Richardson's Forecast: What Went Wrong?

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Operational Numerical Weather Prediction
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Abstract

Richardson's Forecast: What Went Wrong?

The year 1904 was a pivotal one for NWP. In that year, a program for rational weather forecasting was defined by Vilhelm Bjerknes. He showed in principle how the laws of physics could be used to develop a procedure for atmospheric prediction. In the same year, Max Margules demonstrated that any attempt to predict pressure changes using the continuity equation was doomed to failure. A little later, Felix Exner attempted actual calculations of atmospheric changes using a drastically simplified model, with results which were unspectacular but not unreasonable. During the First World War, Lewis Fry Richardson carried out the initial step of a forecast using Bjerknes' approach, but the computed changes were utterly unrealistic. In fact, Margules' work had shown that the bulldozer approach of using the full primitive equations would fail.

The essence of the problem is that pressure changes are determined by the divergence field, which is calculated as a small residue resulting from near-cancellation of large terms, an inherently error-prone process. Richardson's data were contaminated by imbalances which gave rise to spurious large amplitude oscillations, and a computed initial pressure

tendency of 145mb in 6 hours. He also used a large time step which would have resulted in instability of the integration, but this did not affect the initial tendencies.

John von Neumann recognized weather prediction as an ideal problem for an automatic computer. The Princeton group knew of Richardson's work, and carefully considered his disastrous results, which must have been a powerful source of anxiety to them. They struggled for some time to find a means of avoiding the pitfalls encountered by Richardson. There are essentially two ways round the problem: the initial data could be doctored to remove the imbalance, a process called initialization; or the equations themselves could be doctored to eliminate the high frequency spurious gravity wave solutions, a process called filtering.

It appears that Karl-Heinz Hinkelmann was the first to consider the former option. He proposed that if geostrophic winds were assumed initially, the high frequency noise would remain under control. Moreover, the process of geostrophic adjustment, elucidated by Carl Gustav Rossby a decade earlier, implied that the flow would soon adjust to harmony with the pressure field.

Another factor influencing events was the severe limitation on computing power of the ENIAC. The full equations would pose a prohibitive burden on the machine. Von Neumann had been in Göttingen when

Hans Lewy had discovered the numerical stability criterion, and he realized that a small time step would be essential to avoid numerical instability. Thus, a simplified system had two appeals: lesser computational load and freedom from high frequency noise.

Such a filtered system, the quasi-geostrophic system, was devised by Jule Charney. It was free from the high frequency components which had spoiled Richardson's forecast, and held the promise of producing realistic results and allowing a large time step. Thus, the two essential problems with Richardson's approach were circumvented. In fact, an even simpler system, the conservation equation for absolute vorticity, was used for the first numerical forecast. The surprisingly good results from this humble barotropic equation encouraged intensive research which led within a few years to operational NWP.

Anniversaries — General

It's June 15th here in Washington.

But it is already June 16th
in Australia.

So, we can begin celebrating the
100$^{\text{th}}$ Anniversary of Bloomsday.

James Joyce's masterpiece* Ulysses *was set on
16th June, 1904,* Bloomsday*, precisely 100 years ago.

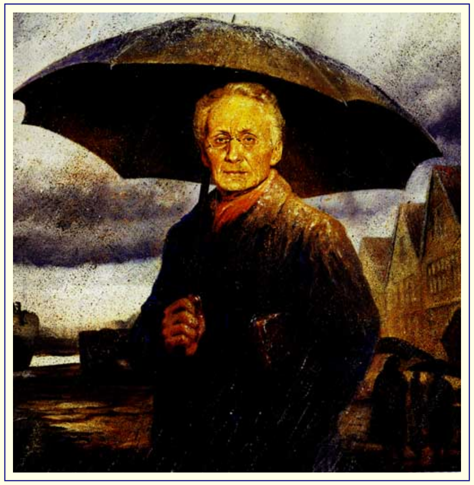
Anniversaries — Meteorological

The year 1904 was a pivotal one for NWP.

- ***Vilhelm Bjerknes*** ***defined a program for rational weather forecasting.***
- ***Max Margules*** ***demonstrated that weather prediction was fraught with danger***
- ***Felix Exner*** ***attempted actual calculations of atmospheric changes.***

Vilhelm Bjerknes (1862–1951)

Vilhelm Bjerknes (1862–1951)



Vilhelm Bjerknes

- **Born March, 1862.**
- **Matriculated in 1880.**
- **Fritjøf Nansen was a fellow-student.**
- **Paris, 1989–90. Studied under Poincaré.**
- **Bonn, 1890–92. Worked with Heinrich Hertz.**
- **Stockholm, 1983–1907.**
- **1898: Circulation theorems**
- **1904: Meteorological Manifesto**
- **Christiania (Oslo), 1907–1912.**
- **Leipzig, 1913–1917.**
- **Bergen, 1917–1926.**
- **1919: Frontal Cyclone Model.**
- **Oslo, 1926**
- **Retired in 1937**
  - **Died, April 9,1951.**

Bjerknes' 1904 Manifesto

To establish a science of meteorology, with the aim of predicting future states of the atmosphere from the present state.

"If it is true, as every scientist believes, that subsequent atmospheric states develop from the preceeding ones according to physical law, then it is apparent that the necessary and sufficient conditions for the rational solution of forecasting problems are the following:

1. **A sufficiently accurate knowledge of the *state* of the atmosphere at the initial time**
2. **A sufficiently accurate knowledge of the *laws* according to which one state of the atmosphere develops from another."**

- ***Step (1) is Diagnostic.***
- ***Step (2) is Prognostic.***

Max Margules (1856–1920)

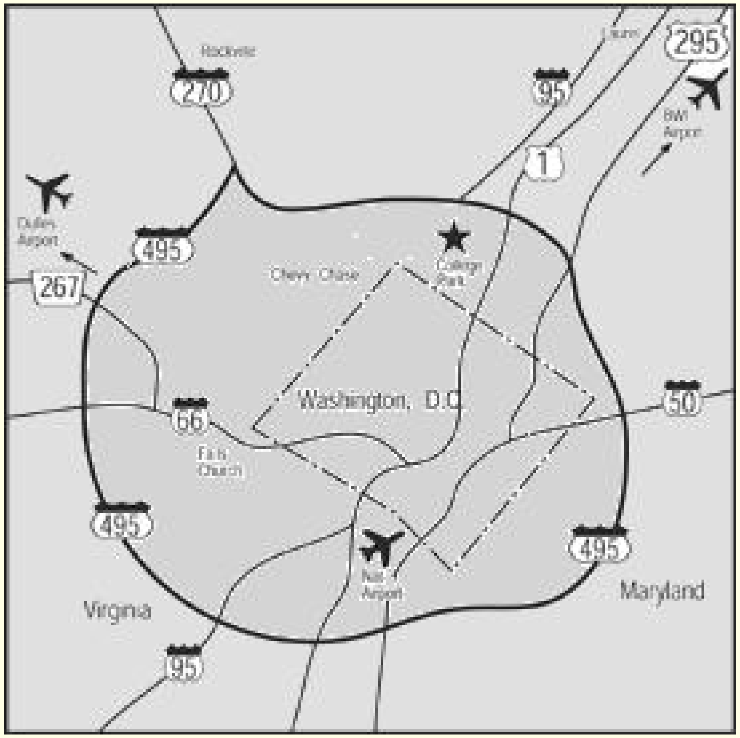
In 1904, Margules published a paper in the *Festschrift* marking the sixtieth birthday of his former teacher, the renowned physicist Ludwig Boltzmann.

Über die Beziehung zwischen Barometerschwankungen und Kontinuitätsgleichung.

"On the Relationship between Barometric Variations and the Continuity Equation."

