regional population's total income, and households are the total regional population. The household demands include demands for land and labor, and household consumption.

Labor supply

Labor supply depends on regional demographics. The SGM tracks population within each region by gender and five-year age cohort. Population data may be read in directly, using projections from either the World Bank or the United Nations for the model base year and for all future SGM time steps. This is the usual way of specifying population in the SGM (see Appendix A for data used).

Alternatively, a demographics module that uses an age-cohort method of calculating population by age and gender for each region can be incorporated. The required input variables are fertility, migration, and mortality rates. The demographics module keeps track of population within each region by gender and five-year age cohorts. Base year populations may evolve by applying assumptions with regard to survival rates, fertility rates, and migration rates.

For any time period, total population (of a region) is given by

$$\text{POP}_{\text{tot}} = \sum_{\text{age}=1}^{\text{NAGE}} \left(\text{POP}_{\text{age,males}} + \text{POP}_{\text{age,females}} \right)$$

<div align="right">Eq. 141</div>

where
  $\text{POP}_{\text{age,,males}}$ is the number of males in age group age,
  $\text{POP}_{\text{age, females}}$ is the number of females in age group age, and
  NAGE is the number of age groups defined, which equals 16.

The fraction of the population that is either over 15 years old or under 65 is calculated to obtain the fraction of the population potentially in the labor force as wage earners. Thus,

$$working\_age\_pop_{tot} = \sum_{age=4}^{age=13}\left(POP_{age,males} + POP_{age,females}\right)$$

Eq. 142

or

$$working\_age\_pop_{ng} = \sum_{age=4}^{age=13}\left(POP_{age,ng}\right)$$

Eq. 143

where
   the male and female population is summed between ages 16 and 65, that is age groups 4 through 13, and
   ng is either the male or female population.

Labor supply is gender-specific, which is denoted by the subscript ng. Labor supply for the projections is calculated in the SGM using the following equation:

$$ED_{i=25=lbs, j=27=hh} = -1 \bullet \sum_{ng} working\_age\_pop_{ng} \bullet LB_{0,ng} \bullet [1 - \exp(LB_{1,ng} \bullet \frac{P_{i=25=lbs,t}}{Pi_{i=ETE, j=27, jj, t}})]$$

Eq. 144

where
   when j equals 27, representing the household sector, jj equals one by definition,
   ng is either male or female,
   $LB_{0,ng}$ is the maximum potential share of the gender-specific working age population employed in any given year,
   working_age_pop$_{ng}$ is the gender-specific working age population (ages 15 to 64) (calculated from the age-specific numbers of people, (see Equations 145 and 146 or read in), and

$$\left[1 - \exp(LB_{1,ng} \bullet \frac{P_{i=25=lbs,t}}{Pi_{i=ETE, j=27, jj, t}})\right] \quad \text{denotes the labor supply responsiveness,}$$

   where
      $P_{i=25,t}$ is the average annual wage rate (calculated initially from the input-output table by

dividing total labor supplied ($P_{i=25=lbs,t} = \dfrac{XS_{i=25=lbs}}{ED_{i=25=lbs, j=27=hh}}$ see Equation 1 and 7) and equals the

market price wage rate during the model solution process, and
      $Pi_{i=ETE, j=27, jj, t}$ denotes the price of the numeraire sector.

The labor supply function parameters *LB* are determined as follows: the maximum potential share of the gender-specific population in the labor force for the base year *LB$_{0,ng}$* is either fixed by an exogenous parameter or calculated from data:

$$LB_{0,ng} = \frac{working\_age\_pop_{ng}}{POP_{tot}}$$

Eq. 145

$LB_{1,ng}$ is a labor supply function coefficient in the labor supply responsiveness equation, which can be formulated as follows and although gender-specific is equal in this case for both male and female:

$$LB_{1,ng} = \frac{\log\left(1 - \frac{ED_{i=25=lbs, j=27=hh}}{\left(working\_age\_pop_{tot}\right) \bullet LB_{0,ng}}\right)}{P_{i=25=lbs, t}}$$ Eq. 146

where

$Ed_{i=25=lbs,j=27=hh}$ is the total demand for labor supply (see also Equation 152 below), which is set exogenously for the base year, and calculated for projections as in Equation 147.

Alternatively, to make calibration to a prescribed path simpler, the labor supply function coefficient $LB_{1,ng}$ can be calculated as a ratio of the labor supply and the working-age population. Under this alternative labor supply elasticity is in effect eliminated.

$$LB_{1,ng} = \frac{ED_{i=25=lbs. j=27=hh}}{working\_age\_pop_{tot}}$$ Eq. 147

where

$ED_{i=25,j=27}$ is the total labor supply to be provided by households and is determined by the wage rate (see Equation 144).

Labor produced, $SalesV_{i=25=lbs}$, is set equal to the negative value of labor supply demand such that the market calculations can be performed when solving the model for supply and demand

$$SalesV_{i=25=lbs} = -1 \bullet ED_{i=25=lbs, j=27=hh}$$ Eq. 148

The fraction of labor demand that is the demand for household's own labor $LB_{2,2}$ is calculated as follows:

$$LB_{2,2} = \frac{X_{i=25=lbs, j=27=hh}}{ED_{i=25=lbs, j=27=hh} \bullet P_{i=25=lbs, t}}$$ Eq. 149

where

$X_{i=25,j=27}$ is labor demand by households in the base year,

$Ed_{i=25,j=27}$ is total labor supplied by households, and

$P_{i=25,t}$ is the market price of the average annual wage rate during the model solution process, and $Ed_{i=25,j=27*} P_{i=25,t}$ is total payment for labor.

The number of households, *Nhh,* in a regional population can be calculated based on the average number of people per household, pp_{hh}, as input, in the following equation:

$$Nhh = \frac{POP_{tot}}{pp_{hh}}$$ Eq. 150

where
    Nhh is the number of households,
    POP_{tot} is the total population in a region, and
    pp_{hh} is the average number of persons per household.

Alternatively, if the number of households, *Nhh,* are known, the number of people per household, *LB$_{2,1}$,* can be calculated:

$$LB_{2,1} = \frac{POP_{tot}}{Nhh}$$
        Eq. 151

Note that next the *LB$_{1,1}$*, *LB$_{1,2}$*, *LB$_{2,1}$*, and *LB$_{2,2}$* values are set constant over time, initially (after the base year calculations), but are then recalculated when projections are made.

Land (agricultural land) supply

Households supply land on the basis of a land rental rate, where the land supply denotes the total land area that could be managed for agricultural or other purposes. SGM 2003 has one land type.[41] Demand for land supply in the base year is regional input data (see Appendix A for data examples) and is calculated for projections in a parallel fashion to the demand for supply of household labor:

$$ED_{i=24=ls,\,j=27=hh} = -1 \bullet TLA \bullet R_0 \bullet [1 - \exp(R_1 \bullet \frac{P_{i=24=ls,t}}{Pi_{i=ETE,\,j=27=hh,\,jj,t}})]$$
        Eq. 152

where
    TLA is the total land area that could be managed for agricultural or other purposes,
    R_0 is the maximum potential share of land supplied to the market for the base year, and

$$\left[1 - \exp(R_1 \bullet \frac{P_{i=24=ls,t}}{Pi_{i=ETE,\,j=27=hh,\,jj,t}}) \right]$$ denotes the land supply responsiveness,

    where
        $P_{i=24,t}$ is the average annual rental rate of a unit of land, initially calculated from the input-output table information ($P_{i=24=ls,t} = \frac{XS_{i=24=ls}}{ED_{i=24=ls,\,j=27=hh}}$; see Equations 1 and 6), and equals the

    market price land rental during the model solution process, and
        $Pi_{i=ETE,j=27=hh,jj,t}$ is the price of the numeraire sector.

The land supply function coefficients *R* are determined as follows:
The maximum usable land, expressed as a fraction of land in a region *R$_0$* is input data for the base year (see Appendix A for values).

R_1 is a land supply function coefficient in the land supply responsiveness equation.

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41 The land supply coefficients *R* have, therefore, no *j* subscripts

$$R_1 = \frac{\log\left(1 - \frac{ED_{i=24=ls, j=27=hh}}{TLA \Big/ R_0}\right)}{P_{i=24=ls, t}}$$

Eq. 153

where

TLA is total regional available land area,

R_0 is the maximum useable land for the region, which is input data for the base, and

$P_{i=24, t}$ is the market price of the land rent during the model solution process,

$ED_{i=24=ls, j=27=hh}$ represents the demand for land use or land supply, which for the base year is regional input data, and is calculated for projections as described above (Equation 152).

Produced availability of land, $SalesV_{i=24=ls}$**,** is set equal to the negative value of the demand for land supply such that the market calculations can be performed when solving the model for supply and demand.

$$SalesV_{i=24=ls} = -1 \bullet ED_{i=24=ls, j=27=hh}$$

Eq. 154

When land rental income is calculated, the relationship, R_2, between land demanded and land rental price is important; R_2 is calculated as follows from the initial elements of the input-output matrix:

$$R_2 = \frac{X_{i=24=ls, j=27=hh}}{Nhh \bullet P_{i=24=ls, t}}$$

Eq. 155

where

$X_{i=24, j=27}$ is the land demanded by households in the base year,

Nhh is the number of households at time t, and

$P_{i=24, t}$ is the market price of land rent during the model solution process.

Note that the R_0, R_1, and R_2 values are set constant over time, initially (after the base year calculations), but are then recalculated when projections are made.

Personal income or household income

Personal income *Pinc* (Equation 156 and updated in Equations 171-175 and 188) equals the sum of retained income *HHinc1* (Equations 157-160), labor income *HHinc2* (Equation 166) and land rental income *HHinc3* (Equation 168) minus personal income tax (calculated by means of the tax rate coefficient *PItr*) to which government transfers *GovTr* (Equation 170) are added and personal savings (Equation 179) are subtracted (Equation 180). Personal income calculated thus, is saved in a household consumption variable *Cons_t* (Equation 181) to be part of GNP balancing at time *t* (Equation 236).

Personal income, government transfers and savings are calculated sequentially, starting with personal income based on retained income *HHinc1*, labor income *HHinc2*, land rental income *HHinc3* and taxation on personal income *PItr* (Equation 157). Retained income *HHinc1* may be impacted by the recycling of carbon revenues, depending on if and which carbon policy is implemented (Equations 171-175). Next, government transfers are calculated based on this

income, but without taxation (Equation 170). Depending on if and which carbon fee policies are implemented, household income is updated based on recycling of carbon revenues. Household savings *HHinc8* are calculated based on taxed income; they are in effect demand for capital by households (Equation 176) and dependent on the capital interest rate $P_{i=26}$. Savings are subtracted from income. After these updates, household consumption is set equal to household income. Household income is once more updated in Equation 188 by labor (*HHinc9)* and land (*HHinc10)* expenditures (see Equations 189 and 190). A budget constraint calculation based on that updated household income (Equation 191) limits all household demands $ED_{i,j}$ before a model solution can be sought.

Thus, the first step in calculating household income is as follows:

$$\text{Pinc} = (\text{HHinc}_1 + \text{HHinc}_2 + \text{HHinc}_3) \bullet (1 - \text{PItr}) + \text{GovTr}_t \qquad \text{Eq. 156}$$

where
    PItr is the personal income tax rate,
    HHinc1 is retained household income,
    HHinc2 is labor income,
    HHinc3 is land rental income, and
    GovTr$_t$ are government transfers in the base year,
which are all explained below.

HHinc₁ is the **retained household income**, which in the base year (see Equation 157) is calculated from the production sector- and subsector-specific retained household earnings parameters $RE_{0,j}$ and the demands by the production sectors and subsectors for capital $X_{i=26,j,jj}$, restrained by taxes *Citr* and *Xitc$_j$* and the market interest rate $P_{i=26}$. In the projections retained income is based on profits in the production sectors (see Equation 160).

For the base year:

$$\text{HHinc}_1 = \sum_{j=1}^{22} \left[\left(\sum_{jj=1}^{6} X_{i=26,j,jj} \right) \bullet \left((1 - \text{Citr}) + \text{Xitc}_j \right) \bullet \left(1 - \left(RE_{0,j} \bullet (1 - \exp(RE_{1,j} \bullet P_{i=26,t})) \right) \right) \right]$$

$$\text{Eq. 157}$$

where for each production sector
    $\sum X_{i=26,j,jj}$ is the summed subsector demand for capital by the production sector in the base year,
    Citr is the corporate income tax rate (see Appendix A for values),
    Xitc$_j$ is the production sector and/or subsector tax credit rate or dividend, which can be
initiated by setting a switch and which impact is dependent on demand. Tax credits are not activated in the reference case. The equation is shown for potential implementation, replacement or elaboration:

$$\text{Xitc}_j = \sum_{jj=1}^{n} \left[\text{XITCrate}_{j,jj} \bullet \text{Kdem}_{j,jj} \right] \qquad \text{Eq. 158}$$

where
        XITCrate$_{is}$ the investment tax credit rate (set to zero), and

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Kdem$_{j,jj}$ is the sector and subsector specific demand (see Equation 133 when final capital demand for each production sector and subsector has been determined and Equation 71 under "Checks on depletable resources,")

P$_{i=26,t}$ is the capital cost interest rate,

RE$_{0,j}$ is the maximum retained earnings for the base year, which is a production sector-specific input parameter,

RE$_{1,j}$ is the production sector-specific retained household earnings parameter, which is (for the base year 1990) calibrated against input data (through the Total corporate Retained Earning parameter, TREte).

$$RE_{1,j} = \frac{\log\left[1 - \dfrac{TREte}{\sum\limits_{j} X_{i=26=capital,j} \bullet RE_{0,j} \bullet (1 - Citr)}\right]}{P_{i=26}}$$ Eq. 159

where

TREte is the total corporate retained earnings, which is read-in for calibration in the base year and calculated for the projections (see Equations 161-164 below),

X$_{i=26,j}$ denotes the demand for capital by each production sector, and demands by investment, households, government and trade in the base year.

The *RE$_{0,j}$* and *RE$_{1,j}$* values are set constant over time after the base year calculations but are then recalculated when projections are made.

For projections, retained income *HHinc$_1$* is based on profits in the production sectors:

$$HHinc_1 = \sum_{j=1}^{22}\left[\left(\sum_{jj=1}^{6}\pi_{j,jj}\right) \bullet \left((1 - Citr) + Xitc_j\right) \bullet \left(1 - \left(RE_{0,j} \bullet (1 - \exp(RE_{1,j} * P_{i=26,t}))\right)\right)\right]$$ Eq. 160

where

$\pi_{j,jj}$ is the production sector or subsector profit summed over the actively operating vintages:

$$\pi_{j,jj} = \sum_{v=t-3}^{t-1} PctDeplete \bullet \pi O_{j,jj,v} + \pi N_{j,jj,v}$$

where

PctDeplete is calculated as not equal to one (see Equation 67) if depletable resources have reached a depletion level when old vintages are operated,

$\pi O_{j,jj,v}$ is the profit from operating old vintage production sectors or subsectors (see Equations 57 and 58),

$\pi N_{j,jj,v}$ is the profit from operating new vintage production sectors or subsectors (see Equation 72),

and

$RE_{1,j}$ are the production sector- and subsector-specific retained household earnings, which are recalculated through Equation 159 with a recalculated total corporate retained earnings (TREte) where

$$TREte = \sum_{j=1}^{22} \left[REte_j \right]$$
Eq. 161

where

$$REte_j = \left[\left(\sum_{jj=1}^{6} \pi_{j,jj} \right) \bullet (1 - Citr) \bullet Xitc_j \right] \bullet \left(RE_{0,j} \bullet (1 - \exp(RE_{1,j} \bullet P_{i=26,t})) \right)$$
Eq. 162

Retained income *HHinc1* might be impacted by the recycling of carbon revenues, depending on if and which carbon policy is implemented (Equations 157-160).

If carbon pricing is simulated, corporate earnings have to be adjusted in accordance with the carbon policy implemented. Among the options explored are the impacts of carbon policies on corporate earnings, *e.g.*, carbon revenue recycling of 60% towards industry and 40% to households (*ICRBfeeOPT=4*) or all are recycled to industry (*ICRBfeeOPT=6*).

Case 4: 60% of the domestic carbon fees are recycled towards industry and 40% to households.

$$TREte = TREte + 0.6 \bullet CrbFeeTot_t + 0.6 \bullet ExIBT_t$$
Eq. 163

Case 6: all of the carbon fees are recycled to industry if the $CrbFeeRcyPct_3$ is on.

$$TREte = TREte + CrbFeeRcyPct_3 \bullet CrbFeeTot_t + CrbFeeRcyPct_3 \bullet ExIBT_t$$
Eq. 164

where
  $CrbFeeRcyPct_3$ refers to a fraction.
  CrbFeeTot is based on domestic carbon fees and carbon permit trade (see Equation 101),
  ExIBT is the amount of IBT tax collected above the base year, which remains independent of any carbon fee recycling. No increases in IBT taxes occur if TxIBT is constant over time.

$$ExIBT_t = \sum_{j=1}^{22} \left(SalesP_j \bullet (TxIBT_{j,t} - TxIBT_{j,t=0}) \right) \text{ (see Equation 209) and}$$

where
  $TxIBT_j$ is the production sector-specific indirect business tax, and
  $SalesP_j$ is gross production summed over all active vintages and production subsectors in

monetary terms: $SalesP_j = \sum_{v=t-3}^{t} \left[\sum_{jj=1}^{n} \left(PRDV_{j,jj,v} \bullet Pr_{j,t} \right) \right]$ (see Equation 208).

The **tax on profits** produced in the production sectors *Tax₃,ⱼ₌₁:₂₂* is kept track of in a tax array as follows:

$$Tax_{3,j=1:22} = Citr \bullet \pi_j - Xitc_j$$
Eq. 165

where

Xitc$_j$ is production sector and/or sub sector tax credit rate or dividend (see Equation 158), and
Citr is the corporate income tax rate, and
π_j are the production sector profits after summing over the operating vintages and subsectors.

HHinc$_2$ is **labor income,** which is estimated based on demand for labor supply, social security taxes and the market price for labor:

$$\text{HHinc}_2 = (1 - \text{SStr}) \bullet -1 \bullet \text{ED}_{i=25=lbs, j=27=hh} \bullet P_{i=25=lbs, t} \qquad \text{Eq. 166}$$

where
SStr is the social security tax rate, which is a regional input parameter for the base year,
ED$_{i=25,j=27}$ is the labor supply demanded, which is described above, including its changes when projections are calculated, and
P$_{i=25,t}$ is the market price of labor rental.

The **social security tax** is kept track of in a tax array as *Tax$_{2,j=27}$* and is also called *HHinc$_5$*

$$\text{Tax}_{2, j=27=hh} = \left[\text{SStr} \bullet -1 \bullet \text{ED}_{i=25=lbs, j=27=hh} \bullet P_{i=25=lbs, t} \right] \qquad \text{Eq. 167}$$

HHinc$_3$ is **land rent income**, which is calculated based on demand for land supply *Ed$_{i=24,j=27}$* (Equation 152) and the market price of land rental *P$_{i=24=ls}$*. Land supply may change over time through the land-use modifying parameter *R$_2$* (see Equation 155) as do the number of households *Nhh* (Equation 150). Note that SGM 2003 has one land type.

$$\text{HHinc}_3 = -1 \bullet \text{ED}_{i=24=ls, j=27=hh} \bullet P_{i=24=ls, t} \qquad \text{Eq. 168}$$

where
Ed$_{i=24,j=27}$ is the demand for land supply, which is described above (Equation 152), including its changes when projections are calculated,
P$_{i=24,t}$ is the market price for land rental during the model solution process,
Nhh is the number of households, which may also change over time, and
R$_2$ is a parameter that is based on land demanded by households and land rental price (see Equation 155), which determines land rent income.

The **personal income tax** *Tax$_{3,j=27}$* is also called *HHinc$_4$*.

$$\text{Tax}_{3, j=27=hh} = (\text{HHinc1} + \text{HHinc2} + \text{HHinc3}) \bullet \text{Pitr} \qquad \text{Eq. 169}$$

where
PItr is the personal income tax rate.

GovTr$_t$ are **government transfers** in Equation 156, which are either input parameters (see Appendix A for base year values) or calculated. *GovTr* are also called *HHinc$_6$*.

When government transfers are calculated for the base year (for *t=0*) and for projections, they are calculated based on per capita income *Pinc* (Equation 156), percent of population in young and

old age groups *YngOld*, and the fraction of the population working *working_age_pop*. Government transfers are assumed to be untaxed.

$$GovTr_t = P_{i=ETE,t} \bullet \left(\begin{array}{l} POP_{tot} \bullet GovTr_0 \bullet \left(\dfrac{(HHinc_1 + HHinc_2 + HHinc_3)}{POP_{tot}} \right)^{g1} \bullet \\ \left(\dfrac{-1 \bullet ED_{i=25=lbs, j=27=hh}}{working\_age\_pop} \right)^{g2} \bullet YngOld^{g3} \end{array} \right) \qquad \text{Eq. 170}$$

where
the exponents g1, g2, and g2 equal zero in SGM 2003, and
YngOld denotes the non-employable population, that is the population under age 15 and over age 65, and the other parameters are described above.

Depending on if and which carbon fee policies are implemented, household income is updated based on recycling of carbon revenues. If carbon pricing is implemented household income has to be adjusted in accordance with the carbon policy. *CrbFeeTot_t* represents the carbon fee at time *t*. The different carbon policy case impacts on household income (*ICRBfeeOPT=case*) are shown below:

Case 3: all carbon fees are returned to household income.

$$Pinc = Pinc + CrbFeeTot_t \qquad \text{Eq. 171}$$

where
$CrbFeeTot_t$ is based on domestic carbon fees and carbon permit trade,

Case 4: 40% of the carbon fees are returned to household and 60% to industry.

$$Pinc = Pinc + 0.4 \bullet CrbFeeTot_t \qquad \text{Eq. 172}$$

Case 5: domestic fees are returned to consumers, permits are returned to the deficit
  (a) if carbon trading is implemented and carbon trade provides for revenue households will receive the difference between domestic carbon fees and the carbon trading fees.

$$Pinc = Pinc + CrbFeeTot_t - CrbTrade_t \qquad \text{Eq. 173}$$

where
$CrbFeeTot_t$ is based on domestic carbon fees and carbon permit trade,
$CarbTrade_t$ refers to the carbon permit trade.

  (b) If carbon trading is a net loss, households receive the domestic carbon fees.

$$Pinc = Pinc + CrbFeeTot_t \qquad \text{Eq. 174}$$

Case 6: all of the carbon fees, determined by *CrbFeeRcyPct_2*, are recycled to households.

$$Pinc = Pinc + CrbFeeRcyPct_2 \bullet CrbFeeTot_t \qquad \text{Eq. 175}$$

where

CrbFeeRcyPct$_2$ refers to a fraction.

Savings supply

The savings supply or capital supply requires the determination of personal income *Pinc*, which includes the income from (a) providing labor services, (b) land rental, (c) retained corporate and personal earnings, (d) government transfers, and (e) possibly, recycled carbon revenue. Household saving supply for the base year is an input parameter, and changes over time are determined by using a simple exponential model. Thus,

$$ED_{i=26,j=27=hh} = -1 \bullet Pinc \bullet S_0 \bullet (1 - S_1 \bullet \exp(S_2 \bullet P_{i=26}))$$ Eq. 176

where
    Pinc is personal income (see Equation 156),
    S_0 is the household savings function coefficient in the base year, and
    $\left[1 - S_1 \bullet \exp(S_2) \bullet P_{i=26,t}\right]$ denotes the saving supply responsiveness, where
    S_1 and S_2 are described below, and
    $P_{i=26,t}$ is the capital cost interest rate and equals the market price during the model solution process.

The **savings function parameters** are determined as follows:

S_0 = the household function coefficient in the base year
$S_1 = 1$ Eq. 177
and

$$S_2 = \frac{\log\left(1 - \dfrac{ED_{i=26=capital.j=27=hh}}{Pinc \bullet S_0}\right)}{P_{i=26,t}}$$ Eq. 178

where
    $ED_{i=26,j=27}$ is the household saving supply (Equation 176),
    Pinc is the taxed personal income to which government transfers are added (Equation 156),
    S_0 is the household savings function coefficient in the base year, and
    $P_{i=26,t}$ is the capital cost interest rate which equals the market price during the model solution process.

