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Selenodynamical parameters from analysis of LLR observations of 1970–2001

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Красинский Г. А.. Определение селенодинамических параметров из анализа лунных лазерных измерений дальности Луны 1970–2001 гг..

Ключевые слова: Вращение Луны, лазерные измерения дальности.

Проведена обработка лазерных измерений дальности Луны 1970–2001 гг. с целью уточнения значений параметров системы Земля-Луна. Модель вращения Луны учитывает эффекты, обусловленные упругостью, приливной диссипацией энергии, фрикционным взаимодействием жидкого ядра и мантии. Для учета влияния жидкого ядра, аппроксимируемого трехосным эллипсоидом, развита модель, основанная на теории Пуанкаре, но учитывающая особенности вращения Луны. Совокупность оцениваемых селенодинамических параметров включает числа Лава h_2, l_2, k_2, Q-фактор диссипации, нормированный момент инерции, коэффициент фрикционной связи, гармоники второго и третьего порядков лунного потенциала и три параметра, характеризующие жидкое ядро. Для всех параметров кроме трех последних найденные оценки представляются вполне надежными. Таким образом убедительных доказательств прямого влияния жидкого ядра на вращение Луны получить не удалось. Для параметра Q (связанного с непосредственно оцениваемым углом запаздывания приливов δ соотношением $Q = 1/2\delta$) оценки находятся в интервале от $Q = 13$ до $Q = 18$ в зависимости от варианта решения, что довольно близко к полученному значению $Q = 11.032 \pm 0.004$ для Земли. Найденная величина Q для Луны указывает на значительную диссипацию энергии в Луне (сопоставимую с земной) несмотря на отсутствие на ней океанов, вносящих, как принято считать, основной вклад в приливную диссипацию Земли. Таким образом, эта гипотеза представляется сомнительной. Анализ временного поведения невязок выявляет их увеличение после марта 1998 г. Этот эффект удается устранить только включением координат отражателей после этой даты в качестве независимо оцениваемых параметров. Разности координат отражателей до и после указанной даты оказались не зависящими от отражателя (в пределах точности оценивания), что, по видимому, может быть интерпретировано как свидетельство внезапного смещения на несколько сантиметров центра масс Луны относительно ее коры.

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Keywords: Rotation of the Moon, Selenodynamics, Lunar Laser Ranging.

LLR observations of 1969–2001 are processed to estimate a set of parameters of the Earth-Moon system. The dynamical model accounts for effects of elasticity of the lunar body, tidal dissipation in the Moon, and friction coupling between the lunar mantle and its fluid core. A Poincare's type model is developed to describe effects of the fluid core assumed to be a three-axis ellipsoid. Estimated selenodynamical parameters include Love number h_2, l_2, k_2, dissipation factor Q, undimensional moment of inertia, coefficients of the lunar gravitational potential of the orders 2 and 3, coupling parameter κ and three parameters describing the fluid core. Except these three parameters the obtained estimates seem to be reliable. So no evidences of direct effects of the fluid core is found. For the dissipation factor Q (defined by the relation $Q = 1/2\delta$ where δ is the tidal lag) the estimates vary in the range from $Q = 13$ to $Q = 18$ depending on solution. They are of the same order as the value $Q = 11.032 \pm 0.004$ obtained for the Earth, which means that the tidal dissipation in the Moon is close to that in the Earth (notwithstanding that there are no oceans on the Moon to contribute to the dissipation). Thus the wide spread opinion that the largest contribution to the dissipation of energy in the Earth is due to ocean tides becomes doubtful. Analysis of residuals reveals a sharp change of their time behavior after March 1998 which effect could not be modeled by other ways but including corrections to coordinates of the reflectors after this date as independent solve-for parameters. Because the corrections derived for the all four observed reflectors appear to be rather close it is conjectured that near this date a jump of a few centimeters in the position of the lunar barycenter with respect to the lunar crust has occurred.

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Contents

1. Introduction

It is well known that the complicated non-rigid structure of the Earth strongly influences its rotation. Resulted effects are clearly detected in the time behavior of the Earth's orientation parameters derived by regular monitoring of the Earth's rotation (mainly by VLBI technics). At present the analogous subtle effects in the rotation of the Moon also may be studied making use of lunar laser ranging measurements (LLR) provided by regular observational programs started in 1969. A state-of-art analysis of the LLR data with applications to lunar rotation is given in (Dickey et al, 1994, [2]), and (Williams et al, 2001, [11]).

Unlike the problem of the Earth's rotation no monitoring of rotational parameters of the Moon is yet possible, thus the case of the Moon seems even more complicated then that of the Earth. The small effects to be studied are detectable only if a sophisticate dynamical model both of the orbital and rotational motions of the Moon is constructed by simultaneous integration of equations of motion of the major planets and the Moon (including equations of the lunar rotation). High accuracy of the LLR data invokes dynamical theories of the adequate precision. The works cited above made use of dynamical models of the well-known series of Developed Ephemerides constructed and supported by Jet Propulsion Laboratory (the ephemerides DE403 or the more advanced version DE405, see [8]). The present study uses as DE405 so the dynamical model EPM (Ephemerides of Planets and Moon) which is under developing in Institute of Applied Astronomy. For applications of EPM to planet problems see [9]. Analysis of LLR data depends not only on a dynamical model but on partial derivatives respectively to a number of parameters many of which also need numerical integration to be calculated. These partials are not distributed along with DE ephemerides and have been obtained by numerical integration with the help of EPM in the frame of a supporting software. An earlier version of the lunar dynamical model (in which no effects of dissipation in the lunar rotation have been taken into account) is described in [1]. In the present work the following improvements of this model have been carried out:

1. Torques due to elasticity of the lunar mantle and those caused by the lag of the tides in the lunar body are modeled as it is described in [3].

