DETERMINATION OF V_{ub}

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The precise determination of the magnitude of V_{ub} with a robust, well-understood uncertainty remains one of the key goals of the heavy flavor physics programs, both experimentally and theoretically. Because $|V_{ub}|$, the smallest element in the CKM mixing matrix, provides a bound on the upper vertex of one of the triangles representing the unitarity property of the CKM matrix, it plays a crucial role in the examination of the unitarity constraints and the fundamental questions on which the constraints can bear (see the minireviews on the CKM matrix [1] and on CP violation [2] for details). Investigation of these issues require measurements that are precise and that have well-understood uncertainties.

The charmless semi-leptonic (s.l.) decay channel $b \to u\ell\overline{\nu}$ provides the cleanest path for the determination of $|V_{ub}|$. However, the theory for the heavy-to-light $b \to u$ transition cannot be as well constrained as that for the heavy-to-heavy $b \to c$ transition used in the determination of $|V_{cb}|$ (see the $|V_{cb}|$ minireview [3]). The extraction of $|V_{ub}|$ and the interplay between experimental measurements and their theoretical interpretation are further complicated by the large background from $b \to c\ell\overline{\nu}$ decay, which has a rate about 60 times higher than that for charmless s.l. decay. Measurements based both on exclusive decay channels and on inclusive techniques have been, and are being, pursued.

The last several years have seen significant developments in both the theoretical framework and the experimental techniques used in the study of $b \to u\ell\overline{\nu}$. The inclusive theory has progressed significantly in categorization of the corrections to the base theory still needed, in their relative importance in different regions of phase space, and in the determination of some of them. Recent work on exclusive processes bolsters confidence in the current uncertainties for the form factor calculations needed to extract $|V_{ub}|$. Experimentally, we have new inclusive and exclusive measurements that minimize dependence on detailed modeling of the signal process to separate signal from the $b \to c\ell\overline{\nu}$ background, have well-defined sensitivities in particular regions of phase space and have improved signal-to-background ratios. These improvements provide us with a first opportunity to develop a method for a robust determination of $|V_{ub}|$ with complete error estimates, including constraints on hitherto unquantified contributions. We review the current determinations of $|V_{ub}|$, focusing primarily on these recent developments. An average of the inclusive information from all regions of phase space remains, unfortunately, beyond our reach because of the potentially sizable corrections for which we lack estimates. Rather, we combine the inclusive results to obtain a central value and, in particular, a more complete evaluation of the uncertainty than has been possible in the past.

Inclusive measurements of $b \to u\ell\overline{\nu}$:

Theoretically, issues regarding the calculation of the total semileptonic partial width $\Gamma(B \to X_u\ell\nu)$ via the operator product expansion (OPE) are well-understood [4–9]. The OPE is both a nonperturbative power series in $1/m_b$ and a perturbative expansion in α_s. At order $1/m_b^2$, it predicts

$$\begin{aligned}\Gamma(B \to X_u\ell\nu) = \frac{G_F^2|V_{ub}|^2}{192\pi^3}\, m_b^5 \\ \times\left[1 - \frac{9\lambda_2 - \lambda_1}{2m_b^2} + \ldots - \mathcal{O}(\frac{\alpha_s}{\pi})\right]\end{aligned} \quad , \tag{1}$$

where λ_2 parameterizes the hyperfine interaction between the heavy quark and the light degrees of freedom and λ_1 is related to the Fermi momentum of the heavy quark. The perturbative corrections are known to order α_s^2 [10]. The OPE is alternatively expressed in terms of the nonperturbative parameters μ_π^2 and μ_G^2, which are closely related to λ_1 and λ_2, respectively, but differ significantly in their infrared treatment. Within the OPE, the importance of a proper field theoretic treatment of the parameters is paramount both for the total rate and for the restricted phase space studies discussed below. The treatment of the quark mass and its associated uncertainties are particularly important given the strong mass dependance of the width. Such considerations have led to useful definitions like the kinematic mass, which are discussed in detail in Ref. 11 and Ref. 12.

The error induced by uncertainties in the nonperturbative parameters $\lambda_{1,2}$ is relatively small, and an evaluation [13] by the LEP VUB working group yielded

$$|V_{ub}| = 0.00445 \left(\frac{\mathrm{B}(b \to u\ell\overline{\nu})}{0.002} \frac{1.55\mathrm{ps}}{\tau_b} \right)^{1/2} \times (1 \pm 0.020_{\mathrm{OPE}} \pm 0.052_{m_b}) \ . \tag{2}$$

The quoted uncertainty is dominated by the uncertainty in the b quark mass, for which $m_b^{1S}(1 \ \mathrm{GeV}) = 4.58 \pm 0.09$ GeV was assumed. The value and uncertainty are in good agreement with a recent survey [14]. No weak annihilation uncertainties (discussed below) are included in the quoted OPE error. Use of the quark-level OPE for prediction of moments of the true inclusive spectra has generated concern regarding potential violation of the underlying assumption of global quark-hadron duality. This concern has been confronted by theoretical wisdom [15] supporting global duality for the inclusive $b \to u\ell\overline{\nu}$ transition, and new data both support this assumption and allow placement of quantitative limits on the violation. In particular, the exclusive and inclusive extractions of $|V_{cb}|$ agree to $(0.8 \pm 1.6) \times 10^{-3}$ [3]. Taking the uncertainty as an upper bound on the global duality violation for $|V_{ub}|$ shows that it should not exceed $\simeq$4%. Using this as bound as an uncertainty estimate for duality effects in the partial width prediction brings the total uncertainty on $|V_{ub}|$ to 6.8%. The agreement of the OPE parameters extracted using moments of different distributions in s.l. decays further supports the small scale for duality violation effects.