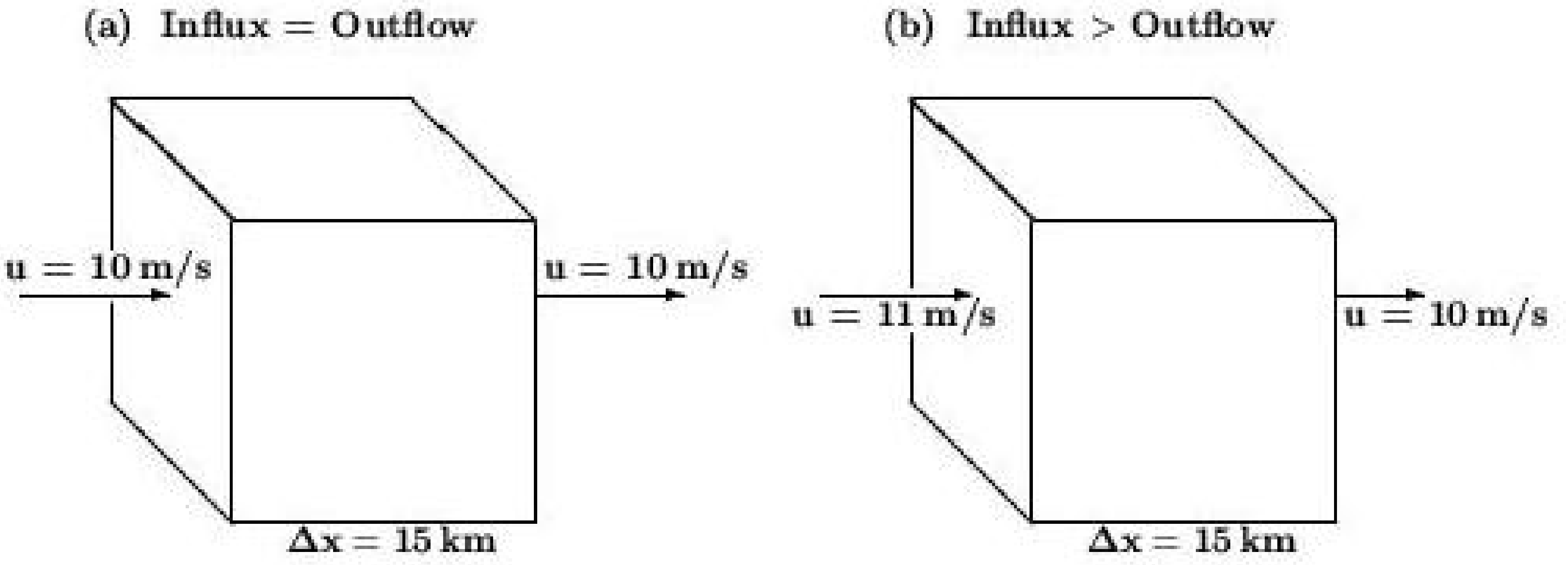
- *Margules considered the possibility of predicting pressure changes by means of the continuity equation.*
- *He showed that, to obtain an accurate estimate of the pressure tendency, the winds would have to be known to a precision quite beyond the practical limit.*
- *He showed that forecasting synoptic changes by this means was doomed to failure.*
- *He concluded that weather forecasting was* immoral and damaging to the character of a meteorologist *(Fortak, 2001).*

Tendency from Continuity Equation



- **The Capital Beltway: a 100 km-long Interstate freeway encircling Washington, DC.**
- **It is roughly a square of side 15km.**
- **It is analogous to a cell of a finite difference model of the atmosphere.**

A Box of Air over the Beltway



Influx equals Outflow:
Pressure remains unchanged.

Influx exceeds Outflow:
Pressure will rise.

Pressure Tendency

Assume a westerly wind over the beltway

$$u > 0 \,, \qquad\qquad v = 0 \,.$$

Assume also that the surface pressure is initially 1000 hPa.

Using Conservation of Mass, a simple *back-of-the-envelope* calculation yields the following amazing result:

- **If the speed on the western side *exceeds* that on the east by *just* 1 m/s, the pressure tendency is about 7 Pa/s.**
- **If this influx continues, the pressure will *double* in about 4 hours.**

Conclusion:

We must apply the Continuity Equation with great care.

Felix Maria Exner (1876–1930)

A first attempt at calculating synoptic changes using physical principles was made by Felix Exner, working in Vienna.

Exner followed a radically different line from Bjerknes.

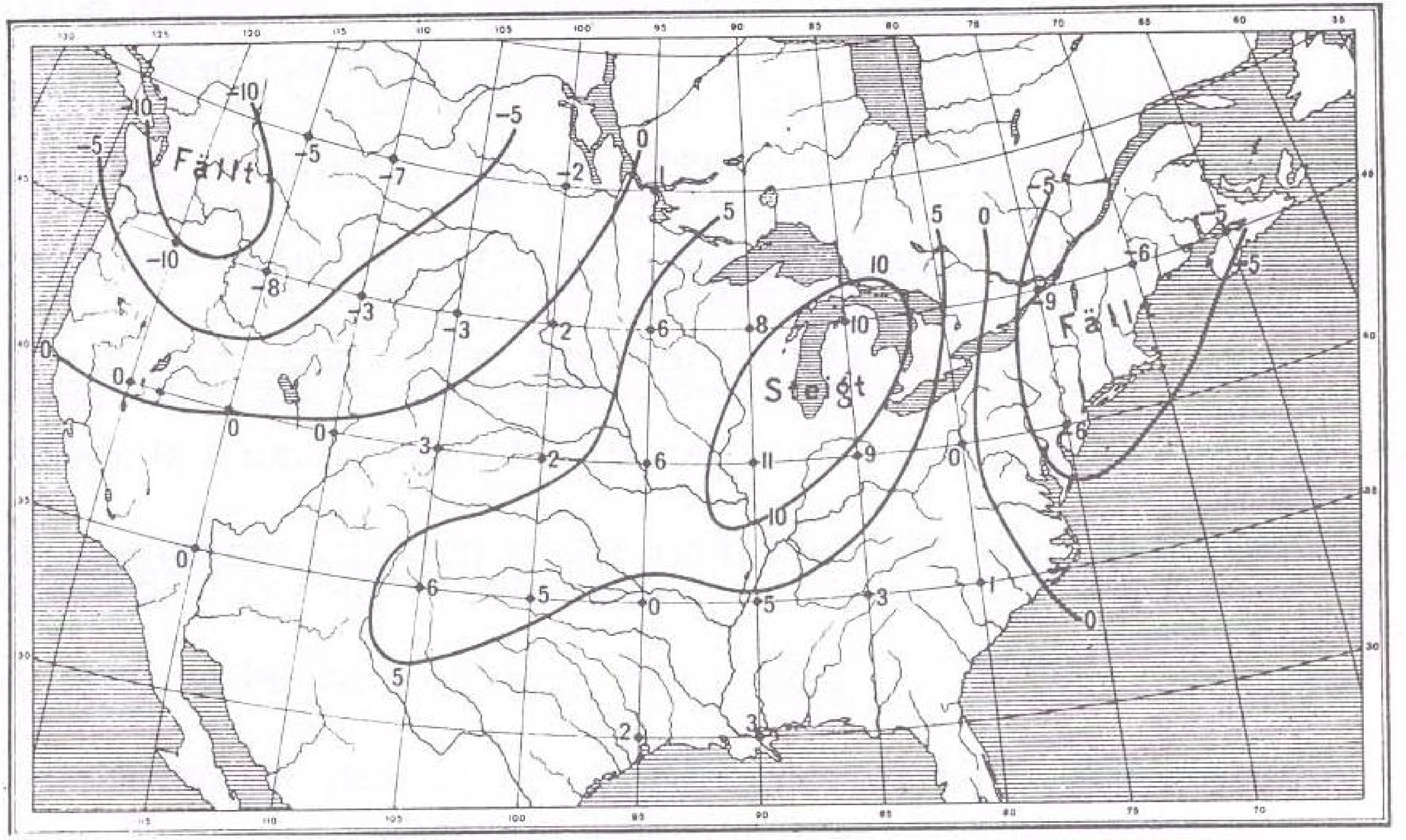
He did *not* make direct use of the continuity equation.

His method was based on a system reduced to the essentials.