2. Poincare's method to model the motion of a rigid body with a fluid core is applied to the case of the Moon.
3. Frictional interaction between the core and the mantle is taken into consideration after [11].

2. Observations and ephemerides

In the present analysis the LLR observations of the time interval 1970–2001 are discussed. They have been carried out mainly at McDonald Observatory (Texas), and CERGA station (France). In McDonald at different epochs three different sites were activated; they are referred here as McDonald, MLRS1, and MLRS2. There exists also a set of observations of two year duration made at Haleakala Observatory (Hawai). Number N of the observations at each site is given in Table 1. About one hundred observations appeared to be either roughly erroneous or of a poor quality (with residuals about 20 nsec for the epoch after 1985). On the whole in this study 14612 observations have been used.

Table 1. Distribution of LLR observations.

Station	Time span	N
McDonald	1970 Mar - 1985 Jun	3439
MLRS1	1985 Jan - 1988 Jan	275
MLRS2	1988 Aug - 2001 Dec	2416
Haleakala	1989 Nov - 1990 Aug	694
CERGA	1988 Jan - 2001 Dec	7788
Total	1970 Mar - 2001 Dec	14612

In LLR analysis a number of parameters under estimation appear to be strongly correlated and may be reliably estimated only because not one but four reflectors could be observed. The latest reflector installed in the course of the Apollo program is Apollo 15; it is of the best quality and thus most often observable. About 78% of the LLR data are obtained for this reflector (numbers of rangings to Apollo 11, Apollo 14, Apollo 15 and Lunachod 2 are 1407, 1410, 11369, and 427 correspondingly). Unfortunately such disparity of the distribution deteriorates reliability of the estimates of a number of selenodynamical parameters.

A priory errors of observations before 1998 were calculated from published data applying the algorithm by Newhall (1995, personal communication). For observations after this date the *a priory* error of each observation is taken from publications when it exceeds 0.1 nsec, and set equal to 0.1 nsec in the opposite case. Before 1998 the observations are obtained by request from the observatories, later on they have been retrieved from FTP server $cddisa.gsfc.nasa.gov/pub/slr/llmpt$.

The dynamical model EPM has been constructed by simultaneous numerical integration of equations of orbital motion of the Moon, major planets, five biggest asteroids, and the lunar rotation. The integration includes reduced equations of 293 asteroids; it is important for the major planets, but not for the Moon. So in this study to save computing time perturbations only from 5 biggest asteroids are accounted for. Potential of the Moon is calculated up to the 4-th order of the zonal index, that of the Earth includes the 2-th order harmonics c_{20} and c_{22}.

To model effects of the fluid core the Poincare's method has been accommodated by accounting peculiarities of the rotational dynamics of the Moon. Some details are given in Appendix, here only a brief summary is outlined for understanding the discussion of the results of LLR processing given beneath. As in the case of the Earth, a resonance between the frequency of free core oscillations with the rotational frequency (one month for the Moon) takes place. Due to this resonance the model of the lunar rotation is very sensitive to the polar and equatorial dynamical flattenings β_c, γ_c of the core defined by the relations

$$\beta_c = \frac{C_c - A_c}{B_c}, \ \ \gamma_c = \frac{B_c - A_c}{C_c},$$

where A_c, B_c, C_c are moments of inertia of the core. Unlike the Earth's case the equatorial flattening of the core cannot be ruled out. As at present there is no reliable information on values of β_c, γ_c, it is not yet possible to carry out rigorous integration of the combined equations of rotation of the Moon and its core. Nevertheless one can model the main contribution from the core by a simplified model of Appendix. In this model the equations of lunar rotations include some additional terms depending on three parameters ν_1, ν_2, ν_3 which depend on γ_c, β_c and the ratio $\rho = C_c/C$ of the moments of inertia of the core and the Moon in the following way:

$$\nu_1 = G\rho\gamma_c,\ \ \nu_2 = G\rho\beta_c,\ \ \nu_3 = G^2\rho\gamma_c,$$

where the "gain-factor" G is

$$G = \frac{n}{\dot{\Omega} + n(\beta_c - 0.5\gamma_c)},$$

$n, \dot{\Omega}$ being the mean motion and the rate of nodes of the lunar orbit.

Meaning of the gain-factor may be understood taking in mind the conventional approach to modeling the fluid core effects in the theory of the Earth's rotation. For the Earth the equatorial flattenings γ_c is set equal to zero, n is the angular velocity of the Earth, and the value $\nu_c = n\beta_c$ is the frequency of so called free core nutation (one revolution in 430 days) while for the Moon this frequency is given by the relation $\nu_c = n(\beta_c - 0.5\gamma_c)$ that enters the expression for G. To account for effects of the fluid core of the Earth a transfer function has to be applied: it multiplies any rigid body nutational harmonics of the frequency ν by a factor which is proportional to $1/(\nu - \nu_c)$ (see for instance [7]). In the case of the Moon the largest harmonics corresponds to motion of the lunar nodes and that explains the origin of the gain-factor G given above.

The parameters ν_1, ν_2, ν_3 enter the equations of rotation in different ways and can be estimated simultaneously with other parameters of the Earth-Moon dynamical system. Then applying the relations $G = \nu_3/n_1, \gamma_c/\beta_c = \nu_1/\nu_2$ it is possible to derive all three parameters involved γ_c, β_c and ρ. If the combination $(2\beta_c - \gamma_c)$ is small in comparison with $|\dot{\Omega}/n| \approx 1/216$ (as it is naturally to suggest) then for G we could expect the estimate $G \approx -200$ from the analysis of LLR data.

Tidal perturbations in the lunar orbital motion caused by tidal dissipation on the Earth's body is computed by the model with a constant lag.