While theoretically the total inclusive rate would allow determination of $|V_{ub}|$ to better than 10%, experimentally the much more copious $b \to c\ell\overline{\nu}$ process makes a measurement over the full phase space unrealizable. To overcome this background, inclusive $b \to u\ell\overline{\nu}$ measurements utilize restricted regions of phase space in which the $b \to c\ell\overline{\nu}$ process is kinematically highly suppressed. The background is forbidden in the regions of large charged lepton energy $E_\ell > (M_B^2 - M_D^2)/(2M_B)$ (the endpoint), low hadronic mass $M_X < M_D$ and large dilepton mass $q^2 > (M_B - M_D)^2$. Extraction of $|V_{ub}|$ from such a measurement requires knowledge of the fraction of the total $b \to u\ell\overline{\nu}$ rate that lies within the utilized region of phase space, which complicates the theoretical issues and uncertainty considerably. Ref. 16 and Ref. 17 discuss the issues in detail.

CLEO [18], BaBar [19], and Belle [20] have all presented recent measurements of the $b \to u\ell\overline{\nu}$ rate near the endpoint. The results, which are for integrated ranges in the $\Upsilon(4S)$ rest frame, are summarized in Table 1. Experimentally, these measurements must contend with a large background from continuum e^+e^- annihilation processes. Suppression of these backgrounds introduces significant efficiency variation with the q^2 of the decay, which introduces model dependence. Greater awareness of this issue has resulted in more sophisticated suppression methods in these recent measurements and thus in over a factor of three reduction in the model dependence of the measured rates relative to earlier measurements [21, 22]. Future measurements, either using fully-reconstructed B-tag samples that would remove the problem or a modest q^2 binning, would essentially eliminate the remaining model dependence.

Table 1: Partial branching fractions for $b \to u\ell\overline{\nu}$ within the charged lepton momentum range ($\Upsilon(4S)$ frame) from 2.6 GeV/c down to the indicated minimum. The estimated fraction f_E of the total $b \to u\ell\overline{\nu}$ rate expected to lie in that range is also given. The dagger (†) indicates the quantity that received the QED radiative correction appropriate to the indicated mix of electrons and muons, which has not always been treated self-consistently in the literature.

$p_\ell^{\rm min}$ (GeV/c)	$\Delta\mathcal{B}_u(p)$ (10^{-4})	f_E	
2.0	$4.22 \pm 0.33 \pm 1.78$	$\dagger 0.266 \pm 0.041 \pm 0.024$	CLEO (e,μ)
2.1	$3.28 \pm 0.23 \pm 0.73$	$\dagger 0.198 \pm 0.035 \pm 0.020$	CLEO (e,μ)
2.2	$2.30 \pm 0.15 \pm 0.35$	$\dagger 0.130 \pm 0.024 \pm 0.015$	CLEO (e,μ)
2.3	$1.43 \pm 0.10 \pm 0.13$	$\dagger 0.074 \pm 0.014 \pm 0.009$	CLEO (e,μ)
	$\dagger 1.52 \pm 0.14 \pm 0.14$	$0.078 \pm 0.015 \pm 0.009$	BaBar (e)
	$1.19 \pm 0.11 \pm 0.10$	$\dagger 0.072 \pm 0.014 \pm 0.008$	Belle (e)
2.4	$0.64 \pm 0.07 \pm 0.05$	$\dagger 0.037 \pm 0.007 \pm 0.003$	CLEO (e,μ)

BaBar [23] and Belle [24] have presented new analyses of the low M_X region [25–28]. They also utilize a moderate (only $\sim 10\%$ loss) $p_\ell > 1.0$ GeV/c requirement. This technique was pioneered by Delphi at LEP [29], but the achievable resolution and signal-to-background ratio were lower compared to those obtained at the B factories. Because of experimental resolution on M_X, the $b \to c\ell\overline{\nu}$ background smears below its theoretical lower limit of $M_X = M_D$, so experiments must impose more stringent M_X requirements which are theoretically more problematic. The BaBar and Belle analyses are based on "B-tag" samples of fully reconstructed hadronic decays and $D^{(*)}\ell\nu$ decays, respectively. In both cases, M_X is calculated directly from the particles remaining after removal of tag and lepton contributions. The BaBar analysis, in particular, reveals a beautiful $b \to u\ell\overline{\nu}$ signal with an unsurpassed signal to background ratio of about 2:1 in the region $M_X < 1.55$ GeV/c, which rivals that of current exclusive analyses. This analysis demonstrates the anticipated power of a large fully reconstructed sample, both in the signal to background levels and in the excellent resolution that can be achieved. The efficiency versus M_X appears reasonably uniform, and the signal yield fitting procedure minimizes the dependence of the extracted rate on the modeling of the detailed $b \to u\ell\overline{\nu}$ dynamics. Both allow for improved determination of $|V_{ub}|$ as theory advances.

Determination of the fraction of the $b \to u\ell\overline{\nu}$ rate in the p_ℓ endpoint or the low M_X region requires resummation of the OPE to all orders in $E_X\Lambda_{\rm QCD}/M_X^2$ [30–34]. The resummation results, at leading-twist order, in a nonperturbative shape function $f(k_+)$, where $k_+ = k^0 + k_\parallel$ and $k^\mu = p_b^\mu - m_b v^\mu$ is the residual b quark momentum after the "mechanical" portion of momentum is subtracted off. Spatial components $k_\parallel$ and $k_\perp$ are defined relative to the $m_b v^\mu - q^\mu$ (roughly the recoiling u quark) direction. At this order, effects such as the "jiggling" of $k_\perp$ are ignored and the differential partial width is given by the convolution of the shape function with the parton level differential distribution. Because the shape function depends only on parameters of the B meson, this leading order description holds for any B decay to a light quark. It holds, in particular, for $B \to s\gamma$, which can provides an estimation of $f(k_+)$ via the shape of the photon energy (E_γ) spectrum [31, 32]. In addition to the increased uncertainty on $|V_{ub}|$ from the m_b and b quark kinetic energy contributions that results from the restriction of phase space, higher twist contributions and unknown power corrections of order $\Lambda_{\rm QCD}/M_B$ [35, 36] also contribute to the uncertainty, as will be discussed further below.

