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Modelling form factors in HQET

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Abstract

We present a simple and straightforward method for relating the form factors in HQET, as defined by the covariant trace formalism, to the overlaps of the rest frame wave functions of the light degrees of freedom. We also point out several inconsistencies present in recent calculations of the radiative rare B decays, and also show how these can be fixed even within the framework of the non-relativistic quark model.

1 Introduction

It is by now a well established fact that hadronic systems containing a single heavy quark ($m_Q \gg \Lambda_{QCD}$) admit additional symmetries which are not present in the full QCD Lagrangian [1]. The light degrees of freedom (LDF) in such hadrons typically have four-momenta small compared with the heavy quark mass. For these systems it is appropriate to adopt an effective theory (HQET) in which the heavy quark mass goes to infinity, with its four-velocity fixed [2, 3]. Since an infinitely massive heavy quark does not recoil from the emission and absorption of soft ($E \approx \Lambda_{QCD}$) gluons, and since magnetic interactions of such a quark are negligible ($\sim \frac{1}{m_Q}$), the strong interactions of the heavy quark are independent of its mass and spin. Because of this, HQET leads to relations between different form factors describing transitions in which a hadron containing a heavy quark Q and moving with four-velocity v^μ, decays into another hadron containing a heavy quark Q', and moving with four-velocity v'^μ. In this way the number of independent form factors for these decays is significantly reduced. For example, the six form factors describing semileptonic decays $B \to D^{(*)} e\bar{\nu}_e$ are, in the heavy quark limit, reduced to a single unknown form factor, the Isgur-Wise function (IW) $\xi(v \cdot v')$.

Since the unknown Lorentz invariant form factors describing a particular decay cannot be calculated from first principles, one still has to rely on some model of strong interactions in order to estimate them. In general these form factors will be related to the overlaps of the wave functions of the LDF in the hadrons before and after the decay. However, one has to be careful in identifying form factors directly with the overlap of the two wave functions. Depending on the definition of the particular form factor, there may be a coefficient of proportionality involved. If these coefficients are not taken into account significantly incorrect results can be obtained no matter which model for the wave functions one uses

In [4] it was observed that the heavy quark limit implies a simple formula for the wave function of any particle containing one heavy quark. Based on this we offer a straightforward method for relating the form factors, as defined within the framework of the trace formalism [5, 6, 7], to overlaps of wave functions of the LDF before and after the decay. Even though we are interested here only in mesons, it is obvious that an analogous calculation can be easily done for baryons. The paper is organized as follows: in Section 2 we review basic definitions of the covariant trace formalism. Section 3 describes how one can identify the form factors in terms of overlaps of the LDF. We also give results for the several cases of interest (transitions of 0^- state into excited states). As a simple application of the results of this paper we consider the non-relativistic quark model in Section 4. Among other things, we also make a few comments on some recent calculations of rare B decays into K-resonances [8, 9]. Our conclusions are summarized in Section 5.

2 Covariant representation of states

The counting of the number of independent form factors is most conveniently done within the framework of the trace formalism, which was formulated in [5, 6] and generalized to excited states in [7]. Following [7], and using notation of [8], the lowest lying mesonic states can be described as follows:

$$
\begin{aligned}
C(v) &= \frac{1}{2}\sqrt{m}(\not{v}+1)\gamma_5 \;, && J^P = 0^- \;,\; j=\frac{1}{2} \;, && (1)\\
C^*(v,\epsilon) &= \frac{1}{2}\sqrt{m}(\not{v}+1)\;\not{\epsilon} \;, && J^P = 1^- \;,\; j=\frac{1}{2} \;, && (2)\\
E(v) &= \frac{1}{2}\sqrt{m}(\not{v}+1) \;, && J^P = 0^+ \;,\; j=\frac{1}{2} \;, && (3)\\
E^*(v,\epsilon) &= \frac{1}{2}\sqrt{m}(\not{v}+1)\gamma_5\;\not{\epsilon} \;, && J^P = 1^+ \;,\; j=\frac{1}{2} \;, && (4)\\
F(v,\epsilon) &= \frac{1}{2}\sqrt{m}\sqrt{\frac{3}{2}}(\not{v}+1)\gamma_5[\epsilon^\mu-\frac{1}{3}\;\not{\epsilon}(\gamma^\mu-v^\mu)] \;, && J^P = 1^+ \;,\; j=\frac{3}{2} \;, && (5)\\
F^*(v,\epsilon) &= \frac{1}{2}\sqrt{m}(\not{v}+1)\gamma_\nu\epsilon^{\mu\nu} \;, && J^P = 2^+ \;,\; j=\frac{3}{2} \;, && (6)
\end{aligned}
$$

$$G(v,\epsilon) = \frac{1}{2}\sqrt{m}\sqrt{\frac{3}{2}}(\not{v}+1)[\epsilon^\mu - \frac{1}{3}\ \not{\epsilon}(\gamma^\mu + v^\mu)]\ , \qquad J^P = 1^-\ ,\ j = \frac{3}{2}\ , \quad (7)$$

$$G^*(v,\epsilon) = \frac{1}{2}\sqrt{m}(\not{v}+1)\gamma_5\gamma_\nu\epsilon^{\mu\nu}\ , \qquad J^P = 2^-\ ,\ j = \frac{3}{2}\ . \quad (8)$$

