The Deep Space Optical Channel: II. Wave Propagation Effects

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Propagation effects of optical waves in the deep space optical channel with a space-based receiver are analyzed. It is found that these effects are negligible for all practical interplanetary systems designs.

I. Introduction

The basic considerations involved in the design of deep space optical communication systems are described in Ref. 1. Besides assessing noise effects (Ref. 2), one must also analyze the wave propagation aspects of the optical channel in order to evaluate their impact on the overall system performance.

The purpose of this report is to summarize the wave propagation characteristics which may affect the optical communication link. Since we assume that the receiving station of the link is located outside the earth atmosphere, propagation effects through the atmosphere are not analyzed here. It is found that propagation effects are negligible in all practical situations. If propagation through the atmosphere were to be included, this result would not be correct. This is one of the advantages of locating the receiver outside the earth atmosphere. (Other advantages include the reduction of the total noise in the system.)

II. Formulas for Calculating the Propagation of Electromagnetic Waves in the Optical Channel

The medium through which the light propagates in a deep space link is basically an inhomogeneous plasma. The general formulas of electromagnetic wave propagation in plasmas are complicated (Ref. 3), but when dealing with optical frequencies (i.e., $3.10^{14} \leqslant f \leqslant 10^{15}$ Hz, corresponding to $0.3 \leqslant \lambda \leqslant 1$ μm) and with typically encountered plasmas, we can greatly simplify the calculations.

The basic reason for this possible simplification is that the region of frequencies that we are interested in lies far above any other relevant frequency of physical significance. We are mainly referring to the plasma resonance frequency, given by

$$f_p \cong 8.9 \sqrt{N} \text{ (Hz)} \tag{1}$$

(where N is the elctron density in the plasma in m^{-3}), to the cyclotron resonance frequency of magnetic fields present in the plasma, and to the rate of collisons of an electron in the plasma with other particles, denoted by $1/\tau_s$. These conditions are discussed in more detail in Ref. 3. For operations close to any of the above frequencies, the interaction between the medium and the field is stronger and hence its consequences are more complicated to describe and evaluate.

Under the above assumptions, the field amplitude of a plane wave propagation in a plasma is typically described by the following exponential:

$$e^{-\frac{1}{2}\alpha Z} e^{i[2\pi f t-(k+\delta k)Z]} \tag{2}$$

where α, the intensity (or power) attenuation coefficient, is approximately given by (Refs. 3, 4)

$$\alpha = \frac{1}{c}\left(\frac{f_p}{f}\right)^2 \frac{1}{\tau_s} \text{ (m}^{-1}\text{)} \tag{3}$$

where f is the optical signal frequency, c is the velocity of light, k is the wave-vector of the field, defined as

$$k = 2\pi \frac{f}{c} \, (\mathrm{m}^{-1}) \tag{4}$$

and the phase shift imposed upon the wave is associated with the following change in the wave-vector (Ref. 3)

$$\delta k = -\frac{\pi f}{c}\left(\frac{f_p}{f}\right)^2 \, (\mathrm{m}^{-1}) \tag{5}$$

The basic propagation configuration is shown in Fig. 1. We have an input beam characterized by an input power $P_{in}(W)$, pulsewidth τ_{in} (sec) and a beam divergence angle θ_{in} (rad), propagating a distance L(m) in a plasma with a density $N(\mathrm{m}^{-3})$, temperature T(K) and density variation $\delta N/N$. The emerging signal has output power P_{out} (W), pulsewidth τ_{out} (sec) and a beam divergence angle θ_{out} (rad). From this and the previous assumptions, the following propagation effects will be considered:

A. Attenuation

The intensity attenuation coefficient α, defined as

$$\alpha \equiv \frac{1}{L} \ln \frac{P_{in}}{P_{out}} \tag{6}$$

can be calculated using Eq. (3). From Ref. 4 we can obtain the following approximate expression for the frequency of collisions $1/\tau_s$, a parameter which is needed for the calculation of α:

$$\frac{1}{\tau_s} = 10^3 N T^{-3/2} (21.4 + 5.44 \ln T - 1.81 \ln N) \tag{7}$$

B. Temporal Dispersion

The general formula relating the optical signal pulsewidths at the input and output is

$$\tau_{out} = \left[\tau_{in}^2 + \left(\frac{1}{2\pi}\frac{L}{\lambda}\frac{f_p^2}{f^4}\frac{1}{\tau_{in}}\right)^2\right]^{1/2} \text{(sec)} \tag{8}$$

The derivation of this equation is similar to the derivation of the dispersion in optical fibers (Ref. 5).