Note that the S_0, S_1, and S_2 values are set constant over time, initially (after the base year calculations), but are then recalculated when projections are made.

Household *Saving* (=*HHinc$_8$*) is set equal to (negative) household savings.

$$Saving = -1 \bullet ED_{i=26=capital, j=27=hh}$$ Eq. 179

where
    $ED_{i=26,j=27}$ is the household savings supply (see Equation 176).

Personal income is updated next for savings expenditures. After this update, the variable represents consumption expenditures.

$$\text{Pinc} = \text{Pinc} - \text{Saving}$$

Eq. 180

Household consumption, *Cons_t*, particpating in the GNP accounting as illustrated in Equation 236 below, thus accounts for personal (household) income that can be used for commodity consumption including land and labor expenditures, given that they are not subtracted as of yet (see Equations 188-190 below).

$$\text{Cons}_t = \text{Pinc}$$

Eq. 181

To obtain the total demanded savings supply, the corporate retained earnings are added.

$$\text{ED}_{i=26=\text{capital},\,j=27=\text{hh}} = \text{ED}_{i=26=\text{capital},\,j=27=\text{hh}} - \text{TREte}$$

Eq. 182

Production of capital by households *SalesV_{i=26}* is set equal to the updated produced savings supply.

$$\text{SalesV}_{i=26} = -1 \bullet \text{ED}_{i=26=\text{capital},\,j=27=\text{hh}}$$

Eq. 183

Total household demand for variable factor inputs

Household demands will be constraint by prices paid for the household variable factor inputs (i=1:22) at time t and by household income after accounting for (subtracting) household expenditures for labor and land (Equation 188). The total costs of meeting these variable-factor household demands are therefore calculated next:

$$\text{DemTot}_{j=27=\text{hh}} = \sum_{i=1}^{22} \left[\text{ED}_{i,\,j=27=\text{hh}} \bullet \text{Pi}_{i,\,j=27=\text{hh},\,jj,\,t} \right]$$

Eq. 184

These summed household demands as calculated in Equation 184 are to be used to scale the demands for inputs by households (see the budget restraints simulation expressed in Equation 191).

Household demand projections

Each consumption good (each i) in household consumption (demand) is assumed to have an income and a price elasticity of demand[42]:

$$\text{ED}_{i=1:22,\,j=27=\text{hh}} = \text{E}_{0,i} \bullet \left(\frac{\text{Pinc}}{\text{Pi}_{i=\text{ETE},\,j=27=\text{hh},\,jj,\,t}} \right)^{\varepsilon_{1,i}} \bullet \left(\frac{\text{Pi}_{i,\,j=27=\text{hh},\,jj,\,t}}{\text{Pi}_{i=\text{ETE},\,j=27=\text{hh},\,jj,\,t}} \right)^{\varepsilon_{2,i}}$$

Eq. 185

where

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42 The constant elasticity equation is found in Edmonds and Reilly (1985:248-249).

Pinc refers to the updated, by savings expenditures, personal income (see Equation 180),

$E_{0,i}$ is a scaler for household demand for supplies (i) which incorporates income and price elasticities:

$$E_{0,i} = \left(\frac{X_{i,j=27=hh}}{Pinc} \right)^{\varepsilon_{1,i}} \bullet P_{i,t}^{\varepsilon_{2,i}}$$

Eq. 186

where

$X_{i,j=27=hh}$ is the household demand for supplies in the base year,

Pinc is household income (Equation 180),

$Pi_{i=ETE,j=27,jj,t}$ is the price of the numeraire sector,

$Pi_{i,j=27,jj,t}$ is the price of paid for inputs with regard to the household sector,

$P_{i,t}$ is the market price, which equals one in the base year and equals the market price during the model solution process,

$\varepsilon_{1,j}$ is the income elasticity,

$\varepsilon_{2,j}$ is the supply price elasticity.

Note that the $E_{0,i}$ values are set constant over time first; then household (*j=27=hh*) fuel use changes of oil, gas, coal and electricity (*i=3, 4, 5, 8*) over time are calculated by means of the updated scaler which is calculated as follows:

$$E_{0,hh,t} = \frac{E_{0,hh,t-1}}{Factor_{i,hh,t}}$$

(unitless)   Eq. 187

where

$$Factor_{i,hh,t} = (1 + HHAEEI_{i,hh})^{Nstep} \qquad \text{(see Equation 19)}$$

where

$HHAEEI_{i,hh}$ is a technical change parameter for household fuel use.

Labor, *HHinc9*, and **land**, *HHinc10*, **expenditures** are subtracted from personal income such that personal income *Pinc* denotes personal income available to purchase produced commodities. Note that consumer purchases of housing are not subtracted from income in SGM 2003.

$$Pinc = Pinc - HHinc_9 - HHinc_{10}$$

Eq. 188

where

$$HHinc_9 = \left(-1 \bullet Pi_{i=25=lbs,j=27=hh,jj,t} \bullet LB_{2,2} \bullet ED_{i=25=lbs,j=27=hh}\right)$$

Eq. 189

are the labor expenditures, and

$$HHinc_{10} = \left(Pi_{i=24=ls,j=27=hh,jj,t} \bullet R_2 \bullet Nhh\right)$$

Eq. 190

are the land expenditures,

where

$Pi_{i=25,j=27,jj,t}$ is the price of labor rental with regard to the household sector, and

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$Pi_{i=24,j=27,jj,t}$ is the price of land rental with regard to the household sector.

Budget constraints need to be satisfied before a model solution can be sought. Therefore, household demand is restrained by the summed shared demands of all inputs as follows:

$$ED_{i=1:22,j=27=hh} = \frac{ED_{i,j} \bullet Pinc}{DemTot_{j=27}}$$

Eq. 191

where total demand by the household sector from the production sectors *DemTot_j* was calculated in Equation 184, total household income *Pinc* was calculated in Equation 156 and updated in Equation 182, and demand by the household sector *ED_{i,j}* for industry goods was calculated in Equation 185.

Finally, the demand for household labor, as determined by the household labor fraction *LB_{2,2}* is subtracted from the total labor supply $Ed_{i=25,j=27}$ such that the labor supply available to the rest of the economy is determined.

$$ED_{i=25=lbs,j=27=hh} = ED_{i=25=lbs,j=27=hh} \bullet (1 - LB_{2,2})$$

Eq. 192

Additional household expenditures

Adjustment factors, *WHSL_i*, for the supply sectors (representing distribution and markup factors) are calculated. All variables in this equation are described above.

$$WHSL_i = \left[\frac{(adj_{i,j=27=hh,t} - 1) \bullet [(P_{i,t} + Tr_{i,t} \bullet Ex\,Im_i) \bullet Txpro_{i,j=27=hh} + Txadd_{i,j=27=hh}] \bullet}{ED_{i,j=27=hh} / Pi_{i=ETE,j=27=hh,jj,t}} \right]$$

Eq. 193

where
   v refers to all actively operating vintages (v=t-3:t-1 for old vintages and v=t for new vintages),
   n is the number of subsectors with n ≤ jj,
   j is the number of production sectors,
   i refers to input sectors,
   adj_{i,j,jj,t}, is a time-period-dependent adjustment factor that reflects markups and intra-regional transport costs,
   P_{i,t} is the market price;,,
   Tr_{i,t} is a time-period-dependent supply sector transportation cost factor,
   ExIm_i is the transportation cost rate parameter,
   Txpro_{i,j,jj} is a proportional tax rate on the ith product,
   Txadd_{i,j,jj} is an additive tax on the ith product,
   EDV_{i,j,jj,v} denotes demand by a vintaged production sector or subsector, and
   Pi_{i=ETE,j,jj,t} is the price paid by the producer for Everything Else input.

Transportation costs for the household market for imports and exports *TRNP_i* are calculated as follows:

$$TRNP_i = Tr_{i,t} \bullet Ex\,Im_i \bullet \left. ED_{i,j=27=hh} \middle/ Pi_{i=ETE,j=27=hh,jj,t} \right.$$

Eq. 194

Taxes on households *Tax₁,ⱼ₌₂₇* are calculated as follows:

$$Tax_{1,j=27=hh} = \sum_{i=1}^{24} \left(P_{i,t} + Tr_{i,t} \bullet Ex\,Im_i \bullet abs(ED_{i,j=27=hh}) \right) \bullet \left(Txpro_{i,j=27=hh} - 1 \right)$$

Eq. 195

Household subsidies *Subsiᵢ* equal household taxes *Tax₁,ⱼ* but for the sign of household demand:

$$Subsid_i = -Tax_{1,j=27=hh}$$

Eq. 196

Taxes on households per unit production *TaxpUⱼ₌₂₇₌ₕₕ* and subsidies to supplies per unit supply are calculated when the additive taxes are greater than zero.

$$TaxpU_{j=27=hh} = \sum_{i=1}^{24} \left(Txadd_{i,j=27=hh} \bullet ED_{i,j=27=hh} \right)$$

Eq. 197

Subsidies per unit supply *SubsidpUᵢ* equal the taxes on the household sector:

$$SubsidpU_i = SubsidpU_i - Txadd_{i,j=27=hh} \bullet ED_{i,j=27=hh}$$

Eq. 198

Carbon permit fees *CPermF₅,ⱼ* are a function of the energy demands and the carbon fees per dollar of energy use, which in turn are a function of (1) the global warming potential and (2) the emission coefficient of the fuel combusted and demanded by households (see Equation 13 for the calculation of *Cpfᵢ*). Thus,

$$CPermF_{5,j=27=hh} = \sum_{i=1}^{24} \left(Cpf_i \bullet ED_{i,j=27=hh} \right)$$

Eq. 199

Government

Production of government services

The government is modeled as potentially producing up to three government services in a nested production function hierarchy:

- general government services,
- national defense, and
- education.

The number of government services modeled in an SGM region is determined by regional data availability.

Government services are assumed to be produced with fixed input-output coefficients.

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The reference case of SGM 2003 contains only one general government service and the government preference function *GF* is, consequently, set to one, given that no government subsectors are simulated. In the equations below we show, however, the potential implementation of different government subsectors.

$$GF_{0:t} = 1$$

<div align="right">Eq. 200</div>

Government taxes and subsidies *TaxGov_{itc=1:5}* are calculated in a manner analogous to taxes and subsidies related to the production sectors. However, they are applied in a model solution iteration as paid to the government sector itself.

$$TaxGov_{itc} = \sum_{i=1}^{25} \left(P_{i,t} + Tr_{i,t} \bullet Ex\,Im_i \bullet ED_{i,j=26=gv} \bullet \left(Txpro_{i,j=26=gv} - 1 \right) \right)$$

<div align="right">Eq. 201</div>

where
   when j=gv equals 26 and jj equals one by definition,
   itc is the indicator for 5 potential taxes,
   $Txpro_{i,j=26=gv}$ is a proportional tax rate on the government sector which equals one for all supplies in the reference case, resulting in zero taxes in the reference case (note that when j equals 26, jj equals one by definition),
   $P_{i,t}$ is the market price, which equals one in the base year and equals the market price during the model solution process,
   $Tr_{i,t}$ is the time-dependent transportation cost factor (see Equation 9)
   $Ex\,Im_i$ is the transportation cost rate parameter (these input parameters currently equal one,
   $ED_{i,j=26=gv}$ is the demand by the government sector for capital supply.

Government supply subsidies *SubsidGov* equal government production taxes *TaxGov_{itc}*.

$$SubsidGov = -TaxGov_{itc}$$

<div align="right">Eq. 202</div>

Taxes on production per unit production *TaxGovpU_{itc}* and subsidies to supplies per unit supply are calculated when the additive taxes are greater than zero,

$$TaxGovpU_{itc} = \sum_{i=1}^{25} \left(Txadd_{i,j=26=gv} \bullet ED_{i,j=26=gv} \right)$$

<div align="right">Eq. 203</div>

where
   $Txadd_{i,j=26=gv}$ is an additive tax on government supplies.

Government subsidies per unit supply *SubsidpU* equal the taxes on the government sector.

$$SubsidpU = TaxGovpU_{itc}$$

<div align="right">Eq. 204</div>

The carbon permit fee *CPermF_{5,j=26}* is a function of the energy demand by government and the carbon fee per dollar of energy production, which is a function of the global warming potential and the emission coefficient (based on the carbon content) of the fuel combusted (see Equation 10 for the calculation of *Cpf_i*). Thus,

<div align="center">95</div>

$$CPermF_{5,j=26=gv} = \sum_{i=1}^{25} \left(Cpf_i \bullet ED_{i,j=26=gv} \right)$$

Eq. 205

Total government taxes are the summed production sector taxes to which various other taxes are added as described in detail below:

$$TotTax_t = \sum_{j=1}^{27} \left(Tax_{1,j} + Tax_{2,j} + Tax_{3,j} + Tax_{4,j} + CPermF_{5,j} \right)$$

Eq. 206

where

$Tax_{1,j=1:22}$ for production sectors are calculated when operating old and new vintages (Equation 84 for v=t-3:t),

$Tax_{1,j=26}$ equals $TaxGov_1$ (see Equation 201) calculated during the previous model solution iteration, ($Tax_{2,j=26}$ equals $TaxGov_2$ which equals $TaxGov_1$

$Tax_{1,j=27}$ equals taxes on household related to exports and imports (Equation 195),

$Tax_{2,j=1:22}$ for production sectors when labor taxes (i=25) are calculated (Equation 85 for v=t-3:t),

$Tax_{2,j=27}$ is the labor income tax related to the social security tax rate (Equation 167),

$Tax_{3,j=1:22}$ is the production sectors (j) tax on profits (Equation 165),

$Tax_{3,j=27}$ is the tax on personal income and personal income tax rate (Equation 169),

$Tax_{4,j=1:22}$ equals the taxes on the production sector-specific sales:

$$Tax_{4,j} = SalesP_j \bullet TxIBT_j$$

Eq. 207

where

$TxIBT_j$ is the production sector-specific indirect business tax, and

$SalesP_j$ is gross production summed over all active vintages and production subsector in monetary terms.

$$SalesP_j = \sum_{v=t-3}^{t} \left[\sum_{jj=1}^{n} \left(PRDV_{j,jj,v} \bullet Pr_{j,t} \right) \right]$$

Eq. 208

where

$PRDV_{j,jj,v}$ is vintage and sector (and subsector, if active) specific gross production,

$Pr_{j,t}$ is the price received for the commodity produced.

The amount of IBT tax collected above the base year (*ExIBT_t*) is based on changes in the production sector-specific business tax. If no changes in these taxes occur the *ExIBTt* values are zero.

$$ExIBT_t = \sum_{j=1}^{22} \left(SalesP_j \bullet \left(TxIBT_{j,t} - TxIBT_{j,t=0} \right) \right)$$

Eq. 209

$CPermF_{5,j=1:22}$ are carbon fees calculated for old vintages and new vintage production sectors when fossil fuels are produced (Equation 90 for v=t-3:t),

$CPermF_{5,j=25}$ are carbon fees based on the per dollar energy use as a function of energy demand for investments and Cpf_i, a carbon permit fee on supplies per dollar of production based on the global warming potential and the emission coefficient of a fuel (see Equation 140).

CPermF$_{5,j=26}$ are carbon fees based on the per dollar energy use as a function of energy demand by government and Cpf$_{i,}$, a carbon permit fee on supplies per dollar of production based on the global warming potential and the emission coefficient of a fuel (see Equation 205).

CPermF$_{5,j=27}$ are carbon fees based on the per dollar energy use as a function of energy demand by households and Cpf$_{i,}$, a carbon permit fee on supplies per dollar of production based on the global warming potential and the emission coefficient of a fuel (see Equation 199).

Total government subsidies are calculated by summing the supply sector subsidies towards operating old and new vintages and the government subsidies:

$$
\text{Subsid}_i = \sum_{j=1}^{22}\left[-1 \bullet \sum_{v=t-3}^{t}\left[\sum_{jj=1}^{n}\left(P_{i,t} + Tr_{i,t} \bullet \text{Ex Im}_i \bullet \text{EDV}_{i,j,jj,v}\right)\bullet\left(\text{Txpro}_{i,j,jj} - 1\right)\right]\right] +
$$

$$
\sum_{j=26}^{27}\left(P_{i,t} + Tr_{i,t} \bullet \text{Ex Im}_i \bullet \text{ED}_{i,j=26=gv}\right)\bullet\left(\text{Txpro}_{i,j=26=gv} - 1\right)
$$

Eq. 210

where

the potential subsidies to the supply sectors for operating old vintages and new vintages are summed, v=t-3:t=1 (see Equations 86 for v=t-3:t), and

the last part of the equation are the government and household subsidies (is SubsidGov which are the government production taxes (see Equation 202)).

Government transfers are calculated based on per capita income, percent of population in young and old age groups, and the fraction of the population working. Government transfers are assumed to be untaxed (see Equation 170 under "Household demands")

Government deficits are input data with negative values. Deficits are calculated relative to the price of the numeraire:

$$
\text{GovDeft}_t = \text{GovDeft}_t \bullet \text{Pi}_{i=ETE,j=26=gv,jj,t}
$$

Eq. 211

where

GovDeft are input data (see Appendix A), and

Pi$_{i=ETE,j=26=gv,jj,t}$ is the price of the numeraire sector.

Government resources *GovMt* can now be calculated as equal to tax collections plus a government deficit less government transfers. In the base year this amounts to the sum of all inputs (supplies)

$$
\text{GovMt}_{t=0} = \sum_{i=0}^{22}\sum_{gvjj=1}^{1} X_{j=26=gv,jj=gvjj}
$$

Eq. 212

In the projections at each point in time *t* the government balance is calculated. This government balance at time *t* is part of the GNP accounting

$$\text{GovMt}_t = \text{TotTax}_t - \text{Subsidy}_t - \text{GovTtr}_t + \text{GovDeft}_t \qquad \text{Eq. 213}$$

where

TotTax_t denotes the summed taxes on active old and new vintage production sectors in combination with various other taxes, *e.g.*, investment, government and household taxes (Equation 208),

Subsidy_t denotes the summed supply sector subsidies (Equation 86 for old vintages and new vintages production sectors) in combination with household subsidies (Equation 196 and government subsidies (Equation 202 (see Equation 210; summing takes place over the 26 supply

sectors: $\text{Subsidy}_t = \sum_{i=1}^{26} \text{Subsid}_i$

GovTr_t denotes the demographics and income dependent government transfers (Equation 170), and

GovDeft_t denotes the government deficits, which have negative values (Equation 211).

Depending on if and which carbon fee policies are implemented, government income is updated based on recycling of carbon revenues.

If carbon pricing is implemented the government deficits $GovDeft_t$ and government resources $GovMt_t$ have to be adjusted in accordance with the carbon policy. The variable $CrbFeeTot_t$ represents the carbon permit fees at time t. The different carbon policy case impacts on the government deficits and resources *ICRBfeeOPT=case 2 through 6* are shown below:

Case 2: all carbon fees produced from government resources are returned to deficit reductions. Government deficits have negative values and carbon fees and therefore have to be subtracted.

$$\text{GovMt}_t = \text{GovMt}_t - \text{CrbFeeTot}_t \qquad \text{Eq. 214}$$

$$\text{GovDeft}_t = \text{GovDeft}_t - \text{CrbFeeTot}_t \qquad \text{Eq. 215}$$

where

CrbFeeTot_t is based on domestic carbon fees and carbon permit trade,

Case 3 and 4: All carbon revenues taken from government resources are returned to other sectors.

$$\text{GovMt}_t = \text{GovMt}_t - \text{CrbFeeTot}_t \qquad \text{Eq. 216}$$

Case 5: Domestic carbon fees are returned to consumers and permits are returned to the deficit.

(a) If carbon trading is implemented and carbon trade provides for revenue, government deficits will receive the carbon fees traded in order to reduce these deficits and carbon domestic fees will be subtracted from government resources to provide for the carbon fees to be returned to households.

$$\text{GovMt}_t = \text{GovMt}_t - \text{CrbFeeTot}_t \qquad \text{Eq. 217}$$

$$\text{GovDeft}_t = \text{GovDeft}_t - \text{CrbTrade}_t \qquad \text{Eq. 218}$$

where

CrbFeeTot_t is based on domestic carbon fees and carbon permit trade,

CarbTrade$_t$ refers to the carbon permit trade.

(b) If carbon trading is a net loss, government resources provide for the domestic carbon fees, but deficits cannot be reduced.

$$\text{GovMt}_t = \text{GovMt}_t - \text{CrbFeeTot}_t \qquad\qquad \text{Eq. 219}$$

Case 6: Government resources will produce part of the carbon fees (see also Equation 165 where *CrbFeeRcyPct$_2$*CrbFeeTot$_t$* is added to household income).

$$\text{GovMt}_t = \text{GovMt}_t - \left(\text{CrbFeeRcyPct}_2 + \text{CrbFeeRcyPct}_3\right) \bullet \text{CrbFeeTot}_t \qquad \text{Eq. 220}$$

Demands for goods and services by the government

Calculation of **government consumption $GC_{i,gv}$ for the base year** of each supply and government sector interaction is determined by government demand and is calculated as follows:

$$\text{GC}_{i,j=26=gv,t=0} = \sum_{gvjj=1}^{1} \left(\frac{X_{i,j=26=gv,gvjj} \Big/ \sum_{i} X_{i,j=26=gv,gvjj}}{P_{i,t}} \right) \qquad\qquad \text{Eq. 221}$$

Note that next the $GC_{i,gv}$ values are set constant over time first and then government ($gv=j=26$) fuel use changes for oil, gas, coal, and electricity ($i=3, 4, 5, 8$) over time are calculated as follows:

$$\text{GC}_{i,j=26=gv,t} = \frac{\text{GC}_{i,j=26=gv,t-1}}{\text{Factor}_{i,j=26=gv,t}} \qquad\qquad \text{(unitless)} \qquad \text{Eq. 222}$$

where

$$\text{Factor}_{i,j=gv,t} = (1 + \text{GVAEEI}_{i,j=gv})^{\text{Nstep}} \qquad \text{(see Equation 21)}$$

where

$\text{GVAEEI}_{i,j=gv}$ is a technical change parameter for government fuel use.

Expenditures by the government subsectors *gvjj* towards government consumption is calculated as follows when based on fixed government subsector weights:

$$\text{PGC}_{j=26=gv} = \sum_{gvjj=1}^{1} \left[\sum_{i=1}^{25} \left(\text{GC}_{i=26=capital,j=26=gv,gvjj} \bullet \text{Pi}_{i,j=26=gv,gvjj,v=t}\right) + \text{GC}_{i=26=capital,j=26=gv,gvjj} \bullet \text{Pi}_{i=ETE,j=26=gv,gvjj,v=t} \right] \qquad \text{Eq. 223}$$

The scheme for allocation of the subsector purchases across production sectors is simulated as follows:

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$$\mathrm{GE}_{\mathrm{gvjj}} = \mathrm{GovMt}_t \bullet \frac{\mathrm{PGC}^{-\mathrm{sig}}_{\mathrm{j=26=gvgvjj}} \bullet \mathrm{GF}^{\mathrm{sig}}_{\mathrm{j=26=gv,gvjj}}}{\displaystyle\sum_{\mathrm{gvjj=1}}^{1}\left(\mathrm{PGC}^{-\mathrm{rho}\bullet\mathrm{sig}}_{\mathrm{j=26=gv,gvjj}} \bullet \mathrm{GF}^{\mathrm{sig}}_{\mathrm{j=26=gv,gvjj}}\right)}$$

Eq. 224

where

GovMt$_t$ is the government resource balance (Equation 220),
rho equals zero resulting in a value of PGC$^{-\mathrm{rho*sig}}$ of one,
    sig equals one,
    PGC$_{\mathrm{j=gv,gvjj}}$ denotes the government expenditures (Equation 223), and
    GF$_{\mathrm{j=26=gv,gvjj}}$, the government preference function, equals government one, given that there are no government subsectors.