Here m and v are the mass and the four-velocity of the heavy meson, and j denotes the total spin of the LDF. Also, ϵ^μ is the polarization vector for spin 1 states (satisfying $\epsilon \cdot v = 0$), while the tensor $\epsilon^{\mu\nu}$ describes a spin 2 object ($\epsilon^{\mu\nu} = \epsilon^{\nu\mu}\ ,\ \epsilon^{\mu\nu}v_\nu = 0\ ,\ \epsilon^\mu{}_\mu = 0$). These eight states form four doublets: (C, C^*) is the $L = 0$ doublet, (E, E^*) and (F, F^*) are the two $L = 1$ doublets, and (G, G^*) is the $L = 2$ doublet.

Matrix elements of a heavy quark current $J(q) = \bar{Q}'\Gamma Q$ between the physical meson states can be calculated simply by taking the trace ($\omega = v \cdot v'$),

$$\langle\Psi'(v')|J(q)|\Psi(v)\rangle = \mathrm{Tr}[\bar{M}'(v')\Gamma M(v)]\mathcal{M}(\omega)\ , \quad (9)$$

where M' and M denote appropriate matrices from (1)-(8), $\bar{M} = \gamma^0 M^\dagger \gamma^0$, and $\mathcal{M}(\omega)$ represents the LDF. Again following [7, 8], we define the IW functions for the transitions of a 0^- ground state into an excited state by

$$\mathcal{M}(\omega) = \begin{cases} \xi_C(\omega)\ , & 0^-_{\frac{1}{2}} \to (0^-_{\frac{1}{2}}, 1^-_{\frac{1}{2}})\ , \\ \xi_E(\omega)\ , & 0^-_{\frac{1}{2}} \to (0^+_{\frac{1}{2}}, 1^+_{\frac{1}{2}})\ , \\ \xi_F(\omega)v_\mu\ , & 0^-_{\frac{1}{2}} \to (1^+_{\frac{3}{2}}, 2^+_{\frac{3}{2}})\ , \\ \xi_G(\omega)v_\mu\ , & 0^-_{\frac{1}{2}} \to (1^-_{\frac{3}{2}}, 2^-_{\frac{3}{2}})\ . \end{cases} \quad (10)$$

The vector index in the last two definitions will be contracted with the one in the representations of excited states (5)-(8). We now show how one can define the IW functions given above in terms of the overlaps of the wave functions of the initial and final states of the LDF.

3 Defining IW functions

It has been pointed out [4] that the assumption of the heavy quark limit implies a simple formula for the wave function of any particle containing one very heavy quark (with total angular momentum J and its projection λ),

$$\Psi^{(\alpha)}_{J\lambda}(v) = \sum_{\lambda_j,\lambda_Q} \langle j,\lambda_j;\frac{1}{2},\lambda_Q|J\lambda\rangle \Phi_{j\lambda_j}(v) u_{\lambda_Q}(v) \ . \tag{11}$$

In this formula (α) refers to all other quantum numbers of the meson, $u_{\lambda_Q}(v)$ is the free Dirac bispinor describing a heavy quark with spin $\frac{1}{2}$, helicity λ_Q, and velocity v (and normalized to $\bar{u}u = 2m$). $\Phi_{j\lambda_j}$ is the wave function of the LDF with total angular momentum j (in the rest frame of the particle), and its projection λ_j. For a meson, this is the wave function of the light antiquark.

From (11) it can be immediately seen that matrix elements of the heavy quark currents $J(q) = \bar{Q}'\Gamma Q$ are linear combinations of matrix elements

$$\langle \Phi'_{j'\lambda_{j'}}(v')|\Phi_{j\lambda_j}(v)\rangle \bar{u}'_{\lambda_{Q'}}(v')\Gamma u_{\lambda_Q}(v) \ . \tag{12}$$

For a given Γ, $\bar{u}'\Gamma u$ is a product of known matrices, and therefore all the unknown dynamics is contained in the overlaps of LDF wave functions $\langle\Phi'|\Phi\rangle$. We choose the spin projection axis (z) as the velocity direction of meson Ψ' as seen in the rest frame of meson Ψ. From the independence of the overlap on the direction of the x-axis we then have

$$\langle \Phi'_{j'\lambda_{j'}}|\Phi_{j\lambda_j}\rangle = 0 \ , \ \text{if } \lambda_{j'} \neq \lambda_j \ . \tag{13}$$

Similarly, from the independence of the overlap on the orientation of the y axis,

$$\langle \Phi'_{j'\lambda_{j'}}|\Phi_{j\lambda_j}\rangle = \eta\eta'(-1)^{j'-j}\langle \Phi'_{j',-\lambda_{j'}}|\Phi_{j,-\lambda_j}\rangle \ , \tag{14}$$

where η and η' are the orbital parities of the initial and final state of the LDF. In the case of interest to us, for the change of the orbital angular momentum from L to L', $\eta\eta' = (-1)^{L+L'}$. Equations (13) and (14) have been given in [4].