Differentiating Eq. (8) with respect to τ_{in}, we find that the shortest pulse that can be achieved is

$$\tau_{in}\,(\mathrm{min}) = \sqrt{\frac{1}{2\pi}\frac{L}{\lambda}\frac{f_p}{f^2}} \;\text{(sec)} \tag{9}$$

which results

$$\tau_{out}\,(\mathrm{min}) = \sqrt{2}\,\tau_{in}\,(\mathrm{min}) \tag{10}$$

C. Beam Divergence

The beam divergence angle is given by the following formula:

$$\theta_{out} = [\theta_{in}^2 + \langle(\delta\theta)^2\rangle]^{1/2} \tag{11}$$

$\delta\theta$ is a random variable related to the fluctuations in the index of refraction of the medium, as explained in the following paragraph.

First we denote by L_c the scale of the fluctuations, which is roughly a characteristic distance over which the parameters of the medium change. For the interplanetary medium, L_c is of the order of several hundreds of kilometers (Ref. 6).

When the condition $L_c \gg \sqrt{\lambda L}$ applies, we can use the simple laws of geometrical optics to analyze the effect of the randomness of the medium on the optical wave. Since this condition applies in our case, we obtain (Ref. 7).

$$\langle(\delta\theta)^2\rangle \cong \langle(\delta n)^2\rangle \frac{L}{L_c} \tag{12}$$

The next step is to relate the refractive index fluctuations $\langle(\delta n)^2\rangle$ to the plasma density fluctuations $\langle(\delta N)^2\rangle$. The index of refraction of a plasma at $f > f_p$ is (Ref. 8)

$$\epsilon = n^2 = 1 - \left(\frac{f_p}{f}\right)^2 = 1 - \frac{1}{4\pi^2}\frac{q^2}{\epsilon_0 m f^2} N \tag{13}$$

so

$$\langle(\delta N)^2\rangle = \frac{1}{4}\left(\frac{f_p}{f}\right)^4 \frac{\langle(\delta N)^2\rangle}{N^2} \tag{14}$$

Combining Eqs. (12) and (14), we obtain the desired expression for $\langle \delta\theta^2 \rangle$, which is to be used in Eq. (11):

$$\langle \delta\theta^2 \rangle = \frac{1}{4}\left(\frac{f_p}{f}\right)^4 \left(\frac{L}{L_c}\right)\left(\frac{\langle (\delta N)^2 \rangle}{N^2}\right) \tag{15}$$

In the next section we will apply the formulas derived above to some envisioned scenarios that occur in deep space missions.

III. Wave Propagation Effects in the Optical Channel

In this section we will apply the formulas derived in the last section to find how the optical wave is modified during propagation. Three propagation cases, shown in Fig. 2, are considered. They include (1) propagation through the interplanetary medium, (2) propagation through a big planet's (Jupiter) magnetosphere, and (3) propagation through the Sun's corona. The relevant parameters and the results for each case are given in Table 1. Values assumed for the calculations are $f = 4 \cdot 10^{14}$ Hz (corresponding to $\lambda = 0.75$ μm), $L/L_c \cong 2 \cdot 10^7$ and $\langle (\delta N)^2 \rangle / N^2 \cong 1$. Values of N and T for these examples were taken from Refs. 8 and 9. Most of the values assumed are on the worst-case side. Analyzing the results, we see that the power attenuation is very small, less than 0.5% in the worst-case scenario. The temporal dispersion is less than 0.1 ps, and the addition to the beam spread angle is less than 1 nrad. It is interesting to note that propagation through a planet's magnetosphere has the largest effect. This is not true if we consider propagation paths closer to the Sun than the 0.1 AU distance assumed in this report. However, in this case noise problems due to the radiation of the Sun will become very severe (Ref. 2), rendering the link impractical.

To summarize, in all three cases the beam spread and power attenuation are always negligible, and temporal dispersion is negligible for pulses longer than 10^{-13} sec.

Other propagation effects not related to plasma phenomena are light scattering and absorption by the interplanetary dust. It can be shown that these effects are also negligible. For example, only 0.25% of the sunlight is scattered in its propagation to earth (Ref. 9), in spite of the fact that the region near the sun has the highest concentration of interplanetary dust.

IV. Conclusion

Propagation effects of optical waves in the deep space to near space link have been analyzed. It has been found that for practical wave propagation considerations, the medium can be taken as a free-space medium.

References

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Table 1. Parameters and results for wave propagation in the deep space link (all values are approximate)

Propagation medium	Conditions				Results		
	Distance L, m	Plasma frequency f_p, Hz	Electron density N, m^{-3}	Electron temperature T, K	Power attenuation exponent αL	Temporal dispersion τ_{out}(min), sec	Beam spread $\sqrt{\langle(\delta\theta)^2\rangle}$, rad
Interplanetary medium	$\leqslant 10^{13}$	$2 \cdot 10^4$	10^5	10^4	$4 \cdot 10^{-16}$	$3 \cdot 10^{-15}$	$6 \cdot 10^{-17}$
Planet's magnetosphere	$\leqslant 10^9$	10^8	$3 \cdot 10^{12}$	10^5	$5 \cdot 10^{-3}$	10^{-13}	10^{-9}
Sun's corona at distance 0.1 AU from the Sun	$\leqslant 10^{10}$	10^7	$3 \cdot 10^{10}$	10^6	$3 \cdot 10^{-8}$	$5 \cdot 10^{-14}$	10^{-11}