Government demand for each supply sector $ED_{i=1:25,j=gv}$ is then calculated based on government purchases. Capital demand $ED_{i=26,j=gv}$ by the government is treated separately. Note that in that case the price of the numeraire sector is required. Also note that government saves automatically, so government capital does not, in this version, increase demand for savings.

$$\mathrm{ED}_{\mathrm{i=1:25,j=26=gv}} = \sum_{\mathrm{gvjj=1}}^{1}\mathrm{GE}_{\mathrm{j=26=gv,gvjj}} \bullet \mathrm{GC}_{\mathrm{i,j=26=gv,gvjj}}$$

Eq. 225

and add government deficit to the demand for savings

$$\mathrm{ED}_{\mathrm{i=26=capital,j=26=gv}} = \sum_{\mathrm{gvjj=1}}^{1}(\mathrm{GE}_{\mathrm{j=26=gv,gvjj}} \bullet \mathrm{GC}_{\mathrm{i=capital,j=26=gv,,gvjj}} \bullet \mathrm{PGC}_{\mathrm{j=26=gv,gvjj}}) + \mathrm{GovDef}_t$$

Eq. 226

and update the demand of the Everything Else sector

$$\mathrm{ED}_{\mathrm{i=ETE,j=26=gv}} = \mathrm{ED}_{\mathrm{i=ETE,j=26=gv}} + \sum_{\mathrm{gvjj=1}}^{1}(\mathrm{GE}_{\mathrm{j=26=gv,gvjj}} \bullet \mathrm{GC}_{\mathrm{i=ETE,j=26=gv,gvjj}})$$

Eq. 227

Imports and exports

Trade is kept track of as an demand array element denoted as $ED_{i,j=24=exim}$. If trade is fixed, trade quantities can be input values ($Trade_{0,i=23}$ data are in Appendix A), and trade demand can be set equal to these initial trade values. The SGM offers users the option to "read in" carbon trade (into either $Trade_{0,i=23}$ or into an additional trade vector $Trade_{1,i=23}$).

$$\mathrm{ED}_{\mathrm{i=1:26,j=24=exim}} = \mathrm{Trade}_{\mathrm{0,i=1:26}}$$

Eq. 228

Over time or with open markets, trade may be assumed to be the difference between production and summed domestic demand (see Equation 231), although this assumption remains part of the market solution process. In the reference case markets are not open across the regions and trade is fixed for the base year, but trade in oil and gas is calculated as in Equation 229 as the difference between demand and supply for the projections.

$$\text{Trade}_{1,i=1:22} = -\sum_{j=1}^{27} \text{ED}_{i=1:22,j} - \text{PRD}_j \qquad \text{Eq. 229}$$

where
  $\text{Ed}_{i,j}$ represents the demand matrix[43] (see Equation 80), and
  PRD_j represents the production vector (vector of outputs j) (see Equation 106).

To calculate the trade balance, the relevant exchange rates have to be used:

$$\text{Trade}_{0,i=1:25} = \text{Trade}_{0,i=1:25} \bullet \text{ExchRate}_{\text{region}} \qquad \text{Eq. 230}$$

and/or $\text{Trade}_{1,i=1:25} = \text{Trade}_{1,i=1:25} \bullet \text{ExchRate}_{\text{region}} \qquad \text{Eq. 231}$

The export-import balance will be determined by calculating the monetary equivalent.

$$\text{Exp Im}_t = \sum_{i=1}^{25} \left[\text{Trade}_{0,i=1:25} \bullet P_{i,t} \right] \qquad \text{Eq. 232}$$

and/or

$$\text{Exp Im}_t = \sum_{i=1}^{25} \left[\text{Trade}_{1,i=1:25} \bullet P_{i,t} \right] \qquad \text{Eq. 233}$$

If carbon pricing is simulated, the export-import balance has to be adjusted in accordance with the carbon policy implemented. This implies that the carbon trade has to be subtracted from the net exports

$$\text{Exp Im}_t = \text{Exp Im}_t - \text{Trade}_{1,i=23} \bullet P_{i=23,t} \qquad \text{Eq. 234}$$

and the carbon traded has to be converted back to tons of carbon *TgC* (from Equation 232):

$$\text{Trade}_{1,i=23} = \text{Trade}_{1,i=23} / \text{ExchRate}_{\text{regionl}} \qquad \text{Eq. 235}$$

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43 In the SGM code the *ED* matrix at a certain stage has diagonal elements calculated as $Ed_{i,j}$-PRD_j; in this documentation the $Ed_{i,j}$ matrix refers to the demand matrix.

Chapter 9. GNP accounting

To ensure that the production and demand are consistent throughout the model, the SGM relies on simple economic relations for the accounting framework. GNP can be given as the price weighted sum of net production of all products, which in turn is equal to the disposition of net output among final demand sectors (Edmonds et al. 1993:303; Kim 1995:36)

$$GNP = \sum_{i=1}^{N} Pi_{i,j} \bullet \left(PRD_j - \sum_{j=1}^{N} ED_{i,j} \right) = KTot_t + Cons_t + GovMt_t + Ex\,Im_t \qquad \text{Eq. 236}$$

where at each point in time t

 i is an input product identifier,

 j is the output product identifier,

 N is the number of factors of either i or j; N=27 for inputs (22 variable inputs plus a carbon option (i=23) plus land, labor, and capital as fixed inputs (M=3) and indirect business taxes and subsidies as final input; N=22 for outputs (22 production sectors; sector 23 is the carbon market);

 $Pi_{i,j}$ is the price paid by producers for inputs,

 PRD_j is gross domestic production,

 $ED_{i,j}$ is the demand for product i in the production of product j,

and

 $Ktot_t$ is total investment demand (see Equation 136)

 $Cons_t$ is household consumption, which is based on retained income, labor income and related costs, land rental income, government transfers and with savings subtracted (see Equation 181),

 $GovMt_t$ is government consumption, which is based on taxes, subsidies, government transfers and government deficits (see Equation 220),

 $ExpIm_t$ is net export, which is based on exogenous input, energy balances and time-dependent market prices (see Equation 234).

Alternatively, consistent with the input-output accounting framework, GNP can be expressed as the sum of payments to the factors of production (Edmonds et al. 1993:303; Kim 1995:36):

$$GNP = \sum_{j=1}^{N} \left(Pr_j \bullet PRD_j - \sum_{i=1}^{N} Pi_{i,j} \bullet ED_{i,j} \right) = \sum_{j=1}^{N} \left(TXibt_j + \pi_j + \sum_{i=N+1}^{N+M} Pi_{i,j} \bullet ED_{i,j} \right) \qquad \text{Eq. 237}$$

where

 i is a row factor identifier (input),

 j is a column factor identifier (output),

 Pr_j is the price received for commodities produced,

 PRD_j is gross domestic production,

 $Pi_{i,j}$ is the price paid by producers for inputs,

 $ED_{i,j}$ is the demand for product i in the production of product j,

and in the last part of the identity;

 $TXibt_i$ is the indirect business tax,

 π_i represent the profits including net interest and capital consumption allowances,

 M is the number of primary factors of production (labor, land, capital).

Chapter 10. The solution procedure

The fundamental problem that the model must address is finding a set of prices that is consistent with demands by producers and consumers for goods, services, and primary factors of production when assumptions about change are made in forecasting.

Thus, for any period, equilibrium will exist when a set of prices is found for which all excess demands are zero. This set of prices is not unique. Walras' Law guarantees that if an equilibrium set of prices exists, any positive scalar multiple of those prices is also an equilibrium set of prices. Any commodity can be chosen as a numeraire, and its price can be determined arbitrarily; in SGM the commodity of the Everything Else sector is the numeraire, and its price is set to one. The number of independent market prices is always one less than the number of markets. Similarly, since the unit of measure chosen for commodities is also arbitrary, units can be chosen such that all equilibrium prices are unity in one period.

When the model solution is reached, there will be a set of prices and quantities of the n-commodity market such that all n equations in the equilibrium condition will be simultaneously satisfied.

At each step in the solution process, trial prices are already known because they are set by the previous time-period prices. To calculate excess demand – the difference between demand and supply – both demand and supply are determined iteratively. **Demand** $EDV_{i,j,jj,v}$ by a vintaged production sector or subsector j or jj for input i is calculated, if the production function is assumed to be a CES function, as dependent on the variable inputs (expressed in the variable $Z_{j,jj}$), prices (prices received for the commodity $Pr_{j,t}$ and prices paid for supplies $Pi_{i,j,jj,v}$), capital $KA_{j,jj,v}$ and the various technical coefficients ($\alpha_{i,j,jj,v}$ and $\alpha cst_{i,j,jj,v}$) and the production sector-specific elasticities (expressed as a function of σ, in the form of ρ or μ).

$$EDV_{i,j,jj,v} = \left(\alpha_{i=26,j,jj,v} \bullet KA_{j,jj,v}{}^{\rho}\right)^{1/\rho} \bullet Z_{j,jj,v}{}^{-1/\rho} \bullet \left(\frac{\alpha_{i,j,jj,v}^{1/(1-\rho)}}{Pi_{i,j,jj,t}^{1/(1-\rho)}}\right) \quad \text{(see Equations 61 and 76)}$$

for the CES production function implementation for old and new vintages, respectively)

$$\text{where } Z_{j,jj} = 1 - (\alpha_{0,j,jj,v} \bullet Pr_{j,t})^{\mu} \bullet \left(\sum_{i=1}^{N-1} \alpha cst_{i,j,jj,v} \bullet Pi_{i,j,jj,t}^{-\mu}\right) \quad \text{(see Equation 49)}$$

The annual vintage-specific **gross production** $PRDV_{j,jj,v}$ by a production sector or subsector is calculated through the CES production function as dependent on the same variables.

$$PRDV_{j,jj,v} = \alpha_{0,j,jj,v} \bullet Z_{j,jj,v}{}^{-1/\rho} \bullet \left(\alpha_{i=26,j,jj,v} \bullet KA_{j,jj,v}{}^{\rho}\right)^{1/\rho} \quad \text{(see Equations 63 and 77 for the}$$
CES production function implementation for old and new vintages, respectively)

Demand is calculated, if the production function represents a Leontief technology (in the SGM when the elasticity of substitution ($\sigma 2$) is smaller than 0.05) as dependent on gross production and the Leontief technical coefficients,

$$EDV_{i,j,jj,v} = \frac{\lambda_{i,j,jj,v}}{\lambda_{i=26,j,jj,v}} \bullet KA_{j,jj,v}$$ (see Equations 64 and 78 for the Leontief production

function implementation for old and new vintages, respectively)

where the annual vintage-specific gross production ($PRDV_{j,jj,v}$) is calculated as dependent on the Leontief technical coefficients and capital.

$$PRDV_{j,jj,v} = \frac{\lambda_{0,j,jj,v}}{\lambda_{i=26,j,jj,v}} \bullet KA_{j,jj,v}$$ (see Equations 65 and 79 for the Leontief production

function implementation for old and new vintages, respectively).

In the case of *putty-clay* behavior a sector's vintaged and projected technical coefficients for the Leontief production function ($\lambda_{i,j,jj,v}$ for *i=1:25* and *j=1:22*) are based on the vintage technical coefficients calculated for the CES production function.

$$\lambda_{i,j,jj,v} = \alpha_{0,j,jj,v}^{\rho1/(1-\rho1)} \bullet \left(\alpha_{i,j,jj,v} \bullet \frac{Pr_{j,t}}{Pi_{i,j,jj,t}} \right)^{1/(1-\rho)}$$ see Equation 44 for the variable factors

and Equation 45 for the fixed factor capital

$$.\lambda_{i=26=capital,j,jj,v} = \alpha_{0,j,jj,v}^{\rho1/(1-\rho1)} \bullet \left(\alpha_{i=26,j,jj,v} \bullet \frac{Pr_{j,t}}{\pi_{j,jj,v}/Ka_{j,jj,v}} \right)^{1/(1-\rho1)}$$

Vintage and subsector-production-specific demands were summed over the operating vintages and subsectors (if operating) (*i=1:25; j=1:22*). Thus, $ED_{i=1:25,j=1:22}$ represent the demands[44] by the production sectors *j* for inputs, *i*, and those for carbon (i=23), land (i=24), labor (i=25) by each production sector.

$$ED_{i=1:25,j=1:22} = \sum_{v=t-3}^{t} \left[\sum_{jj=1}^{n} EDV_{i,j,jj,v} \right]$$ (see Equation 80)

where *n* is the number of subsectors with $n \leq jj$, and *v* denotes the operating old and new vintages.

For the final demand sectors, demand equations are described in a series of equations in the "Final Demand" sections: "Investments", "Households", "Government" , and "Imports and Exports" and summarized below.

$ED_{i=1:26,j=24=exim}$ represents demands by trade (see Equation 228),
$ED_{i=1:22,j=25=inv}$ represents redistribution of demands for investment (Equation 134),

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44 In the SGM code the *ED* matrix at a certain stage has diagonal elements calculated as $Ed_{i,j}-PRD_j$; in this documentation the $Ed_{i,j}$ matrix refers to the demand matrix.

$ED_{i=26,j=25=inv}$ represents demands for capital by investment (Equation 135),

$ED_{i=2=ETE,j=25=inv}$ represents demands for the Everything Else sector after adjustments for supply sector transportation costs and additional adjustment factors (Equation 137),

$ED_{i=1:25,j=26=gv}$ represents demands by the government sector (Equation 225), including those for carbon, land, and labor,

$ED_{i=2=ETE,j=26=gv}$ represents demands for the Everything Else sector by the government sector (Equation 227)

$ED_{i=26,j=26=gv}$ represents demands for capital by the government sector (Equation 226)

$ED_{i=1:22,j=27=hh}$ represents demands for supplies by households; these are income- and price-elasticity dependent (Equation 185 and constrained for overall budget: Equation 191),

$ED_{i=24=ls,j=27=hh}$ represents demand for land supply by households (Equation 152)

$ED_{i=25=lbs,j=27=hh}$ represents demand for labor supply by households (Equations 144 and 192),

$ED_{i=26=capital,j=27=hh}$ represents demand for saving supplies by households (Equations 176 and corrected for retained income: Equation 182).

Budget constraints for households with regard to demands are expressed in Equation 191.

Gross production for each sector summed over active vintages and active production subsectors was retained in a production sector vector to be part of the model solution algorithms.

$$SalesV_{i=1:23} = PRD_{j=1:23} = \sum_{v=t-3}^{t}\left[\sum_{jj=1}^{n}PRDV_{j=1:23,jj,v}\right] \qquad \text{(see Equation 106)} \qquad \text{Eq. 238}$$

where
   j=i=1:23 denote the product markets (see Figure 1) Note that when carbon prices are zero a market for carbon (for j=23) is not operated.

Supplies of land, labor and savings were set equal to the negative values of the demands for land, labor and savings when describing the household sector and retained in *SalesV*. *SalesV$_{i=24:26}$* denotes the supplies to the factor markets provided by labor, land and Oher Value Added (see Figure 1):

$SalesV_{i=24=ls}$ (i=24; m=23; land supply: see Equation 154);
$SalesV_{i=25=lbs}$ (i=25; m=24; labor supply: see Equation 148);
$SalesV_{i=26=capital}$ (i=26; m=25; savings supply: see Equation 183)

These production sector vector elements are multiplied by conversion vectors and stored as the market-supply vector:

$$MrkPrd_m = SalesV_i \bullet PRconvrt_i \qquad\qquad \text{Eq. 239}$$

where
   PRconvrt converts monetary values to energy quantities such that an energy balance will be obtained in the market solution.

Excess demand is calculated across all markets as the difference between demand and gross production, which determines the supply to the demand sectors, and stored as a market-excess demand vector ($MrkEd_m$)

$$\text{MrkEd}_m = \sum_{j=1}^{27} \left(\text{Ed}_{i,j} \bullet \text{PRconvrt}_i \right) - \text{Mrk Prd}_m \qquad \text{Eq. 240}$$

where the supply sector indicators, $i=1:23$, and the production sector indicators, $j=1:23$, map on the market indicator m.

Note that the market solutions are calculated in either monetary or energy units through *PRconvrt*; in addition, if exchange rates are involved due to trade or multiple interacting regions, demand $ED_{i,j}$ and gross production PRD_j are converted by the appropriate exchange rates $ExchRate_{region}$.

Both demands and supplies are stored after a log transformation in a vector $Xvec_{inewt}$ to be checked if they fall within the elasticity bounds and meet the solution criteria for convergence.

$$\text{Xvec}_{inewt} = \text{Log}(\text{MrkEd}_m + \text{Mrk Prd}_m) - \text{Log}(\text{Mark Prd}_m) \qquad \text{Eq. 241}$$

If not, then for the next iteration a *W* matrix is inverted to calculate the new prices based on the necessary delta of the price perturbation.

$$W_{i,j} = \left[\frac{\text{DemTmp} - (\text{MrkEd}_m + \text{Mrk Prd}_m)}{\text{MrkEd}_m + \text{Mrk Prd}_m} - \frac{\text{SupTmp} - \text{Mrk Prd}_m}{\text{Mrk Prd}_m} \right] \Bigg/ \frac{\text{delta}}{P_{i,t} - \text{delta}}$$

$$\text{Eq. 242}$$

where
    DemTmp equals $MrkEd_m + MrkPrd_m$ and
    SupTmp equals $MrkPrd_m$ which are values from the previous solution attempt.

The new prices feed back to the demand and supply equations and the process is reiterated until a price perturbation is found for an equilibrium solution. The process is a derivative-based Newton-Raphson search procedure (see Appendix B). The procedure converges very quickly once prices approach their equilibrium values. A simple sector-by-sector line search is used first to bring prices that are far from equilibrium close to their equilibrium values (see Appendix B).

Figure 13 illustrates, for one region, the extent to which the model solution searches for the solutions of fossil fuel investments. Table 4 illustrates the variable values at the end of the last iteration of the market solving algorithms when carbon policies are based on carbon emission limits at randomly chosen time period 7.
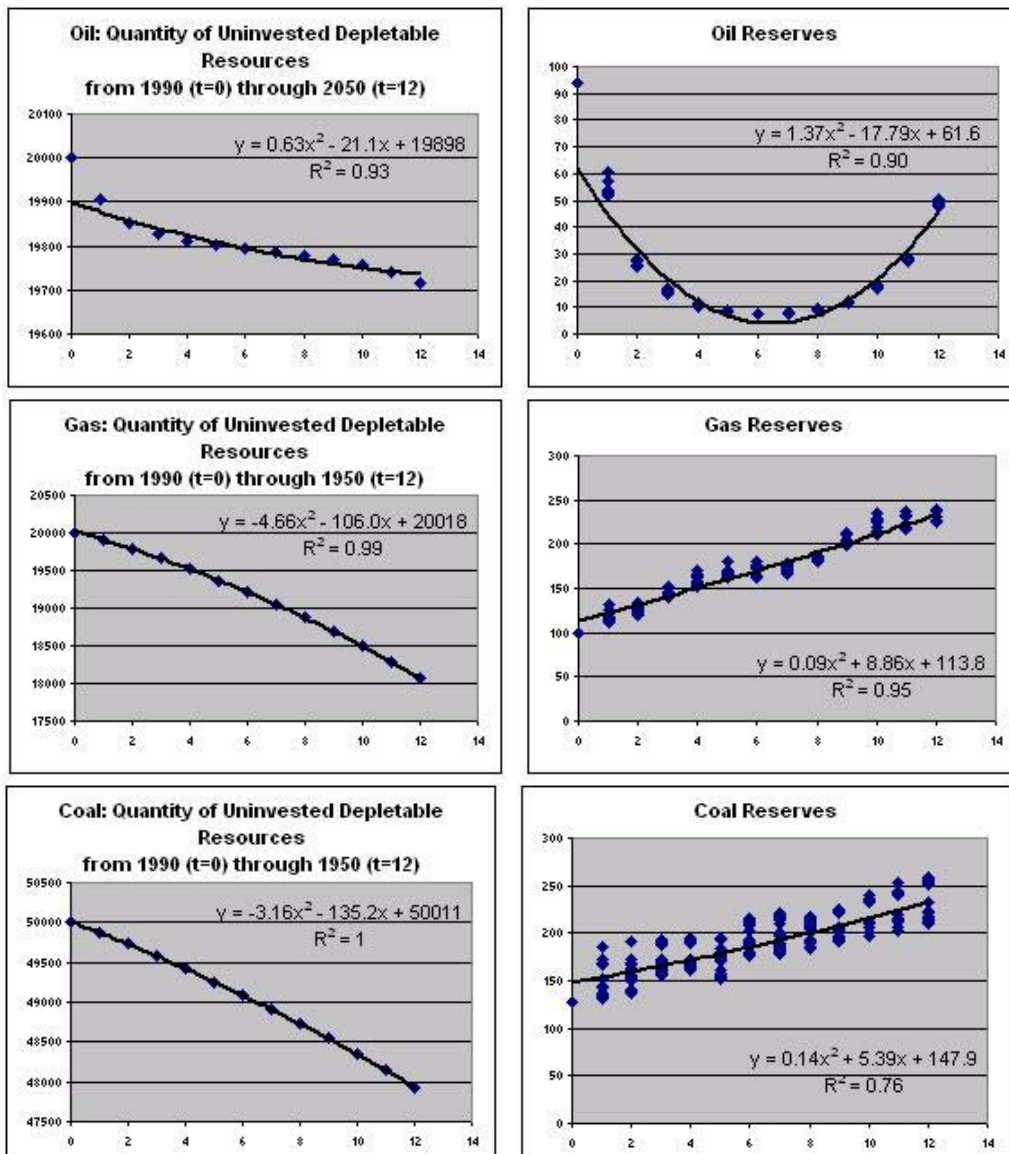
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Figure 13 Quantities of uninvested depletable resources and invested capital in depletable resources for one region in a case other than the reference case illustrating the well-behaved nature of the SGM

Table 4 Variable values at the end of the last iteration of the market solving algorithms when carbon policies are based on carbon emission limits

For t=7 when carbon policies are based on carbon equivalent emissions and carbon	Mrk (m)	(j)	MrkEd = Excess Demand	MrkPrd (note energy in ExaJoules)	P_i	SalesV (note energy in monetary

fees of $100 ton C (see Figure 18) Region = USA						values)
Other Ag.	1	1	0.02328	138868.0	0.91327	138868.0
C.Oil Prd.	2	3	0.00000	20.8	1.00000	47777.9
N.Gas Prd.	3	4	0.00000	21.8	1.00000	50024.8
Coal Prd.	4	5	0.00000	27.8	0.89636	27122.8
Coke Prd.	5	6	0.00000	1.2	2.17083	5933.4
[empty]	6	7	0.00000	0.0	1.00000	0.0
Elec Gen.	7	8	0.00003	20.7	1.38707	425025.0
Ref Oil	8	9	0.00000	41.9	1.72712	176722.0
Gas T&D	9	10	0.00001	36.9	1.59899	176302.0
Wood Prd.	10	11	0.14703	439236.0	0.83425	439236.0
Chemicals	11	12	0.29714	616838.0	0.85995	616838.0
Cement	12	13	0.01484	143155.0	0.91768	143155.0
Steel	13	14	0.02649	153190.0	0.90411	153190.0
NFMetals	14	15	0.03776	128960.0	0.76174	128960.0
OthInd	15	16	6.44723	5948400.0	0.85386	5948400.0
PassTran	16	17	0.92306	328508.0	1.00436	328508.0
FrghtTran	17	18	0.55866	425260.0	0.95212	425260.0
Grains and Oil Seeds	18	19	0.05353	99969.7	0.84195	99969.7
Animal Products	19	20	-0.04671	196859.0	0.67687	196859.0
Forestry	20	21	-0.21275	9553.9	0.84770	9553.9
Food Processing	21	22	-2.90778	1038580.0	0.72144	1038580.0
Carbon market for fixed carbon fee of $100 per ton carbon	22	23	0.00081	2440.6	165.66	2440.6
	23	24	0.00000	0.0	0.00000	0.0
Land	24	25	-0.00001	170365.0	60.62	170365.0
Labor	25	26	71.01880	2269550.0	0.08938	2269550.0
OVA (other value added)	26	2	0.00000	14169100.	1.00000	14169100.

Chapter 11. Calibration procedure

All SGM regions are calibrated to match base year energy consumption, carbon emissions, and economic activity. 1990 is the current model base year, and one model diagnostic is the comparison between base year model output and base year data. Base year model output should exactly match base year data from energy balance tables and input-output tables (see also Appendix A).

Some SGM regions are also calibrated to match external projections on energy consumption and economic output, usually from an official government source, from the present to 2020 or beyond. This is especially true of SGM-USA, where the Annual Energy Outlook, published by the US Energy Information Administration, provides projections to 2025. We use a sequential procedure for baseline calibration of gross national product (GNP), electricity generation, and fossil fuel consumption.