Ideally, $|V_{ub}|$ would be determined without introduction of an intermediate extracted shape function through the use of appropriately weighted spectra [32,37–41]. This would avoid introduction of an element of model dependence. For the lepton spectrum, for example, one would take

$$\left|\frac{V_{ub}}{V_{tb}V_{ts}^*}\right|^2 = \frac{3\alpha}{\pi}\, K_{\rm pert}\frac{\widehat{\Gamma}_u(E_0)}{\widehat{\Gamma}_s(E_0)} + O(\Lambda_{\rm QCD}/M_B)\ , \qquad (3)$$

where $K_{\rm pert}$ is a calculable perturbative kernel, and $\widehat{\Gamma}_u(E_0)$ and $\widehat{\Gamma}_s(E_0)$ are appropriately weighted integrals over, respectively, the lepton energy and photon energy spectra above the minimum cutoff energy E_0. Practical application of this approach awaits measurement of the lepton momentum spectrum in the B, not the $\Upsilon(4S)$, rest frame, which B-tag methods will permit in the future. Similar expressions exist for integration over the low hadronic mass region [38,41], so, in principle, current M_X analyses should be able to take such an approach. Experimental efficiency and lepton momentum cutoffs must, however, be incorporated into the integrals. To date, experiments have instead introduced intermediate shape functions of a variety of forms into the inclusive analyses, as discussed below.

A third way to isolate the charmless s.l. signal is to use a selection based on the q^2 of the leptonic system. Restriction of phase space to regions of large q^2 also restores the validity of the OPE [42,43] and suppresses shape function effects. Taking only the region kinematically forbidden to $b \to c\ell\overline{\nu}$, $q^2 > (M_B–M_D)^2$, unfortunately introduces a low mass scale [44,45] into the OPE and the $1/m^3$ uncertainties blow up to be of order $(\Lambda_{\rm QCD}/m_c)^3$. However, a combination of M_X

with looser q^2 requirements can suppress both $b \to c\ell\overline{\nu}$ background experimentally and shape function effects theoretically. Furthermore, the q^2 requirement moves the parton level pole away from the experimentally feasible M_X requirement. The shape function effects, while suppressed, cannot be neglected. One drawback of the q^2 requirement is the elimination of higher energy hadronic final states, which may exacerbate duality concerns.

A recent Belle analysis [46] has been performed in this region. Belle employs a $p_\ell > 1.2$ GeV/c requirement in the $\Upsilon(4S)$ rest frame, and an "annealing" procedure to separate reconstructed particles into signal and "other B" halves. They then examine the integrated rate in the region $M_X < 1.7$ GeV and $q^2 > 8$ GeV^2 to extract $|V_{ub}|$, which again has the desired effect of minimizing dependence of the analysis on detailed $b \to u\ell\overline{\nu}$ modeling. The signal to background ratio of the annealing technique, about 1:6 for the Belle analysis, is significantly degraded relative to that of the hadronic B-tag technique. As we mentioned in the previous review, control of background subtractions of this size requires extreme care and careful scrutiny of the associated systematic issues. Belle finds the rate $\Delta\mathcal{B}$ in that region of phase space to be

$$\Delta\mathcal{B} = (7.37 \pm 0.89_{\rm stat} \pm 1.12_{\rm sys} \pm 0.55_{c\ell\nu} \pm 0.24_{u\ell\nu}) \times 10^{-4} \ . \quad (4)$$

An analysis of this restricted region of phase space, for which the shape function influence is significantly reduced [42], with the significantly cleaner B tag technique should be a priority for both B experiments.

Each analysis discussed here has relied on an intermediate shape function to evaluate the fraction of the inclusive rate that lies in its restricted region of phase space. The endpoint analyses have used rate fractions [18] based on intermediate shape functions derived from the CLEO $b \to s\gamma$ photon spectrum. Several two-parameter ansaetze [47,48], $F[\Lambda^{SF}, \lambda_1^{SF}]$, were implemented as the form of the shape function. These parameters are related to the HQET parameters of similar name, and play a similar role in evaluation of the rates. At this time, however, we do not know the precise relationship between the shape function parameters, or the moments of the shape function, and the HQET nonperturbative parameters $\overline{\Lambda}$ and λ_1 [49,50]. The fact that Λ^{SF} and λ_1^{SF} depend on the functional ansatz while the HQET parameters depend on the renormalization scheme underscores the current ambiguity.