As an illustrative example, we choose the $0^- \to 0^+$ transitions and axial-vector current ($\Gamma = \gamma^\mu\gamma_5$). From (11) we have

$$\Psi^{(\pm)}_0 = \frac{1}{\sqrt{2}}[\Phi^{(\pm)}_{\frac{1}{2},\frac{1}{2}} u_{-\frac{1}{2}} - \Phi^{(\pm)}_{\frac{1}{2},-\frac{1}{2}} u_{\frac{1}{2}}] \ , \tag{15}$$

where $+$ and $-$ refer to 0^+ and 0^- states, respectively. Also, from (13) and (14) one can see that

$$\langle \Phi'^{(+)}_{\frac{1}{2},-\frac{1}{2}}(v')|\Phi^{(-)}_{\frac{1}{2},-\frac{1}{2}}(v)\rangle = -\langle \Phi'^{(+)}_{\frac{1}{2},\frac{1}{2}}(v')|\Phi^{(-)}_{\frac{1}{2},\frac{1}{2}}(v)\rangle \ , \tag{16}$$

and all other overlaps are zero. Therefore, it immediately follows that

$$\langle 0^+, v'|\Gamma|0^-, v\rangle = \frac{1}{2}[\bar{u}'_{-\frac{1}{2}}\Gamma u_{-\frac{1}{2}} - \bar{u}'_{\frac{1}{2}}\Gamma u_{\frac{1}{2}}]\langle \Phi'^{(+)}_{\frac{1}{2},\frac{1}{2}}(v')|\Phi^{(-)}_{\frac{1}{2},\frac{1}{2}}(v)\rangle \ . \tag{17}$$

Now choosing $\Gamma = \gamma^3\gamma_5$ and evaluating (17) in the rest frame of 0^- meson, where $v^\mu = (1,0,0,0)$ and $v'^\mu = (\omega, 0, 0, \sqrt{\omega^2-1})$, one obtains

$$\langle 0^+, v'|\gamma^3\gamma_5|0^-, v\rangle = -\sqrt{mm'}\sqrt{2}\sqrt{\omega+1}\langle \Phi'^{(+)}_{\frac{1}{2},\frac{1}{2}}(v')|\Phi^{(-)}_{\frac{1}{2},\frac{1}{2}}(v)\rangle \ . \tag{18}$$

On the other hand, using (1), (3) and (10) inside (9) one finds

$$\langle 0^+, v'|\gamma^\mu\gamma_5|0^-, v\rangle = \sqrt{mm'}(-v^\mu + v'^\mu)\xi_E(\omega) \ , \tag{19}$$

which specialized to the rest frame of 0^- state yields

$$\langle 0^+, v'|\gamma^3\gamma_5|0^-, v\rangle = \sqrt{mm'}\sqrt{\omega^2-1}\xi_E(\omega) \ . \tag{20}$$

Comparing (18) and (20) we obtain (apart from the irrelevant overall sign)

$$\xi_E(\omega) = \sqrt{\frac{2}{\omega-1}}\langle \Phi'^{(+)}_{\frac{1}{2},\frac{1}{2}}(v')|\Phi^{(-)}_{\frac{1}{2},\frac{1}{2}}(v)\rangle \ . \tag{21}$$

Of course, in order to obtain this formula for ξ_E (with the same overall sign), we could have chosen any other component and any other current giving a non-vanishing matrix element. Also, instead of $0^- \to 0^+$ transitions, we could have chosen $0^- \to 1^+$ transitions, and specialize to any of the three possible polarizations of the 1^+ state. Finally, any other reference frame besides the rest frame of 0^- state should yield the same expression.