Various technical parameters are available in SGM to influence the time path of model output, especially autonomous time trends governing the efficiency of inputs in production processes. The first step in baseline calibration is to match GDP projections by adjusting an autonomous labor efficiency improvement parameter. The second step is to match projections of electricity generation, in units of billion kilowatt-hours, by adjusting an autonomous electricity efficiency improvement parameter in all model activities that use electricity. Third, the mix of fossil fuels within electricity generation is adjusted by varying the time path of the cost to produce electricity using oil, natural gas, or coal. Fourth, fossil fuel consumption outside of electricity generation is adjusted using fossil fuel efficiency improvement parameters in all model activities that use fossil fuels. These adjustments in efficiency and cost parameters are not independent, so the baseline calibration process is repeated at least once.

This calibration process is used for baseline energy consumption and carbon emissions, but another set of parameters governs the slope of the marginal abatement cost curves. For all production processes in SGM except electricity, the elasticity of input substitution determines the rate that other inputs can substitute for carbon-intensive fuels, as the price of carbon changes relative to other prices. The electricity sector in SGM is actually a collection of production processes that represent different ways of generating electricity. A parameter in the logit sharing mechanism, that determines the market share of generating technologies, governs the rate that one technology can substitute for another as relative costs change. A set of marginal abatement cost curves is constructed by running a series of constant-carbon-price experiments in SGM, with a carbon price introduced in the 2005 time step and held constant thereafter. If any of these parameters are changed, they will affect baseline calibration and the baseline calibration process must be repeated. Conversely, some of the baseline calibration parameters mentioned earlier, especially cost parameters in the electricity sector, influence the slope of the marginal abatement cost curves.

Chapter 12. The energy balance and emissions

One of the main functions of the SGM is to balance energy demand and supply and quantify the resulting emissions by

- accounting for baseline greenhouse gas emissions over time for a country or group of countries;
- finding the least-cost path to meet any particular emissions constraint;
- providing a measure of the carbon price, in dollars per metric ton;
- providing a measure of the overall cost of meeting an emissions target.

Fossil fuel emissions

Emission calculations, in the SGM, are based on the amount of fossil fuel combusted and fuel-specific emission coefficients.

Total demand for each of the fossil fuels is calculated first. These total demands for each region need to account for trade, for both raw fuels (oil, coal, and gas) and for refined fuels (refined oil and Gas T&D). The product of fuel-specific total demand and related emission coefficients and global warming potential (*GWP*) results in total carbon emissions after accounting for the fixed factor energy trade when regional markets are closed ($ED_{i=1:26, j=24=exim} = Trade_{0,i=1:26}$; see Equations 228). Thus,

$$TotDem_{i=1:22} = \sum_{j=1}^{27} ED_{i,j} - ED_{i,j=24}$$

Eq. 243

where

$Ed_{i,j}$ represents the demand for supply i by production sector j which equals the summed over the subsectors and its operating vintages' demands (see Equation 80).

Emissions for **oil** combustion are calculated as follows:

$$EmissionsC_{j=3=oil} = EMC_i \bullet GWP_{i=3} \bullet$$
$$\left(0.97 \bullet PRconvrt_{fn=3} \bullet TotDem_{i=3} - PRconvrt_{fn=9} \bullet Trade_{0,i=9=refinedoil}\right)$$

Eq. 244

where,

EMC_i is the oil-specific carbon content in million tons Carbon per exajoule of the energy source,

GWP_i is the global warming potential,

$TotDem_i$ is the oil-specific total demand,

$PRconvrt_{fn}$ is the energy conversion factor, which converts the relative price in monetary units to physical energy units; the factor of 0.97 accounts for the fact that not all oil products are combusted, *e.g.*, tar and lubricants; *PRconvert* for each supply sector in each region is expressed in exajoules per million 1990 regional currency, and

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Trade$_{0,i=9}$ is either a fixed trade amounts (net exports) or calculated as the difference between production and the summed domestic demand.

Emissions for **gas** and **coal** combustion are calculated similarly, but for the 0.97 factor:

$$\text{EmissionsC}_{j=4=\text{gas}} = \text{EMC}_i \bullet \text{GWP}_{i=4} \bullet$$
$$\left(\text{PRconvrt}_{\text{fn}=4} \bullet \text{TotDem}_{i=4} - \text{PRconvrt}_{\text{fn}=10} \bullet \text{Trade}_{0,i=10=\text{Gas}-\text{T\&D}}\right)$$

Eq. 245

where

    EMC$_i$ is the gas-specific carbon content in million tons C per exajoule of the energy source,

    GWP$_i$ is the global warming potential,

    TotDem$_i$ is the gas-specific total demand,

    PRconvrt$_i$ is the energy conversion factor, which converts the relative price in monetary units to physical energy units, and

    Trade$_{0,i=10}$ is a fixed trade amount (net exports) or calculated as the difference between production and the summed domestic demand.[45]

$$\text{EmissionsC}_{j=5=\text{coal}} = \text{EMC}_5 \bullet \text{GWP}_{i=5} \bullet \left(\text{PRconvrt}_{\text{fn}=5} \bullet \text{TotDem}_{i=5}\right)$$

Eq. 246

where

    EMC$_5$ is the coal-specific carbon content in million tons Carbon per exajoule of the energy source,

    GWP$_i$ is the global warming potential,

    TotDem$_i$ is the coal-specific total demand, and

    PRconvrt$_i$ is the energy conversion factor, which converts the relative price in monetary units to physical energy units.

The summed CO_2 emissions are then

$$\text{EmTot}_t = \sum_{j=3}^{5} \text{EmissionsC}_i$$

Eq. 247

Other greenhouse gas emissions and non-greenhouse gases

All other potential emissions (*CH₄, N₂O, HFC, PFC, and SF6*) are calculated based on "emission activities". "Emission activity" or *EmAct$_{ix}$* is a term used to describe the potential of emissions to be generated during a production process either as an industrial or an energy production process, or in agriculture. Emission activities are directly related to demand. Thus,

$$\text{EmAct}_{ix} = \text{PRconvrt}_{ix} \bullet \text{TotDem}_{ix}$$

Eq. 248

where

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45 In the code the calculations are made twice: in the energy balance subroutine and in the GHG emissions subroutines; in the energy balance subroutine oil emissions are calculated by multiplying with the 0.97 factor, which accounts for the fact that some oil products are not combusted, *e.g.*, tar, lubricants, etc.; in the GHG emission subroutine emissions are calculated without that factor.

ix is mapped on the supply sector index i,

$PRconvrt_i$ is the emission activity energy conversion factor, which converts the relative price in monetary units to physical energy units, and

$TotDem_{ix}$ is emission activity demand (see Equation 243).

With correct mapping of *EMC*, *GWP* and *PRconvrt* parameters towards the emission activities, emissions of non- greenhouse gases and other than CO_2 greenhouse gases can be calculated. Total demand for each region accounts for trade. Thus, carbon-equivalent emissions are calculated based on a production sector's emission activity, which is the product of the total demands by a production sector and a conversion factor that converts monetary units to energy units (see Equation 248), multiplied by the emission activities' emission coefficient *EMC* and the emissions' global warming potential coefficient *GWP*:

$$\text{Emissions}_j = \text{EMC}_i \bullet \text{GWP}_i \bullet \left(\text{PRconvrt}_i \bullet \text{TotDem}_i \right)$$
<div align="right">Eq. 249</div>

If these emissions are greenhouse gases, they can be added to the total emissions. If they are non-greenhouse gases they are kept separate.

Emissions of gases, like *CO, NO_X, SO_X, VOC, PM10* and emissions of black carbon can potentially be simulated in a similar fashion if and when necessary data are available linking production processes and related emissions.

Mitigation and marginal abatement cost curves

The SGM has options of calculating consequences of mitigation policies. For carbon dioxide and carbon-equivalent emissions we have obtained marginal abatement cost curves (MACCs)[46]. These marginal abatement curves are generated exogenously, based on the relationships between carbon prices and the percents reduction in emissions, which are specific to each source and type of emission. Cost curves are defined for a variable number of points relating the carbon price (level) to a corresponding percent (fraction) reduction in emissions. The model finds the percentage reduction in emissions through linear interpolations between carbon prices. Thus, when testing the impacts of carbon prices, the SGM solves for the emission levels. Alternatively, when setting carbon dioxide emission limits or carbon-equivalent emission limits as model input, the SGM market calculates the carbon fees. Obtained revenue, in both cases, can be recycled to government, households and/or industry. These options are explained in "Carbon policies" and summarized in Appendix A. Table 5 lists the relationships between emission sources, the drivers and the control options.

Table 5 Emissions sources, drivers and control options

Activity	Emission	Driver	Mitigation option
Oil Combustion	CO_2	Total Combustion	Endogenous
Gas Combustion	CO_2	Total Combustion	Endogenous
Coal Combustion	CO_2	Total Combustion	Endogenous
Coal Production	CH_4	Coal	MAC

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[46] DeAngelo et al. (2004); Delhotal et al. (2004); Schaefer et al. (2004); Scheele and Kruger (2004)

Enteric	CH$_4$	Ag Production	MAC
NatGasSys	CH$_4$	Gas Trans & Dist	MAC
OilSys	CH$_4$	Oil Production	MAC
Manure	CH$_4$	Ag Production	MAC
OthAgMeth	CH$_4$	Ag Production	MAC
ManureN	N$_2$O	Ag Production	MAC
SoilN	N$_2$O	Ag Production	MAC
Landfills	CH$_4$	Everything Else	MAC
OthNonAgMeth	CH$_4$	Everything Else	MAC
Wastewater	CH$_4$	Everything Else	MAC
HFC23	HFC-23	Everything Else	MAC
ODSSub	HFC's	Everything Else	MAC
IndProcsN	N$_2$O	Everything Else	MAC
MobileN	N$_2$O	Everything Else	MAC
StationaryN	N$_2$O	Everything Else	MAC
Aluminum	PFC's	Everything Else	MAC
Semiconductor	PFC's	Everything Else	MAC
Mg	SF$_6$	Everything Else	MAC
ElecDist	SF$_6$	Electricity	MAC
Soil and Forest Sequestration	CO$_2$	NA	Sequestration function[47]

The SGM modeling team has experimented with regard to joint product production in the gas production sector (methane and natural gas). Emission mitigation technology involves multiple sectors in this case. The level of mitigation might be determined by an expected profit rate calculation done on the margins between mitigation levels provided (Mark Jacobsen 6/12/00 & 7/13/00). The energy sectors are made to bear the mitigation costs. In the joint-product mitigation case only activities in the left column (see lists below) are mitigated and the ones in the right column are not mitigated. The fact that the industrial processes and landfills are not undergoing mitigation in this case drives the price for mitigating other sectors up when emission constraints are severe (see Figure 23 in Chapter thirteen).

Mitigated activities **No mitigation**
Oilcomb   Landfills
Gascomb   OthNonAgMeth
Coalcomb   Wastewater
CoalPr   HFC23
Enteric   ODSSub
NatGasSys   IndProcsN
OilSys   MobileN
Manure   StationaryN

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47 In SGM 2004 sequestration offsets are an option based data thanks to personal communication with Bruce A. McCarl : mccarl@tamu.edu

OthAgMeth   Aluminum
ManureN   Semiconductor
SoilN   Mg
ElecDist

Display of results

For carbon dioxide and carbon-equivalent emissions, we usually display the results as marginal abatement cost curves (MACCs). To calculate marginal abatement curves as output, the simplest method is to obtain a reference case through calibration without carbon fees imposed, and then setting carbon permit fees exogenously and evaluating the results with regard to GHG emissions. The marginal abatement curves can be generated by plotting emissions over time for each of the imposed carbon fees, *e.g.* Figure 14. Alternative ways of displaying results are shown in Chapter thirteen.
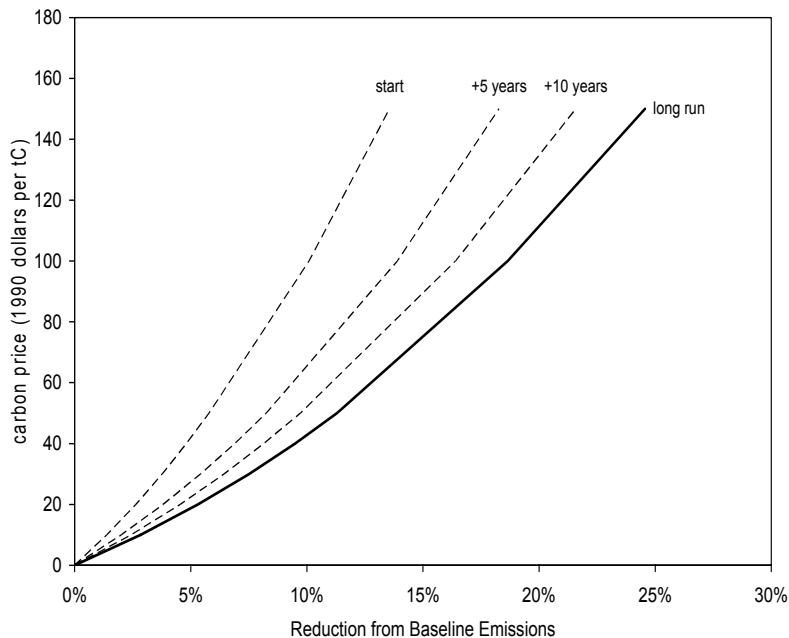
Figure 14 Marginal abatement cost curves for the United States energy system generated with a series of constant-carbon-price experiments.

Chapter 13. Reference case results

Projections/Validation/Calibration for the USA

This chapter contains reference case results and briefly reviews calibration and validation. In addition, selected results of carbon policy implementations are presented. The simulation results are not analyzed in depth.

The first step in baseline calibration is to match GNP projections to an exogenous source (usually the Annual Energy Outlook) by adjusting the autonomous labor efficiency improvement parameter. GNP projections obtained from the SGM simulations can also be evaluated against historical data for validation. Results for the reference case for the US are shown in Figure 15.

**Historical and Projected Normalized GNP
(against 1990 values)**
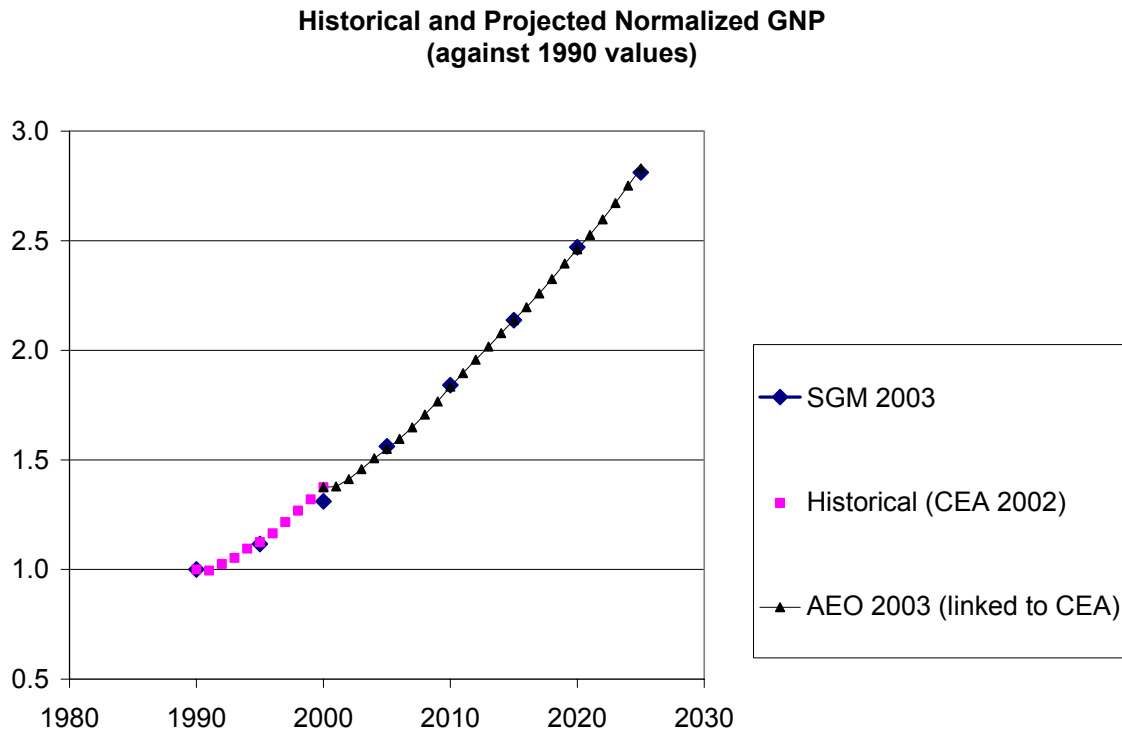
Figure 15 Historical and projected normalized GNP

A set of curves can be constructed by running a series of constant-carbon-price experiments in the SGM, *e.g.*, with a carbon price introduced in the 2005 time step and held constant thereafter. CO_2 emissions in units of million tons of carbon in the reference case and as response to carbon fees are shown in Figure 16.

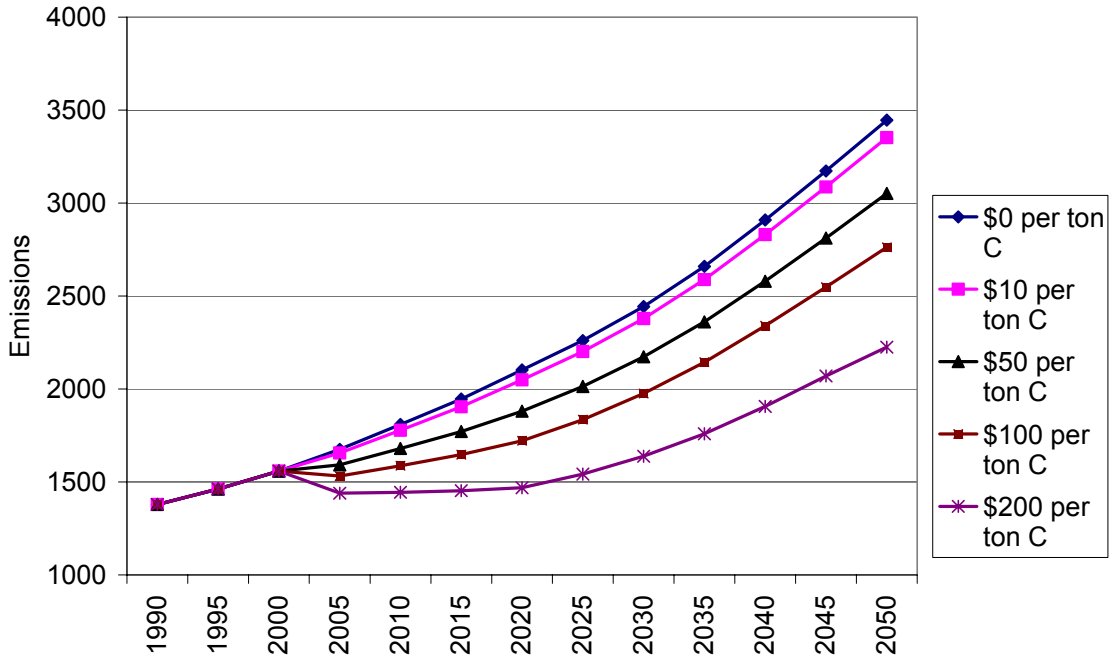**Response of Carbon Dioxide Emissions to Carbon Prices**

Figure 16 CO_2 emissions in units of million tons of carbon in the reference case with a zero carbon fee and the responses of CO_2 emissions to carbon fees of $10, $50, $100, and $200 per ton carbon

The SGM has the option tracking GHG emissions from processes in the agricultural sector, the Everything Else sector, from the electricity generation and distribution sector, and from the distributed gas production sector (see Table 65). In that case, not only carbon dioxide emissions are calculated, but also methane, nitrous oxide, HFCs, PFCs and SF6 emissions in units of carbon-equivalents. Carbon-equivalent emissions are calculated based on a production sector's emission activity, which is the product of the total demands by a production sector and a conversion factor that converts monetary units to energy units, multiplied by the emission activities' emission coefficient (*EMC*) and the emissions' global warming potential coefficient (*GWP*) (see Equation 249).

Figure 17 illustrates the response of the total of carbon-equivalent (CE) emissions to a carbon fee of $100 per ton carbon compared to the response to a carbon fee of $100 per ton carbon when only CO_2 emissions are impacted.

Figure 18 illustrates the primary fossil fuel-specific carbon dioxide emission responses to a carbon price of $100 per ton of carbon. Coal emissions respond most strongly, followed by emissions generated from oil combustion; emissions due to gas combustion are least affected.

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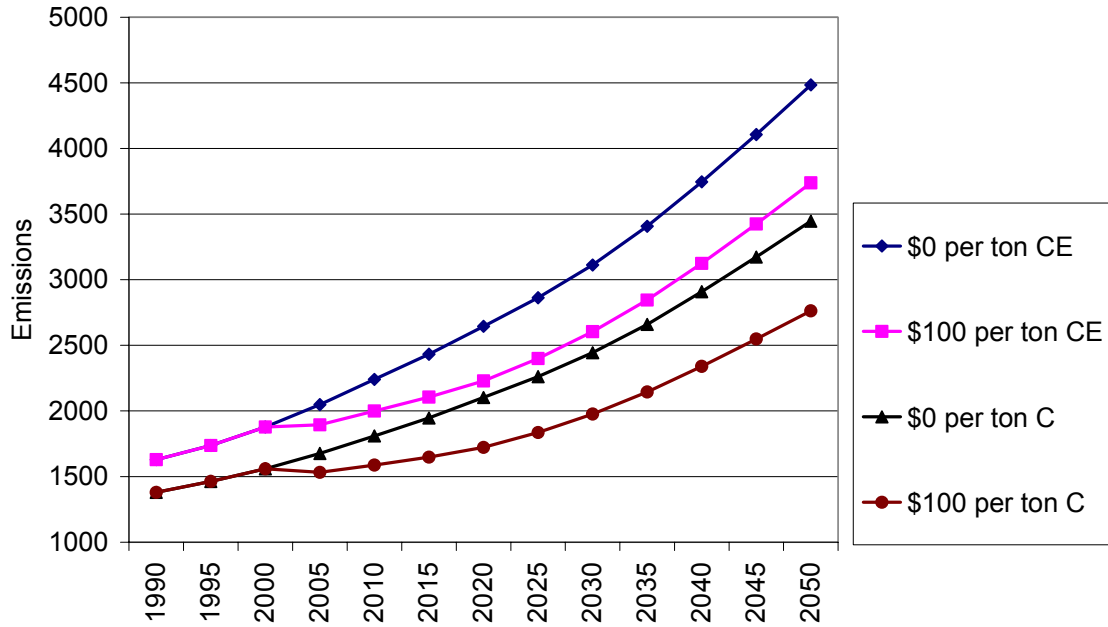**Response of Carbon Dioxide and Carbon-Equivalent Emissions to Carbon Prices**

Figure 17 Response of CO_2 and carbon-equivalent emissions in units of million tons of carbon to carbon fees of $100 per ton carbon

**Response of Carbon Dioxide Emissions from Oil, Coal, and Gas
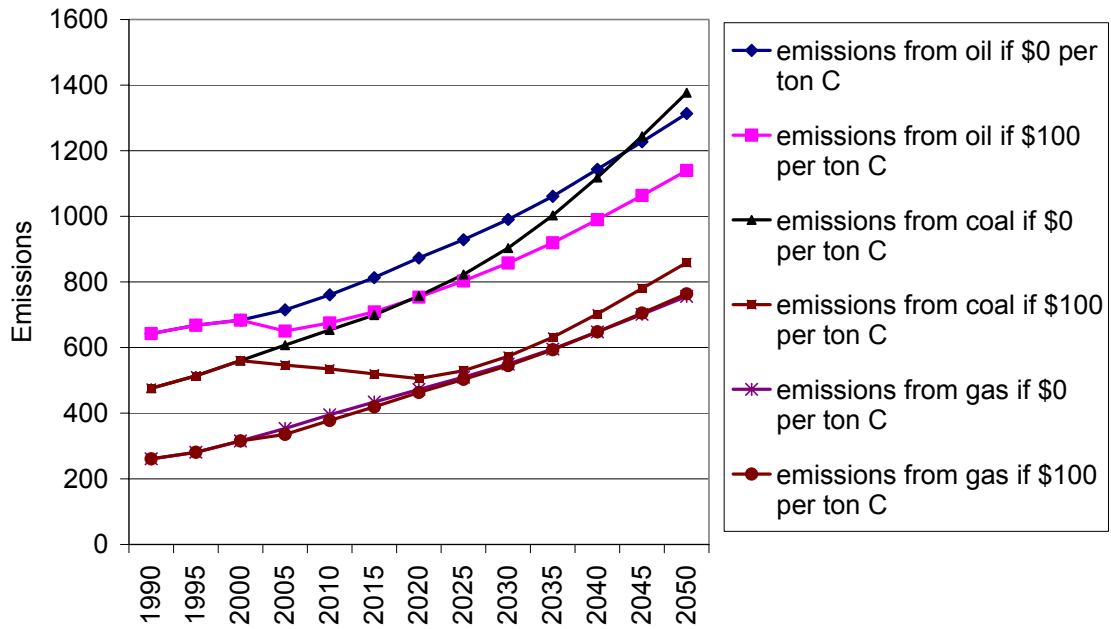Combustion to Carbon Fees**

Figure 18 Response of CO_2 emissions in units of million tons of carbon from oil, gas, and coal combustion to carbon fees of zero versus $100 per ton carbon

Figure 19 illustrates the same for all carbon-equivalent emissions from the energy producing sectors.