Exner's Method

- ***Exner assumed that the atmospheric flow is geostrophically balanced and that the thermal forcing is constant in time.***
- ***He deduced a mean zonal wind from temperature observations.***
- ***He then derived a prediction equation representing advection of the pressure pattern with constant westerly speed, modified by diabatic heating.***
- ***His method yielded a realistic forecast in the case illustrated in his paper.***

Exner’s Forecast

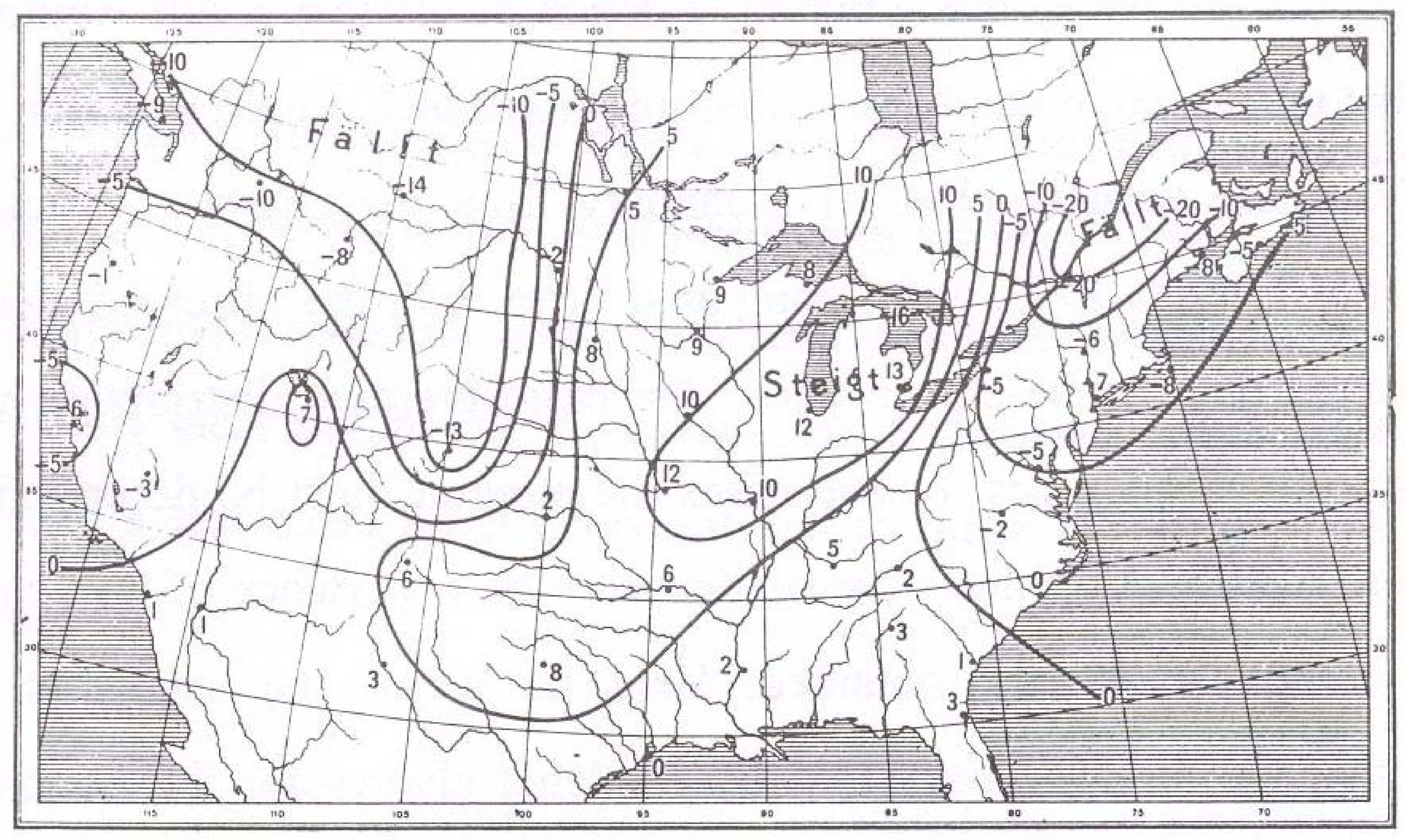


Calculated Pressure Change

between 8pm and 12pm on 3 January, 1895

Hundreths of an inch. [*Steigt*=rises; Fällt=falls].

Verification



Observed Pressure Change
between 8pm and 12pm on 3 January, 1895
Hundreths of an inch. [*Steigt*=rises; Fällt=falls].

Richardson's Reaction

Exner's work deserves attention as a first attempt at systematic, scientific weather forecasting.

The only reference by Richardson to the method was a single sentence in his book *Weather Prediction by Numerical Process* (p. 43):

"F. M. Exner has published a prognostic method based on the source of air supply."

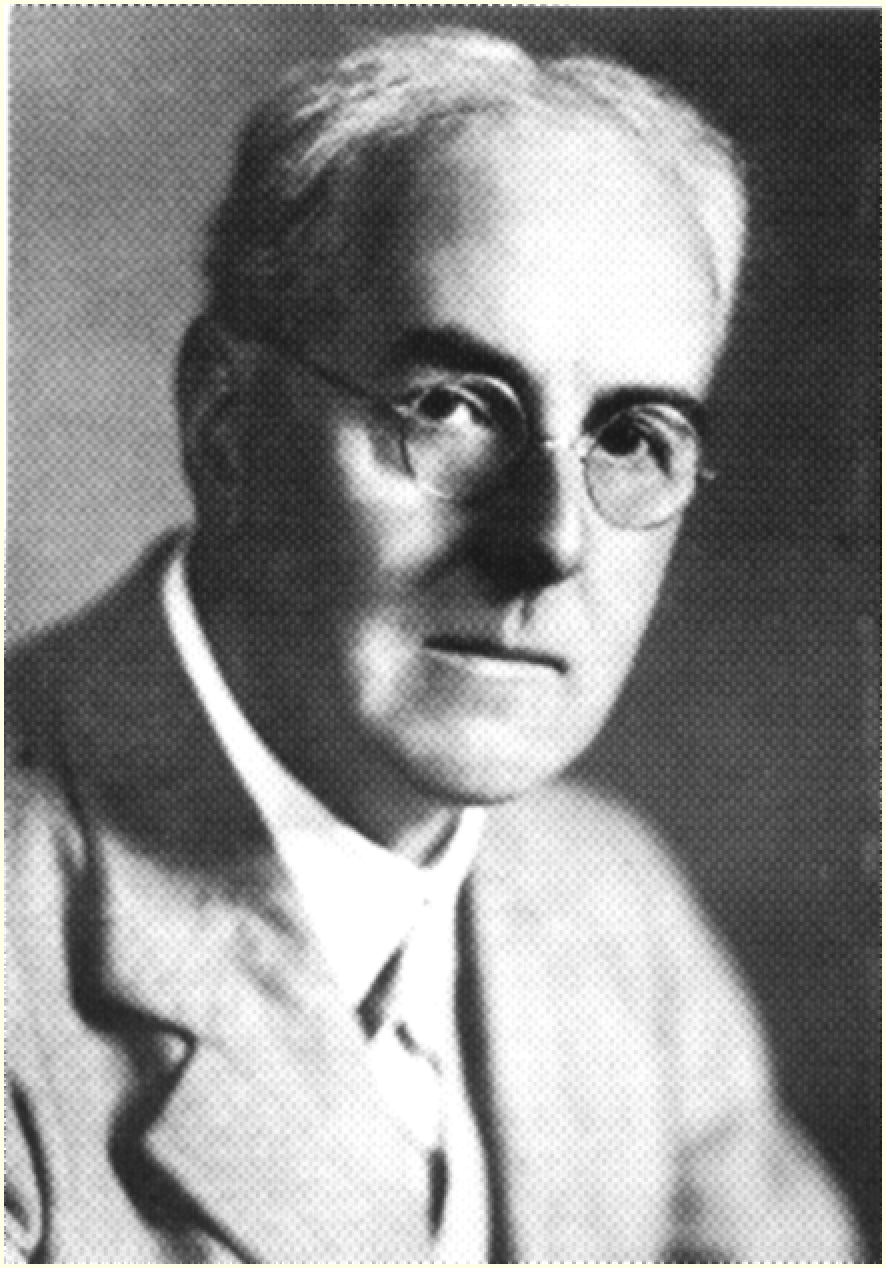
It would appear from this that Richardson was not particularly impressed by it!