With the limited E_γ statistics available, there exists a strong correlation between the two parameters because of the interplay between the effective b quark mass (controlled by Λ^{SF}) and the effective b quark kinetic energy (controlled by λ_1^{SF}) in determining the *mean* of the E_γ spectrum. No external constraints that could break the correlation, such as $m_b^{(1S)}$ or measured $b \to c\ell\overline{\nu}$ moments, have been input into the $b \to s\gamma$ fits because of their unknown relationship to the shape function parameters. The resulting effective m_b mass range contributing to the uncertainties (± 200 MeV) is therefore much larger than the current m_b uncertainty. Given the current independence of the shape function and m_b determinations and the broad effective m_b range sampled in the shape function, we do not consider it necessary to treat the E_γ-derived phase space fractions and the partial width (Eq. (2)) as positively correlated [46]. As the data statistics increase, it will become possible to constrain the shape function parameters directly from distributions in $b \to u\ell\overline{\nu}$, such as the M_X spectrum, thus removing the uncertainties introduced from their derivation from another class of decays. Furthermore, once the renormalization behaviour of the shape function and the relationship of its moments to the HQET parameters becomes known, the powerful constraints from the kinematic mass of the b quark and from moments information in the $b \to c\ell\nu$ system can be incorporated into a shape function derivation based either on $b \to s\gamma$ or $b \to u\ell\overline{\nu}$.

Two alternate approaches to the shape function evaluation have been taken in experimental studies so far. In their low-M_X analysis [23], BaBar has evaluated the phase space fraction using the same $f(k_+)$ parameterizations noted above, but has substituted HQET parameters derived from studies of spectral moments of the $b \to s\gamma$ and $b \to c\ell\overline{\nu}$ processes. The Belle M_X-q^2 analysis [46](discussed below) uses the calculation of Bauer *et al.* [42] based on a form with a single parameter

$a = \Lambda^{SF}/\lambda_1^{SF}$, which was estimated from the $m_b^{(1S)}$ mass and from typical estimates for λ_1. The uncertainties in the different rate fractions in the momentum endpoint, the low M_X and the M_X-q^2 regions are strongly correlated, and the values and uncertainties are sensitive to the theoretical assumptions made. Hence, a common theoretical scheme must be chosen for meaningful comparison of the extracted values of $|V_{ub}|$. Given the *ad hoc* nature of the association of shape function parameters with the HQET parameters in the $\overline{MS}$ or the $\Upsilon(1S)$ mass scheme [7], and the difficulty in evaluating the uncertainty in such an association, we have chosen to extract $|V_{ub}|$ from all of the measurements discussed using the $b \to s\gamma$-derived shape function.

Table 2: Summary of inclusive $|V_{ub}|$ measurements. The last five measurements are incorporated into the analysis presented below. The errors in the first group are the experimental and theoretical uncertainties. The errors in the second group are from the statistical, experimental systematic, E_γ-based rate fraction, and $\Gamma_{\rm tot}$ uncertainties. The two groups are *not* directly comparable as they have not been evaluated with identical theoretical inputs.

	$\|V_{ub}\|(10^{-3})$	
ALEPH [53]	$4.12 \pm 0.67 \pm 0.76$	neural net
L3 [54]	$5.70 \pm 1.00 \pm 1.40$	cut and count
DELPHI	$4.07 \pm 0.65 \pm 0.61$	M_X
OPAL [55]	$4.00 \pm 0.71 \pm 0.71$	neural net
LEP Avg.	$4.09 \pm 0.37 \pm 0.56$	
CLEO [56]	$4.05 \pm 0.61 \pm 0.65$	$d\Gamma/dq^2dM_X^2dE_\ell$
Belle	$5.00 \pm 0.64 \pm 0.53$	M_X, $D^{(*)}\ell\nu$ tag
CLEO	$4.11 \pm 0.13 \pm 0.31 \pm 0.46 \pm 0.28$	$2.2 < p < 2.6$
BaBar	$4.31 \pm 0.20 \pm 0.20 \pm 0.49 \pm 0.30$	$2.3 < p < 2.6$
Belle	$3.99 \pm 0.17 \pm 0.16 \pm 0.45 \pm 0.27$	$2.3 < p < 2.6$
Belle	$4.63 \pm 0.28 \pm 0.39 \pm 0.48 \pm 0.32$	$M_X < 1.7$, $q^2 > 8$
BaBar	$4.79 \pm 0.29 \pm 0.28 \pm 0.60 \pm 0.33$	$M_X < 1.55$

The full set of inclusive $|V_{ub}|$ results is summarized in Table 2, which is an updated version of the Heavy Flavors Averaging Group summary [51]. All endpoint results have QED radiative corrections applied correctly. The listed uncertainties do not include contributions for potentially large theoretical corrections that have been categorized but remain incalculable (see below). The last five results in the table, which we will use below, have been updated to a common framework based on the CLEO E_γ-derived shape function. The rate fractions [52] for the BaBar M_X analysis (f_M) and the Belle $M_X - q^2$ analysis f_{qM} are $f_M = 0.55 \pm 0.14$ and $f_{qM} = 0.33 \pm 0.07$. The central values for these and for the endpoint fractions (Table 1) correspond to an exponential shape function ansatz [48] and $(\lambda_1^{SF}, \overline{\Lambda}^{SF}) = (-0.342, 0.545)$, with small corrections related to background subtractions in the $b \to s\gamma$ spectrum. The errors are dominated by the statistical uncertainty in the $f(k_+)$ fit to the E_γ spectrum, but include contributions from experimental systematics, α_s uncertainties and modeling. Incorporation of results beyond those used here will require significant input from the experimental analyses, and is left to the HFAG.

Combining inclusive information:

Evaluation of the total uncertainty on $|V_{ub}|$ remains problematic because of a variety of theoretical complications. A recent review [16] discusses these issues in detail. There are three main contributions. The first arises from subleading (higher twist) contributions to the shape function resummation [57–61]. These involve incorporation of effects such as the variation of $k_\perp$, and are not universal for all B decay processes. Hence with the use of $b \to s\gamma$ to obtain a shape function, there are two contributions, one from subleading contributions to the use of a shape function in $b \to u\ell\overline{\nu}$ process itself, and the second from the different corrections in $b \to s\gamma$ from which the shape function is obtained. These contributions are potentially large, since they are of order $\Lambda_{\rm QCD}/m_b$. Indeed, a partial estimate of these effects [59] for the momentum endpoint region finds corrections that are similar in size to the total uncertainties of those analyses. Recent work indicates that the subleading contributions for $u\ell\nu$ and $s\gamma$ may partially cancel in the low M_X and the low M_X-high q^2 regions [61].