Let us summarize the results obtained following the simple proceedure outlined above for several cases of interest. We emphasize that all the results given here were explicitly verified for the choices of $\Gamma = \gamma^\mu, \gamma^\mu\gamma_5, \gamma^\mu\gamma^\nu, \gamma^\mu\gamma^\nu\gamma_5$, and in two convenient reference frames: besides the rest frame of the 0^- meson, we have also used the Breit frame ($\mathbf{v} = -\mathbf{v}'$), in which $v^\mu = (\sqrt{\frac{\omega+1}{2}}, 0, 0, -\sqrt{\frac{\omega-1}{2}})$ and $v'^\mu = (\sqrt{\frac{\omega+1}{2}}, 0, 0, \sqrt{\frac{\omega-1}{2}})$. Polarization vectors describing spin 1 states ($\epsilon \cdot v' = 0$) were the standard ones, $\epsilon^{(\pm)} = \mp\frac{1}{\sqrt{2}}(0, 1, \pm i, 0)$ in both frames, $\epsilon^{(0)} = (\sqrt{\omega^2-1}, 0, 0, \omega)$ in the rest frame of 0^-, and $\epsilon^{(0)} = (\sqrt{\frac{\omega-1}{2}}, 0, 0, \sqrt{\frac{\omega+1}{2}})$ in the Breit frame. Let us also, for the sake of simplicity, define

$$\langle\Phi'|\Phi\rangle \equiv \langle\Phi'^{(\alpha')}_{j',\frac{1}{2}}(v')|\Phi^{(\alpha)}_{\frac{1}{2},\frac{1}{2}}(v)\rangle \ , \tag{22}$$

and state our results in terms of this overlap:

- $0^-_{\frac{1}{2}} \to (0^-_{\frac{1}{2}}, 1^-_{\frac{1}{2}})$ transitions. In this case we obtain

$$\xi_C(\omega) = \sqrt{\frac{2}{\omega+1}}\langle\Phi'|\Phi\rangle \ . \tag{23}$$

  This expression was given in [10, 11].

- $0^-_{\frac{1}{2}} \to (0^+_{\frac{1}{2}}, 1^+_{\frac{1}{2}})$ transitions. Here, as shown in the previous section,

$$\xi_E(\omega) = \sqrt{\frac{2}{\omega-1}}\langle\Phi'|\Phi\rangle \ . \tag{24}$$

- $0^-_{\frac{1}{2}} \to (1^+_{\frac{3}{2}}, 2^+_{\frac{3}{2}})$ transitions. In this case we have

$$\xi_F(\omega) = \sqrt{\frac{3}{\omega-1}}\frac{1}{\omega+1}\langle\Phi'|\Phi\rangle \ . \tag{25}$$

- $0^-_{\frac{1}{2}} \to (1^-_{\frac{3}{2}}, 2^-_{\frac{3}{2}})$ transitions. Here,

$$\xi_G(\omega) = \sqrt{\frac{3}{\omega+1}}\frac{1}{\omega-1}\langle\Phi'|\Phi\rangle \ . \tag{26}$$

We now proceed to relate the overlaps $\langle\Phi'|\Phi\rangle$ to the actual wave functions describing the LDF in the rest frame of the particle. Following [11, 12], in the valence quark approximation, we can write for the LDF wave function in the rest frame of the particle

$$\Phi^{(0)}(x) = \phi^{(0)}(\mathbf{x})e^{-iE_{\bar{q}}t} \ , \tag{27}$$

where $E_{\bar{q}}$ denotes the energy of the LDF. The LDF wave function of a meson moving with (ordinary) velocity $\boldsymbol{\beta}$ along the z axis (laboratory frame) is then given by

$$\Phi(x') = S(\boldsymbol{\beta})\Phi^{(0)}(x) \ , \tag{28}$$

with $x' = \Lambda^{-1}(\boldsymbol{\beta})x$ being the laboratory frame, x the rest frame of the meson, and $S(\boldsymbol{\beta})$ is the wave function Lorentz boost.

Since IW functions are Lorentz invariant they can be calculated in any frame. Particularly convenient is the Breit frame, in which the two mesons move with equal and opposite velocities. As noted in [11], the wave functions relevant for the overlap $\langle\Phi'|\Phi\rangle$ are at $t' = 0$ in the Breit frame. Therefore, denoting the three-velocity of the final meson as $\boldsymbol{\beta}$, by the use of (28) we have

$$\begin{aligned}
\langle\Phi'(v')|\Phi(v)\rangle &= \int d^3x'\Phi'^{\dagger}(x')\Phi(x')|_{t'=0} \\
&= \int d^3x'\Phi'^{(0)\dagger}(x_+)S^{\dagger}(\boldsymbol{\beta})S(-\boldsymbol{\beta})\Phi^{(0)}(x_-)|_{t'=0} \\
&= \int d^3x'\Phi'^{(0)\dagger}(x_+)\Phi^{(0)}(x_-)|_{t'=0} \ .
\end{aligned} \tag{29}$$