Partials of rangings respectively to dynamical parameters of the orbital and rotational model of the Moon are computed mostly by integration of variational equations; in a few cases they have been obtained by integration of the rigorous system of equations with slightly varied values of the parameter under study.

The LLR dataset has been also processed with the help of DE405 lunar ephemerides making use of the partials obtained with EPM. However nominal values of many of the estimated parameters in DE405 are not known; that is why only corrections to such parameters could be determined. Moreover we might implement the improved values of dynamical parameters only to EPM, but not to DE405. In Solution A values of 65 parameters listed in Table 2 have been improved, all of them being then fed back to EPM by iterations.

Table 2. List of estimated parameters.

1-6	Lunar orbital state vector for the epoch JD 2446000.5
7-12	Euler's angles and their time derivatives for the same epoch
13	Lag of the Moon's body tides
14-16	Lunar Love numbers k_2, h_2, l_2
17	Lag of the Earth's body tides
18-29	Harmonics of lunar potential from c_{20} to s_{33}
30-38	Coordinates of reflectors A11, A14, L2
39	Coordinates X of A15
40-54	Coordinates of 5 stations
55-56	Corrections to orientation of the Earth's equator ϵ, ϕ
57-58	Secular trends $\dot{\epsilon}, \dot{\phi}$
59-60	Secular trends in sidereal angles of the Earth and Moon
61	Undimensional lunar moment of inertia $g = C/mR^2$
62-64	Moon's core factors ν_1, ν_2, ν_3
65	Moon's core-mantle coupling factor κ

Some comments have to be done.

1. As lunar rangings are invariant relatively to rotations of the Earth-Moon system as whole, all set of parameters of orientation of this system cannot be determined simultaneously. Due to this reason two coordinates of the most often observable reflector Apollo 15 have been fixed (longitude and latitude). Values of these two parameters were obtained from a simplified solution made as the first step, in which lunar librations have not being improved.
2. LLR observations are sensitive to the Earth's gravitational constant Gm_E. However our experience has shown that the observable effect reduces to scaling of distances and cannot be reliably separated from corrections to the X coordinate of the reflectors. Thus the value Gm_E has not been not included to the list of estimated parameters.
3. For meaning of the coupling parameter κ see Appendix.

3. Discussion of results

The estimates of selenodynamical parameters (which are of primary interest of this study) are given in Table 4 both for EPM and DE405 as Solution A (65 parameters mentioned above). As improved values of dynamical parameters of DE405 could not be derived, only the obtained corrections to them are presented being marked by the prime symbol $'$. corrections are presented being marked by the prime symbol $'$. Post-fit residuals for EPM are presented in Fig 1. Analogous residuals for DE405 appear to be about 5 % less noisy; visually two plots could not be distinguished and thus the plot for DE405 is not given.

Because the EPM model has been implemented by the obtained corrections the post-fit residuals practically coincide with differences $O-C$ computed with the improved model. For DE405 a similar work could not be carried out in the full scale and only estimates of coordinates of the reflectors and Love numbers h_2, l_2 of the Moon have been incorporated. Corresponding differences O-C are presented in Fig. 3. The plot demonstrates that the lunar ephemerides DE405 need considerable corrections as it was naturally to expect because these ephemerides have been completed a number of years ago on the base of less set of the LLR data than used in the present work.

The plot of LLR residuals in Fig 1 shows that some sharp change of them has occurred after March 1998 (residuals for DE405 demonstrate quite the same type of the behavior). Being independent of the model applied such effect cannot be explained by any drawbacks of the dynamical models. Checking the algorithms and software used for reduction of LLR data has not revealed errors which could explain this effect. Experiments have proved that the degradation of fitting for the epoch after March 1998 disappears if coordinates of the all four reflectors after this epoch are estimated as independent parameters. Because the time span after March 1998 is comparatively small it appears possible to estimate all three coordinates of Apollo 15 for this epoch (unlike for the previous one). Indeed strong correlations with lunar librations do not arise as the librations are mainly depend on the observations of the previous epoch. In this way an alternative solution has been produced (Solution B; see Table 4) in which coordinates of the reflectors after March 1998 are determined independently. For this solution the noticed above peculiarities of the post-fit residuals vanish and the overall fitting becomes considerably better (see Fig. 2 where the plot is given for EPM, and Fig. 4 for DE405).

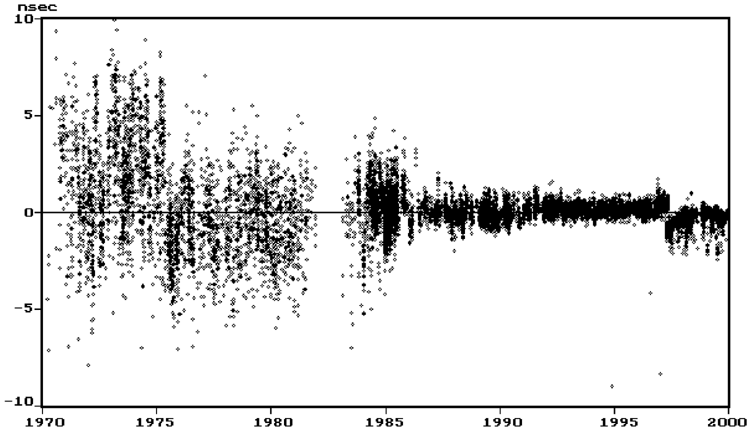

Fig. 1. EPM residuals, Solution A.