The second contribution, from "weak annihilation" processes, is formally of order $(\Lambda_{\rm QCD}/m_b)^3$ but receives a large multiplicative enhancement of $16\pi^2$ [62,63]. The contribution, which requires factorization violation to be nonzero, is expected to be localized near $q^2 \sim m_b^2$, and this localization can result in a further enhancement of the effect on $|V_{ub}|$. For the endpoint region, which sees about 10% of the total rate, an effect on the total rate of 2-3% (corresponding to factorization violation of about 10%), produces an effect on the measured rate of 20-30%.

Finally, there are unknown contributions from potential violation of local quark hadron duality. The true differential distribution cannot be predicted via the OPE—the resonant substructure is not described. However, spectra integrated over a sufficiently broad range should be better described.

The problems just outlined present a challenge to the averaging of the various inclusive results. Results with a potentially large bias might be included with neither a correction nor an appropriate uncertainty due to these effects. The resulting $|V_{ub}|$ determination would be potentially biased and the attached uncertainty unreliable. This motivated us not to provide an average result in the first edition of this review two years ago.

As an alternative, we here choose measurements in the region of phase space that appears to have the best compromise of the affects discussed to obtain an estimate of $|V_{ub}|$. Measurements from the other regions of phase space, which have increased sensitivity to one or more of the corrections, then provide limits on the uncertainties from these effects and thereby allow as complete as possible an estimation of the theoretical uncertainty, as first proposed by Ref. 52. At this time, the low M_X, high q^2 region appears to be the best motivated choice. It has reduced (though by no means negligible) corrections from the shape function and thus also from the subleading contributions to the shape function. Yet it integrates over a sufficient fraction of the spectrum to dilute weak annihilation contributions and concerns on local quark hadron duality.

While this choice is at present subjective, it offers the advantage of a reduction of the shape function influence coupled with the ability to bound the remaining theoretical uncertainties. In the opinion of the reviewers, this is a reasonable tradeoff for the statistical loss relative to the low M_X region. We expect that each experiment will perform an improved combination of information from the different regions of phase space where the experimental and theoretical correlations can be made manifest more straightforwardly.

We further stress that we view all three regions as equally crucial in this combination of information, as a more complete evaluation of the inclusive uncertainty than has previously existed is necessary for proper use of the inclusive results. The choice of the phase space region should not be misconstrued as a preference of experimental technique. Indeed, we look forward to a similar (or improved) analysis when a sample of clean results based on fully tagged B samples have been obtained for all regions of phase space.

At present only Belle [46] has contributed a result for this region of phase space, so for now we take this result as the "central value":

$$\begin{aligned}|V_{ub}|/10^{-3} = 4.63 \pm 0.28_{\mathrm{stat}} \pm 0.39_{\mathrm{sys}} \pm 0.48_{\mathrm{f_{qM}}} \pm 0.32_{\Gamma\mathrm{thy}} \\ \pm \sigma_{\mathrm{WA}} \pm \sigma_{\mathrm{SSF}} \pm \sigma_{\mathrm{LQD}} \ . \end{aligned} \tag{5}$$

Additional measurements by the B factories of the rate in this region of phase space will soon improve the experimental uncertainties.

We must determine the last three uncertainties for weak annihilation (WA), subleading shape function corrections (SSF) and local quark hadron duality (LQD). The measurements from other regions of phase space are crucial for this task.

We assume that the WA contribution is largely contained within each of the $p_\ell > 2.2$ GeV/c, the $M_X < 1.55$ GeV and the combined $M_X < 1.7$ GeV, $q^2 > 8$ GeV^2 regions. The contribution will be most diluted in the low M_X region, with the rate fraction $f_M = 0.55 \pm 0.14$, and most concentrated in the endpoint region, with the rate fraction $f_e = 0.14 \pm 0.03$ (without radiative corrections). It is simple to show that for a neglected WA contribution, a comparison of $|V_{ub}|$ from these two regions would predict the bias in the M_X, q^2 region (with rate fraction $f_{qM} = 0.33 \pm 0.07$) to be

$$[(1 - f_{qM})/f_{qM}][f_e f_M/(f_M - f_e)] \approx 0.39 \tag{6}$$

of the observed difference. Comparison of the endpoint result from CLEO and the low M_X result from BaBar, taking into consideration the almost total correlation in the shape function and Γ_{tot} uncertainties, yields $\Delta|V_{ub}|/10^{-3} = 0.69 \pm 0.53$. There is not sufficient sensitivity to draw conclusions regarding the presence of a WA component, but we can place a bound. We take the larger of the error and central value and scale according to Eq. (6) to obtain

$$\sigma_{\rm WA} \approx 0.27\ . \tag{7}$$

To estimate the uncertainty from the subleading corrections to the shape function, we assume that subleading corrections will scale like the fractional change in the predicted rate $(\Delta\Gamma/\Gamma)$ with and without convolution of the parton-level expression with the shape function. As the base comparison, we take the low M_X region, with $(\Delta\Gamma/\Gamma)_M = 0.15$, and compare to the combined M_X, q^2 region, with $(\Delta\Gamma/\Gamma)_{qM} = -0.075$). The shifts again depend on the shape function modeling, and the quoted values correspond to the $f(k_+)$ from the best fit to the CLEO E_γ spectrum . The theory uncertainties are again correlated, and we find $\Delta|V_{ub}|/10^{-3} = 0.16 \pm 0.63$. Scaling the uncertainty of the comparison by $|(\Delta\Gamma/\Gamma)_{qM}/(\Delta\Gamma/\Gamma)_M| = 0.49$, we have