Response of Carbon-Equivalent Emissions by Various Processes to Carbon-Equivalent Prices
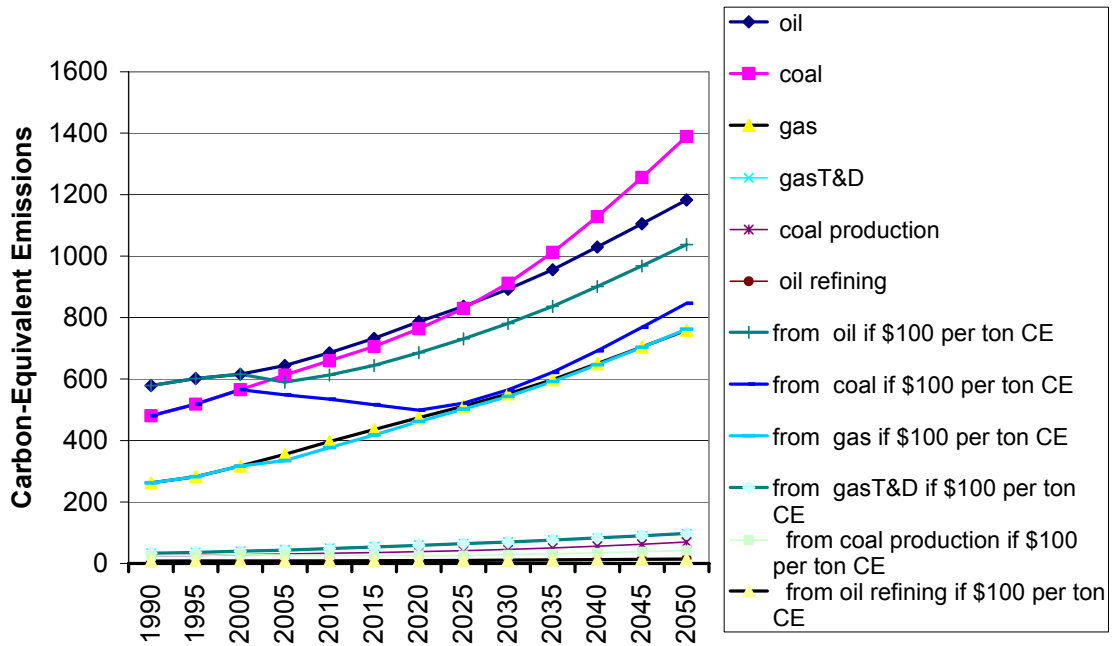

Figure 19 Response of carbon-equivalent emissions in units of million tons of carbon from energy production, energy transformation, agricultural and other production processes to carbon fees of zero versus $100 per ton carbon

When agricultural practices respond to carbon prices, the largest response is due to nitrous oxide emissions from fertilizer application, followed by methane emissions from enteric fermentation, followed by manure management. In addition landfill management impacts overall carbon equivalent emissions greatly (see Figure 20). Figure 21 shows the responses of the various industrial processes to carbon prices.
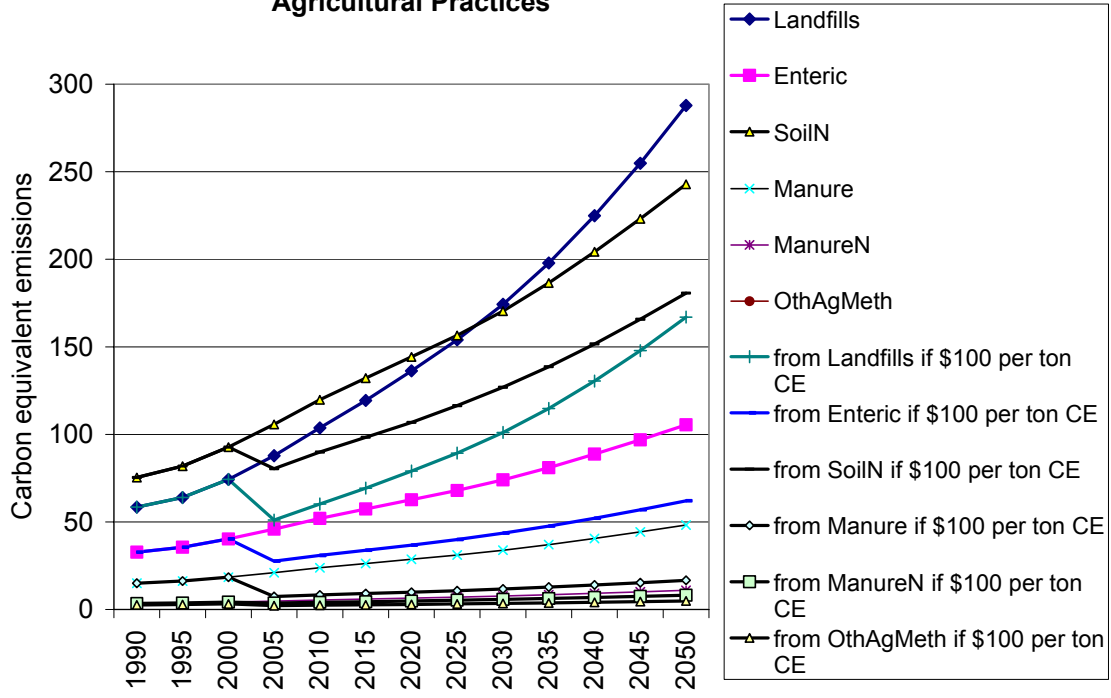
Figure 20 Response of carbon-equivalent emissions in units of million tons of carbon from agricultural processes to carbon fees of zero versus $100 per ton carbon

Response to Carbon Prices of Carbon-Equivalent Emissions from Various Industrial Processes
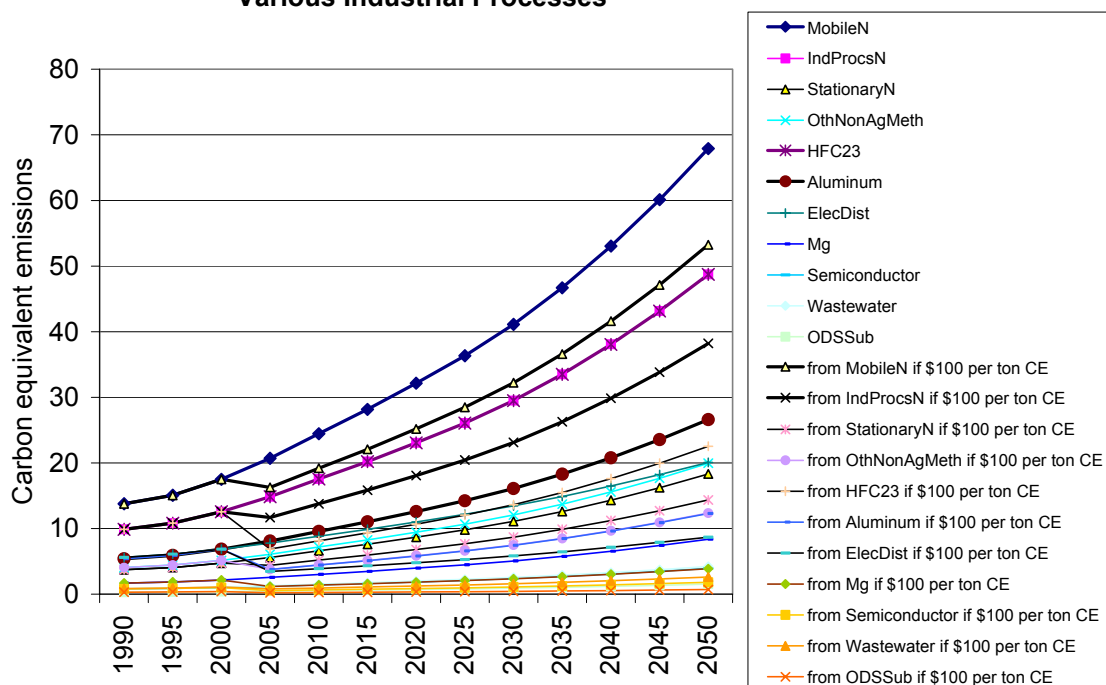

Figure 21 Carbon-equivalent emissions in units of million tons of carbon from industrial processes responding to carbon fees

The SGM has the option of setting emission limits and letting the model solve for the carbon fee that would need to be imposed to reach that goal. Figure 22 illustrates stabilization of GHG emissions. Figure 23 illustrates the carbon fees (market prices of carbon) calculated.

To meet desired carbon emission targets, three options can be exercised: (1) reduce carbon dioxide emissions from primary fuel combustion, (2) reduce emissions through mitigation of all carbon-equivalent emission processes, (3) reduce emissions through mitigation of a limited number of carbon-equivalent emission processes, that is from energy production processes and agricultural processes only (see list below).

Carbon prices respond accordingly. Figure 23 shows all three responses. The high carbon price is paired with carbon-equivalent emissions that exclude those of landfills and industrial processes such that the burden of emission reductions falls on the energy sector and the remaining agricultural processes. When the carbon-equivalent emission reduction burden is spread over all carbon-equivalent emissions the resulting carbon price is very similar to when only carbon dioxide emissions are targeted. Note that after the year 2045 emission restraints were allowed to be relaxed resulting in an immediate decrease of the carbon price.

Mitigated activities	No mitigation
Oilcomb	Landfills
Gascomb	OthNonAgMeth
Coalcomb	Wastewater
CoalPr	HFC23
Enteric	ODSSub
NatGasSys	IndProcsN
OilSys	MobileN
Manure	StationaryN
OthAgMeth	Aluminum
ManureN	Semiconductor
SoilN	Mg
ElecDist	

**Attaining Emission Stabilization Levels of Carbon Dioxide and Carbon·
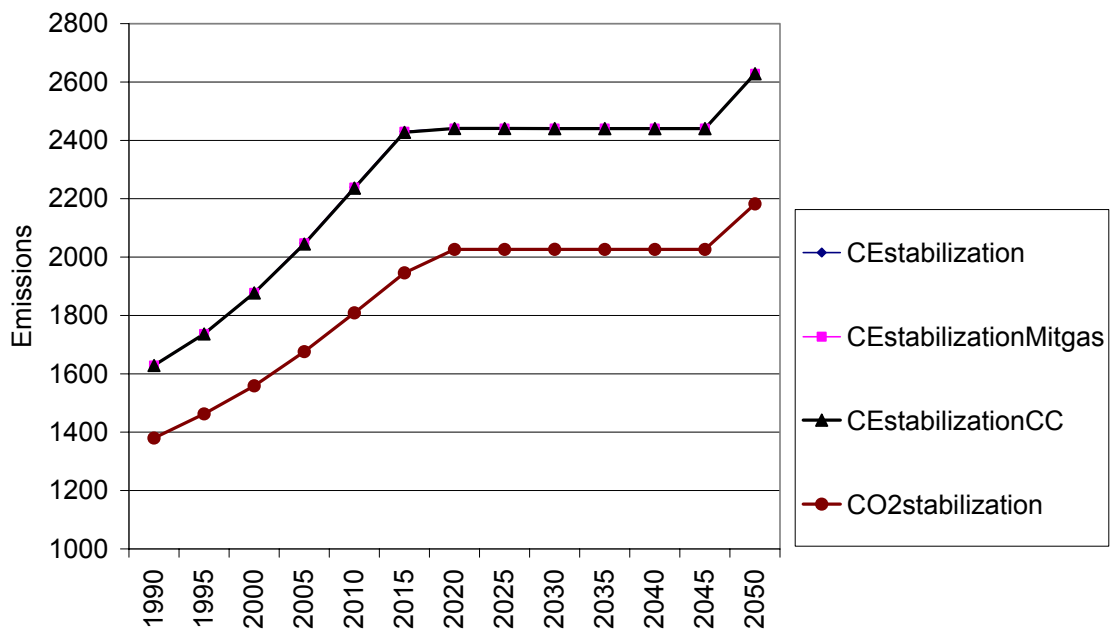Equivalent Emissions**

Figure 22 Emission stabilization levels in units of million tons of carbon set exogenously and reached as model output

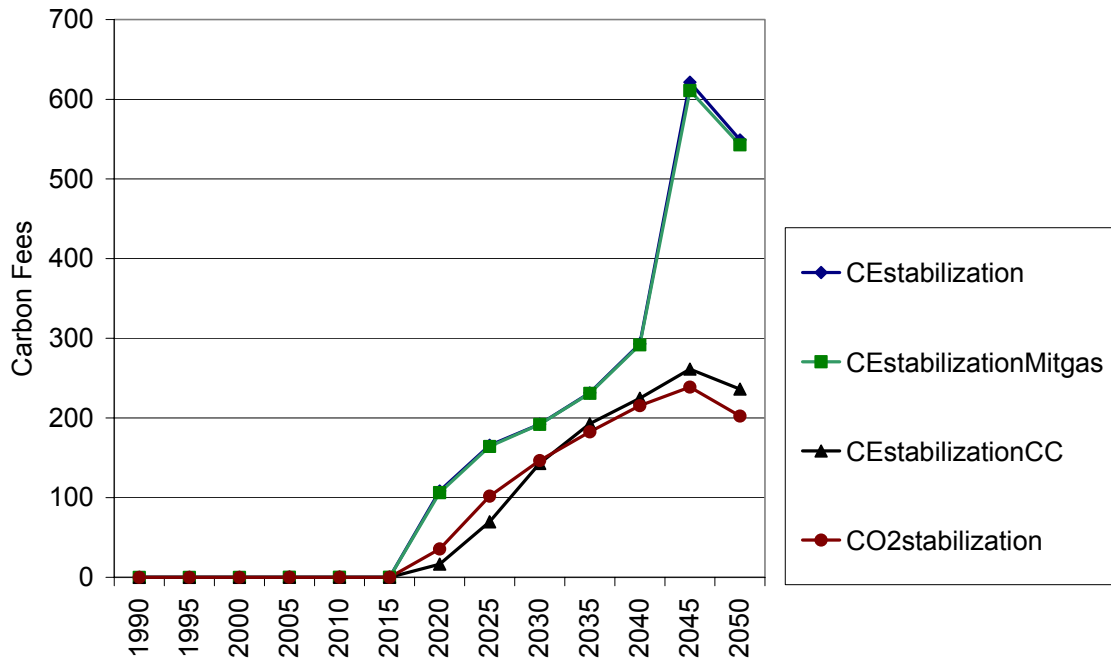**Market Prices for Carbon to Attain Emission Stabilization Levels**

Figure 23 Market prices for carbon when exogenously set emission stabilization levels are attained. Note that the steep curve is for the carbon price when only energy production and agricultural carbon-equivalent emissions are mitigated but for landfills; no industrial emissions are mitigated.

Figure 24 illustrates the various emission pathways resulting from the carbon-equivalent mitigation strategies for the energy-production sectors. Figure 25 illustrates the emission pathways resulting from the carbon-equivalent mitigation strategies for the agricultural processes

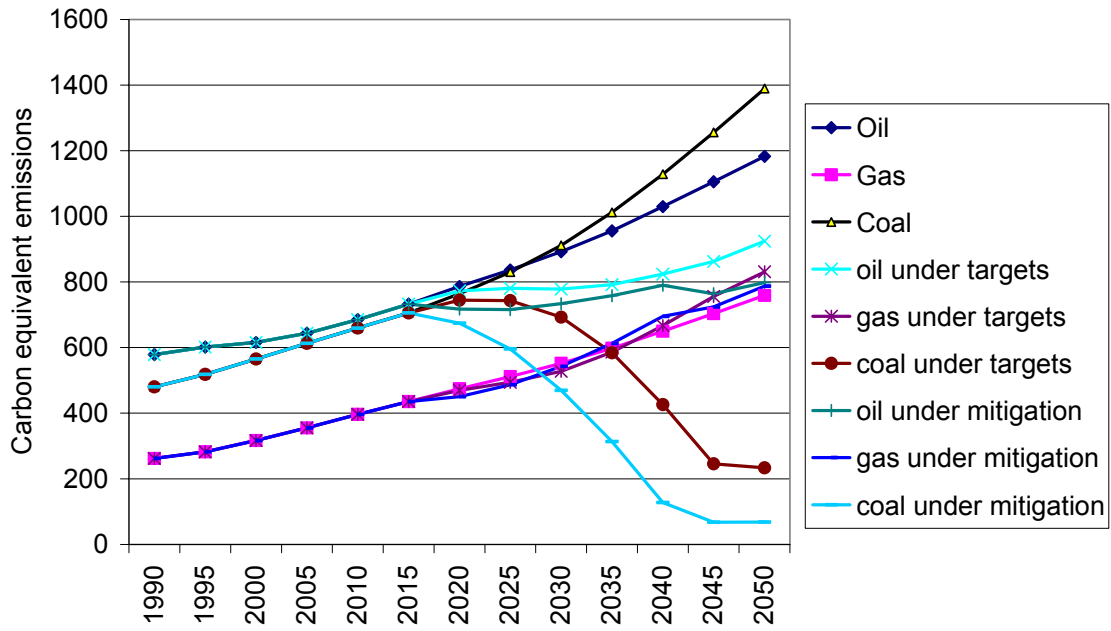**Pathways of Mitigation Through Inclusion or Exclusion of Various Sectors**

Figure 24 Emission pathways due to the energy sectors when carbon-equivalent emission limits are imposed

Response of Carbon-Equivalent Emissions from Agricultural Practices to Carbon Prices
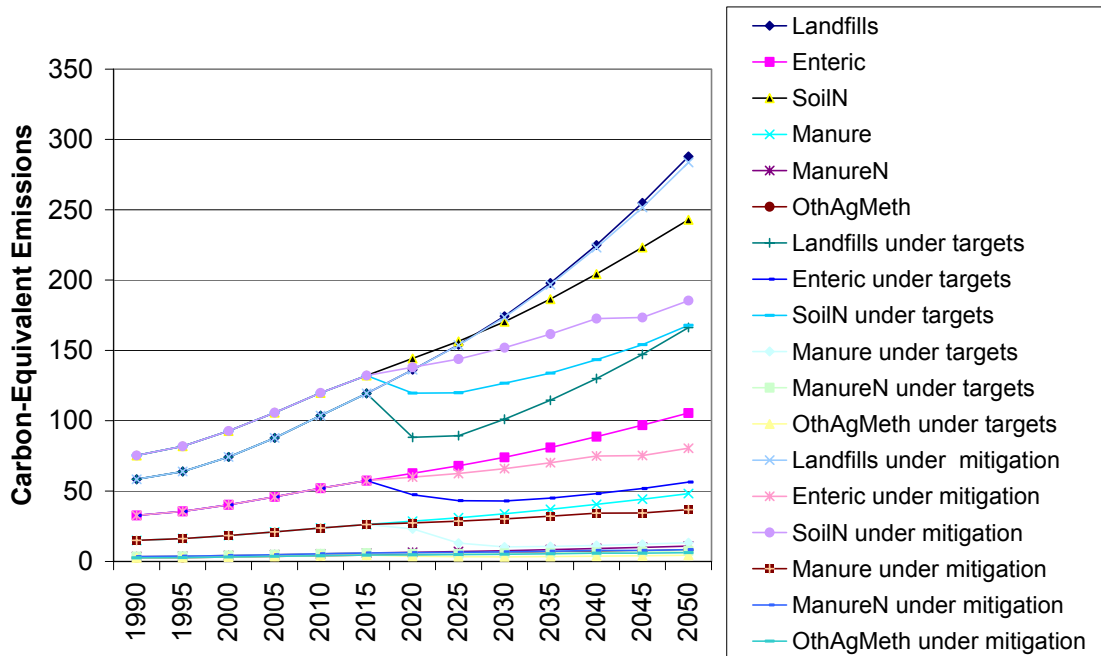

Figure 25 Emission pathways due to the agricultural sector when carbon-equivalent emission limits are imposed

Thus, when mitigation is implemented in the energy sectors and some of the agricultural sectors and not in landfill management nor in the other sectors, we find the energy sectors and the mitigation-active agricultural sectors much more impacted compared to when all mitigation options are activated. A considerably higher carbon price results.

Technical scale coefficients ($\alpha_{i,j,v}$) that relate "inputs to production" and the "production process" are partially determined by the elasticities of substitution in the production equations and partially dependent on the market prices resulting from solving for excess demand close to zero in the model solution process (see section on "Technical change"). Figures 26 and 27 illustrate both aspects for $\alpha_{i=1,j=1}$ − that is, for the technical scale coefficient that relate agricultural input to agricultural production. In the reference case, the technical scale coefficients are vintage dependent. When carbon prices change over time due to imposed carbon emission limits, the technical scale coefficients are affected due to changes in prices paid. Note that when emission limits are relaxed from year 2045 to 2050, responses can be seen in the technical scale coefficients as illustrated in Figure 27.

**The Technical Scale Coefficients over Time That Relate Agricultural
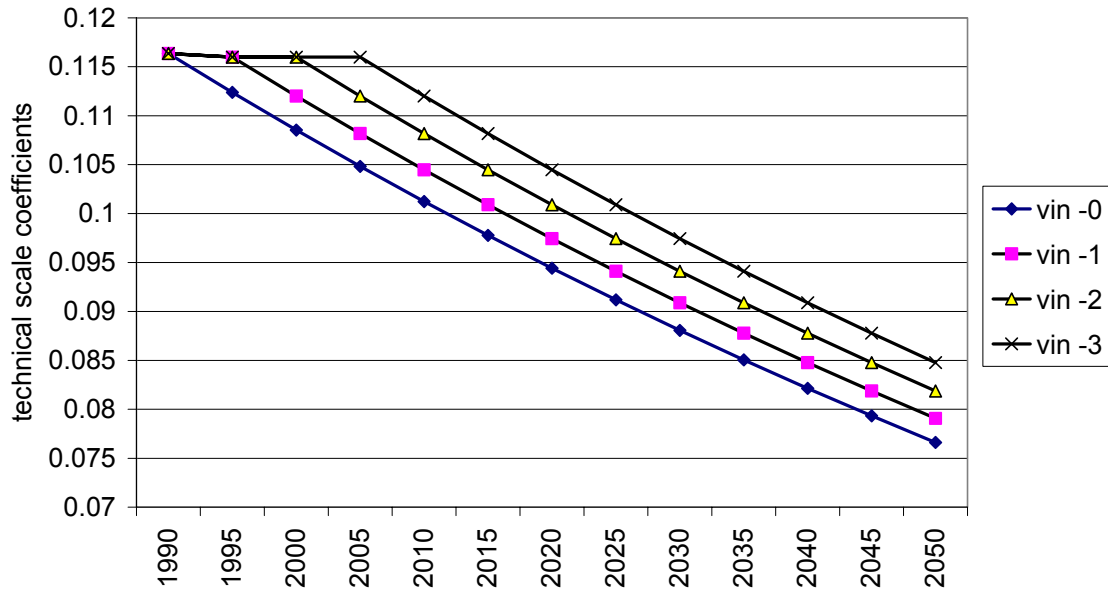Input to Production in the Reference Case**

Figure 26 The vintage-specific (vin-3 through vin-0) technical scale coefficients over time that
relate the agricultural input to production in the reference case

The Technical Scale Coefficients When Mitigation Is Applied for All Emitting Sectors or for Industry Only
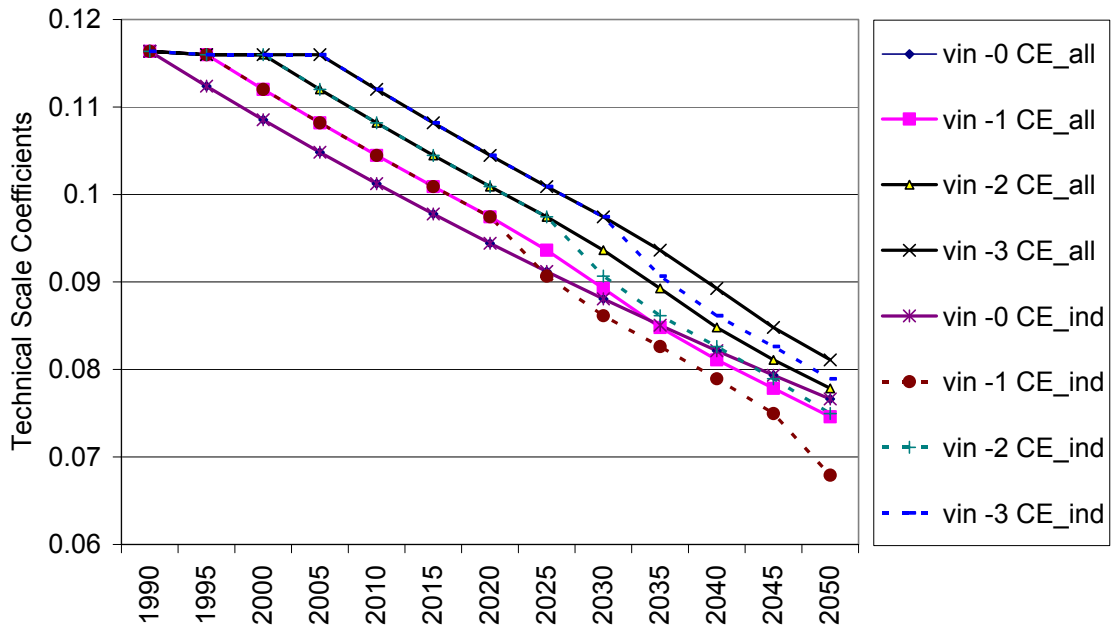

Figure 27 The vintage-specific technical scale coefficients (vin-3 through vin-0) over time that relate the agricultural input to production. The vintage-specific technical scale coefficients are shown (1) when all sectors are impacted by a carbon price and all carbon-equivalent emissions are mitigated (CE_all) and (2) when all sectors are impacted by a carbon price and the energy production sectors and a limited number of agricultural processes are mitigated (CE_ind) (no landfill mitigation and no industrial processes mitigation).