In this expression x_+ and x_- denote the rest frames of the final (moving in the $+z$ direction) and initial meson (moving in the -z direction), respectively. To obtain the last equation we have used the fact that Lorentz boosts satisfy $S^{\dagger}(\boldsymbol{\beta}) = S(\boldsymbol{\beta}) = S^{-1}(-\boldsymbol{\beta})$, so that boost factors cancel out. Also, we have ($\beta = |\boldsymbol{\beta}|$)

$$x_{\pm}|_{t'=0} = \Lambda(\pm\boldsymbol{\beta})x'|_{t'=0} = (\mp\gamma\beta z', x', y', \gamma z') \ , \tag{30}$$

where, in terms of ω,

$$\gamma = \sqrt{\frac{\omega+1}{2}} \,, \tag{31}$$

$$\beta = \sqrt{\frac{\omega-1}{\omega+1}} \,. \tag{32}$$

Using (27) and (30) in (29) we find

$$\langle\Phi'(v')|\Phi(v)\rangle = \int d^3x' \phi'^{(0)\dagger}(x',y',\gamma z')\phi^{(0)}(x',y',\gamma z')e^{-i(E_{\bar{q}}+E'_{\bar{q}})\gamma\beta z'} \,. \tag{33}$$

Finally, after rescaling the z' coordinate ($z' \to \frac{1}{\gamma}z'$), renaming integration variables, and using kinematical identities (31) and (32), we obtain

$$\langle\Phi'(v')|\Phi(v)\rangle = \sqrt{\frac{2}{\omega+1}}\int d^3x \phi'^{(0)\dagger}(\mathbf{x})\phi^{(0)}(\mathbf{x})e^{-iaz} \,, \tag{34}$$

where

$$a = (E_{\bar{q}}+E'_{\bar{q}})\sqrt{\frac{\omega-1}{\omega+1}} \,. \tag{35}$$

This formula was first obtained in [11] for the semileptonic $B \to D^{(*)}e\bar{\nu}_e$ decays (where $E'_{\bar{q}} = E_{\bar{q}}$), and was already used for the calculation of the corresponding Isgur-Wise function [11]-[15].

To illustrate the use of the results obtained in this section, we consider the non-relativistic quark model as one simple example.

4 Non-relativistic quark model

Assuming that we can describe heavy-light mesons using a simple non-relativistic potential model, the rest frame LDF wave functions (with angular momentum j and its projection λ_j), can be written as

$$\phi^{(\alpha L)}_{j\lambda_j}(\mathbf{x}) = \sum_{m_L,m_s} R_{\alpha L}(r)Y_{Lm_L}(\Omega)\chi_{m_s}\langle L,m_L;\frac{1}{2},m_s|j,\lambda_j;L,\frac{1}{2}\rangle \,, \tag{36}$$

where χ_{m_s} represent the rest frame spinors normalized to one, $\chi^\dagger_{m'_s}\chi_{m_s} = \delta_{m'_s,m_s}$, and α represents all other quantum numbers.

Explicitly, taking into account Clebsch-Gordan coefficients, the states that we need are

$$\begin{aligned}
\phi^{(\alpha 0)}_{\frac{1}{2}\frac{1}{2}}(\mathbf{x}) &= R_{\alpha 0} Y_{00}\chi_{\frac{1}{2}} \ , & (37)\\
\phi^{(\alpha 1)}_{\frac{1}{2}\frac{1}{2}}(\mathbf{x}) &= R_{\alpha 1}[\sqrt{\frac{2}{3}}Y_{11}\chi_{-\frac{1}{2}} - \sqrt{\frac{1}{3}}Y_{10}\chi_{\frac{1}{2}}] \ , & (38)\\
\phi^{(\alpha 1)}_{\frac{3}{2}\frac{1}{2}}(\mathbf{x}) &= R_{\alpha 1}[\sqrt{\frac{1}{3}}Y_{11}\chi_{-\frac{1}{2}} + \sqrt{\frac{2}{3}}Y_{10}\chi_{\frac{1}{2}}] \ , & (39)\\
\phi^{(\alpha 2)}_{\frac{3}{2}\frac{1}{2}}(\mathbf{x}) &= R_{\alpha 2}[\sqrt{\frac{3}{5}}Y_{21}\chi_{-\frac{1}{2}} - \sqrt{\frac{2}{5}}Y_{20}\chi_{\frac{1}{2}}] \ . & (40)
\end{aligned}$$

Now, using the well known expression

$$e^{-ikz} = \sum_{l=0}^{\infty}(2l+1)(-i)^l j_l(kr)\sqrt{\frac{4\pi}{2l+1}}Y_{l0} \ , \qquad (41)$$

together with the wave functions given above, the overlap expression (34) gives[1]