$$\sigma_{\rm SSF} \approx 0.31\ . \tag{8}$$

Finally, we must make an estimate of the local duality uncertainty. We assume that a potential violation will scale with the fraction of rate f in a given region as $(1 - f)/f$. This form ranges from no "local" violation for integration of the full phase space ($f = 1$), to large uncertainty for use of a very localized region of phase space ($f \to 0$). The estimate derives from comparison of the CLEO $p_\ell > 2.2$ GeV/c analysis ($f \sim 0.14 \pm 0.03$) to the average of the BaBar and Belle $p_\ell > 2.3$ GeV/c analyses ($f \sim 0.07 \pm 0.02$). The subleading correction estimates of Ref. 59 are applied to minimize potential cancelation between duality violation and subleading corrections. This yields $(|V_{ub}|^{2.3} - |V_{ub}|^{2.2} + 0.27)/10^{-3} = 0.29 \pm 0.38$, where the 0.27 is the estimate of the relative subleading correction. With our scaling assumption, we then apply a scale factor s of

$$s = \frac{(1 - f_{qM})/f_{qM}}{(1 - f_{2.3})/f_{2.3} - (1 - f_{2.2})/f_{2.2}} \approx 0.29 \qquad (9)$$

to the uncertainty in this difference estimate. Our local duality estimate therefore is

$$\sigma_{\rm LQD} \sim 0.11.$$

From this analysis, we finally obtain

$$\begin{aligned} |V_{ub}|/10^{-3} = 4.63 &\pm 0.28_{\rm stat} \pm 0.39_{\rm sys} \pm 0.48_{f_{qM}} \pm 0.32_{\Gamma{\rm thy}} \\ &\pm 0.27_{\rm WA} \pm 0.31_{\rm SSF} \pm 0.11_{\rm LQD} \ , \end{aligned} \qquad (10)$$

for a total theory error of 15% and total precision of 18%. Given that the uncertainties are dominated by experimental limits, addition in quadrature seems appropriate. Note that these estimates apply *only* in the combined low M_X, high q^2 region of phase space. The limits presented here can be improved both in robustness, through more sophisticated scaling estimates, and in magnitude, through additional and improved $|V_{ub}|$ measurements and through inputs from other sources. The consistency of the values of $|V_{ub}|$ extracted with different inclusive methods and the stability of the results over changes in the selected region of phase space will provide increasing confidence in the reliability of the extracted results and of their estimated uncertainties. Improvement of the $b \to s\gamma$ photon energy spectrum is key until a self-consistent extraction of the shape function from $b \to u\ell\overline{\nu}$ transition becomes available. Comparisons of the D^0 versus D_s semileptonic widths and of the rates for charged versus neutral B mesons can provide estimates of the weak annihilation contributions [63]. Finally, improved theoretical guidance concerning the scaling of the effects over phase space would allow development of a simultaneous extraction of $|V_{ub}|$ and the corrections, with all experimental information contributing directly to $|V_{ub}|$.

Exclusive measurements of $b \to u\ell\overline{\nu}$:

Reconstruction of exclusive $b \to u\ell\overline{\nu}$ channels provides powerful kinematic constraints for suppression of the $b \to c\ell\overline{\nu}$ background. For this suppression to be effective, an estimate of the four momentum of the undetected neutrino must be provided. The measurements to date have made use of detector hermeticity and the well-determined beam parameters to define a missing momentum that is used as the neutrino momentum. Signal-to-background ratios (S/B) of order two have been achieved in these channels.

To extract $|V_{ub}|$ from an exclusive channel, the form factors for that channel must be known. The form factor normalization dominates the uncertainty on $|V_{ub}|$. The q^2-dependence of the form factors, which is needed to determine the experimental efficiency, also contributes to the uncertainty, but at a much reduced level. For example, the requirement of a stiff lepton for background reduction in these analyses introduces a q^2-dependence to the efficiency. In the limit of a massless charged lepton (a reasonable limit for the electron and muon decay channels), the $B \to \pi\ell\nu$ decay depends on one form factor $f_1(q^2)$:

$$\frac{d\Gamma(B^0 \to \pi^- \ell^+ \nu)}{dy\, d\cos\theta_\ell} = |V_{ub}|^2 \frac{G_F^2 p_\pi^3 M_B^2}{32\pi^3} \sin^2\theta_\ell |f_1(q^2)|^2 \ , \qquad (11)$$

where $y = q^2/M_B^2$ and θ_ℓ is the angle between the charged lepton direction in the virtual W $(\ell + \nu)$ rest frame and the direction of the virtual W. For the vector meson final states ρ and ω, three form factors A_1, A_2, and V are necessary (see *e.g.* reference [64]).

Calculation of these form factors constitutes a considerable theoretical industry, with a variety of techniques now being employed. Form factors based on lattice QCD calculations [65–77] and on light cone sum rules [78–87] currently have uncertainties in the 15% to 20% range. A variety of quark model calculations exists [88–102]. Finally, a number of other approaches [103–109], such as dispersive bounds and experimentally-constrained models based on Heavy Quark Symmetry, seek to improve the q^2 range where the form factors can be estimated, without introducing significant model dependence.