The second step in the calibration process is to match projections of electricity generation, in units of billion kilowatt hours, by adjusting an autonomous electricity efficiency improvement parameter in all model activities that use electricity. The results, shown in Figure 28 illustrate a combination of validation against historical data and the results of calibration of projections in model-to-model output comparisons.

Historical and Projected Electricity Generation in the U.S.A.
(bilion KWh)
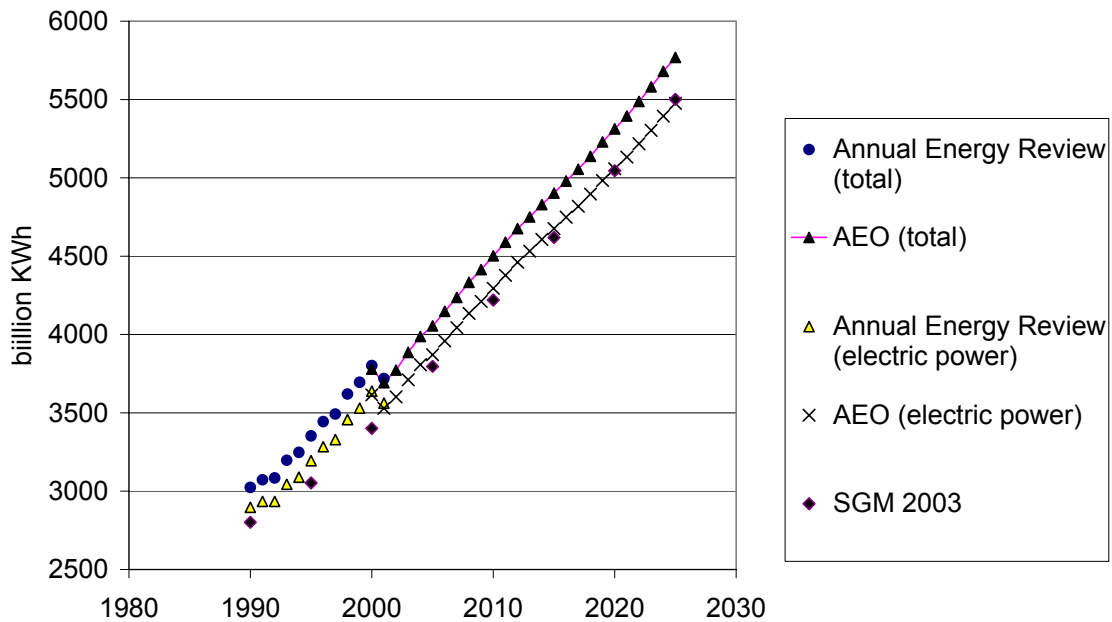
Figure 28 Historical and projected electricity generation in the USA. in billion kilowatt hours

The total amount of CO_2 emissions generated by the electricity production sector is directly related to the total amount of electricity generated, but more specifically by what fuels are combusted. The mix of fossil fuels within electricity generation is adjusted by varying the time path of the cost to produce electricity using oil, natural gas, or coal in the calibration process. Consequences of these adjustments manifest themselves not only in the total amount of CO_2 emitted but also in the fuel-specific emission balance (see Figure 29). Moreover, when carbon fees are imposed, this emission balance is affected also. Figures 30 and 31 illustrate the CO_2 emissions over time from the combustion of fossil fuels for electricity generation for the reference case and for when $100 per ton carbon fees are imposed.

Historical and Projected Carbon Dioxide Emissions from the Combustion of Fossil Fuels for Electricity Generation

Figure 29 Historical and projected CO_2 emissions in units of million tons of carbon from the combustion of fossil fuels from electricity generation

Carbon Dioxide Emissions From the Combustion of Fossil Fuels for Electricity Generation When No Carbon Fees Are Imposed

Figure 30 CO_2 emissions in units of million tons of carbon from the combustion of fossil fuels from electricity generation

Carbon Dioxide Emissions From the Combustion of Fossil Fuels for Electricity Generation When Carbon Fees Are Imposed of $100 per Ton Carbon
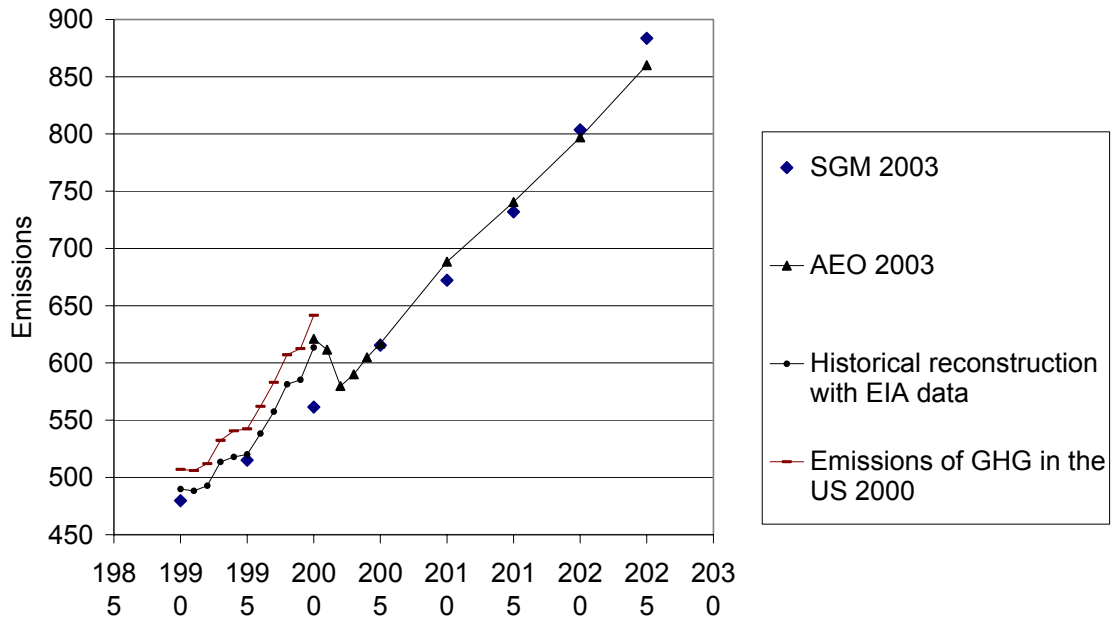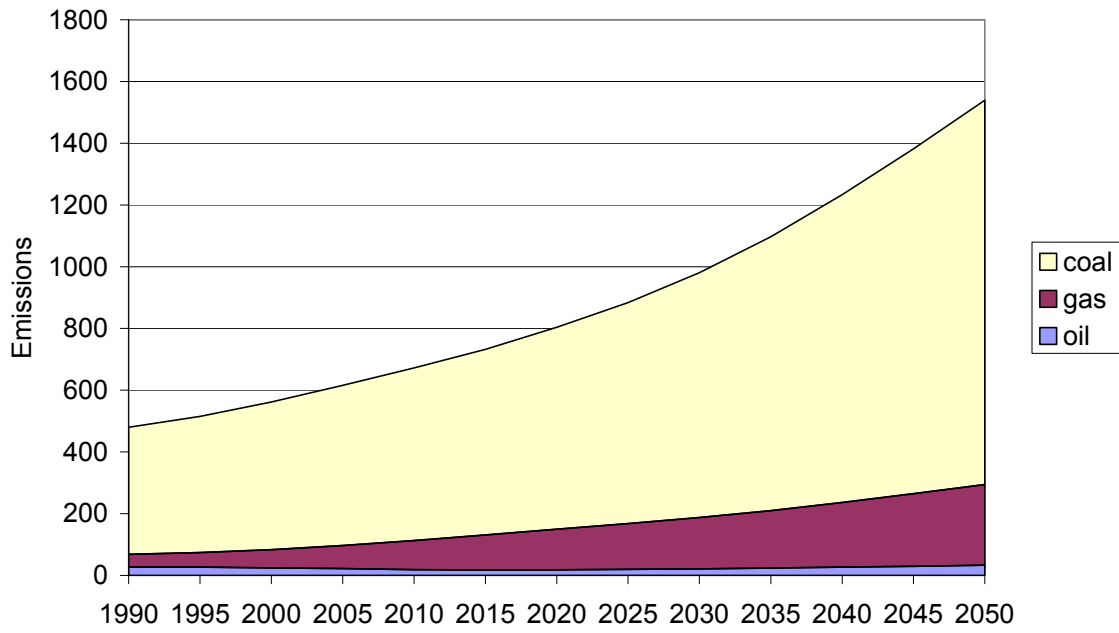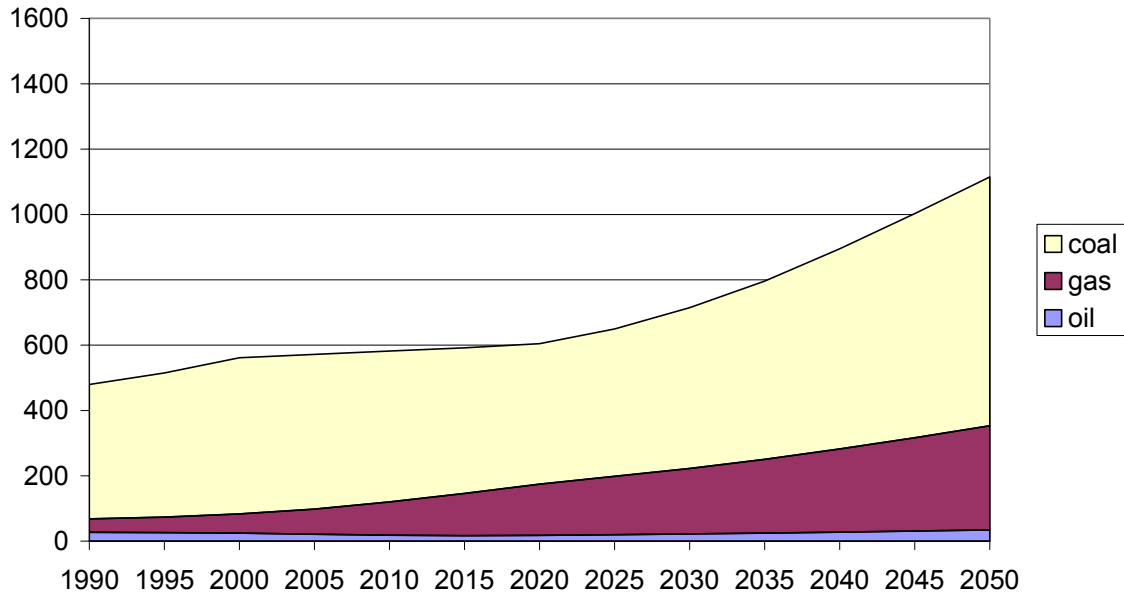

Figure 31 CO_2 emissions in units of million tons of carbon from the combustion of fossil fuels from electricity generation when carbon fees are imposed of $100 per ton carbon

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List of Equations

$$XS_{i,v} = \sum_{j=1}^{22}\left[\sum_{jj=1}^{n}X_{i,j,jj,v}\right] \qquad \text{Eq. 1}$$

$$XS_{j,v} = \sum_{i=1}^{26}\left[\sum_{jj=1}^{n}X_{i,j,jj,v}\right] + X_{i=27,j,jj,v} \qquad \text{Eq. 2}$$

$$q_{j,jj,v} = \alpha_{0,j,jj,v}\left[\sum_{i=1}^{N-1}\alpha_{i,j,jj,v}X_{i,j,jj,v}{}^{\rho} + \alpha_{i=26,j,jj,v}X_{i=26,j,jj,v}{}^{\rho}\right]^{1/\rho} \qquad \text{Eq. 3}$$

$$0 \le \sigma_{j,v=old} \le \sigma_{j,v=new} < 1 \qquad \text{Eq. 4}$$

$$q_{j,jj,v} = \lambda_{0,j,jj,v}\min\left(\frac{X_{1,j,jj,v}}{\lambda_{1,j,jj,v}},\dots,\frac{X_{i,j,jj,v}}{\lambda_{i,j,jj,v}}\dots\frac{X_{N,j,jj,v}}{\lambda_{N,j,jj,v}}\right) \qquad \text{Eq. 5}$$

$$P_{i=24=ls,t} = \frac{XS_{i=24=ls}}{ED_{i=24=ls,j=27=hh}} \qquad \text{Eq. 6}$$

$$P_{i=25=lbs,t} = \frac{XS_{i=25=lbs}}{ED_{i=25=lbs,j=27=hh}} \qquad \text{Eq. 7}$$

$$Pr_{j,t=0} = \frac{\sum_{i=1}^{N+1}X_{i,j,jj,v} - X_{i=N+1,j,jj,v}}{\sum_{i=1}^{N+1}X_{i,j,jj,v}} \qquad \text{Eq. 8}$$

$$Pi_{i,j,jj,t} = adj_{i,j,jj,t} \bullet \left(P_{i,t} \bullet Ex\,Im_i \bullet Txpro_{i,j,jj} + Txadd_{i,j,jj} + Cpf_i\right) \qquad \text{Eq. 9}$$

$$Cpf_i = P_{i=23,t} \bullet ExchRate_{region} \bullet \sum_{fn}EMC_{fn} \bullet PRconvrt_i \bullet GWP_{fn} \qquad \text{Eq. 10}$$

$$adj_{i,j,jj,t} = Ad_{i,j,jj} \bullet (1 - \frac{T}{Tadjz_{i,j,jj}}) + Adjz_{i,j,jj} \bullet \frac{T}{Tadjz_{i,j,jj}} \qquad \text{Eq. 11}$$

$$Pi_{i=capital,j,jj,t} = wedge_{i=capital,j,jj,t} + P_{i=26} \qquad \text{Eq. 12}$$

$$Pr_{j,t} = \frac{P_{i,t} + Tr_{j,t} \bullet Ex\,Im_j}{1 + TxIBT_j} \qquad \text{Eq. 13}$$

$$Tr_{j,t} = Tri_j \bullet (1 - \frac{T}{Ttrz_j}) + Trz_j \bullet \frac{T}{Ttrz_j} \qquad \text{Eq. 14}$$

$$Pie_{i,j,jj} = Pi_{i,j,jj,t} \bullet \left(\frac{1}{\left(1 + Pi_{i=26,j,jj,t}\right)} \right)^{0.5} \bullet \left(1 + \frac{1}{Pi_{i=26,j,jj,t}} \right) \bullet \left(1 - \left(\frac{1}{\left(1 + Pi_{i=26,j,jj,t}\right)} \right)^{T_{exp}} \right) \qquad \text{Eq. 15}$$

$$Pe_{i,j,jj} = Pr_{j,t} \bullet \left(\frac{1}{\left(1 + Pi_{i=26,j,jj,t}\right)} \right)^{0.5} \bullet \left(1 + \frac{1}{Pi_{i=26,j,jj,t}} \right) \bullet \left(1 - \left(\frac{1}{\left(1 + Pi_{i=26,j,jj,t}\right)} \right)^{T_{exp}} \right) \qquad \text{Eq. 16}$$

$$a_{i,j} = \frac{\lambda_{i,j}}{\lambda_{0,j}} \qquad \text{Eq. 17}$$

$$a_{i,j}(p) = \left[\alpha_{0,j} \bullet \alpha_{i,j} \right]^{\rho/(1-\rho)} \bullet \left[\frac{p_j}{p_i} \right]^{1/(\rho-1)} \qquad \text{Eq. 18}$$

$$Factor_{i,j,jj,t} = Factor_{i,j,jj,t-1} \bullet (1 + TECHNN)^{Nstep} \qquad \text{(unitless) Eq. 19}$$

$$Factor_{i,hh,t} = (1 + HHAEEI_{i,hh})^{Nstep} \qquad \text{(unitless) Eq. 20}$$

$$Factor_{i,gv,t} = (1 + GVAEEI_{i,gv})^{Nstep} \qquad \text{(unitless) Eq. 21}$$

$$\alpha_{i,j,jj,v} = \left(\frac{X_{i,j,jj,v} / P_{i,t}}{X_{i=ETE,j,jj,v} / P_{i=ETE,t}} \right)^{\frac{\rho}{\mu}} \bullet \frac{P_{i,t}}{P_{i=ETE,t}} \qquad \text{Eq. 22}$$

$$\alpha_{0,j,jj,v} = \left(\frac{P_{0,j,jj}}{P_{i=ETE,t}} \right)^{-1/\rho} \bullet \left(\sum_{i=1}^{N+1} X_{i,j,jj,v} \right)^{-1/\mu} \bullet \left(\frac{X_{i=ETE,j,jj,v}}{P_{i=ETE,t}} \right)^{1/\mu} \qquad \text{Eq. 23}$$

$$P_{0,j,jj} = \frac{\displaystyle\sum_{i=1}^{N+1} X_{i,j,jj,v} - X_{i=N+1,j,jj,v}}{\displaystyle\sum_{i=1}^{N+1} X_{i,j,jj,v}} \qquad \text{Eq. 24}$$

$$\alpha_{i=26,j,jj,v} = \left(\frac{\displaystyle\sum_{i=1}^{N+1} X_{i,j,jj,v}}{\alpha_{0,j,jj,v}} \bullet \frac{1}{\displaystyle\sum_{v=-3}^{0} KAprior_{j,jj,v}} \right)^{\rho} \bullet Z_{j,jj,v} \qquad \text{Eq. 25}$$

$$Z_{j,jj,v} = 1 - (\alpha_{0,j,jj,v} P_{0,j,jj})^{\mu} \bullet \sum_{i=1}^{N-1} \left(P_{i,t} \bullet \frac{\alpha_{i,j,jj,v}}{\alpha_{i=ETE,j,jj,v}} \right) \qquad \text{Eq. 26}$$

$$\alpha_{i,j,jj,v} = \frac{\alpha_{i,j,jj,v}}{\displaystyle\sum_{i=0}^{N=26} \alpha_{i,j,jj,v}} \qquad \text{for i=1:26 and j=1:22} \qquad \text{Eq. 27}$$

$$\alpha_{0,j,jj,v} = \left(\frac{\alpha_{0,j,jj,v}}{\sum_{i=0}^{N=26} \alpha_{i,j,jj,v}} \right)^{1/\rho} \qquad \text{Eq. 28}$$

$$\alpha_{i,j,jj,v} = \alpha_{i,j,jj,v-1} \bullet \text{Factor}_{i,j,jj,t}{}^{\rho} \qquad \text{Eq. 29}$$

$$\alpha_{i,j,jj,v} = \alpha_{0,j,jj,v}{}^{\rho} \bullet \alpha_{i,j,jj,v} \qquad \text{Eq. 30}$$

$$\alpha\text{cst}_{i,j,jj,v} = \left(\alpha_{i,j,jj,v} \right)^{1/(1-\rho)} \qquad \text{Eq. 31}$$

$$\alpha_{0,j,jj,v} = 1 \qquad \text{Eq. 32}$$

$$\alpha\text{cst}_{0,j,jj,v} = \alpha_{0,j,jj,v}{}^{\sigma \bullet \rho} = 1 \qquad \text{Eq. 33}$$

$$\alpha_{i=26,j,jj,v} = \alpha_{i=26,j,jj,v}^{(1-\rho2)/(1-\rho1)} \bullet \left(\frac{\text{Pr}_{j,t}}{\pi_{j,jj,v} / \text{KA}_{j,jj,v}} \right)^{(\rho1-\rho2)/(1-\rho1)} \qquad \text{Eq. 34}$$

$$\text{KA}_{j,jj,v} = \text{KAprior}_{j,jj,v} \qquad \text{Eq. 35}$$

$$\alpha_{i,j,jj,v} = \alpha_{i,j,jj,v}^{(1-\rho2)/(1-\rho1)} \bullet \left(\frac{\text{Pr}_{j,t}}{\text{Pi}_{i,j,jj,t}} \right)^{(\rho1-\rho2)/(1-\rho1)} \qquad \text{Eq. 36}$$

$$\alpha\text{cst}_{i,j,jj,v} = \left(\alpha_{i,j,jj,v} \right)^{1/(1-\rho2)} \qquad \text{Eq. 37}$$

$$\alpha\text{cst}_{0,j,v} = \alpha_{0,j,v}{}^{\sigma2 \bullet \rho2} = 1 \qquad \text{Eq. 38}$$

$$\lambda_{i,j,jj,v} = \frac{X_{i,j,jj,v} / \text{Pi}_{i,j,jj,t=0}}{\sum_{i=1}^{N} X_{i,j,jj,v} / \text{Pr}_{j,t=0}} \qquad \text{Eq. 39}$$

$$\lambda_{0,j,jj,v} = 1 \qquad \text{Eq. 40}$$

$$\lambda_{i=26=\text{capital},j,jj,v} = \frac{\sum_{v=-3}^{0} \text{KAprior}_{j,jj,v}}{\sum_{i=1}^{N} X_{i,j,jj,v} / \text{Pr}_{j,t}} \qquad \text{Eq. 41}$$

$$\lambda_{0,j,jj,v} = 1 \qquad \text{Eq. 42}$$

$$\lambda_{i,j,jj,t} = \frac{\lambda_{i,j,jj,t-1}}{(1+\gamma_i)^{\text{Nstep}}} \qquad \text{Eq. 43}$$

$$\lambda_{i,j,jj,v} = \alpha_{0,j,jj,v}{}^{\rho1/(1-\rho1)} \bullet \left(\alpha_{i,j,jj,v} \bullet \frac{\text{Pr}_{j,t}}{\text{Pi}_{i,j,jj,t}} \right)^{1/(1-\rho)} \qquad \text{Eq. 44}$$

$$\lambda_{i=26=capital,j,jj,v} = \alpha_{0,j,jj,v}^{\rho1/(1-\rho1)} \bullet \left(\alpha_{i=26,j,jj,v} \bullet \frac{Pr_{j,t}}{\pi_{j,jj,v} / Ka_{j,jj,v}} \right)^{1/(1-\rho1)} \qquad \text{Eq. 45}$$