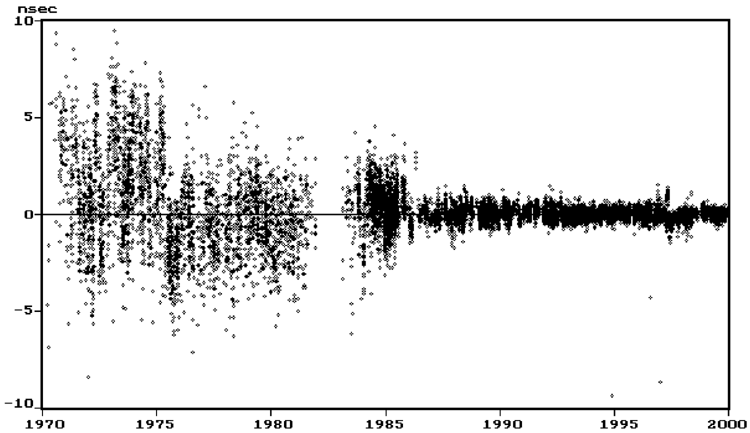

Fig. 2. EPM residuals, Solution B.

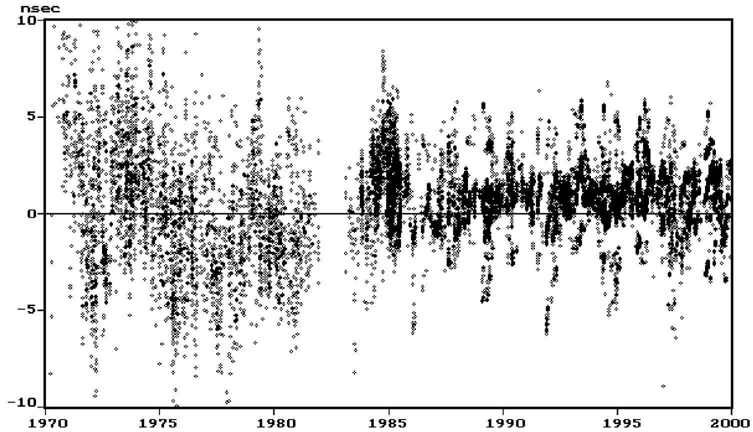

Fig. 3. DE405 prefit residuals.

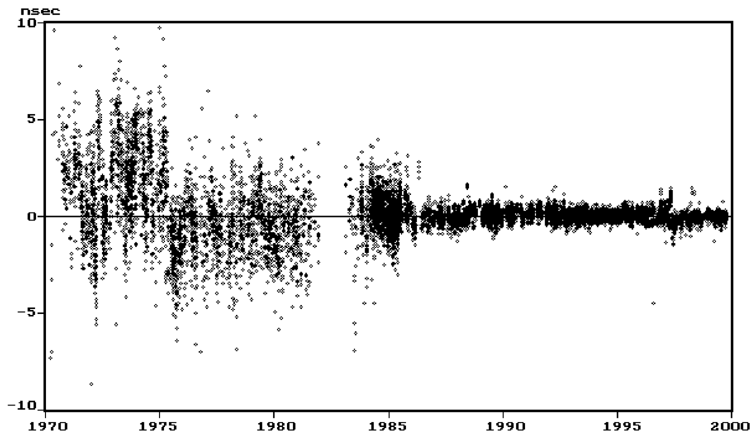

Fig. 4. DE405 residuals, Solution B.

In Table 3 the differences D_x, D_y, D_z between the two sets of coordinates of the reflectors related to the epoch before and after March 1998 are given for the DE405 model. One can see that the corrections for each reflectors are close. That makes plausible a conjecture that a sharp jump of the lunar crust (to which the reflectors are rigidly fixed) as whole relatively to the barycenter of the Moon took place at March 1998. Similar effects of a smaller scale are known to exist for the Earth from analysis of satellite ranging data.

Table 3. Jump in coordinates of reflectors after March 1998 (m).

D_x	D_y	D_z	
-0.171 ± 0.054	0.066 ± 0.083	-0.308 ± 0.077	Apollo 11
-0.192 ± 0.053	0.110 ± 0.104	-0.021 ± 0.096	Apollo 14
-0.090 ± 0.081	0.237 ± 0.086	-0.604 ± 0.256	Lunachod 2
-0.117 ± 0.049	0.270 ± 0.035	-0.240 ± 0.027	Apollo 15

By averaging the values given in Table 5 we have the following estimates of the jumps D_x, D_y, D_z in each of the coordinates X, Y, Z (in meters):

$$
\begin{aligned}
D_x &= -0.15 \pm 0.04 \\
D_y &= 0.23 \pm 0.07 \\
D_z &= -0.23 \pm 0.07.
\end{aligned}
$$

The analogous jumps in coordinates obtained with the EPM model are not so consistent but do not contradict to the DE405-based estimates:

$$
\begin{aligned}
D_x &= -0.17 \pm 0.03 \\
D_y &= 0.37 \pm 0.21 \\
D_z &= -0.10 \pm 0.26.
\end{aligned}
$$

Comparing the residuals obtained with EPM and DE405 as well as the formal errors of the estimates one can see that DE405 provides somewhat better fitting (about 5-10 %). Origin of these systematic errors in EPM is not yet clear. They are probably due to some mismodeling of lunar rotations because the experiments in which the orbital motion of the Moon is taken from EPM but its rotational motion from DE405 indeed improve fitting.

In Table 4 there are also presented LLR-based results from [2] and from the more recent publication [11]. For the aims of comparison we present our estimates of the lunar tidal lag δ also in terms of the quality factor Q defined as $Q = 1/2\delta$. Comparing the values of Q in papers [2], [11] with the given there time delays $\tau = \delta/n$ (n is the Moon's mean motion), it becomes clear that another definition has been used in these works: $Q = 1/\delta$. We prefer the conventional definition and thus the values Q in Table 4 taken from [2], [11] are transformed to be in accordance with it. Statistical errors in Table 4 are given only for the estimated parameters, but not for the derived ones.