$$\begin{aligned}
\langle\Phi^{\alpha' 0}_{\frac{1}{2}\frac{1}{2}}(v')|\Phi^{\alpha 0}_{\frac{1}{2}\frac{1}{2}}(v)\rangle &= \sqrt{\frac{2}{\omega+1}}\langle j_0(ar)\rangle^{\alpha'\alpha}_{00} \ , & (42)\\
\langle\Phi^{\alpha' 1}_{\frac{1}{2}\frac{1}{2}}(v')|\Phi^{\alpha 0}_{\frac{1}{2}\frac{1}{2}}(v)\rangle &= i\sqrt{\frac{2}{\omega+1}}\langle j_1(ar)\rangle^{\alpha'\alpha}_{10} \ , & (43)\\
\langle\Phi^{\alpha' 1}_{\frac{3}{2}\frac{1}{2}}(v')|\Phi^{\alpha 0}_{\frac{1}{2}\frac{1}{2}}(v)\rangle &= -i\sqrt{2}\sqrt{\frac{2}{\omega+1}}\langle j_1(ar)\rangle^{\alpha'\alpha}_{10} \ , & (44)\\
\langle\Phi^{\alpha' 2}_{\frac{3}{2}\frac{1}{2}}(v')|\Phi^{\alpha 0}_{\frac{1}{2}\frac{1}{2}}(v)\rangle &= \sqrt{2}\sqrt{\frac{2}{\omega+1}}\langle j_2(ar)\rangle^{\alpha'\alpha}_{20} \ , & (45)
\end{aligned}$$

where

$$\langle F(r)\rangle^{\alpha'\alpha}_{L'L} = \int r^2 dr R^*_{\alpha' L'}(r)R_{\alpha L}(r)F(r) \ . \qquad (46)$$

[1]Using the states analogous to (37)-(40), but for $\lambda_j = -\frac{1}{2}$, one can verify that all overlaps indeed satisfy (13) and (14), so that everything is consistent.

At this point, one can readily obtain the values for the IW functions (23)-(26) and their derivatives at the zero recoil point. Let us summarize the results (ignoring irrelevant phase factors and suppressing quantum numbers α' and α):

- $0^-_{\frac{1}{2}} \to (0^-_{\frac{1}{2}}, 1^-_{\frac{1}{2}})$ transitions.

$$\xi_C(\omega) = \frac{2}{\omega+1}\langle j_0(ar)\rangle_{00} \ , \tag{47}$$

$$\xi_C(1) = \langle 1\rangle_{00} \ , \tag{48}$$

$$\xi_C'(1) = -\frac{1}{2} - \frac{1}{12}(E_{\bar{q}} + E'_{\bar{q}})^2 < r^2 >_{00} \ . \tag{49}$$

Note that these expressions include transitions from the ground state into radially excited states. If the two $j = \frac{1}{2}$ states are the same, $\xi_C(1)$ is normalized to one and $E'_{\bar{q}} = E_{\bar{q}}$. Otherwise, $\xi_C(1)$ vanishes because excited states are orthogonal to the ground state. Also note that we reobtain the bound [11] $\xi_C'(1) \leq -\frac{1}{2}$, which is lower than the bound obtained from the Bjorken sum rule [16] ($\xi_C'(1) \leq -\frac{1}{4}$). Both upper bounds follow from the basic principles and are not inconsistent.[2]

[2]The Bjorken sum rule connects $\xi_C'(1)$ with values for form factors describing transitions from S into P-wave states. In our notation, it is given as $\xi_C'(1) = -(\frac{1}{4} + \Delta)$, where $\Delta = \frac{1}{4}\sum_i |\xi_E^{(i)}(1)|^2 + \frac{2}{3}\sum_j |\xi_F^{(j)}(1)|^2 + \ldots$ is a positive quantity. On the other hand, equation (49) does not say anything about other form factors. Assuming that the two lowest P-wave doublets are degenerate (which is true in spinless models), and neglecting contributions from the excited P-wave doublets, one can combine the Bjorken sum rule with (49), (51) and (54). This leads to inequality $\Delta \geq \frac{1}{12}(E_{\bar{q}} + E'_{\bar{q}})^2\langle r\rangle_{10}^2 \geq \frac{1}{4}$, where $E_{\bar{q}}$ and $E'_{\bar{q}}$ are the S and P-wave energies of the LDF, respectively. This resembles the uncertainty principle, and the minimum value for Δ is always larger than $\frac{1}{4}$, thus avoiding conflict between the two limits. We have explicitly checked that the above inequality is satisfied for several "heavy-light systems" for which analytical solutions are possible (e.g. the hydrogen atom). The same conclusion also follows from numerical values of S to P-wave form factors at zero recoil point, obtained in the context of several realistic heavy-light models [17].

- $0^-_{\frac{1}{2}} \to (0^+_{\frac{1}{2}}, 1^+_{\frac{1}{2}})$ transitions.

$$\begin{aligned}
\xi_E(\omega) &= \frac{2}{\sqrt{\omega^2-1}}\langle j_1(ar)\rangle_{10} \ , &(50)\\
\xi_E(1) &= \frac{1}{3}(E_{\bar q}+E'_{\bar q})\langle r\rangle_{10} \ , &(51)\\
\xi'_E(1) &= -\frac{1}{6}(E_{\bar q}+E'_{\bar q})\langle r\rangle_{10} - \frac{1}{60}(E_{\bar q}+E'_{\bar q})^3 < r^3 >_{10} \ \ . &(52)
\end{aligned}$$