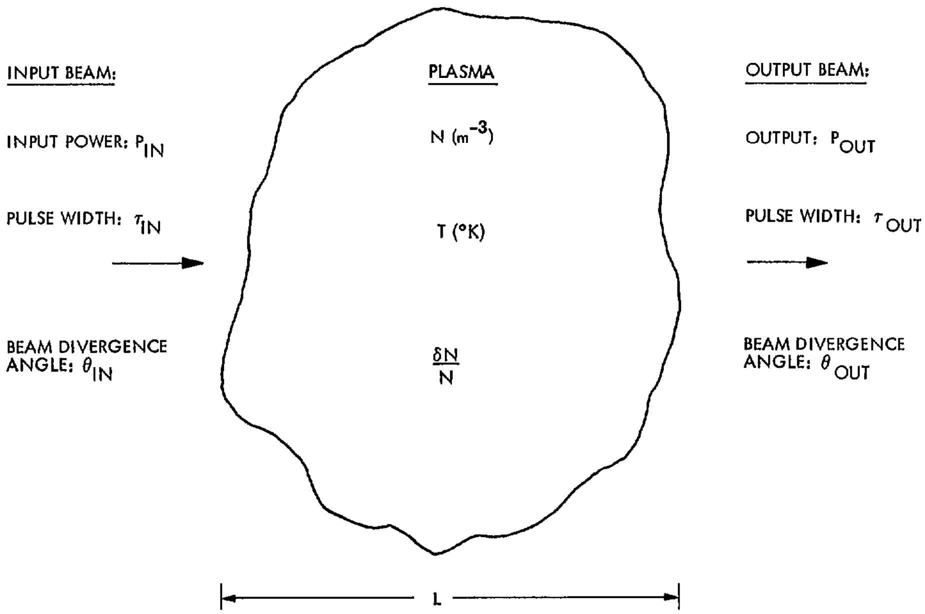

Fig. 1. Propagation configuration

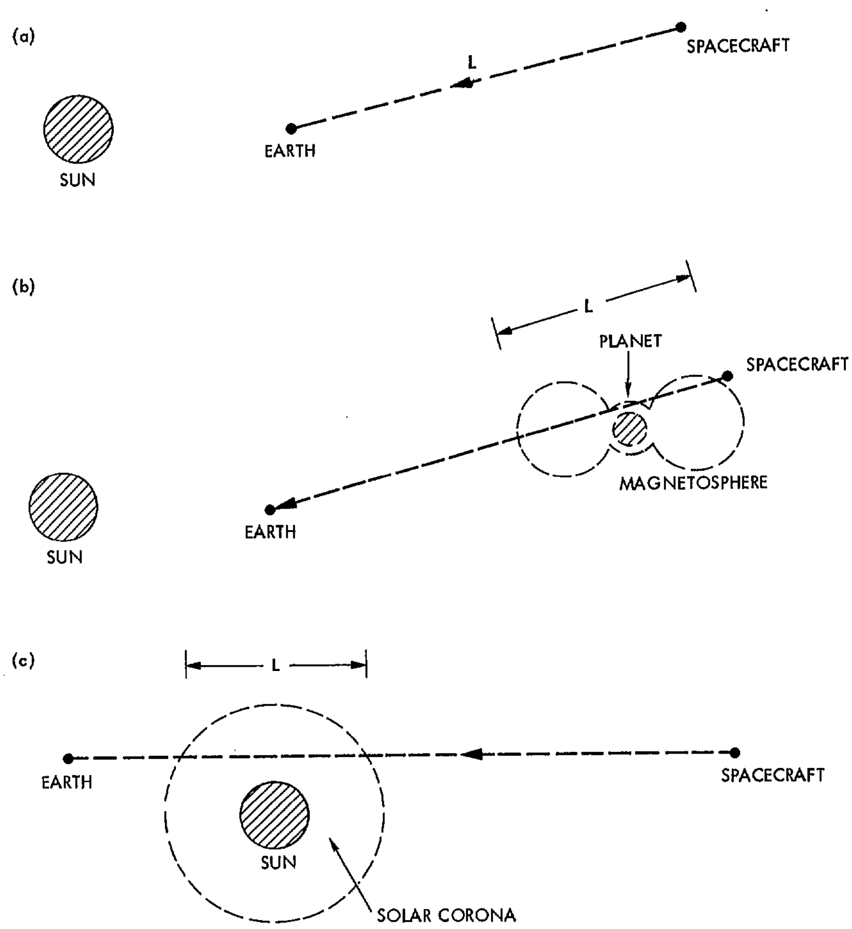

Fig. 2. Three propagation cases (not to scale)