⋆ ⋆ ⋆

However, as we shall shortly see,

- **Exner's forecast was unspectacular but reasonable.**
- **Richardson's forecast was spectacularly unreasonable.**

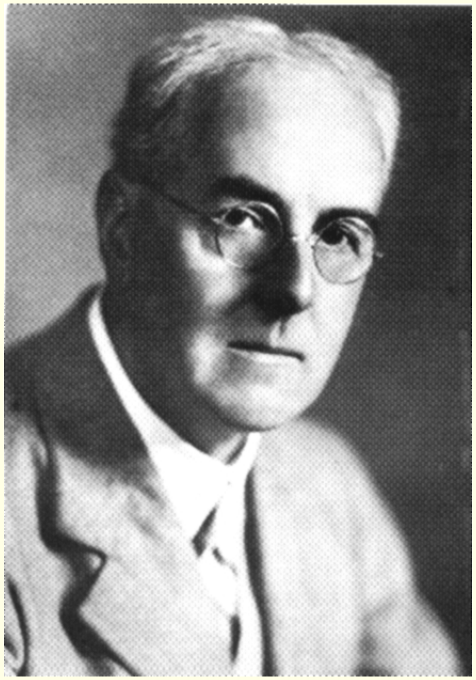
Lewis Fry Richardson, 1881–1953.



Bjerknes believed that for the solution of the forecasting problem graphical or mixed graphical and numerical methods were appropriate.

However, Richardson was bolder — or more foolhardy — than Bjerknes.

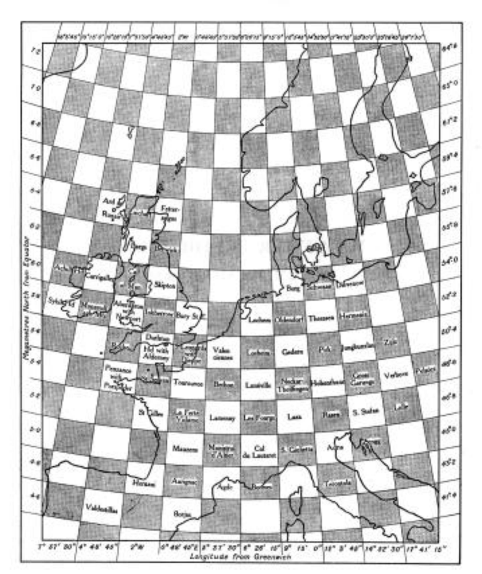
He attempted a bulldozer approach, calculating changes from the full partial differential equations.



- **Born, 11 October, 1881, Newcastle-upon-Tyne**
- **Family background: well-known quaker family**
- **1900–1904: Kings College, Cambridge**
- **1913–1916: Met. Office. Superintendent, Eskdalemuir Observatory**
- **Resigned from Met Office in May, 1916. Joined Friends' Ambulance Unit.**
- **1919: Re-employed by Met. Office**
- **1920: M.O. linked to the Air Ministry. LFR Resigned, on grounds of concience**
- **1922:** *Weather Prediction by Numerical Process*
- **1926: Break with Meteorology.**
  Worked on Psychometric Studies.
  Later on Mathematical causes of Warfare
- **1940: Resigned to pursue "peace studies"**
- **Died, September, 1953.**

Richardson contributed to Meteorology, Numerical Analysis, Fractals, Psychology and Conflict Resolution.

The Finite Difference Scheme



The globe is divided into cells, like the checkers of a chess-board.

Spatial derivatives are replaced by finite differences:

$$\frac{df}{dx} \to \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x} .$$

Similarly for time derivatives:

$$\frac{dQ}{dt} \to \frac{Q^{n+1} - Q^{n-1}}{2\Delta t} = F^n$$

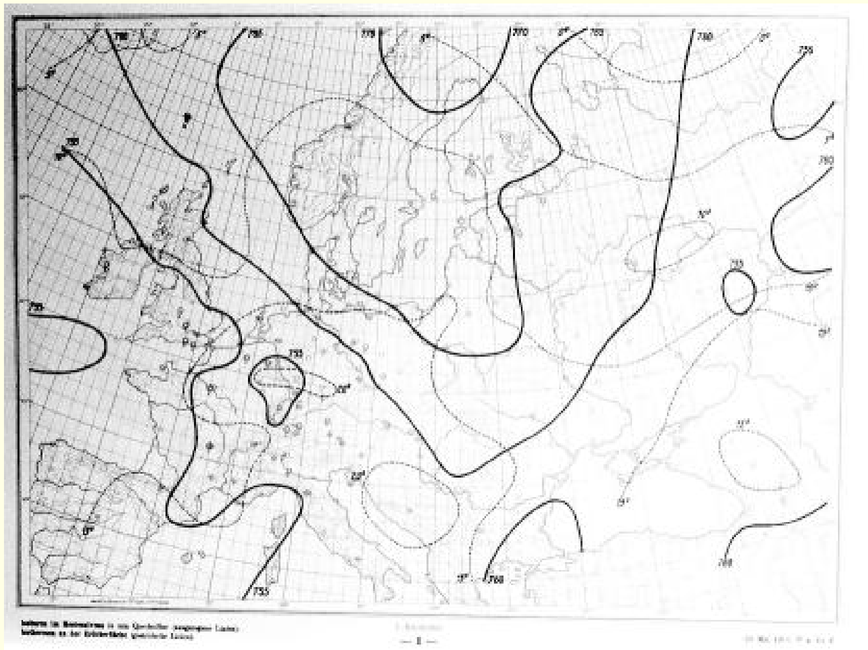
This can immediately be solved for Q^{n+1}:

$$Q^{n+1} = Q^{n-1} + 2\Delta t F^n .$$

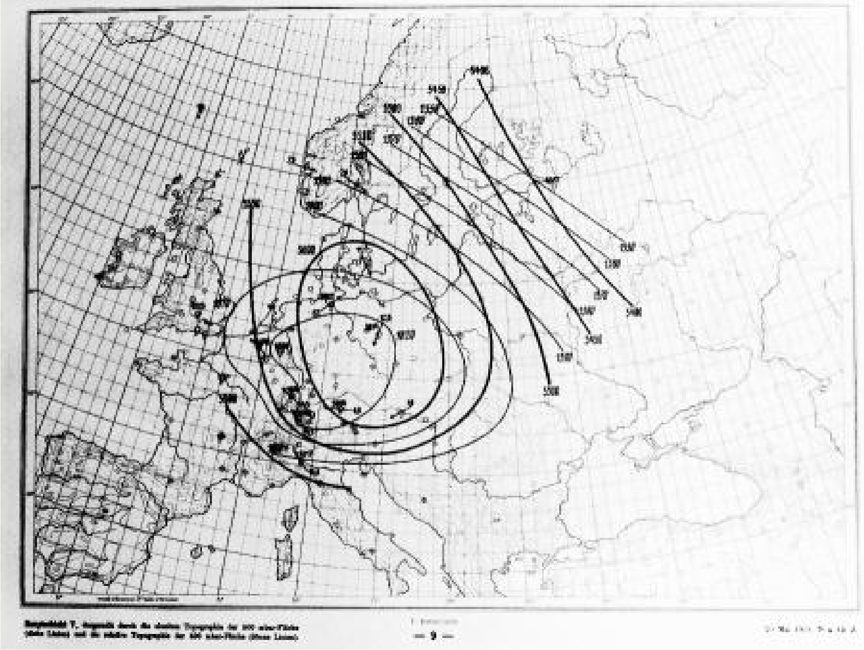
By repeating the calculations for many time steps, we can get a forecast of any length.

Richardson calculated only the initial rates of change.