- $0^-_{\frac{1}{2}} \to (1^+_{\frac{3}{2}}, 2^+_{\frac{3}{2}})$ transitions.

$$\begin{aligned}
\xi_F(\omega) &= \sqrt{\frac{3}{\omega^2-1}}\frac{2}{\omega+1}\langle j_1(ar)\rangle_{10} \ , &(53)\\
\xi_F(1) &= \frac{1}{2\sqrt{3}}(E_{\bar q}+E'_{\bar q})\langle r\rangle_{10} \ , &(54)\\
\xi'_F(1) &= -\frac{1}{2\sqrt{3}}(E_{\bar q}+E'_{\bar q})\langle r\rangle_{10} - \frac{1}{40\sqrt{3}}(E_{\bar q}+E'_{\bar q})^3 < r^3 >_{10} \ \ . &(55)
\end{aligned}$$

- $0^-_{\frac{1}{2}} \to (1^-_{\frac{3}{2}}, 2^-_{\frac{3}{2}})$ transitions.

$$\begin{aligned}
\xi_G(\omega) &= \frac{2\sqrt{3}}{\omega^2-1}\langle j_2(ar)\rangle_{20} \ , &(56)\\
\xi_G(1) &= \frac{1}{10\sqrt{3}}(E_{\bar q}+E'_{\bar q})^2\langle r^2\rangle_{20} \ , &(57)\\
\xi'_G(1) &= -\frac{1}{10\sqrt{3}}(E_{\bar q}+E'_{\bar q})^2\langle r^2\rangle_{20} - \frac{1}{280\sqrt{3}}(E_{\bar q}+E'_{\bar q})^4 < r^4 >_{20} \ \ . &(58)
\end{aligned}$$

Let us briefly compare these results with the formulation of [8], which was also followed by [9]. There, apart from irrelevant phase factors, different IW functions are identified directly as overlaps of the wave functions of the initial and the final state of the LDF in the rest frame of the initial meson. Explicitly (putting a tilde over the form factors to avoid confusion),

$$\tilde{\xi}_C(\omega) \ = \ \langle j_0(\tilde{a}r)\rangle_{00} \ , \qquad (59)$$

$$\tilde{\xi}_E(\omega) = \sqrt{3}\langle j_1(\tilde{a}r)\rangle_{10} \ , \tag{60}$$
$$\tilde{\xi}_F(\omega) = \sqrt{3}\langle j_1(\tilde{a}r)\rangle_{10} \ , \tag{61}$$
$$\tilde{\xi}_G(\omega) = \sqrt{5}\langle j_2(\tilde{a}r)\rangle_{20} \ , \tag{62}$$

with the definition

$$\tilde{a} = E'_{\bar{q}}\sqrt{\omega^2 - 1} \ . \tag{63}$$

This formulation has several difficulties. For example, if one uses harmonic oscillator wave functions (as was done in [8]) in order to estimate overlaps $\langle j_j(\tilde{a}r)\rangle$, then it is easy to show that

$$\tilde{\xi}'_C(1) = -\frac{E'^2_{\bar{q}}}{2\beta_B^2} \ , \tag{64}$$

where β_B is the variational parameter. Using the values from the ISGW model [18] (which is also used in [8]), $\beta_B \approx 0.4\ GeV$ and $E'_{\bar{q}} \approx 330\ MeV$, one gets the slope of the IW function for the semileptonic B decays,

$$\tilde{\xi}'_C(1) \approx -0.34 \ , \tag{65}$$

which is clearly too large [19]. On the other hand, the same wave function used in (49) gives ($E_{\bar{q}} = E'_{\bar{q}} \approx 330\ MeV$)

$$\xi'_C(1) \approx -0.84 \ , \tag{66}$$

a result which is in much better agreement with the data [19]. Furthermore, from (59)-(62) one can see that all form factors (except for $\tilde{\xi}_C$) vanish at the zero recoil point, whereas the Bjorken sum rule [16] requires a nonvanishing P-wave form factors in this limit. It is clear that our form factors (50) and (53) do not suffer from that problem. Also, from (60) and (61) one can see that $\tilde{\xi}_E(\omega) = \tilde{\xi}_F(\omega)$, while (50) and (53) imply that our form factors satisfy

$$\xi_E(\omega) = \frac{\omega + 1}{\sqrt{3}}\xi_F(\omega) \ , \tag{67}$$

and in particular

$$\xi_E(1) = \frac{2}{\sqrt{3}}\xi_F(1) \ . \tag{68}$$

Even though we shall postpone a full account of the radiative rare B decays for later [20], let us just point out that the authors of [8] emphasize a substantially larger branching fraction for the decay $B \to K_2^*(1430)\gamma$ than the one found in [21]. If one includes only the factor of $\sqrt{\frac{3}{\omega-1}}\frac{1}{\omega+1}$ from (25) (which is about $\frac{1}{\sqrt{3}}$ for $\omega \approx 2$), one gets a factor of 3 smaller result than the one quoted in [8], bringing this particular branching ratio into much better agreement with the result of [21].