In the equations of motion of the Moon the tidal lag is multiplied by k_2; so sometimes the effective parameter of dissipation is presented as the product $k_2\delta$ (or as k_2/Q). For the aims of comparison the combination k_2/Q also is presented. In the last line of the table the undimensional moment of inertia of the Moon $g = C/mR^2$ is given. This value was obtained by confronting the estimate of the gravitational coefficients c_{20} derived from its contribution to the orbital motion of the Moon, with the value $\beta = -c_{20}/g$ derived from its impact on the rotation of the Moon.

The errors of our estimates are formal ones given in the sense of one σ. As the residuals still expose signatures due to some unmodeled effects, the formal errors are too optimistic and in order to get more realistic values they have to be multiplied at least by the factor 2. That must be taken in mind when comparing our estimates with those from [2], [11]. Another reason why our formal errors sometimes are much less is the larger duration of our dataset (the end of the time span in [2] is December 1993, that of [11] is July 1998).

The following comments to the results presented in Table 4 are to be done:

Lunar Love number k_2. In paper [2] the estimate k_2=0.0302 is given and discussed in some detail. It is argued that lunar seismic velocity profiles provided by the Apollo mission correspond to smaller value $k_2 \approx 0.021 - 0.024$. The value k_2=0.0285 obtained in Solution B is well in accordance with the estimate k_2=0.0287 given in [11]; so our analysis has confirmed the noticed discrepancy.

Table 4. Selenodynamical parameters derived from LLR.

	EPM-A	EPM-B	DE405-A	DE405-B	Paper [2]	Paper [11]
k_2	0.0381	0.0285	0.0064′	0.0022′	0.0302	0.0287
	±9	±8	±9	±8	±12	±8
h_2	0.0903	0.0861	0.0996	0.0916		0.0340
	±42	±35	±39	±32		±180
l_2	0.0514	0.0426	0.0510	0.0510		
	±33	±27	±31	±25		
Lag,	1.6249	2.0559	0.0001′	0.0004′	2.204	1.518
deg	± 31	± 28	± 29	± 25		± 32
Q	17.630	13.034			13.250	18.870
					± 500	
k_2/Q	0.00215	0.00218			0.002272	0.00152
					± 32	
κ,	-70	-80	-66	-59		1122
$10^{-11}/d$	±6	±5	±5	±5		±257
g	0.39268	0.39223	0.00027′	−0.00018′		0.3940
	±4	±4	±4	±4		

Lunar Love numbers h_2, l_2. The estimates of lunar Love numbers h_2 and l_2 are only slightly sensitive to the used models as well as to the two versions of solution. These estimates seem to be reliable, though our h_2 is 2.5 times larger then that given in [11]. The estimate of l_2 is probably obtained in the first time and we do not aware of other publications to compare with.

Dissipation quality factor Q. Our estimate Q=13.0 for solution B is very close to the value 13.25 given in [2] but somewhat differs from Q=18.870 claimed in the more recent work [11] which is based on a larger set of LLR data (pay attention that in Table 4 the results of [2], [11] are scaled in order to correspond to the conventional definition of Q). Probably the discrepancy could be explained by the fact that in [11] the value of Q has been estimated in several frequency bands and thus Q at the monthly frequency (which value can be compared with our estimate) would diminish.

In this context it seems useful to give the estimate obtained for the Earth's dissipation factor Q_E (and the corresponding tidal lag δ_E) from its effects in the orbital motion of the Moon: $Q_E = 11.032 \pm 0.004$ ($\delta_E = 2.5968 \pm 0.0009$ deg). It corresponds to the secular deceleration of the lunar mean motion $\dot{n} \approx -25''/cy^2$ well confirmed by many studies. One can see that the Moon only slightly less dissipative than the Earth. This result seems important bearing in mind the adopted explanation of dissipative effects in the Earth's rotation as caused by the dissipation in oceans (after the well known monograph by Lambeck [6]). Now this suggestion seems doubtful because absence of oceans on the Moon shows that the strong dissipation originates in the planet body, and probably the same is true for the Earth. Another evidence that the dissipation is generated by the body tides is the surprisingly constant value of Q_E during the last 600 million years as it follows from paleontologic data (for most resent results see [4]). This conclusion may be of a practical importance as the adopted IERS standards for modeling the dynamics of the Earth's satellites deliberately do not include accounting for the lag δ_E of the body tides.

Effects of fluid core. In Solution B an attempt has been undertaken to evaluate parameters ν_1, ν_2, ν_3 and then to derive estimates of the parameters γ_c, β_c and ρ of the fluid core. Unfortunately while the estimates of ν_1, ν_2, ν_3 appear statistically significant

$$\begin{aligned} \nu_1 &= (126 \pm 18) \cdot 10^{-7} \\ \nu_2 &= (147 \pm 88) \cdot 10^{-8} \\ \nu_3 &= (267 \pm 76) \cdot 10^{-6} \end{aligned}$$

they lead to meaningless values of γ_c, β_c and ρ. These values are given here for DE405, those for EPM agree with them. So at present with the accuracies achieved the question whether the effects of the fluid core are detectable is still open.

Coupling factor κ. This parameter is a factor to multiply relative angular velocities of the core χ_x, χ_y in order to calculate the corresponding torque. In accordance with the theory developed in Appendix, on the place of χ_x, χ_y one can take the angular velocities ω_x, ω_y multiplied by the gain-factor G given above. When comparing our estimate of κ

with that from [11] given in Table 4 it is necessary to account that we have used the theoretical value $G = -216$ which corresponds to negligible β_c, γ_c. Reason of the disagreement with the result of [11] is not clear, probably it is due to another scaling.

In Table 5 there are given corrections to some parameters which probably have no clear physical meaning but characterize quality of the dynamical theories in use. They are corrections dϵ, dϕ to orientation of the Earth's equator, centennial rates d$\dot{\epsilon}$, d$\dot{\phi}$ of these values, and corrections to centennial rates of the Earth's and Moon's rotational angles. One can see that all these values are less for DE405 comparing with EPM which fact demonstrates again that in the lunar part of EPM there are still some unmodeled effects.