The Leipzig Charts for 0700 UTC, May 20, 1910



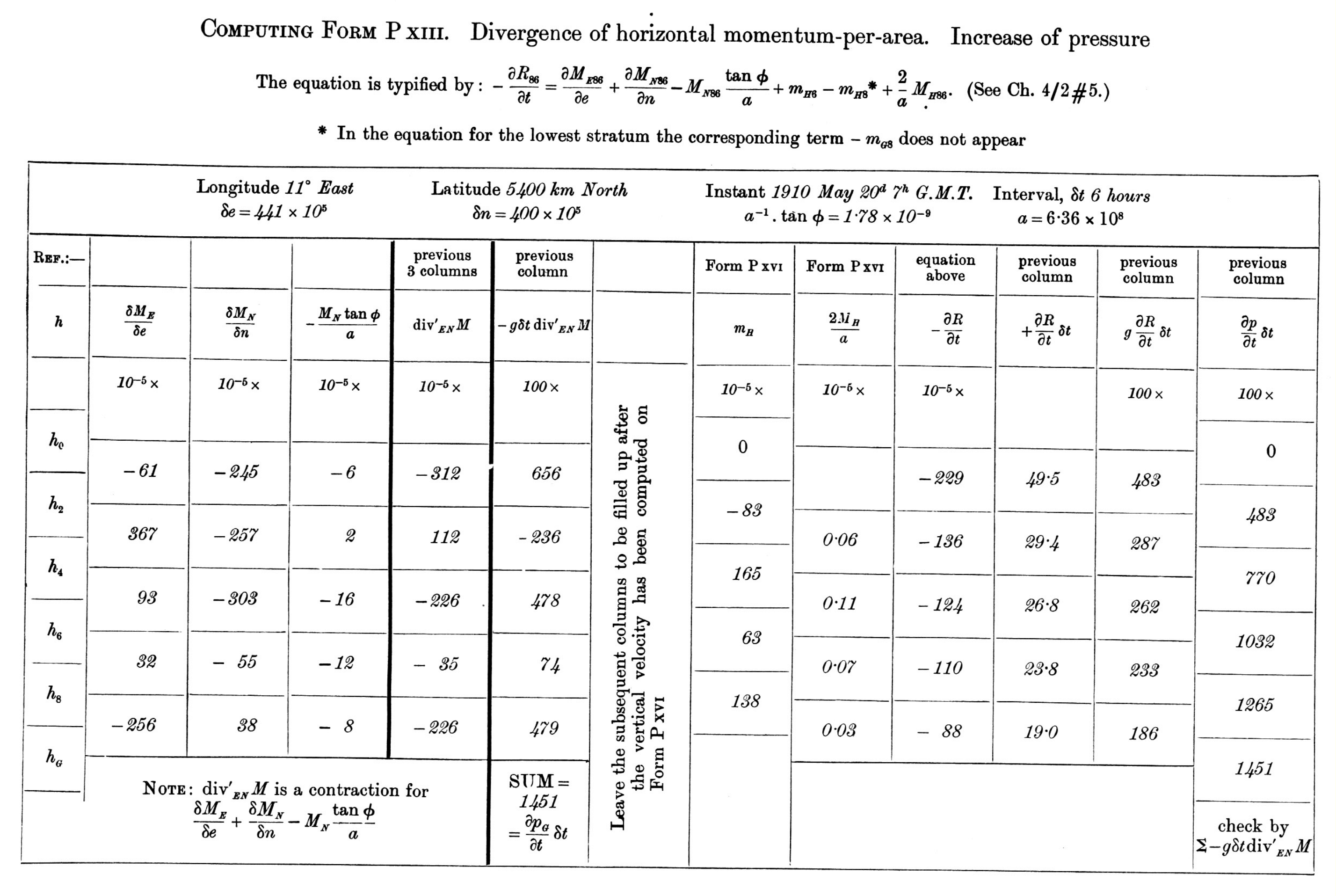
Bjerknes' sea level pressure analysis.



Bjerknes' 500 hPa height analysis.

Some of the initial data for Richardson's "forecast".

Richardson's *Spread-sheet*



COMPUTING FORM P XIII. Divergence of horizontal momentum-per-area. Increase of pressure

The equation is typified by: $-\frac{\partial R_{86}}{\partial t} = \frac{\partial M_{E86}}{\partial e} + \frac{\partial M_{N86}}{\partial n} - M_{N86}\frac{\tan\phi}{a} + m_{H6} - m_{H8}{}^{*} + \frac{2}{a}\dot{M}_{H86}$. (See Ch. 4/2 #5.)

* In the equation for the lowest stratum the corresponding term $-m_{G8}$ does not appear

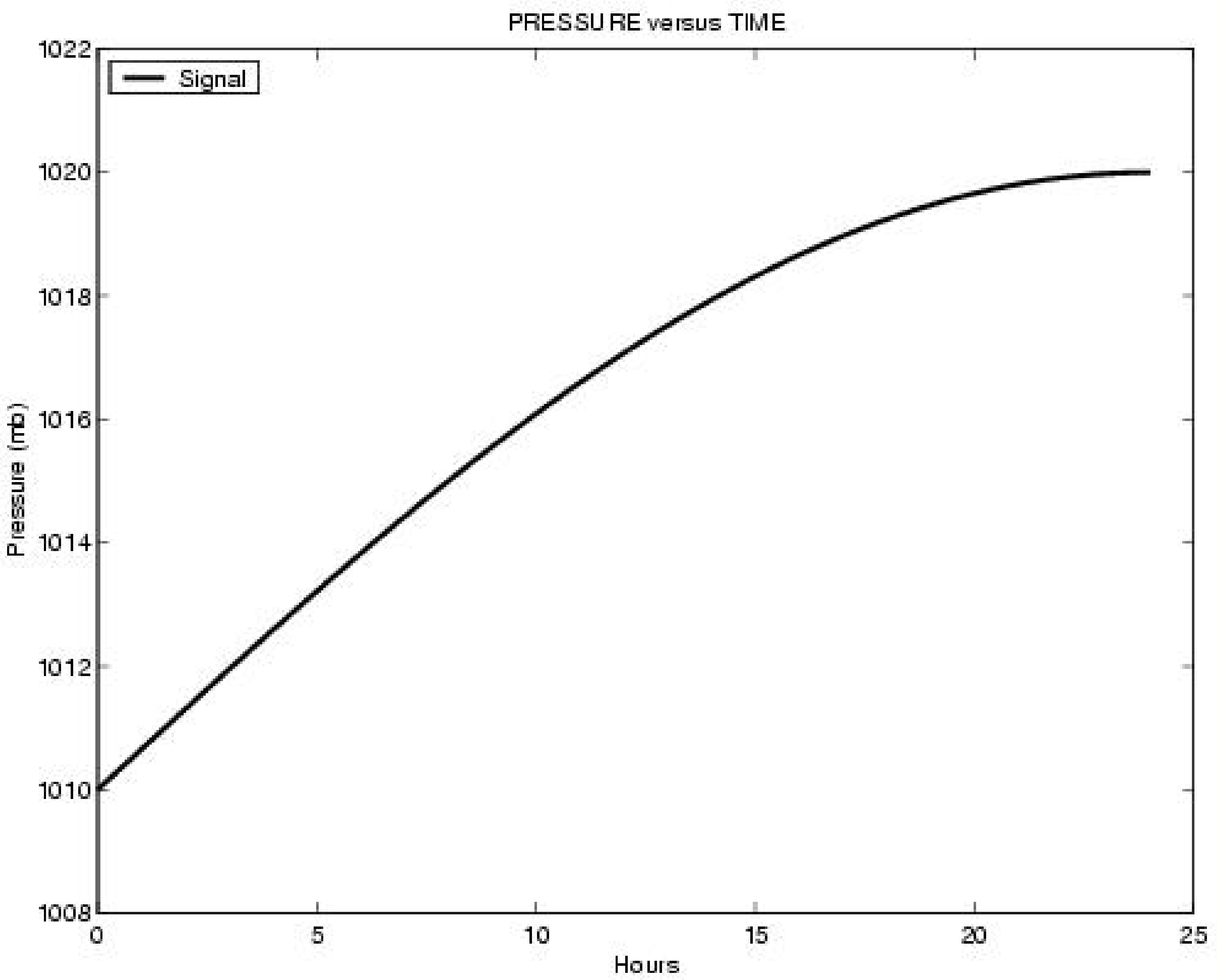
Longitude *11° East* $\delta e = 441 \times 10^5$ | Latitude *5400 km North* $\delta n = 400 \times 10^5$ | Instant *1910 May 20^d 7^h G.M.T.* $a^{-1}.\tan\phi = 1{\cdot}78 \times 10^{-9}$ | Interval, *δt 6 hours* $a = 6{\cdot}36 \times 10^8$