Finally, we can generalize the quark model approach to any model involving the Dirac equation with a spherically symmetric potential. There, the wave function has the form

$$\phi^{(\alpha k)}_{j\lambda_j}(\mathbf{x}) = \left(\begin{array}{c} f^k_{\alpha j}(r)\mathcal{Y}^k_{j\lambda_j}(\Omega) \\ ig^k_{\alpha j}(r)\mathcal{Y}^{-k}_{j\lambda_j}(\Omega) \end{array} \right) \ , \tag{69}$$

where $\mathcal{Y}^k_{j\lambda_j}$ are the usual spherical spinors, $k = l$ ($l = j + \frac{1}{2}$) or $k = -l-1$ ($l = j - \frac{1}{2}$), and α again denotes all other quantum numbers. Using this it can be shown that all the expressions (42)-(45) and (47)-(58) are unchanged, except that the expectation value (46) is replaced by

$$\langle F(r)\rangle^{\alpha'\alpha}_{L'L} \to \langle F(r)\rangle^{\alpha'\alpha}_{j'j} = \int r^2 dr [f^{*k'}_{\alpha' j'}(r) f^k_{\alpha j}(r) + g^{*k'}_{\alpha' j'}(r) g^k_{\alpha j}(r)] F(r) \ . \tag{70}$$

5 Conclusions

We have presented a simple method for relating form factors as defined by the covariant trace formalism [5, 6, 7], to the explicit overlaps of the rest frame wave functions describing the initial and the final states of the light degrees of freedom. We have obtained explicit formulae for several transitions of interest (from 0^- into a few lowest excited states), and have shown how one can apply these expressions in the simple quark model, and in models involving the Dirac equation with spherically symmetric potentials. We have also pointed out several inconsistencies present in recent calculations of radiative rare B decays into higher K-resonances [8, 9], and have shown how these can be fixed even within the same non-relativistic quark model that was used in [8]. A full account of the radiative rare B decays will be presented elsewhere [20].

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References

[1] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989); Phys. Lett. B **237**, 527 (1990).

[2] H. Georgi, Phys. Lett. B **240**, 447 (1990).

[3] There are many excellent reviews on the subject of heavy quark symmetry and effective theory. See, for example, H. Georgi, in Proceedings of TASI-91, edt. by R. K. Ellis et al., World Scientific, 1991; N. Isgur and M. B. Wise, in *Heavy Flavours*, edt. by. A. J. Bouras and M. Lindner, World Scientific, 1992; M. Neubert, Phys. Rept. **245**, 259 (1994).

[4] K. Zalewski, Phys. Lett. B **264**, 432 (1991).

[5] H. Georgi, Nucl. Phys. B **348**, 293 (1991).

[6] J. G. Körner and G. A. Schuler, Z. Phys. C **38**, 511 (1988).

[7] A. F. Falk, Nucl. Phys. B **378**, 79 (1992).

[8] A. Ali, T. Ohl, and T. Mannel, Phys. Lett. B **298**, 195 (1993).

[9] M. R. Ahmady, D. Liu, and A. H. Fariborz, *Estimates of B-Decays into K-resonances and Dileptons*, hep-ph/9506235.

[10] M. Neubert and V. Rickert, Nucl. Phys. B **382**, 97 (1992).

[11] M. Sadzikowski and K. Zalewski, Z. Phys. C **59**, 677 (1993).

[12] H. Høgaasen and M. Sadzikowski, Z. Phys. C **64**, 427 (1994).

[13] M. G. Olsson and S. Veseli, Phys. Rev. D **51**, 2224 (1995).

[14] M. G. Olsson, S. Veseli, and K. Williams, Phys. Rev. D **51**, 5079 (1995).

[15] M. G. Olsson and S. Veseli, Phys. Lett. B**353**, 96 (1995).

[16] J. D. Bjorken, I. Dunietz, and J. Taron, Nucl. Phys. B **371**, 111 (1992).

[17] S. Veseli and M. G. Olsson, *S to P wave form factors in semi-leptonic B decays*, UW-Madison preprint, MADPH-95-907 (hep-ph/9509230), to appear in Z. Phys. C.

[18] N. Isgur, D.Scora, B. Grinstein, and M. B. Wise, Phys. Rev. D **39**, 799 (1989).

[19] B. Barish et al., CLEO Collaboration, Phys. Rev. D **51**, 1014 (1995).

[20] S. Veseli and M. G. Olsson, *Radiative rare B decays revisited*, UW-Madison preprint, MADPH-95-900 (hep-ph/9508255), to appear in Phys. Lett. B.

[21] T. Altomari, Phys. Rev. D **37**, 677 (1988).