Table 5. Corrections to Earth's equator.

Parameter	Unit	EPM	σ	DE405	σ
dϵ	arcs	0.0018	0.0003	0.0007	0.0003
dϕ	arcs	0.0239	0.0005	0.0042	0.0004
d$\dot{\epsilon}$	arcs /cy	0.0089	0.0024	0.0138	0.0021
d$\dot{\phi}$	arcs/cy	-0.2432	0.0049	-0.0116	0.0043
$\dot{s}$, Earth	arcs/cy	-0.0226	0.0018	0.0014	0.0014
$\dot{s}$, Moon	arcs/cy	-0.1165	0.0121	-0.0293	0.0122

4. Conclusive remarks

The up-to-date LLR measurements of high precision provide unique information for selenodynamical studies. Adequate modeling of lunar rotation puts forward a challenge for analysts because LLR residuals still show signals of unknown origin. For instance quite mysterious is a signature (with any of the two dynamical models in use) that gives rise to improbable corrections to the Earth's Love numbers h_2, l_2 (about 0.2 and 0.1 with formal errors as small as a few percents). Including these parameters to the list of solve-for unknowns considerably improves the fitting and diminishes corrections to all lunar Love numbers by the factor

2. At this stage we have failed to find a parameter that could provoke such special signature in the residuals.

Our study shows that the direct impact of the fluid core on the Moon's rotation is probably negligible but the indirect effect of the frictional mantle-core interaction seems to be detectable. Hopefully more certain conclusions might be derived in future with more data from the ongoing LLR programs of CERGA and MLRS. We would like to pay attention of the contributors on a great importance for selenodynamics to obtain more ranging data for other reflectors but Apollo 15.

This study has been carried out by the software package ERA for ephemeris and dynamical astronomy (see [1], [5]). DOS version of the software with the LLR applications developed in this research, may be retrieved by anonymous FTP *quasar.ipa.nw.ru/incoming/era*.

Appendix

Modeling effects of the fluid core of the Moon

Here we follow the approach developed by Poincare for a rigid body with a fluid cavity [10].

The following notations are used:

$A < B < C$ are main moments of inertia of the Moon.
β, γ are combinations of A, B, C that enter the equations of lunar rotation.
θ is nutation angle of the coordinate system fixed to the main axes.
ϕ is precession angle of this system
ψ is rotational angle.
$\omega_x, \omega_y, \omega_z$ are projections of the lunar angular velocity $\boldsymbol{\omega}$ to the axes of the rotating system fixed to the mean axes of inertia.
N_x, N_y, N_z are projections in the same system of the momentum vector $\boldsymbol{N}$ of the perturbing forces from Sun and Earth in the rigid body approximation.

The following relations hold true:

$$\beta = \frac{C-A}{B} = \frac{(4c_{22} - 2c_{20})/g}{(4c_{22} + 2c_{20})/g + 2} \approx -\frac{c_{20}}{g},$$
$$\gamma = \frac{B-A}{C} = 4\frac{c_{22}}{g},$$

where

$$g = \frac{C}{mR^2}$$

is the undimensional moment of inertia (m, R being the lunar mass and its mean radius).

If $\boldsymbol{r}$ is radius vector of a perturbing body in the rotating frame of the mean axes of inertia, $\boldsymbol{\rho}$ is that in the inertial frame the following relation takes place

$$\boldsymbol{r} = P_3(\psi)P_1(\theta)P_3(\phi)\boldsymbol{\rho}, \tag{1}$$

where P_i $(i = 1, 2, 3)$ is rotational matrix for the i-th axis.

In these notations the equations of lunar rotation may written in the form:

$$
\begin{aligned}
\dot{\omega}_x &= \frac{\gamma - \beta}{1 - \beta\gamma}\omega_z\omega_y + \frac{N_x}{A}, \\
\dot{\omega}_y &= \beta\omega_z\omega_x + \frac{N_y}{B}, \\
\dot{\omega}_z &= -\omega_y\omega_x\gamma + \frac{N_z}{C}.
\end{aligned} \tag{2}
$$

At the right parts of these equations the components ω_x, ω_y, ω_z of the angular velocity have to be expressed in terms of the Euler's angles and their time velocities with the help of the Euler's kinematic relations

$$
\begin{aligned}
\omega_x &= \dot{\phi}\sin\theta\sin\psi + \dot{\theta}\cos\psi, \\
\omega_y &= \dot{\phi}\sin\theta\cos\psi - \dot{\theta}\sin\psi, \\
\omega_z &= \dot{\phi}\cos\theta + \dot{\psi}.
\end{aligned} \tag{3}
$$

Rotation of the fluid core in the Poincare model is described by the velocity field that can be characterized by the vector of angular velocity $\boldsymbol{\chi} = (\chi_x, \chi_y, \chi_z)$ relatively to the body fixed rotating frame. If A_c, B_c, C_c $(A_c < B_c < C_c)$ are the main moments of inertia of the fluid core then the following dynamical relations between $\omega_x, \omega_y, \omega_z$ and χ_x, χ_y, χ_z hold true [7]:

$$
\begin{aligned}
&(\dot{\omega}_x + \dot{\chi}_x)A_c + \chi_z(\omega_y + \chi_y)B_c - \chi_y(\omega_z + \chi_z)C_c = 0, \\
&(\dot{\omega}_y + \dot{\chi}_y)B_c + \chi_x(\omega_z + \chi_z)C_c - \chi_z(\omega_x + \chi_x)A_c = 0, \\
&(\dot{\omega}_z + \dot{\chi}_z)C_c + \chi_y(\omega_x + \chi_x)A_c - \chi_x(\omega_y + \chi_y)A_c = 0.
\end{aligned} \tag{4}
$$

