Forward Analysis Strategy (preliminary)

November 11, 2004

Abstract

This note summarizes and extends the discussions on PID we had during the last weeks. Ideas from several people are used.

1 Cross section formula

The differential cross section can be expressed as:

$$\sigma_{i\alpha} = N^{norm} \cdot N_{i\alpha} = N^{norm} \cdot U_{i\alpha}^{j\beta} \cdot N_{j\beta} \tag{1}$$

where $N_{i\alpha}$ is the true particle yield, $N_{j\beta}$ is reconstructed particle yield, $U_{i\alpha}^{j\beta}$ is the unfolding matrix and N^{norm} the normalization factor. The meaning of the indexes is:

$$\begin{aligned} i &= (p,\theta)_{true} \\ \alpha &= PID_{true} \\ j &= (p,\theta)_{rec} \\ \beta &= PID_{rec} \end{aligned}$$

The current forward analysis uses three types of tracks with different momentum resolutions. Now we have two possibilities: either we express the final cross section as a sum of terms for each track types or we compute independent cross sections for each track type and then a weighted sum gives us the final result. The final result must be equivalent but the second option seems preferable because it allows the comparison of the cross sections obtained with different track types. Mathematicaly the second options would read

$$\sigma_{i\alpha}^{(t)} = N^{norm} \cdot U_{i\alpha}^{(t)j\beta} \cdot N_{j\beta}^{(t)} \tag{2}$$

and

$$\sigma_{i\alpha} = \sum_{t=1}^{3} f^{(t)} \cdot \sigma_{i\alpha}^{(t)} \tag{3}$$

where $f^{(t)}$ are the track type fractions:

$$f^{(t)} = \frac{\sum_{j,\beta} N_{j\beta}^{(t)}}{\sum_{j,\beta} N_{j\beta}} \tag{4}$$

In principle the sevaral $\sigma_{i\alpha}^{(t)}$ must differ from each other. This is in fact what provokes the model dependence of the final cross section. Indeed, different models have different track type fractions and then the sum of Eq. 3 gives different results. However, if the track type fractions are obtained from the data, equation 3 should give a nearly model independent result.
Each unfolding matrix $U_{i\alpha}^{(t)j\beta}$ is the product of several terms:

$$U_{i\alpha}^{(t)j\beta} = \frac{N_{i\alpha}}{N_{j\beta}^{(t)}} = \underbrace{\frac{N_{i\alpha}}{N_{i\alpha}^{acc}}}_{1/\varepsilon_{i\alpha}^{acc}} \underbrace{\frac{N_{i\alpha}^{acc}}{N_{i\alpha}^{(t)-trk-prim}}}_{1/\varepsilon_{i\alpha}^{(t)-trk}} \underbrace{\frac{N_{i\alpha}^{(t)-trk-prim}}{N_{i\alpha}^{(t)-trk}}}_{\eta_{i\alpha}^{(t)-prim}} \underbrace{\frac{N_{i\alpha}^{(t)-trk}}{N_{i\alpha}^{(t)-pid}}}_{1/\varepsilon_{i\alpha}^{(t)-pid}} \underbrace{\frac{N_{i\alpha}^{(t)-pid}}{N_{j\beta}^{(t)}}}_{M_{i\alpha}^{(t)j\beta}} \tag{5}$$

$$U_{i\alpha}^{(t)j\beta} = \frac{1}{\varepsilon_{i\alpha}^{acc}} \cdot \frac{1}{\varepsilon_{i\alpha}^{(t)-trk}} \cdot \eta_{i\alpha}^{(t)-prim} \cdot \frac{1}{\varepsilon_{i\alpha}^{(t)-pid}} \cdot M_{i\alpha}^{(t)j\beta} \tag{6}$$

where:

- $N_{i\alpha}$: true primary particle yield
- $N_{i\alpha}^{acc}$: number of primary particles accepted by the dipole
- $N_{i\alpha}^{(t)-trk-prim}$: number of primary particles with momentum reconstructed
- $N_{i\alpha}^{(t)-trk}$: number of particles (primaries or not) with momentum reconstructed
- $N_{i\alpha}^{(t)-pid}$: number of particles (primaries or not) with PID asignment (pass the cut in PID probability).
- $N_{j\beta}^{(t)}$: number of particles with PID asignment in bins of reconstructed momentum and PID.
- $\varepsilon_{i\alpha}^{acc}$: geometrical acceptance.
- $\varepsilon_{i\alpha}^{(t)-trk}$: tracking efficiency

- $\eta_{i\alpha}^{(t)-prim}$: primary purity, that is the fraction of reconstructed particles that are primaries. This quantity accounts for the non-primary background subtraction.
- $\varepsilon_{i\alpha}^{(t)-pid}$: PID efficiency
- $M_{i\alpha}^{(t)j\beta}$: migration matrix from reconstructed bins $j\beta$ to true bins $i\alpha$

The acceptance and the tracking efficiency should be nearly independent of the particle type (independent of α). However this is not completely true. For example, an electron that loses energy would arrive to the dipole with a different angle and momentum that a proton in the same kinematic bin. Thus the geometrical acceptance could be slightly different. We will see later that this is not a problem because the unfolding matrix can be computed in one go, and it will take into account all the dependencies and correlations.