Of particular interest are the light cone sum rules (LCSR) and lattice QCD (LQCD) calculations, which minimize modeling assumptions as they are QCD-based calculations and provide a much firmer basis compared to the quark model calculations for systematic evaluation of the uncertainties. The calculations used in the current results have been summarized nicely in Ref. 11. The LCSR are expected to be valid in the region $q^2 \lesssim 16$ GeV^2. The light cone sum rules calculations use quark-hadron duality to estimate some spectral densities, and offer a "canonical" contribution to the related uncertainty of 10% with no known means of rigorously limiting that uncertainty. The theory community is currently debating the size of potential contributions to the form factors missing from the LCSR approach [110–114] that have been revealed using the newly-developed soft collinear effective theory (SCET) [115–118]. The

$B \to \rho \ell \nu$ form factors, in particular, could be appreciably overestimated, biasing $|V_{ub}|$ low. Two exclusive results will therefore be presented in this review, one based on the full set of exclusive results, and the second based only on results in the $q^2 > 16$ GeV^2 region for $B \to \rho \ell \nu$.

The LQCD calculations that can be applied to experimental $B \to X_u \ell \nu$ decay remain, to date, in the "quenched" approximation (no light quark loops in the propagators), which limits the ultimate precision to the 15% to 20% range. The q^2 range accessible to these calculations has been $q^2 \gtrsim 16$ GeV^2. Significant progress has been made towards unquenched lattice QCD calculations, and a recent comparison [119] of a range unquenched results to experiment shows much better agreement (few percent) than the corresponding quenched results. Work has begun on the unquenched form factors needed for $|V_{ub}|$, though the initial results have been limited to valence quarks closer to the strange quark mass. Nevertheless, initial results [120,121] are compatible with the 15% to 20% uncertainties used for the quenching uncertainty, lending them some validity.

The exclusive $|V_{ub}|$ results are summarized in Table 3. These include a simultaneous measurement of the $B \to \pi \ell \overline{\nu}$

and the $B \to \rho\ell\overline{\nu}$ transitions by CLEO [122], and measurement of the $B \to \rho\ell\overline{\nu}$ rate by CLEO [123] and BaBar [124]. All measurements employ the missing energy and momentum to estimate the neutrino momentum. With that technique, the major background results from $b \to c\ell\overline{\nu}$ decays in events that cannot be properly reconstructed (for example, because of additional neutrinos in the event) and hence which overestimate the neutrino energy. All measurements also employ the isospin relations

$$\Gamma(B^0 \to \pi^- \ell^+ \nu) = 2\Gamma(B^+ \to \pi^0 \ell^+ \nu)$$

and

$$\Gamma(B^0 \to \rho^- \ell^+ \nu) = 2\Gamma(B^+ \to \rho^0 \ell^+ \nu) \tag{12}$$

to combine the charged and neutral decays. These relationships can be distorted by $\rho-\omega$ mixing [125], and all results discussed here allow for this possibility in their systematic evaluation.

In the combined π and ρ measurement, strict event quality requirements were made that resulted in a low efficiency, but a relatively low background to signal ratio over a fairly broad lepton momentum range. The ρ-only analyses employ looser event cleanliness requirements, resulting in a much higher efficiency. The efficiency gain comes at the price of an increased background, and the analyses are primarily sensitive to signal with lepton momenta above 2.3 GeV/c, which is near (and beyond) the kinematic endpoint for $b \to c\ell\overline{\nu}$ decays which are therefore highly suppressed.

The combined π and ρ analysis of CLEO employs relatively loose lepton selection criteria and extracts rates independently in three separate q^2 intervals. Form factor dependence of the rates is then evaluated using models and calculations that exhibit a broad variation in $d\Gamma/dq^2$, which shows that this approach has eliminated model dependence of the rates in $\pi\ell\nu$, and significantly reduced it in $\rho\ell\nu$. To further reduce modeling uncertainties, CLEO then extracts $|V_{ub}|$ using only the LQCD and LCSR QCD-based calculations restricted to their respective valid q^2 ranges, thereby eliminating modeling used for extrapolation. Averages of the CLEO results, with and without the low q^2 region for $\rho\ell\nu$ are listed in Table 3.

A more complete review of recent $B \to X_u \ell\nu$ branching fractions, including analyses too incomplete for inclusion in this $|V_{ub}|$ summary, can be found in Reference [126]. Of note is the recent evidence presented for $B \to \omega\ell\nu$ by Belle [127].

With all results resting on use of detector hermeticity, the potential for significant correlation among the dominant experimental systematics exists [126]. Results from the three measurements have been averaged here assuming full correlation in these systematics. The $\rho\ell\nu$-only results [123,124], which depend more heavily on modeling even for the LCSR and LQCD calculations, are deweighted by 5% in the average. This yields

$$|V_{ub}| = (3.27 \pm 0.13 \pm 0.19^{+0.51}_{-0.45}) \times 10^{-3} \qquad (13)$$

where the errors arise from statistical, experimental systematic and form factor uncertainties, respectively. While similar in precision to the exclusive result in the previous $|V_{ub}|$ minireview, this result relies much less heavily on modeling. Should the LCSR form factors prove to be overestimated, we also provide an average excluding any result using information for $q^2 < 16$ GeV^2 in the $\rho\ell\nu$ modes, with the result

$$|V_{ub}| = (3.26 \pm 0.19 \pm 0.15 \pm 0.04^{+0.54}_{-0.39}) \times 10^{-3} \ , \qquad (14)$$

where the errors arise from statistical, experimental systematic $\rho\ell\nu$ form factor uncertainties, and LQCD and LCSR (treated as correlated), respectively.