$$\pi_{j,jj,v} = Pr_{j,t} \bullet q_{j,jj,v} - \sum_{i=1}^{N-1} Pi_{i,j,jj,t} \bullet X_{i,j,jj,v} \qquad \text{Eq. 46}$$

$$Pi_{i,j,jj,t} = Pr_{j,t} \bullet \frac{\partial q_{j,jj,v}}{\partial X_{i,j,jj,v}} , i=1, \dots N-1 \qquad \text{Eq. 47}$$

$$\pi_{j,jj,v} = \alpha_{0,j,jj,v} \bullet Pr_{j,t} \bullet Z_{j,jj,v}^{-1/\mu} \bullet Y_{j,jj,v}^{1/\rho} \qquad \text{Eq. 48}$$

$$Z_{j,jj,v} = 1 - (\alpha_{0,j,jj,v} \bullet Pr_{j,t})^{\mu} \bullet \left(\sum_{i=1}^{N-M} \alpha cst_{i,j,jj,v} \bullet Pi_{i,j,jj,t}^{-\mu} \right) \qquad \text{Eq. 49}$$

$$Y_{j,jj,v} = \sum_{i=N-M+1}^{N} \left(acst_{i=26,j,jj,v} \bullet KA_{j,jj,j}^{\rho} \right) \qquad \text{Eq. 50}$$

$$\pi_{j,jj,v} = \left[\frac{\lambda_{0,j,jj,v}}{\lambda_{i=capital,j,jj,v}} \bullet Pr_{j,t} - \sum_{i}^{N-1} \left(\frac{\lambda_{i,j,jj,v}}{\lambda_{i=capital,j,jj,v}} \bullet Pi_{i,j,jj,t} \right) \right] \bullet KA_{j,jj,v} \qquad \text{Eq. 51}$$

$$\frac{\partial \pi_{j,jj,v}}{\partial Pi_{i,j,jj,t}} = -X_{i,j,jj,v} \qquad \text{Eq. 52}$$

$$X_{i,j,jj,v} = \left(\alpha_{0,j,jj,v} \bullet Pr_{j,t} \bullet \frac{\alpha_{i,j,jj,v}}{Pi_{i,j,jj,t}} \right)^{\mu/\rho} \bullet \left(\frac{Y_{j,jj,v}}{Z_{j,jj,v}} \right)^{1/\rho} \qquad \text{Eq. 53}$$

$$X_{i,j,jj,v} = \frac{\lambda_{i,j,jj,v}}{\lambda_{0,j,jj,v}} \bullet \frac{\lambda_{0,j,jj,v}}{\lambda_{i=26,j,jj,v}} \bullet KA_{j,jj,v} = \frac{\lambda_{i,j,jj,v}}{\lambda_{i=26,j,jj,v}} \bullet KA_{j,jj,v} \qquad \text{Eq. 54}$$

$$E\pi r_{j,jj,t} = \alpha_{0,j,jj,v} \bullet \frac{Pe_{j,jj}}{Pie_{i=ETE,j,jj,t}} \bullet Z_{j,jj,v}^{(\rho-1)/\rho} \bullet \alpha_{i=26,j,jj,v}^{1/\rho} \qquad \text{Eq. 55}$$

$$Z_{j,jj,v=t} = 1 - (\alpha_{0,j,jj,v} \bullet Pe_{j,jj})^{\rho/(1-\rho)} \bullet \sum_{i=1}^{N-1} \left(\alpha cst_{i,j,jj,v} \bullet Pie_{i,j,jj}^{\rho/(\rho-1)} \right) \qquad \text{Eq. 56}$$

$$E\pi r_{j,jj,t} = \frac{\lambda_{0,j,jj,v}}{\lambda_{i=26=capital,j,jj,v}} \bullet Pe_{j,jj} - \sum_{i}^{N-1} \left(\frac{\lambda_{i,j,jj,v}}{\lambda_{i=26=capital,j,jj,v}} \bullet Pie_{j,jj} \right) \qquad \text{Eq. 57}$$

$$\pi O_{j,jj,v=t-3:t-1} = \alpha_{0,j,jj,v} \bullet Pr_j \bullet \left(Z_{j,jj,v}^{(-1/\rho)} \right)^{(1-\rho)} \bullet \left(\alpha_{i=26,j,jj,v} \bullet KA_{j,jj,v}^{\rho} \right)^{1/\rho} \qquad \text{Eq. 58}$$

$$\pi O_{j,jj,v=t-3:t-1} = \left[\frac{\lambda_{0,j,jj,v}}{\lambda_{i=26,j,jj,v}} \bullet Pr_{j,t} - \sum_{i}^{N-1} \left(\frac{\lambda_{i,j,jj,v}}{\lambda_{i=26,j,jj,v}} \bullet Pi_{i,j,jj,t} \right) \right] \bullet KA_{j,jj,v} \qquad \text{Eq. 59}$$

$$\pi Or_{j,jj,v} = \frac{\pi O_{j,jj,v}}{KA_{j,jj,v}} \qquad \text{Eq. 60}$$

$$EDV_{i,j,jj,v=t-3:t-1} = fac \bullet \left(\alpha_{0,j,jj,v} \bullet Pr_{j,t} \bullet \frac{\alpha_{i,j,jj,v}}{Pi_{i,j,jj,t}} \right)^{1/(1-\rho)} \bullet Z_{j,jj,v}^{-1/\rho} \bullet \left(\alpha_{i=26,j,jj,v} \bullet KA_{j,jj,v}^{\rho} \right)^{1/\rho} ..$$

Eq. 61

$$fac = 3 \bullet \left(\frac{\pi Or_{j,jj,v}}{0.01} \right)^2 - 2 \bullet \left(\frac{\pi Or_{j,jj,v}}{0.01} \right)^3 \qquad \text{Eq. 62}$$

$$PRDV_{j,jj,v=t-3:t-1} = fac \bullet \alpha_{0,j,jj,v} \bullet Z_{j,jj,v}^{-1/\rho} \bullet \left(\alpha_{i=26,j,jj,v} \bullet KA_{j,jj,v}^{\rho} \right)^{1/\rho} \qquad \text{Eq. 63}$$

$$EDV_{i,j,jj,v=t-3:t-1} = PRDV_{j,jj,v} \bullet \frac{\lambda_{i,j,jj,v}}{\lambda_{0,j,jj,v}} \qquad \text{Eq. 64}$$

$$PRDV_{j,jj,v=t-3:t-1} = fac \bullet \frac{\lambda_{0,j,jj,v}}{\lambda_{i=26,j,jj,v}} \bullet KA_{j,jj,v} \qquad \text{Eq. 65}$$

$$Deplete_j = ((2 \bullet PRDV_{j,jj,v}) + (3 \bullet PRDV_{j,jj,v})) \bullet PRconvrt_j \qquad \text{Eq. 66}$$

$$PctDeplete = \frac{DRSVtmp_{j,jj,t}}{Deplete_j} \qquad \text{Eq. 67}$$

$$PRDV_{j,jj,v=t-3:t-1} = PctDeplete \bullet PRDV_{j,jj,v} \qquad \text{Eq. 68}$$

$$\pi O_{j,jj,v=t-3:t-1} = PctDeplete \bullet \pi_{j,jj,v} \qquad \text{Eq. 69}$$

$$DrsVtmp_{j,jj,t} = DrsVtmp_{j,jj,t} - Deplete_j \qquad \text{Eq. 70}$$

$$Kdem_{j,jj} = PctDeplete \bullet Kdem_{j,jj} \qquad \text{Eq. 71}$$

$$\pi Nr_{j,jj,v=t} = \alpha_{0,j,jj,v} \bullet Pr_{j,t} \bullet Z_{j,jj,v}^{-1/\mu} \bullet \alpha_{i=26,j,jj,v}^{1/\rho} \qquad \text{Eq. 72}$$

$$\pi Nr_{j,jj,v=t} = \frac{\lambda_{0,j,jj,v}}{\lambda_{i=26,j,jj,v}} \bullet Pr_{j,t} - \sum_{i}^{N-1} \left(\frac{\lambda_{i,j,jj,v}}{\lambda_{i=26,j,jj,v}} \bullet Pi_{i,j,jj,t} \right) \qquad \text{Eq. 73}$$

$$\pi N_{j,jj,v=t} = \alpha_{0,j,jj,v} \bullet Pr_{j,t} \bullet \left(Z_{j,jj,v}^{(-1/\rho)} \right)^{(1-\rho)} \bullet \left(\alpha_{i=26,j,jj,v} \bullet KA_{j,jj,v}^{\rho} \right)^{1/\rho} \qquad \text{Eq. 74}$$

$$\pi N_{j,jj,v=t} = \left[\frac{\lambda_{0,j,jj,v}}{\lambda_{i=26,j,jj,v}} \bullet Pr_{j,t} - \sum_{i}^{N-1} \left(\frac{\lambda_{i,j,jj,v}}{\lambda_{i=26,j,jj,v}} \bullet Pi_{i,j,jj,t} \right) \right] \bullet KA_{j,jj,v} \qquad \text{Eq. 75}$$

$$EDV_{i,j,jj,v=t} = \left(Pr_{j,t} \bullet \alpha_{0,j,jj,v} \bullet \frac{\alpha_{i,j,jj,v}}{Pi_{i,j,jj,t}} \right)^{1/(1-\rho)} \bullet Z_{j,jj,v}^{-1/\rho} \bullet \left(\alpha_{i=26,j,jj,v} \bullet KA_{j,jj,v}^{\rho} \right)^{1/\rho} \qquad \text{Eq. 76}$$

$$PRDV_{j,jj,v=t} = \alpha_{0,j,jj,v} \bullet Z_{j,jj,v}^{-1/\rho} \bullet \left(\alpha_{i=26,j,jj,v} \bullet KA_{j,jj,v}^{\rho} \right)^{1/\rho} \qquad \text{Eq. 77}$$

$$EDV_{i,j,jj,v=t} = PRDV_{j,jj,v} \bullet \frac{\lambda_{i,j,jj,v}}{\lambda_{0,j,jj,v}} \qquad \text{Eq. 78}$$

$$PRDV_{j,jj,=t} = \frac{\lambda_{0,j,jj,v}}{\lambda_{i=26,j,jj,v}} \bullet KA_{j,jj,v} \qquad \text{Eq. 79}$$

$$ED_{i=1:25,\,j=1:22} = \sum_{v=t-3}^{t}\left[\sum_{jj=1}^{n} EDV_{i,j,jj,v}\right] \qquad \text{Eq. 80}$$

$$Cost_{i,j,jj,v=t-3:t-1} = EDV_{i,j,jj,v} \bullet Pi_{i,j,jj,t} \qquad \text{Eq. 81}$$

$$WHSL_i = \sum_{j=1}^{22}\left[\sum_{v=t-3}^{t}\left[\sum_{jj=1}^{n}\left[\frac{(adj_{i,j,jj,t}-1)\bullet\left[(P_{i,t}+Tr_{i,t})\bullet Ex\,Im_i \bullet Txpro_{i,j,jj}+Txadd_{i,j,jj}\right]\bullet}{EDV_{i,j,jj,v=t-3:t}\Big/Pi_{i=ETE,j,jj,t}}\right]\right]\right]$$

Eq. 82

$$TRNP_i = \sum_{j=1}^{22}\left[\sum_{v=t-3}^{t}\left[\sum_{jj=1}^{n}\left(Tr_{i,t}\bullet Ex\,Im_i \bullet EDV_{i,j,jj,v=t-3:t}\Big/Pi_{i=ETE,j,jj,t}\right)\right]\right] \qquad \text{Eq. 83}$$

$$Tax_{1,j} = \sum_{i=1}^{22}\left[\sum_{v=t-3}^{t}\left[\sum_{jj=1}^{n}\left(P_{i,t}+Tr_{i,t}\bullet Ex\,Im_i \bullet EDV_{i,j,jj,v=t-3:t}\right)\bullet\left(Txpro_{i,j,jj}-1\right)\right]\right] \qquad \text{Eq. 84}$$

$$Tax_{2,j} = \sum_{v=t-3}^{t}\left[\sum_{jj=1}^{n}\left(P_{i=25,t}+Tr_{i=25,t}\bullet Ex\,Im_{i=25}\bullet EDV_{i=25,j,jj,v=t-3:t}\right)\bullet\left(Txpro_{i=25,j,jj}-1\right)\right] \text{Eq. 85}$$

$$Subsid_i = \sum_{j=1}^{22}\left[-1\bullet\sum_{v=t-3}^{t}\left[\sum_{jj=1}^{n}\left(P_{i,t}+Tr_{i,t}\bullet Ex\,Im_i \bullet EDV_{i,j,jj,v=t-3:t}\right)\bullet\left(Txpro_{i,j,jj}-1\right)\right]\right] \text{Eq. 86}$$

$$TaxpU_{1,j} = \sum_{i=1}^{25}\left[\sum_{v=t-3}^{t}\left[\sum_{jj=1}^{n}\left(Txadd_{i,j,jj}\bullet EDV_{i,j,jj,v=t-3:t}\right)\right]\right] \qquad \text{Eq. 87}$$

$$TaxpU_{2,j} = \sum_{v=t-3}^{t}\left[\sum_{jj=1}^{n}\left(Txadd_{i=25,j,jj}\bullet EDV_{i=25,j,jj,v=t-3:t}\right)\right] \qquad \text{Eq. 88}$$

$$SubsidpU_i = \sum_{j=1}^{22}\left[-1\bullet\sum_{v=t-3}^{t}\left[\sum_{jj=1}^{n}\left(Txadd_{i,j,jj}\bullet EDV_{i,j,jj,v=t-3:t}\right)\right]\right] \qquad \text{Eq. 89}$$

$$CPermF_{5,\,j=1:22} = \sum_{i=1}^{25}\left[\sum_{v=t-3}^{t}\left[\sum_{jj=1}^{n}\left(Cpf_i \bullet EDV_{i,j,jj,v=t-3:t}\right)\right]\right] \qquad \text{Eq. 90}$$

$$P_{i=23,t} = CarbVar_t \qquad \text{Eq. 91}$$

$$P_{i=23,t} = 0.1 \qquad \text{Eq. 92}$$

$$\text{CarbonLim}_t = \text{CarbVar}_t \qquad \text{Eq. 93}$$

$$\text{CarbonLim}_t = \text{EmTot}_t \bullet \text{CarbVar}_t \qquad \text{Eq. 94}$$

$$\text{ED}_{i=23,j=23} = -\text{CarbLim}_t \qquad \text{Eq. 95}$$

$$\text{PRD}_{j=23} = \text{CarbLim}_t \qquad \text{Eq. 96}$$

$$\text{ED}_{i=23,j=3,4.5} = \text{EmissionsC}_{j=3,4,5} \qquad \text{Eq. 97}$$

$$\text{ED}_{i=23,j=1:22} = \text{Emissions}_j = \sum_i \text{TotDem}_i \bullet \text{PRconvrt}_i \bullet \text{GWP}_i \bullet \text{EMC}_i \qquad \text{Eq. 98}$$

$$\text{CrbTrade}_t = P_{i=23,t} \bullet \text{ExchRate}_{\text{region}} \bullet (\text{CarbLim}_t - \text{EmTot}_t) \qquad \text{Eq. 99}$$

$$\text{CrbTrade}_t = \text{CrbTrade}_t - \text{CPermF}_{5,j=25=\text{imex}} - \text{CPermF}_{5,j=26=\text{gv}} - \text{CPermF}_{5,j=27=\text{hh}} \qquad \text{Eq. 100}$$

$$\text{CrbFeeTot}_t = \text{CrbFeeTot}_t + \text{CrbTrade}_t \qquad \text{Eq. 101}$$

$$K_{j,t} = \sum_{jj=1}^{n} \text{KAflow}_{j,jj,t} \qquad \text{for the base year} \qquad \text{Eq. 102}$$

$$E\pi r_{j,jj,t=0} = \text{EXPPROF}_{j,jj,t=0} \qquad \text{for the base year} \qquad \text{Eq. 103}$$

$$K_{j,t} = \text{Temp}_j \bullet \left(\frac{\sum_{jj=1}^{n} E\pi r_{j,jj,t}^{\text{rhoinv}+1}}{\sum_{jj=1}^{n} E\pi r_{j,jj,t}^{\text{rhoinv}}} \right)^{\text{rinv}} \qquad \text{Eq. 104}$$

$$\text{Temp}_j = \text{basekap}_j \bullet \text{scaler} \bullet \left[\frac{\text{working\_age\_pop}_t}{\text{working\_age\_pop}_{t-1}} \right]^{\text{accinv}} \bullet \text{margvalue}^{-\text{rinv}} \qquad \text{Eq. 105}$$

$$\text{PRD}_j = \sum_{v=t-3}^{t} \left[\sum_{jj=1}^{n} \text{PRDV}_{j,jj,v} \right] \qquad \text{Eq. 106}$$

$$\text{Qnew}_j = \text{Qproj}_t \bullet \sum_{v=t-4}^{t-1} \left[\sum_{jj=1}^{n} \text{PRDV}_{j,jj,v} \right] - \sum_{v=t-3}^{t} \left[\sum_{jj=1}^{n} \text{PRDV}_{j,jj,v} \right] \qquad \text{Eq. 107}$$

$$\text{TestK}_j = \text{Capq}_{j,jj} \bullet \text{Qnew}_j \qquad \text{for jj=1} \qquad \text{Eq. 108}$$

$$\text{Capq}_{j,jj} = \alpha_{0,j,jj,v}^{\rho/(1-\rho)} \bullet \alpha_{i=26,j,jj,v}^{1/(1-\rho)} \bullet \left(\frac{Pr_{j,t}}{\pi Nr_{j,jj,v=t}} \right)^{1/(1-\rho)} \qquad \text{Eq. 109}$$

$$\text{Share}_{j,jj} = \frac{E\pi r_{j,jj,t}^{\text{rhoinv}}}{\sum_{jj=1}^{n=3} \left(E\pi r_{j,jj,t}^{\text{rhoinv}} \right)} \qquad \text{for j=8 and jj=1:6} \qquad \text{Eq. 110}$$

$$QCap_{j=8} = \sum_{jj=1}^{n=3} \left(\frac{Share_{j,jj}}{Capq_{j,jj}} \right) \qquad \text{Eq. 111}$$

$$TestK_j = Qnew_j - \sum_{jj=1}^{n} \left[\frac{\dfrac{2 \bullet Kaflow_{j,jj,t-1}}{Capq_{j,jj}} + \dfrac{3 \bullet ExoElec_{jj}}{Capq_{j,jj}}}{QCap_j} \right] + \qquad \text{for j=8 \ Eq. 112}$$

$$2 \bullet \sum Kaflow_{j,jj,t-1} + 3 \bullet \left(ExoElec_{jj=5} + ExoElec_{jj=6} \right)$$

$$K_{j,t} = TestK_j \bullet \left(\frac{\sum_{jj=1}^{n} E\pi r_{j,jj,t}^{rhoinv+1}}{\sum_{jj=1}^{n} E\pi r_{j,jj,t}^{rhoinv}} \right)^{rinv} \qquad \text{Eq. 113}$$

$$K_{j,t} = ExoInvst_j \qquad \text{Eq. 114}$$

$$KAflow_{j=8,jj,t} = ExoElec_{jj} \qquad \text{for j=8 and jj=5 or 6} \qquad \text{Eq. 115}$$

$$K_{j=8,t} = K_{j=8,t} + \sum_{jj=1}^{n} ExoElec_{jj} \qquad \text{Eq. 116}$$

$$KAflow_{j,jj,t} = Share_{j,jj,t} \bullet K_{j,t} \qquad \text{Eq. 117}$$

$$KA_{j,jj,t} = 2 \bullet KAflow_{j,jj,t-1} + 3 \bullet KAflow_{j,jj,t} \qquad \text{Eq. 118}$$

$$DrsCtmp_{j,jj} = Drsce_{j,jj} \qquad \text{Eq. 119}$$

$$DrsVtmp_{j,jj,t} = Drsve_{j,jj,t} \qquad \text{Eq. 120}$$

$$Reserve_j = PRconvrt_j \bullet Eqdep_j \qquad \text{Eq. 121}$$

$$Eqdep_j = xnp_j \bullet \alpha_{0,j,jj,v} \bullet Z_{j,jj,v} \bullet \alpha_{i=26,j,jj,v}^{1/(1-\rho)} \bullet KA_{j,jj,t} \qquad \text{Eq. 122}$$

$$Drsce_{j,jj} = DrsCtmp_{j,jj} + Drsce_{j,jj} \bullet \left((1 + Resgro_{j,jj}^{nt*Nstep}) - (1 + Resgro_{j,jj}^{(nt-1)*Nstep}) \right)$$
$$\text{Eq. 123}$$

$$Resgro_{j,jj} = Resgro_{j,jj} \bullet (1 - \frac{T}{Tresgrz_{j,jj}}) + Resgrz_{j,jj} \bullet \frac{T}{Tresgrz_{j,jj}} \qquad \text{Eq. 124}$$

$$DrsVtmp_{j,jj,t} = Reserve_j \qquad \text{Eq. 125}$$

$$DrsCtmp_{j,jj} = DrsCtmp_{j,jj} - Reserve_j \qquad \text{Eq. 126}$$

$$Pct \, Reserves = DrsCtmp_{j,jj} \Big/ Reserve_j \qquad \text{Eq. 127}$$

$$DrsVtmp_{j,jj,t} = Reserve_j \qquad \text{Eq. 128}$$

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$$\text{Eqdep}_{j,jj} = \text{Pct Reserves} \bullet \text{Eqdep}_{j,jj} \qquad \text{Eq. 129}$$

$$\text{KAflow}_{j,jj,t} = \text{Pct Reserves} \bullet \text{KAflow}_{j,jj,t} \qquad \text{Eq. 130}$$

$$\text{KA}_{j,jj,v=t} = \text{Pct Reserves} \bullet \text{KA}_{j,jj,v=t} \qquad \text{Eq. 131}$$

$$\text{Reserve}_j = \text{DrsCtmp}_{j,jj} \qquad \text{Eq. 132}$$

$$\text{Kdem}_{j,jj} = \text{KAflow}_{j,jj,t} \qquad \text{Eq. 133}$$

$$\text{ED}_{i,j=25=\text{inv}} = \sum_{jj=1}^{n} \left[\sum_{j=1}^{22} \text{Kdem}_{j,jj} \bullet \text{CapMat}_{i,j} \right] \qquad \text{Eq. 134}$$

$$\text{ED}_{i=26,j=25=\text{inv}} = \sum_{i=1}^{22} \left[\sum_{jj=1}^{n} \left[\sum_{j=1}^{22} \text{Kdem}_{j,jj} \bullet \text{CapMat}_{i=26,j} \bullet \text{Pi}_{i,j,jj,t} \right] \right] \qquad \text{Eq. 135}$$

$$\text{Ktot}_t = \sum_{i=1}^{22} \left[\sum_{jj=1}^{n} \left[\sum_{j=1}^{22} \text{Kdem}_{j,jj} \bullet \text{CapMat}_{i,j} \right] \right] \qquad \text{Eq. 136}$$

$$\text{ED}_{i=\text{ETE},j} = \text{ED}_{i=\text{ETE},j} + \sum_{i=1}^{22} \text{WHSL}_i + \sum_{i=1}^{22} \text{TRNP}_i \qquad \text{Eq. 137}$$

$$\text{WHSL}_i = \sum_{j=1}^{22} \left[\sum_{v=t-3}^{t} \left[\sum_{jj=1}^{n} \left[\frac{\left(\text{adj}_{i,j,jj,t} - 1\right) \bullet \left[\left(P_{i,t} + Tr_{i,t}\right) \bullet \text{Ex Im}_i \bullet \text{Txpro}_{i,j,jj} + \text{Txadd}_{i,j,jj}\right] \bullet}{\text{EDV}_{i,j,jj,v} \Big/ \text{Pi}_{i=\text{ETE},j,jj,t}} \right] \right] \right] +$$

$$\left[\frac{\left(\text{adj}_{i,j=27=\text{hh},jj=1,t} - 1\right) \bullet \left[\left(P_{i,t} + Tr_{i,t}\right) \bullet \text{Ex Im}_i \bullet \text{Txpro}_{i,j=27=\text{hh},jj=1} + \text{Txadd}_{i,j=27=\text{hh},jj=1}\right] \bullet}{\text{ED}_{i,j=27=\text{hh}} \Big/ \text{Pi}_{i=\text{ETE},j=27=\text{hh},jj=1,t}} \right]$$

$$\text{Eq. 138}$$

$$\text{TRNP}_i = \sum_{j=1}^{22} \left[\sum_{v=t-3}^{t} \left[\sum_{jj=1}^{n} \left(Tr_{i,t} \bullet \text{Ex Im}_i \bullet \text{EDV}_{i,j,jj,v} \Big/ \text{Pi}_{i=\text{ETE},j,jj,t} \right) \right] \right] +$$

$$\left(Tr_{i,t} \bullet \text{Ex Im}_i \bullet \text{ED}_{i,j=27=\text{hh}} \Big/ \text{Pi}_{i=\text{ETE},j=27=\text{hh},jj,t} \right)$$

$$\text{Eq. 139}$$

$$\text{CPermF}_{5,j=25=\text{inv}} = \sum_{i=1}^{22} \left(\text{Cpf}_i \bullet \text{ED}_{i,j=25=\text{inv}} \right) \qquad \text{Eq. 140}$$

$$\text{POP}_{\text{tot}} = \sum_{\text{age}=1}^{\text{NAGE}} \left(\text{POP}_{\text{age,males}} + \text{POP}_{\text{age,females}} \right) \qquad \text{Eq. 141}$$

$$\text{working\_age\_pop}_{\text{tot}} = \sum_{\text{age}=4}^{\text{age}=13} \left(\text{POP}_{\text{age,males}} + \text{POP}_{\text{age,females}} \right) \qquad \text{Eq. 142}$$

$$\text{working\_age\_pop}_{ng} = \sum_{age=4}^{age=13}\left(POP_{age,ng}\right) \qquad \text{Eq. 143}$$

$$ED_{i=25=lbs, j=27=hh} = -1 \bullet \sum_{ng} \text{working\_age\_pop}_{ng} \bullet LB_{0,ng} \bullet [1 - \exp(LB_{1,ng} \bullet \frac{P_{i=25=lbs,t}}{Pi_{i=ETE, j=27, jj, t}})]$$