REF.:—				previous 3 columns	previous column		Form P XVI	Form P XVI	equation above	previous column	previous column	previous column
h	$\frac{\delta M_E}{\delta e}$	$\frac{\delta M_N}{\delta n}$	$-\frac{M_N \tan\phi}{a}$	$\mathrm{div}'_{EN}M$	$-g\delta t\,\mathrm{div}'_{EN}M$		m_H	$\frac{2\dot{M}_H}{a}$	$-\frac{\partial R}{\partial t}$	$+\frac{\partial R}{\partial t}\delta t$	$g\frac{\partial R}{\partial t}\delta t$	$\frac{\partial p}{\partial t}\delta t$
	$10^{-5}\times$	$10^{-5}\times$	$10^{-5}\times$	$10^{-5}\times$	$100\times$	Leave the subsequent columns to be filled up after the vertical velocity has been computed on Form P XVI	$10^{-5}\times$	$10^{-5}\times$	$10^{-5}\times$		$100\times$	$100\times$
h_0							0					0
	−61	−245	−6	−312	656				−229	49·5	483	
h_2							−83					483
	367	−257	2	112	−236			0·06	−136	29·4	287	
h_4							165					770
	93	−303	−16	−226	478			0·11	−124	26·8	262	
h_6							63					1032
	32	−55	−12	−35	74			0·07	−110	23·8	233	
h_8							138					1265
	−256	38	−8	−226	479			0·03	−88	19·0	186	
h_G												1451
	NOTE: $\mathrm{div}'_{EN}M$ is a contraction for $\frac{\delta M_E}{\delta e} + \frac{\delta M_N}{\delta n} - M_N\frac{\tan\phi}{a}$				SUM = *1451* $= \frac{\partial p_G}{\partial t}\delta t$							check by $\Sigma - g\delta t\,\mathrm{div}'_{EN}M$

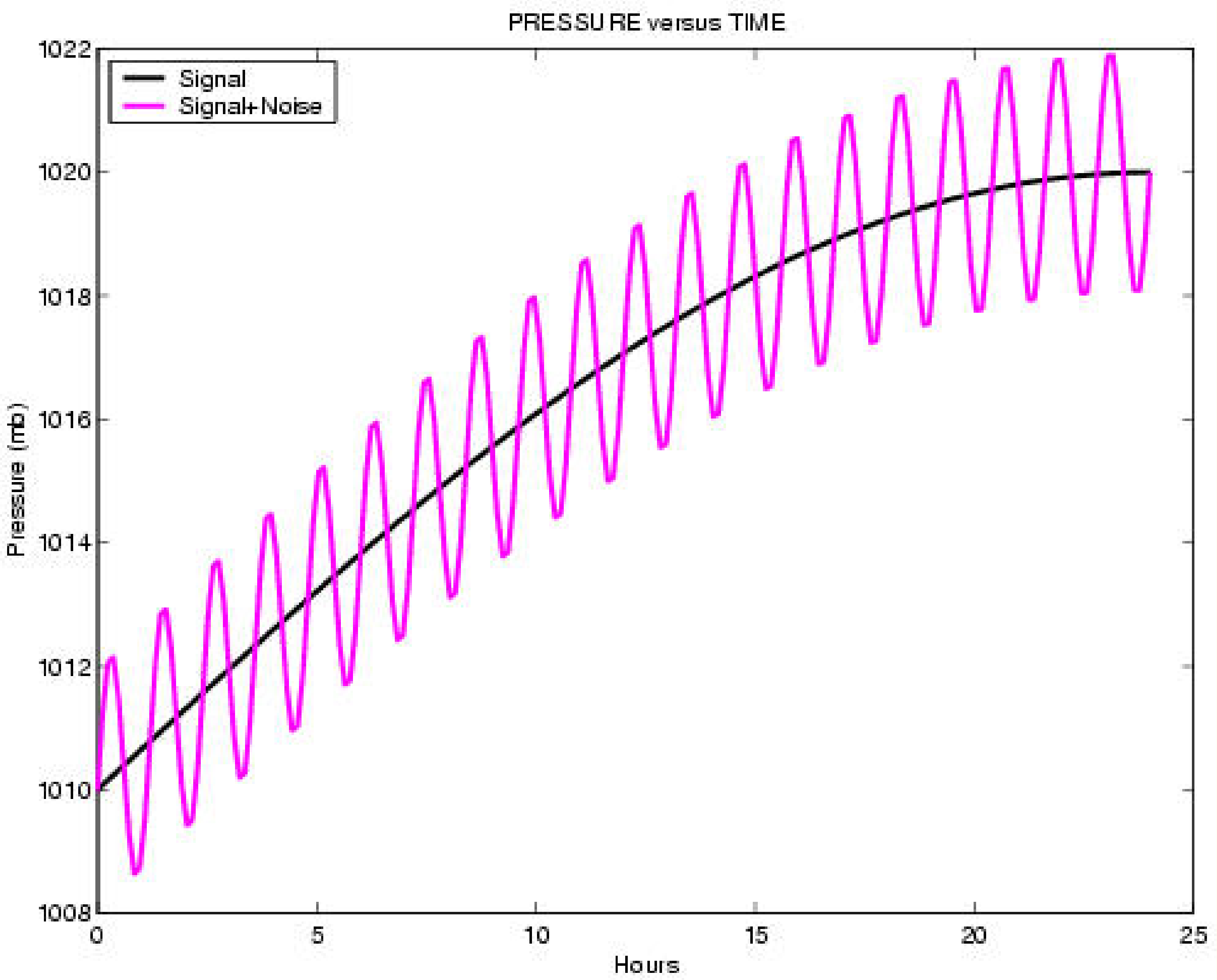
Richardson's Computing Form $\mathrm{P}_{\mathrm{XIII}}$

The figure in the bottom right corner is the forecast change in surface pressure: 145 mb in six hours!