The future for exclusive determinations of $|V_{ub}|$ appears promising. Unquenched lattice calculations are appearing, with very encouraging results. These calculations will eliminate the primary source of uncontrolled uncertainty in these calculations, and have already provided some validity to the quenching uncertainty estimate used in the results presented here. Simultaneously, the B factories are performing very well, and very large samples of events in which one B meson has been fully reconstructed are already being used. This will allow a more robust determination of the neutrino momentum, and should allow a significant reduction of backgrounds and experimental systematic uncertainties. The high statistics should also allow more detailed measurements of $d\Gamma/dq^2$, which have already

Table 3: Summary of all exclusive $|V_{ub}|$ measurements. For the CLEO 00 and BaBar 01 measurements, the errors arise from statistical, experimental systematic and form factor modeling uncertainties, respectively. For the CLEO 03 measurements, the errors arise from statistical, experimental systematic, $\rho\ell\nu$ form factor, and LQCD and LCSR calculation uncertainties, respectively. In the CLEO 03 averages, the LQCD and LCSR uncertainties have been treated as correlated.

mode	$\lvert V_{ub}\rvert(10^{-3})$	q^2 range	FF
CLEO 00 $\rho\ell\nu$	$3.23 \pm 0.24^{+0.23}_{-0.26} \pm 0.58$	all	model survey
BaBar 01 $\rho\ell\nu$	$3.64 \pm 0.22 \pm 0.25^{+0.39}_{-0.56}$	all	model survey
CLEO 03 $\pi\ell\nu$	$3.33 \pm 0.24 \pm 0.15 \pm 0.06\ ^{+0.57}_{-0.40}$	$q^2 < 16$ GeV^2	LCSR
CLEO 03 $\pi\ell\nu$	$2.88 \pm 0.55 \pm 0.30 \pm 0.18\ ^{+0.45}_{-0.35}$	$q^2 > 16$ GeV^2	LQCD
CLEO 03 $\pi\ell\nu$	$3.24 \pm 0.22 \pm 0.13 \pm 0.09^{+0.55}_{-0.39}$	average	
CLEO 03 $\rho\ell\nu$	$2.67 \pm 0.27\ ^{+0.38}_{-0.42} \pm 0.17\ ^{+0.47}_{-0.35}$	$q^2 < 16$ GeV^2	LCSR
CLEO 03 $\rho\ell\nu$	$3.34 \pm 0.32\ ^{+0.27}_{-0.36} \pm 0.47\ ^{+0.50}_{-0.40}$	$q^2 > 16$ GeV^2	LQCD
CLEO 03 $\rho\ell\nu$	$3.00 \pm 0.21^{+0.29}_{-0.35} \pm 0.28^{+0.49}_{-0.38}$	average	
CLEO 03 $\pi + \rho$	$3.17 \pm 0.17^{+0.16}_{-0.17} \pm 0.03^{+0.53}_{-0.39}$	average	
CLEO 03 $\pi + \rho$	$3.26 \pm 0.19 \pm 0.15 \pm 0.04^{+0.54}_{-0.39}$	average	no $\rho\ell\nu$ LCSR

provided a sorely-needed litmus test for the form factor calculations and reduced the form factor shape contribution to the uncertainty on $|V_{ub}|$. Should theory allow use of the full range of q^2 in the extraction of $|V_{ub}|$ [128], the B factories have already logged data sufficient for a 5% statistical determination of $|V_{ub}|$.

For both lattice and the B factories, $\pi\ell\nu$ appears to be a golden mode for future precise determination of $|V_{ub}|$. The one caveat is management of contributions from the B^* pole, but recent work [76] suggests that this problem can be successfully overcome. $B \to \eta\ell\nu$ will provide a valuable cross check. The $\rho\ell\nu$ mode will be more problematic for high precision: the broad width introduces both experimental and theoretical difficulties. Experiments must, for example, assess potential nonresonant $\pi\pi$ contributions, but only crude arguments based on isospin and quark-popping have been brought to bear to date. Theoretically, no calculation, including lattice, has dealt with the width of the ρ. When the lattice calculations become unquenched, the ρ will become unstable and the $\pi\pi$ final state must be faced by the calculations. The methodology for accommodation of high-energy two particle final states on the lattice has yet to be developed. The $\omega\ell\nu$ mode may provide a more tractable alternative to the ρ mode because of the relative narrowness of the ω resonance. Agreement between accurate $|V_{ub}|$ determinations from $\pi\ell\nu$ and from $\omega\ell\nu$ will provide added confidence in both.

Combined results:

The experimental bounds provided for the outstanding uncertainties in the inclusive $|V_{ub}|$ measurements in the low M_X, high q^2 region and theoretical work which clarifies the reliability of the LCSR and quenched LQCD form factors make possible comparison of the inclusive and exclusive determinations of $|V_{ub}|$. Results agree to better than 1.5 times the quadratic combination of the quoted uncertainties. Therefore it becomes feasible to propose an average the inclusive and exclusive results, which have comparable accuracies.

The uncertainties have been combined in quadrature, using the larger (upward) error for the exclusive numbers. The proposed average of the inclusive and exclusive results, with all the exclusive data considered, is

$$|V_{ub}| = (3.67 \pm 0.47) \times 10^{-3} \ . \tag{15}$$

Including in the average only the exclusive analyses based on data with $q^2 > 16$ GeV^2 in the $\rho\ell\nu$ mode, the average becomes $|V_{ub}| = (3.70 \pm 0.49) \times 10^{-3}$, so exclusion of this region, if appropriate, has only a minor effect.

The procedure proposed here results in a value of $|V_{ub}|$ with a 13% uncertainty. With the experimental and theoretical progress expected over the next few years an improvement of the accuracy at the 10% level, and possibly below, appears now realizable.

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