Eq. 144

$$LB_{0,ng} = \frac{\text{working\_age\_pop}_{ng}}{POP_{tot}} \qquad \text{Eq. 145}$$

$$LB_{1,ng} = \frac{\log\left(1 - \dfrac{ED_{i=25=lbs, j=27=hh}}{\left(\text{working\_age\_pop}_{tot}\right) \bullet LB_{0,ng}}\right)}{P_{i=25=lbs,t}} \qquad \text{Eq. 146}$$

$$LB_{1,ng} = \frac{ED_{i=25=lbs. j=27=hh}}{\text{working\_age\_pop}_{tot}} \qquad \text{Eq. 147}$$

$$SalesV_{i=25=lbs} = -1 \bullet ED_{i=25=lbs, j=27=hh} \qquad \text{Eq. 148}$$

$$LB_{2,2} = \frac{X_{i=25=lbs, j=27=hh}}{ED_{i=25=lbs, j=27=hh} \bullet P_{i=25=lbs,t}} \qquad \text{Eq. 149}$$

$$Nhh = \frac{POP_{tot}}{pp_{hh}} \qquad \text{Eq. 150}$$

$$LB_{2,1} = \frac{POP_{tot}}{Nhh} \qquad \text{Eq. 151}$$

$$ED_{i=24=ls, j=27=hh} = -1 \bullet TLA \bullet R_0 \bullet [1 - \exp(R_1 \bullet \frac{P_{i=24=ls,t}}{Pi_{i=ETE, j=27=hh, jj, t}})] \qquad \text{Eq. 152}$$

$$R_1 = \frac{\log\left(1 - \dfrac{ED_{i=24=ls. j=27=hh}}{TLA / R_0}\right)}{P_{i=24=ls,t}} \qquad \text{Eq. 153}$$

$$SalesV_{i=24=ls} = -1 \bullet ED_{i=24=ls, j=27=hh} \qquad \text{Eq. 154}$$

$$R_2 = \frac{X_{i=24=ls, j=27=hh}}{Nhh \bullet P_{i=24=ls,t}} \qquad \text{Eq. 155}$$

$$Pinc = (HHinc_1 + HHinc_2 + HHinc_3) \bullet (1 - PItr) + GovTr_t \qquad \text{Eq. 156}$$

$$HHinc_1 = \sum_{j=1}^{22}\left[\left(\sum_{jj=1}^{6} X_{i=26, j, jj}\right) \bullet \left((1 - Citr) + Xitc_j\right) \bullet \left(1 - \left(RE_{0,j} \bullet (1 - \exp(RE_{1,j} \bullet P_{i=26,t}))\right)\right)\right]$$

Eq. 157

$$Xitc_j = \sum_{jj=1}^{n} \left[XITCrate_{j,jj} \bullet Kdem_{j,jj} \right] \qquad \text{Eq. 158}$$

$$RE_{1,j} = \frac{\log\left[1 - \dfrac{TREte}{\sum\limits_j X_{i=26=capital,j} \bullet RE_{0,j} \bullet (1 - Citr)} \right]}{P_{i=26}} \qquad \text{Eq. 159}$$

$$HHinc_1 = \sum_{j=1}^{22} \left[\left(\sum_{jj=1}^{6} \pi_{j,jj} \right) \bullet \left((1 - Citr) + Xitc_j \right) \bullet \left(1 - \left(RE_{0,j} \bullet (1 - \exp(RE_{1,j} * P_{i=26,t})) \right) \right) \right] \text{Eq. 160}$$

$$TREte = \sum_{j=1}^{22} \left[REte_j \right] \qquad \text{Eq. 161}$$

$$REte_j = \left[\left(\sum_{jj=1}^{6} \pi_{j,jj} \right) \bullet (1 - Citr) \bullet Xitc_j \right] \bullet \left(RE_{0,j} \bullet (1 - \exp(RE_{1,j} \bullet P_{i=26,t})) \right) \qquad \text{Eq. 162}$$

$$TREte = TREte + 0.6 \bullet CrbFeeTot_t + 0.6 \bullet ExIBT_t \qquad \text{Eq. 163}$$

$$TREte = TREte + CrbFeeRcyPct_3 \bullet CrbFeeTot_t + CrbFeeRcyPct_3 \bullet ExIBT_t \qquad \text{Eq. 164}$$

$$Tax_{3,j=1:22} = Citr \bullet \pi_j - Xitc_j \qquad \text{Eq. 165}$$

$$HHinc_2 = (1 - SStr) \bullet -1 \bullet ED_{i=25=lbs,j=27=hh} \bullet P_{i=25=lbs,t} \qquad \text{Eq. 166}$$

$$Tax_{2,j=27=hh} = \left[SStr \bullet -1 \bullet ED_{i=25=lbs,j=27=hh} \bullet P_{i=25=lbs,t} \right] \qquad \text{Eq. 167}$$

$$HHinc_3 = -1 \bullet ED_{i=24=ls,j=27=hh} \bullet P_{i=24=ls,t} \qquad \text{Eq. 168}$$

$$Tax_{3,j=27=hh} = (HHinc1 + HHinc2 + HHinc3) \bullet Pitr \qquad \text{Eq. 169}$$

$$GovTr_t = P_{i=ETE,t} \bullet \left(\begin{array}{l} POP_{tot} \bullet GovTr_0 \bullet \left(\dfrac{(HHinc_1 + HHinc_2 + HHinc_3)}{POP_{tot}} \right)^{g1} \bullet \\ \left(\dfrac{-1 \bullet ED_{i=25=lbs,j=27=hh}}{working\_age\_pop} \right)^{g2} \bullet YngOld^{g3} \end{array} \right) \qquad \text{Eq. 170}$$

$$Pinc = Pinc + CrbFeeTot_t \qquad \text{Eq. 171}$$

$$Pinc = Pinc + 0.4 \bullet CrbFeeTot_t \qquad \text{Eq. 172}$$

$$Pinc = Pinc + CrbFeeTot_t - CrbTrade_t \qquad \text{Eq. 173}$$

$$Pinc = Pinc + CrbFeeTot_t \qquad \text{Eq. 174}$$

$$Pinc = Pinc + CrbFeeRcyPct_2 \bullet CrbFeeTot_t \qquad \text{Eq. 175}$$

$$ED_{i=26,j=27=hh} = -1 \bullet Pinc \bullet S_0 \bullet (1 - S_1 \bullet \exp(S_2 \bullet P_{i=26})) \qquad \text{Eq. 176}$$

$$S_1 = 1 \qquad \text{Eq. 177}$$

$$S_2 = \frac{\log\left(1 - \frac{ED_{i=26=capital, j=27=hh}}{Pinc \bullet S_0}\right)}{P_{i=26,t}} \qquad \text{Eq. 178}$$

$$Saving = -1 \bullet ED_{i=26=capital, j=27=hh} \qquad \text{Eq. 179}$$

$$Pinc = Pinc - Saving \qquad \text{Eq. 180}$$

$$Cons_t = Pinc \qquad \text{Eq. 181}$$

$$ED_{i=26=capital, j=27=hh} = ED_{i=26=capital, j=27=hh} - TREte \qquad \text{Eq. 182}$$

$$SalesV_{i=26} = -1 \bullet ED_{i=26=capital, j=27=hh} \qquad \text{Eq. 183}$$

$$DemTot_{j=27=hh} = \sum_{i=1}^{22}\left[ED_{i, j=27=hh} \bullet Pi_{i, j=27=hh, jj, t}\right] \qquad \text{Eq. 184}$$

$$ED_{i=1:22, j=27=hh} = E_{0,i} \bullet \left(\frac{Pinc}{Pi_{i=ETE, j=27=hh, jj, t}}\right)^{\varepsilon_{1,i}} \bullet \left(\frac{Pi_{i, j=27=hh, jj, t}}{Pi_{i=ETE, j=27=hh, jj, t}}\right)^{\varepsilon_{2,i}} \qquad \text{Eq. 185}$$

$$E_{0,i} = \left(\frac{X_{i, j=27=hh}}{Pinc}\right)^{\varepsilon_{1,i}} \bullet P_{i,t}^{\varepsilon_{2,i}} \qquad \text{Eq. 186}$$

$$E_{0,hh,t} = \frac{E_{0,hh,t-1}}{Factor_{i,hh,t}} \qquad \text{(unitless) Eq. 187}$$

$$Pinc = Pinc - HHinc_9 - HHinc_{10} \qquad \text{Eq. 188}$$

$$HHinc_9 = \left(-1 \bullet Pi_{i=25=lbs, j=27=hh, jj, t} \bullet LB_{2,2} \bullet ED_{i=25=lbs, j=27=hh}\right) \qquad \text{Eq. 189}$$

$$HHinc_{10} = \left(Pi_{i=24=ls, j=27=hh, jj, t} \bullet R_2 \bullet Nhh\right) \qquad \text{Eq. 190}$$

$$ED_{i=1:22, j=27=hh} = \frac{ED_{i,j} \bullet Pinc}{DemTot_{j=27}} \qquad \text{Eq. 191}$$

$$ED_{i=25=lbs, j=27=hh} = ED_{i=25=lbs, j=27=hh} \bullet \left(1 - LB_{2,2}\right) \qquad \text{Eq. 192}$$

$$WHSL_i = \left[\frac{\left(adj_{i, j=27=hh, t} - 1\right) \bullet \left[\left(P_{i,t} + Tr_{i,t} \bullet Ex\,Im_i\right) \bullet Txpro_{i, j=27=hh} + Txadd_{i, j=27=hh}\right] \bullet}{ED_{i, j=27=hh} \Big/ Pi_{i=ETE, j=27=hh, jj, t}}\right] \qquad \text{Eq. 193}$$

$$TRNP_i = Tr_{i,t} \bullet Ex\,Im_i \bullet ED_{i, j=27=hh}\Big/ Pi_{i=ETE, j=27=hh, jj, t} \qquad \text{Eq. 194}$$

$$Tax_{1, j=27=hh} = \sum_{i=1}^{24}\left(P_{i,t} + Tr_{i,t} \bullet Ex\,Im_i \bullet abs(ED_{i, j=27=hh})\right) \bullet \left(Txpro_{i, j=27=hh} - 1\right) \qquad \text{Eq. 195}$$

$$Subsid_i = -Tax_{1, j=27=hh} \qquad \text{Eq. 196}$$

$$TaxpU_{j=27=hh} = \sum_{i=1}^{24} \left(Txadd_{i,j=27=hh} \bullet ED_{i,j=27=hh}\right) \qquad \text{Eq. 197}$$

$$SubsidpU_i = SubsidpU_i - Txadd_{i,j=27=hh} \bullet ED_{i,j=27=hh} \qquad \text{Eq. 198}$$

$$CPermF_{5,j=27=hh} = \sum_{i=1}^{24} \left(Cpf_i \bullet ED_{i,j=27=hh}\right) \qquad \text{Eq. 199}$$

$$GF_{0:t} = 1 \qquad \text{Eq. 200}$$

$$TaxGov_{itc} = \sum_{i=1}^{25} \left(P_{i,t} + Tr_{i,t} \bullet Ex\,Im_i \bullet ED_{i,j=26=gv} \bullet \left(Txpro_{i,j=26=gv} - 1\right)\right) \qquad \text{Eq. 201}$$

$$SubsidGov = -TaxGov_{itc} \qquad \text{Eq. 202}$$

$$TaxGovpU_{itc} = \sum_{i=1}^{25} \left(Txadd_{i,j=26=gv} \bullet ED_{i,j=26=gv}\right) \qquad \text{Eq. 203}$$

$$SubsidpU = TaxGovpU_{itc} \qquad \text{Eq. 204}$$

$$CPermF_{5,j=26=gv} = \sum_{i=1}^{25} \left(Cpf_i \bullet ED_{i,j=26=gv}\right) \qquad \text{Eq. 205}$$

$$TotTax_t = \sum_{j=1}^{27} \left(Tax_{1,j} + Tax_{2,j} + Tax_{3,j} + Tax_{4,j} + CPermF_{5,j}\right) \qquad \text{Eq. 206}$$

$$Tax_{4,j} = SalesP_j \bullet TxIBT_j \qquad \text{Eq. 207}$$

$$SalesP_j = \sum_{v=t-3}^{t} \left[\sum_{jj=1}^{n} \left(PRDV_{j,jj,v} \bullet Pr_{j,t}\right) \right] \qquad \text{Eq. 208}$$

$$ExIBT_t = \sum_{j=1}^{22} \left(SalesP_j \bullet \left(TxIBT_{j,t} - TxIBT_{j,t=0}\right)\right) \qquad \text{Eq. 209}$$

$$Subsid_i = \sum_{j=1}^{22} \left[-1 \bullet \sum_{v=t-3}^{t} \left[\sum_{jj=1}^{n} \left(P_{i,t} + Tr_{i,t} \bullet Ex\,Im_i \bullet EDV_{i,j,jj,v}\right) \bullet \left(Txpro_{i,j,jj} - 1\right) \right] \right] +$$
$$\sum_{j=26}^{27} \left(P_{i,t} + Tr_{i,t} \bullet Ex\,Im_i \bullet ED_{i,j=26=gv}\right) \bullet \left(Txpro_{i,j=26=gv} - 1\right) \qquad \text{Eq. 210}$$

$$GovDeft_t = GovDeft_t \bullet Pi_{i=ETE,j=26=gv,jj,t} \qquad \text{Eq. 211}$$

$$GovMt_{t=0} = \sum_{i=0}^{22} \sum_{gvjj=1}^{1} X_{j=26=gv,jj=gvjj} \qquad \text{Eq. 212}$$

$$GovMt_t = TotTax_t - Subsidy_t - GovTtr_t + GovDeft_t \qquad \text{Eq. 213}$$

$$GovMt_t = GovMt_t - CrbFeeTot_t \qquad \text{Eq. 214}$$

$$GovDeft_t = GovDeft_t - CrbFeeTot_t \qquad \text{Eq. 215}$$

$$GovMt_t = GovMt_t - CrbFeeTot_t \qquad \text{Eq. 216}$$

$$GovMt_t = GovMt_t - CrbFeeTot_t \qquad \text{Eq. 217}$$

$$GovDeft_t = GovDeft_t - CrbTrade_t \qquad \text{Eq. 218}$$

$$GovMt_t = GovMt_t - CrbFeeTot_t \qquad \text{Eq. 219}$$

$$GovMt_t = GovMt_t - (CrbFeeRcyPct_2 + CrbFeeRcyPct_3) \bullet CrbFeeTot_t \qquad \text{Eq. 220}$$

$$GC_{i,j=26=gv,t=0} = \sum_{gvjj=1}^{1} \left(\frac{X_{i,j=26=gv,gvjj} / \sum_{i} X_{i,j=26=gv,gvjj}}{P_{i,t}} \right) \qquad \text{Eq. 221}$$

$$GC_{i,j=26=gv,t} = \frac{GC_{i,j=26=gv,t-1}}{Factor_{i,j=26=gv,t}} \qquad \text{(unitless) Eq. 222}$$

$$PGC_{j=26=gv} = \sum_{gvjj=1}^{1} \left[\sum_{i=1}^{25} \left(GC_{i=26=capital,j=26=gv,gvjj} \bullet Pi_{i,j=26=gv,gvjj,v=t} \right) + GC_{i=26=capital,j=26=gv,gvjj} \bullet Pi_{i=ETE,j=26=gv,gvjj,v=t} \right] \qquad \text{Eq. 223}$$

$$GE_{gvjj} = GovMt_t \bullet \frac{PGC_{j=26=gvgvjj}^{-sig} \bullet GF_{j=26=gv,gvjj}^{sig}}{\sum_{gvjj=1}^{1} \left(PGC_{j=26=gv,gvjj}^{-rho \bullet sig} \bullet GF_{j=26=gv,gvjj}^{sig} \right)} \qquad \text{Eq. 224}$$

$$ED_{i=1:25,j=26=gv} = \sum_{gvjj=1}^{1} GE_{j=26=gv,gvjj} \bullet GC_{i,j=26=gv,gvjj} \qquad \text{Eq. 225}$$

$$ED_{i=26=capital,j=26=gv} = \sum_{gvjj=1}^{1} (GE_{j=26=gv,gvjj} \bullet GC_{i=capital,j=26=gv,,gvjj} \bullet PGC_{j=26=gv,gvjj}) + GovDef_t$$

.... Eq. 226

$$ED_{i=ETE,j=26=gv} = ED_{i=ETE,j=26=gv} + \sum_{gvjj=1}^{1} (GE_{j=26=gv,gvjj} \bullet GC_{i=ETE,j=26=gv,gvjj}) \qquad \text{Eq. 227}$$

$$ED_{i=1:26,j=24=exim} = Trade_{0,i=1:26} \qquad \text{Eq. 228}$$

$$Trade_{1,i=1:22} = -\sum_{j=1}^{27} ED_{i=1:22,j} - PRD_j \qquad \text{Eq. 229}$$

$$Trade_{0,i=1:25} = Trade_{0,i=1:25} \bullet ExchRate_{region} \qquad \text{Eq. 230}$$

$$\text{and/or} \quad Trade_{1,i=1:25} = Trade_{1,i=1:25} \bullet ExchRate_{region} \qquad \text{Eq. 231}$$

$$\operatorname{Exp}\operatorname{Im}_t = \sum_{i=1}^{25}\left[\operatorname{Trade}_{0,i=1:25} \bullet P_{i,t}\right] \qquad \text{Eq. 232}$$

$$\operatorname{Exp}\operatorname{Im}_t = \sum_{i=1}^{25}\left[\operatorname{Trade}_{1,i=1:25} \bullet P_{i,t}\right] \qquad \text{Eq. 233}$$

$$\operatorname{Exp}\operatorname{Im}_t = \operatorname{Exp}\operatorname{Im}_t - \operatorname{Trade}_{1,i=23} \bullet P_{i=23,t} \qquad \text{Eq. 234}$$

$$\operatorname{Trade}_{1,i=23} = \operatorname{Trade}_{1,i=23} / \operatorname{ExchRate}_{regionl} \qquad \text{Eq. 235}$$

$$\operatorname{GNP} = \sum_{i=1}^{N} \operatorname{Pi}_{i,j} \bullet \left(\operatorname{PRD}_j - \sum_{j=1}^{N} \operatorname{ED}_{i,j}\right) = \operatorname{KTot}_t + \operatorname{Cons}_t + \operatorname{GovMt}_t + \operatorname{Ex}\operatorname{Im}_t \qquad \text{Eq. 236}$$

$$\operatorname{GNP} = \sum_{j=1}^{N} \left(\operatorname{Pr}_j \bullet \operatorname{PRD}_j - \sum_{i=1}^{N} \operatorname{Pi}_{i,j} \bullet \operatorname{ED}_{i,j}\right) = \sum_{j=1}^{N} \left(\operatorname{TXibt}_j + \pi_j + \sum_{i=N+1}^{N+M} \operatorname{Pi}_{i,j} \bullet \operatorname{ED}_{i,j}\right) \qquad \text{Eq. 237}$$

$$\operatorname{SalesV}_{i=1:23} = \operatorname{PRD}_{j=1:23} = \sum_{v=t-3}^{t}\left[\sum_{jj=1}^{n} \operatorname{PRDV}_{j=1:23,jj,v}\right] \quad \text{(see Equation 106)} \quad \text{Eq. 238}$$

$$\operatorname{Mrk}\operatorname{Pr}d_m = \operatorname{SalesV}_i \bullet \operatorname{PRconvrt}_i \qquad \text{Eq. 239}$$

$$\operatorname{MrkEd}_m = \sum_{j=1}^{27}\left(\operatorname{Ed}_{i,j} \bullet \operatorname{PRconvrt}_i\right) - \operatorname{Mrk}\operatorname{Pr}d_m \qquad \text{Eq. 240}$$

$$\operatorname{Xvec}_{inewt} = \operatorname{Log}(\operatorname{MrkEd}_m + \operatorname{Mrk}\operatorname{Prd}_m) - \operatorname{Log}(\operatorname{Mark}\operatorname{Pr}d_m) \qquad \text{Eq. 241}$$

$$W_{i,j} = \left[\frac{\operatorname{DemTmp} - (\operatorname{MrkEd}_m + \operatorname{Mrk}\operatorname{Pr}d_m)}{\operatorname{MrkEd}_m + \operatorname{Mrk}\operatorname{Pr}d_m} - \frac{\operatorname{SupTmp} - \operatorname{Mrk}\operatorname{Pr}d_m}{\operatorname{Mrk}\operatorname{Pr}d_m}\right] \bigg/ \frac{\operatorname{delta}}{P_{i,t} - \operatorname{delta}}$$

Eq. 242

$$\operatorname{TotDem}_{i=1:22} = \sum_{j=1}^{27} \operatorname{ED}_{i,j} - \operatorname{ED}_{i,j=24} \qquad \text{Eq. 243}$$

$$\operatorname{EmissionsC}_{j=3=oil} = \operatorname{EMC}_i \bullet \operatorname{GWP}_{i=3} \bullet$$
$$\left(0.97 \bullet \operatorname{PRconvrt}_{fn=3} \bullet \operatorname{TotDem}_{i=3} - \operatorname{PRconvrt}_{fn=9} \bullet \operatorname{Trade}_{0,i=9=refinedoil}\right) \qquad \text{Eq. 244}$$

$$\operatorname{EmissionsC}_{j=4=gas} = \operatorname{EMC}_i \bullet \operatorname{GWP}_{i=4} \bullet$$
$$\left(\operatorname{PRconvrt}_{fn=4} \bullet \operatorname{TotDem}_{i=4} - \operatorname{PRconvrt}_{fn=10} \bullet \operatorname{Trade}_{0,i=10=Gas-T\&D}\right) \qquad \text{Eq. 245}$$

$$\operatorname{EmissionsC}_{j=5=coal} = \operatorname{EMC}_5 \bullet \operatorname{GWP}_{i=5} \bullet \left(\operatorname{PRconvrt}_{fn=5} \bullet \operatorname{TotDem}_{i=5}\right) \qquad \text{Eq. 246}$$

$$\operatorname{EmTot}_t = \sum_{j=3}^{5} \operatorname{EmissionsC}_i \qquad \text{Eq. 247}$$

$$\operatorname{EmAct}_{ix} = \operatorname{PRconvrt}_{ix} \bullet \operatorname{TotDem}_{ix} \qquad \text{Eq. 248}$$

$$\text{Emissions}_j = \text{EMC}_i \bullet \text{GWP}_i \bullet \left(\text{PRconvrt}_i \bullet \text{TotDem}_i \right) \qquad \text{Eq. 249}$$