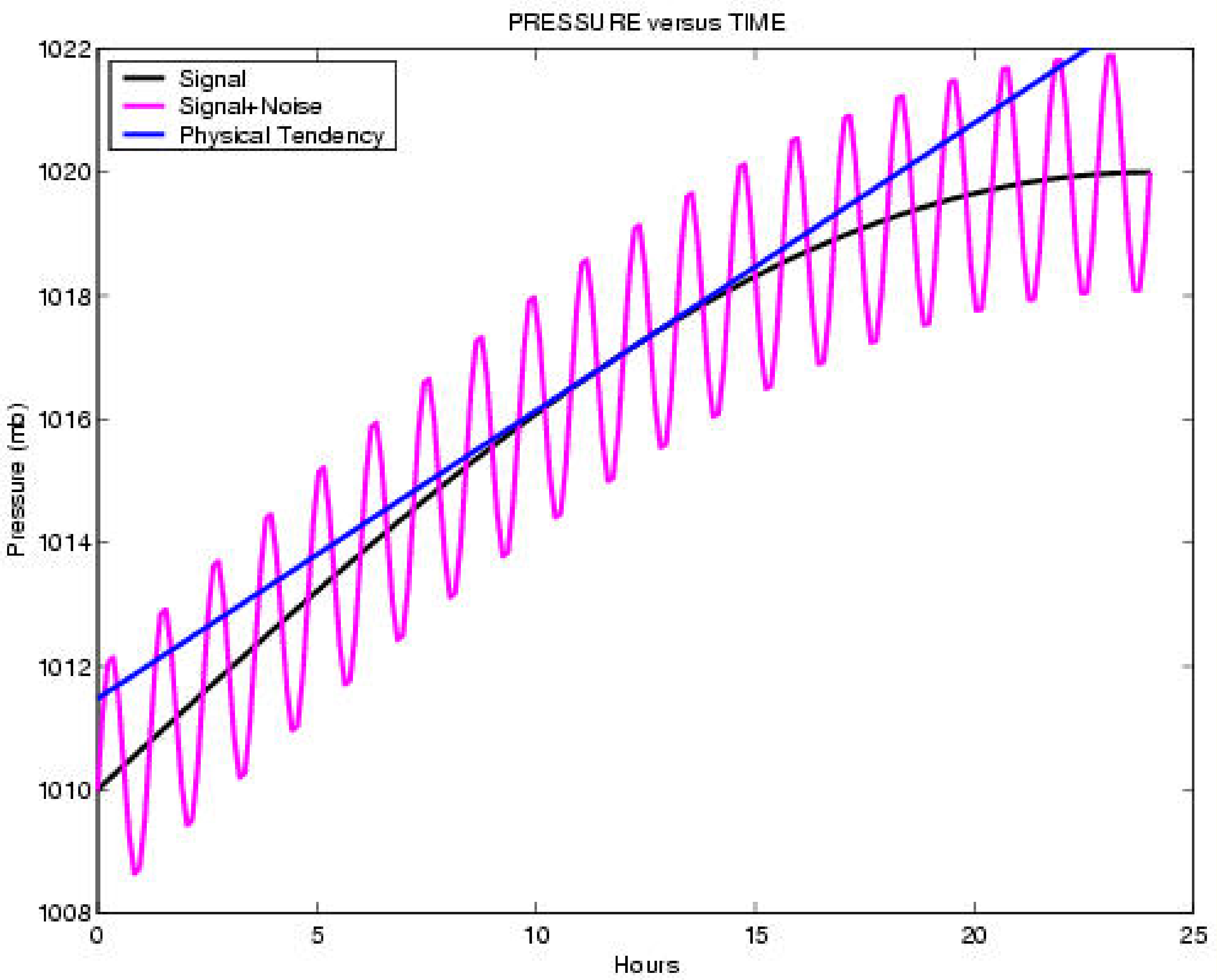
Smooth Evolution of Pressure



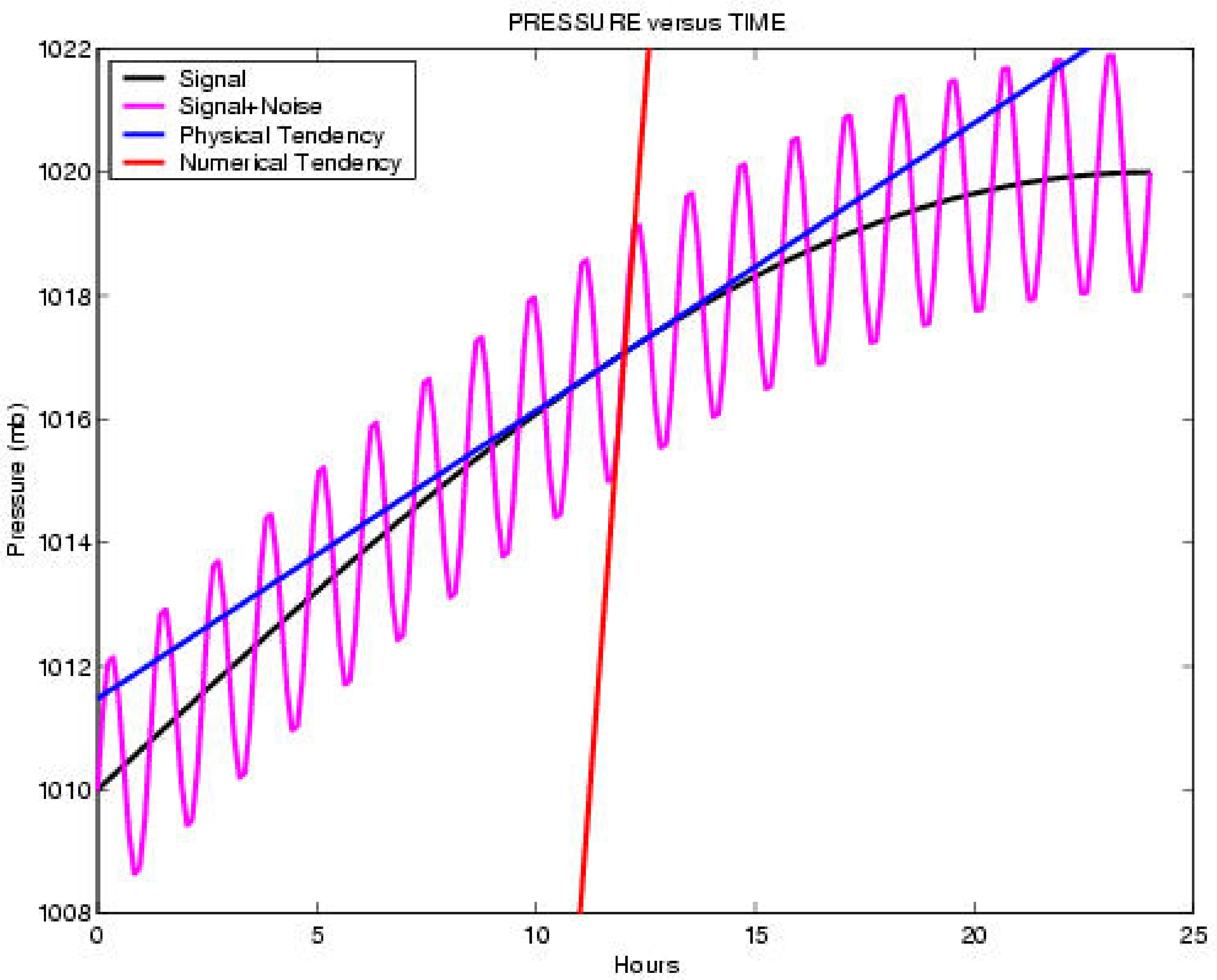
Noisy Evolution of Pressure

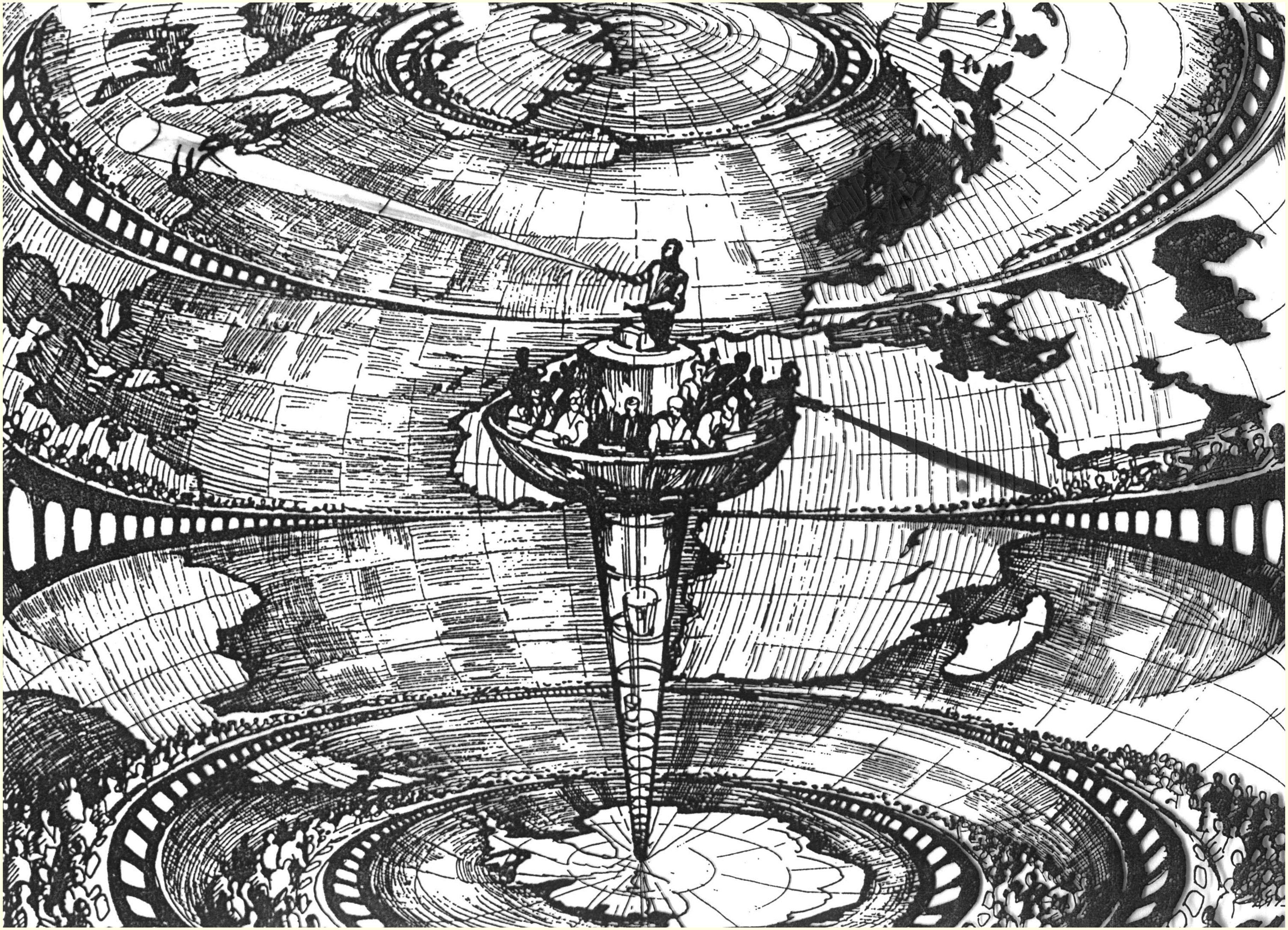


Tendency of a Smooth Signal



Tendency of a Noisy Signal





Richardson's Forecast Factory (A. Lannerback).
Dagens Nyheter, Stockholm. Reproduced from L. Bengtsson, *ECMWF*, 1984

64,000 Computers: The first Massively Parallel Processor

Gravity Waves and NWP

Richardson's calculated pressure tendency is arithmetically correct the resulting pressure change over six hours is preposterous. Why?

- **Richardson extrapolated the *instantaneous* pressure change, assuming it to remain constant over a long time period.**
- **This ignores the propensity of the atmosphere to respond rapidly to changes.**
- **An increase of pressure causes an immediate pressure gradient which acts to resist further change.**
- **The resulting gravity wave oscillations act in such a way as to restore balance.**
- **They result in pressure changes which may be large but which oscillate rapidly in time.**

Margules, 1893, was the first comprehensive study of gravity wave dynamics.

The ineluctable conclusion is that . . .

> ***the instantaneous rate of change is not a reliable indicator of the long-term variation in pressure*.**