All the terms in equation 6 can be computed with the MC. Most of them can be tunned and validated with well known data samples (beam particles and elastic scattering). However some of them cannot be easily tunned. One of these terms is the downstream tracking efficiency, that is, the probability that an accepted particle is reconstructed downstream the dipole. This quantity depends strongly on the hit finding efficiency and also on the matching χ^2 distribution. In particular the matching χ^2 is not easily reproduced by the MC. Thus, the downstream tracking efficiency must be computed partialy with data. The efficiency of the PID detectors (including track-PID matching) is also a delicate ingreedient. However, it can be easily taken into account by tunning the detector pdf's, so that the final PID efficiency is well reproduced by the MC. We will see in section 2.2 that the track-PID matching can be separated from the pure PID detector response.

It is worth noting that at the end, the unfolding matrices can be computed basically in one go, without looking at intermediate efficiencies, purities and migration matrices (previously validated). The unfolding algorithms should give us directly the corrected yield. Probably some correction factors will be applied afterwards (i.e. downstream matching efficiency ratio). We should try to have an scheme in which the MC does most of the work because it is easier and safter to tune the MC than to have 10 different terms in the cross section.

As we can see the analysis is quite simple. Three independent cross sections are calculated for each track type as a product of a normalization factor (common for all), an unfolding matrix, basically computed with MC, and an observed yield. The final cross section is the weighted sum of these three cross sections.

2 Particle identification

2.1 True or reconstructed momentum?

The conditional PID probabilities (pdf's) of the different subdetectors are correlated because of the momentum resolution. Imagine a particle with reconstructed momentum 3 GeV. If the true momentum is 2.2 the particle will leave no signal in the cerenkov and the mass squared measured by the tof wall will be close to the central value. If the true momentum is 3.8 the particle will leave a signal in the cerenkov and the mass squared will be statistically farther from the central value. The conclusion is that the amount of signal in the cerenkov tell us something about the mass square distribution in the tof. This correlation dissapear for infinite resolution. Fortunately, in Harp's forward region the correlation is small because the momentum resolution is good. However, this correlation implies that the product of probabilities in the Bayes formula is only an approximation (a good one). Probably one could leave with this small correlation, because probabilities are not used directly in the cross section formula, but efficiencies and migration. Not taking into account this correlation implies not optimal probabilities, but the cross section will be correct anyway. The effect is similar to what happens if one does not use the iterative PID. Anyway, we will see that for other reasons it is better to use the true momentum.

For a given set of pid measurements $\vec{\mathbf{m}}$ and reconstructed momentum $\vec{\mathbf{p}}_r$ associated to a particle the Bayes equation gives us the posterior probability of being of type α:

$$P_k(\alpha|\vec{\mathbf{m}},\vec{\mathbf{p}}_r) = \frac{P(\vec{\mathbf{m}}|\alpha,\vec{\mathbf{p}}_r)P_{k-1}(\vec{\mathbf{p}}_r,\alpha)}{\sum_\beta P(\vec{\mathbf{m}}|\beta,\vec{\mathbf{p}}_r)P_{k-1}(\vec{\mathbf{p}}_r,\beta)} \tag{7}$$

where α is the true pid and k reffers to the iteration number. $P_k(\alpha|\vec{\mathbf{m}},\vec{\mathbf{p}}_r)$ is the quantity in which we can put a cut (i.e. 90%) to decide the particle type. $P(\vec{\mathbf{m}}|\alpha,\vec{\mathbf{p}}_r)$ is the conditional probability (pdf) and $P_{k-1}(\vec{\mathbf{p}}_r,\alpha)$ is the prior probability of having a particle of type α with momentum $\vec{\mathbf{p}}_r$. This last quantity represents the prior knowledge on the yields corrected by efficiencies and purities, but not by kinematic migration (because it is expressed in terms of reconstructed momentum).

The use of the reconstructed momentum in Eq. 7 has several disadvantages:

- The pdf's of the different subdetectors are not completely independent.

- If the momentum resolution is improved the pdf's must be recalculated.
- One needs a different pdf for each track type.
- The prior is also a function of the reconstructed momentum

Now we have two possibilities. Either we express the posterior probability, $P_k(\alpha|\vec{\mathbf{m}}, \vec{\mathbf{p}}_r)$, in terms of the true momentum or we express only the pdf's, $P(\vec{\mathbf{m}}|\alpha, \vec{\mathbf{p}}_r)$ as a function of the true momentum. In both cases a resolution function $P(\vec{\mathbf{p}}_t|\vec{\mathbf{p}}_r)$ must be introduced. $P(\vec{\mathbf{p}}_t|\vec{\mathbf{p}}_r)$ is just the probability that a particle with recostructed momentum $\vec{\mathbf{p}}_r$ has a true momentum $\vec{\mathbf{p}}_t$. The posterior probability becomes

$$P_k(\alpha|\vec{\mathbf{m}}, \vec{\mathbf{p}}_r) = \int d\vec{\mathbf{p}}_t P_k(\alpha|\vec{\mathbf{m}}, \vec{\mathbf{p}}_t) \cdot P(\vec{\mathbf{p}}_t|\vec{\mathbf{p}}_r) \tag{8}$$