Differential equations (2) have to be modified by adding the perturbing torques that depend on the vector $\boldsymbol{\chi}$. In terms of the moments of inertia A, B, C (not of the parameters of libration β, γ as in equations (2)) the modified equations have the form:

$$
\begin{aligned}
&A\dot{\omega}_x + \omega_z\omega_y(C-B) - B_c\chi_y\omega_z + C_c\chi_z\omega_y + A_c\dot{\chi}_x = N_x,\\
&B\dot{\omega}_y + \omega_x\omega_z(A-C) - C_c\chi_z\omega_x + A_c\chi_x\omega_z + B_c\dot{\chi}_y = N_y,\\
&C\dot{\omega}_z + \omega_y\omega_x(B-A) - A_c\chi_x\omega_y + B_c\chi_y\omega_x + C_c\dot{\chi}_z = N_z.
\end{aligned}
\tag{5}
$$

Expressing $\dot{\chi}_x, \dot{\chi}_y, \dot{\chi}_z$ from equations (4) and inserting the results to equations (5) we obtain:

$$
\begin{aligned}
&\dot{\omega}_x + \omega_z\omega_y\frac{C-B}{A-A_c} + (\chi_y\omega_z + \chi_z\omega_y + \chi_z\chi_y)\frac{C_c-B_c}{A-A_c} = \frac{L_x}{A-A_c},\\
&\dot{\omega}_y + \omega_x\omega_z\frac{A-C}{B-B_c} + (\chi_z\omega_x + \chi_x\omega_z + \chi_x\chi_z)\frac{A_c-C_c}{B-B_c} = \frac{L_y}{B-B_c},\\
&\dot{\omega}_z + \omega_y\omega_x\frac{B-A}{C-C_c} + (\chi_x\omega_y + \chi_y\omega_x + \chi_y\chi_x)\frac{A_c-C_c}{C-C_c} = \frac{L_z}{C-C_c}.
\end{aligned}
\tag{6}
$$

In terms of the parameters of libration β, γ for the Moon as whole, and of the analogous parameters β_c, γ_c for the fluid core

$$
\beta_c = \frac{C_c - A_c}{B_c} \;\; \gamma_c = \frac{B_c - A_c}{C_c},
$$

and of the ratios ρ_A, ρ_B, ρ_C of the corresponding moments of inertia defined by the relations

$$
\begin{aligned}
\rho_A &= \frac{A_c}{A} \equiv \frac{C_c}{C}\left(\frac{1-\beta_c\gamma_c}{1+\beta_c}\right)\left(\frac{1-\beta\gamma}{1+\beta}\right),\\
\rho_B &= \frac{B_c}{B} \equiv \frac{C_c}{C}\left(\frac{1+\gamma_C}{1+\beta_C}\right)\left(\frac{1+\gamma}{1+\beta}\right),\\
\rho_C &= \frac{C_c}{C},
\end{aligned}
$$

we can rewrite equations (6) in the form:

$$
\dot{\omega}_x \quad + \quad \omega_z\omega_y\frac{\beta-\gamma}{1-\beta\gamma}q_A + Q_{yz}\frac{(\beta_c-\gamma_c)}{1-\beta_C\gamma_C}\rho_A q_A = \frac{L_x}{A}q_A,
$$

$$
\begin{aligned}
\dot{\omega}_y &- \omega_x\omega_z\beta q_B + Q_{xz}\beta_C\rho_B q_B = \frac{L_y}{B}q_B, \qquad (7)\\
\dot{\omega}_z &+ \omega_y\omega_x\gamma q_C + Q_{xy}\gamma_C\rho_C q_C = \frac{L_z}{C}q_C,
\end{aligned}
$$

with the notations

$$
\begin{aligned}
Q_{yz} &= \chi_y\omega_z + \chi_z\omega_y + \chi_z\chi_y,\\
Q_{xz} &= \chi_z\omega_x + \chi_x\omega_z + \chi_x\chi_z, \qquad (8)\\
Q_{xy} &= \chi_x\omega_y + \chi_y\omega_x + \chi_y\chi_x,
\end{aligned}
$$

and

$$
q_A = \frac{1}{1-\rho_A},\ q_B = \frac{1}{1-\rho_B},\ q_C = \frac{1}{1-\rho_C}. \qquad (9)
$$

Equations (4) may be rewritten now in the following way:

$$
\begin{aligned}
\dot{\chi}_x &+ \frac{1+\gamma_c}{1+\beta_c}\chi_z(\omega_y+\chi_y) - \frac{1+\beta_c}{1-\beta_c\gamma_c}\chi_y(\omega_z+\chi_z) + \dot{\omega}_x = 0\\
\dot{\chi}_y &+ \frac{1+\beta_c}{1+\gamma_c}\chi_x(\omega_z+\chi_z) - \frac{1-\beta_c\gamma_c}{1+\gamma_c}\chi_z(\omega_x+\chi_x) + \dot{\omega}_y = 0 \qquad (10)\\
\dot{\chi}_z &+ \frac{1-\beta_c\gamma_c}{1+\beta_c}\chi_y(\omega_x+\chi_x) - \frac{1+\gamma_c}{1+\beta_c}\chi_x(\omega_y+\chi_y) + \dot{\omega}_z = 0
\end{aligned}
$$

Thus we have obtained a close system of differential equations (3), (7), (10) to describe rotation of the Moon with a fluid core. Because the information on numerical values of the parameters of the core is rather uncertain, in this work we consider equations (10) for the lunar rotation under the Cassini laws. In the simplest way the Cassini laws may be formulated in the ecliptical inertial frame. So temporarily the Euler's angles $\phi,\ \theta,\ \psi$ at the right parts of relations (3) will be considered as defined in the ecliptical system. For the Cassini laws we have $\theta = \mathrm{const},\ \dot{\psi} = \dot{\Omega}$ ($\dot{\Omega}$ is the secular motion of the lunar node along the ecliptic), $\dot{\phi} + \dot{\psi} = n$ (n is the lunar mean motion), $\omega_z = n - \dot{\phi} + \dot{\psi} \approx n$. Then Euler's relations (3) reduce to the form:

$$
\omega_x = \dot{\Omega}\sin\theta\sin\psi,
$$

$$
\begin{aligned}
\omega_x &= \dot{\Omega} \sin\theta \cos\psi, \qquad (11)\\
\omega_z &= n.
\end{aligned}
$$

For the time derivatives $\dot{\omega}_x, \dot{\omega}_y$ we obtain:

$$
\begin{aligned}
\dot{\omega}_x &= \dot{\Omega}(n - \dot{\Omega}) \sin\theta \cos\psi,\\
\dot{\omega}_y &= -\dot{\Omega}(n - \dot{\Omega}) \sin\theta \sin\psi.
\end{aligned}
$$

These relations for $\dot{\omega}_x, \dot{\omega}_y$ have to be inserted to equations (10). Assuming $\chi_z << n$ (which estimate is justified beneath) equations (10) may be written in the following form (neglecting the squares of β_c, γ_c):

$$
\begin{aligned}
\dot{\chi}_x &- n_1\chi_y + \dot{\omega}_x = 0,\\
\dot{\chi}_y &+ n_2\chi_x + \dot{\omega}_y = 0,
\end{aligned}
$$

where $n_1 = n\ (1 + \beta_c),\ n_2 = n\ (1 + \beta_c - \gamma_c)$.

These equations have the particular solution for the forced oscillations:

$$
\begin{aligned}
\chi_x &= -\frac{2\dot{\Omega}(n-\dot{\Omega})^2}{(n-\dot{\Omega})^2 - n_1 n_2} \sin\theta \sin\psi \approx \frac{2\dot{\Omega}n}{2\dot{\Omega} + n(2\beta_c - \gamma_c)} \sin\theta \sin\psi, \qquad (12)\\
\chi_y &= -\frac{2\dot{\Omega}(n-\dot{\Omega})^2}{(n-\dot{\Omega})^2 - n_1 n_2} \sin\theta \cos\psi \approx \frac{2\dot{\Omega}n}{2\dot{\Omega} + n(2\beta_c - \gamma_c)} \sin\theta \cos\psi. \qquad (13)
\end{aligned}
$$

Expressions (11), (12) and (13) have to be used in the equation for $\dot{\chi}_z$ that has the form:

$$
\dot{\chi}_z + (1 - \beta_c)\chi_y\omega_x - (1 + \gamma_c - \beta_c)\chi_x\omega_y - \chi_x\chi_y\gamma_c = 0.
$$

After integration we derive the inequality

$$
|\chi_z| < \left(\frac{\dot{\Omega}}{2\dot{\Omega} + (2\beta_c - \gamma_c)}\right)^2 n \sin^2\theta \max(\beta_c, \gamma_c) << n,
$$

which justifies the derivation of relations (12), (13).

Making use of (11) expressions (12), (13) may be rewritten in the form:

$$\chi_x = \frac{2n}{2\dot{\Omega} + n(2\beta_c - \gamma_c)}\omega_x, \tag{14}$$

$$\chi_y = \frac{2n}{2\dot{\Omega} + n(2\beta_c - \gamma_c)}\omega_y. \tag{15}$$

In this invariant form they are valid in any inertial reference frame in which the Euler's angles are defined. Now we can substitute the derived expressions for $\chi_x,\ \chi_y$ to equations (7) to obtain modified Euler's equations which take into account the main effects of the fluid core of the Moon:

$$\begin{aligned}
\dot{\omega}_x &+ \omega_z\omega_y\left(\frac{\beta-\gamma}{1-\beta\gamma} + G\rho\frac{\beta_c-\gamma_c}{1-\beta_C\gamma_C}\right)\frac{1}{1-\rho} = \frac{L_x}{A(1-\rho)},\\
\dot{\omega}_y &- \omega_x\omega_z\left(\beta + G\rho\beta_C\right)\frac{1}{1-\rho} = \frac{L_y}{B(1-\rho)},\\
\dot{\omega}_z &+ \omega_y\omega_x\left(\gamma + G\rho\left(2+G\right)\gamma_C\right)\frac{1}{1-\rho} = \frac{L_z}{C(1-\rho)},
\end{aligned} \tag{16}$$

in which the undimensional constant G is defined as

$$G = \frac{2n}{2\dot{\Omega} + n(2\beta_c - \gamma_c)} \tag{17}$$

and the approximation $\rho_A = \rho_B = \rho_C = \rho$ is used.

Processing of LLR observations the parameters β, γ, and $\rho, \beta_c, \rho\gamma_c$ can be estimated simultaneously as the torques L_x, L_y, L_z at the right hand of (16) depend on β, γ but not on ρ, β_c, γ_c. We assume that the combination $2\beta_c - \gamma_c$ has to be small in comparison with $|2\dot{\Omega}/n| \approx 0.01$; then for physically meaningful estimates we must expect $G \approx -200$ and of course $\rho > 0$.

In order to model the dissipative coupling between the core and mantle after [2] we add components L_x^{mc}, L_y^{mc} of the coupling torque to the right hands of the first two equations (16):

$$L_x^{mc} = \kappa\chi_x, L_y^{mc} = \kappa\chi_y,$$

where κ is a coupling factor.

Using the approximation (14), (4) we have

$$L_x^{mc} = G\kappa\omega_x, L_y^{mc} = G\kappa\omega_y,$$

where G is given by Equation (17).

While processing LLR data these relations have been used assuming $G = -216$.

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