Now the Bayes formula can be applied to $P_k(\alpha|\vec{\mathbf{m}}, \vec{\mathbf{p}}_t)$:

$$P_k(\alpha|\vec{\mathbf{m}}, \vec{\mathbf{p}}_t) = \frac{P(\vec{\mathbf{m}}|\alpha, \vec{\mathbf{p}}_t) P_{k-1}(\vec{\mathbf{p}}_t, \alpha)}{\sum_\beta P(\vec{\mathbf{m}}|\beta, \vec{\mathbf{p}}_t) P_{k-1}(\vec{\mathbf{p}}_t, \beta)} \tag{9}$$

but the final decission is taken from $P_k(\alpha|\vec{\mathbf{m}}, \vec{\mathbf{p}}_r)$ (see Eq. 8), because obviously, only the reconstructed momentum is known. In equation 9 the prior depends also on the true momentum what means that before doing a second iteration one should do the full unfolding.

The second option consists in expresing as a function of the true momentum only the pdf's:

$$P(\vec{\mathbf{m}}|\vec{\mathbf{p}}_r, \alpha) = \int d\vec{\mathbf{p}}_t P_k(\vec{\mathbf{m}}|\vec{\mathbf{p}}_t, \alpha) \cdot P(\vec{\mathbf{p}}_t|\vec{\mathbf{p}}_r) \tag{10}$$

This second option has the advantage that the prior depends on the reconstructed momentum, so the kinematic migration do not need to be taken into account before the second iteration. However, if one needs to correct by efficiencies and PID migration anyway it seems natural to do the full unfolding. Moreover, the first option should be more precise because more information is used.

2.2 Angular dependece

The angular dependence of the pdf's complicates things. In general this dependence will be highly dominated by matching efficiency and not by detector response. Thus it could be interesting to split the pdf's in two parts:

$$P(\vec{\mathbf{m}}|\alpha, \vec{\mathbf{p}}_t) = P(\vec{\mathbf{m}}|\alpha, p_t)^{resp} \cdot P(\vec{\mathbf{m}}|p_t, \theta_t)^{match} \tag{11}$$

where the first term is related with the detector response (including hardware inefficiency) and the second term accounts for the matching efficiency. Again this has the advantage that the detector response pdf's do not need to be updated every time the reconstruction improves.

The matching efficiency is not easily reproduced by the MC. Therefore it could be interesting to correct $P(\vec{\mathbf{m}}|p_t,\theta_t)^{match}$

2.3 Iterative procedure

Equation 9 requires the true particle yields $P_k(\vec{\mathbf{p}}_t,\alpha)$ as input. Obviously the yields are not known a priory. The initial input, $P_0(\vec{\mathbf{p}}_t,\alpha)$, could be given by the MC:

$$P_0(\vec{\mathbf{p}}_t,\alpha) = \frac{N_{i\alpha}^{MC}}{\sum_{i\alpha} N_{i\alpha}^{MC}} \tag{12}$$

Then one runs over the selected data sample with a cut in probability (fixed or momentum dependent) so that a reconstructed raw yield, $(N_{j\beta}^{(t)})_1$, is obtained for each track type. Afterwards the full unfolding procedure is applied to obtain the corrected yields:

$$(N_{i\alpha})_1 = \sum_{t=1}^{3} f^{(t)} \cdot (U_{i\alpha}^{(t)j\beta})_1 \cdot (N_{j\beta}^{(t)})_1 \tag{13}$$

These are used now to compute again the probabilities

$$P_1(\vec{\mathbf{p}}_t,\alpha) = \frac{(N_{i\alpha})_1}{\sum_{i\alpha}(N_{i\alpha})_1} \tag{14}$$

that are introduced into the Bayes equation 9. Finally the analysis is re-run with a new probability cut (probably different). The new raw yield is corrected:

$$(N_{i\alpha})_2 = \sum_{t=1}^{3} f^{(t)} \cdot (U_{i\alpha}^{(t)j\beta})_2 \cdot (N_{j\beta}^{(t)})_2 \tag{15}$$

and that's the end of the story.

3 Systematic errors

The main sources of systematics of the unfolding matrices are:

- Primary purity. This can be computed comparing several models.

- Tracking efficiency. In principle, the track type splitting solves this problem. However a residual effect may remain. Again, one could compute this systematic using different hadron generators.
- PID efficiency. This systematic is associated with a wrong description of the PID detector response and track-PID matching efficiency.
- The kinematic part of the migration matrix is affected by momentum and angular resolutions, which must be well reproduced by the MC. Residual effects can be studied variating the MC resolutions within reasonable limits (drift distance resolution functions). The PID part of the migration matrix will be related with the pdf's and the probability cuts. Both can be variated.

In summary:

- Compute the unfolding matrices with different hadron generators
- Variate the drift distance resolution functions
- Variate the pdf's (mainly hardware and matching inefficiencies)