- **To obtain an accurate prediction, it is necessary to proceed incrementally: the *time step has to be short enough* to allow the adjustment to take place.**
- **Gravity-wave oscillations may be present, but they need not spoil the forecast: *they may be regarded as noise* super-imposed on the long-term synoptic evolution.**
- **They may also be effectively removed by a minor adjustment of the data, *viz.*, known as *initialization*.**

Modern numerical forecasts are made <u>using the continuity equation</u> in the manner that Margules regarded as impossible, but <u>initialization</u> controls gravity wave noise and a <u>small time step</u> ensures that the calculations remain stable.

Richardson on Smoothing

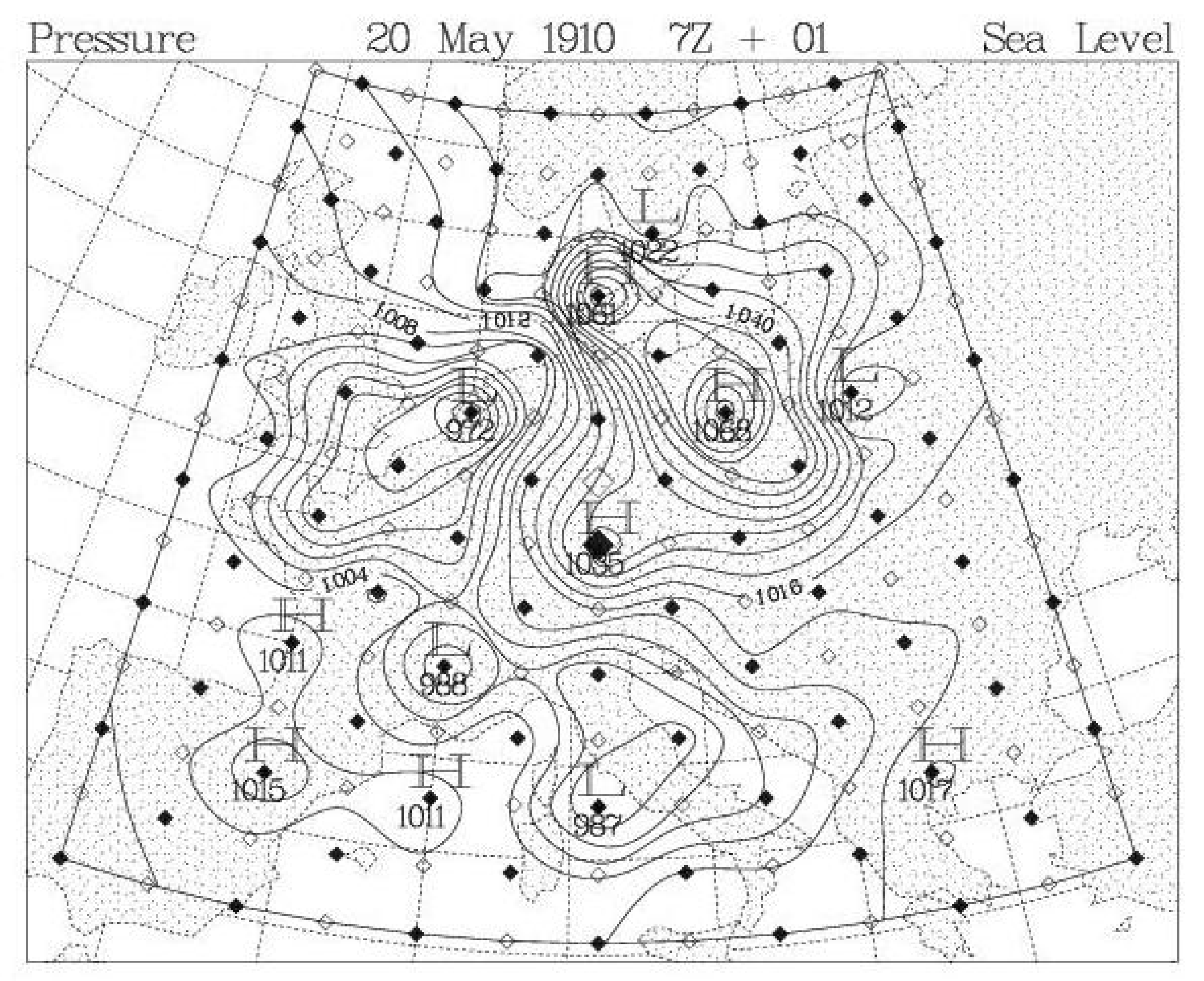
"The scheme of numerical forecasting has been developed so far that it is reasonable to expect that when the smoothing of Ch. 10 has been arranged, it may give forecasts agreeing with the actual smoothed weather."

Richardson devoted a short chapter of his book to the problem of smoothing the initial data for the forecast, in which he outlines five smoothing methods:

- **A. Space Means.**
- **B. Time Means.**
- **C. Potential Function.**
- **D. Stream Function.**
- **E. Smoothing during the Forecast.**

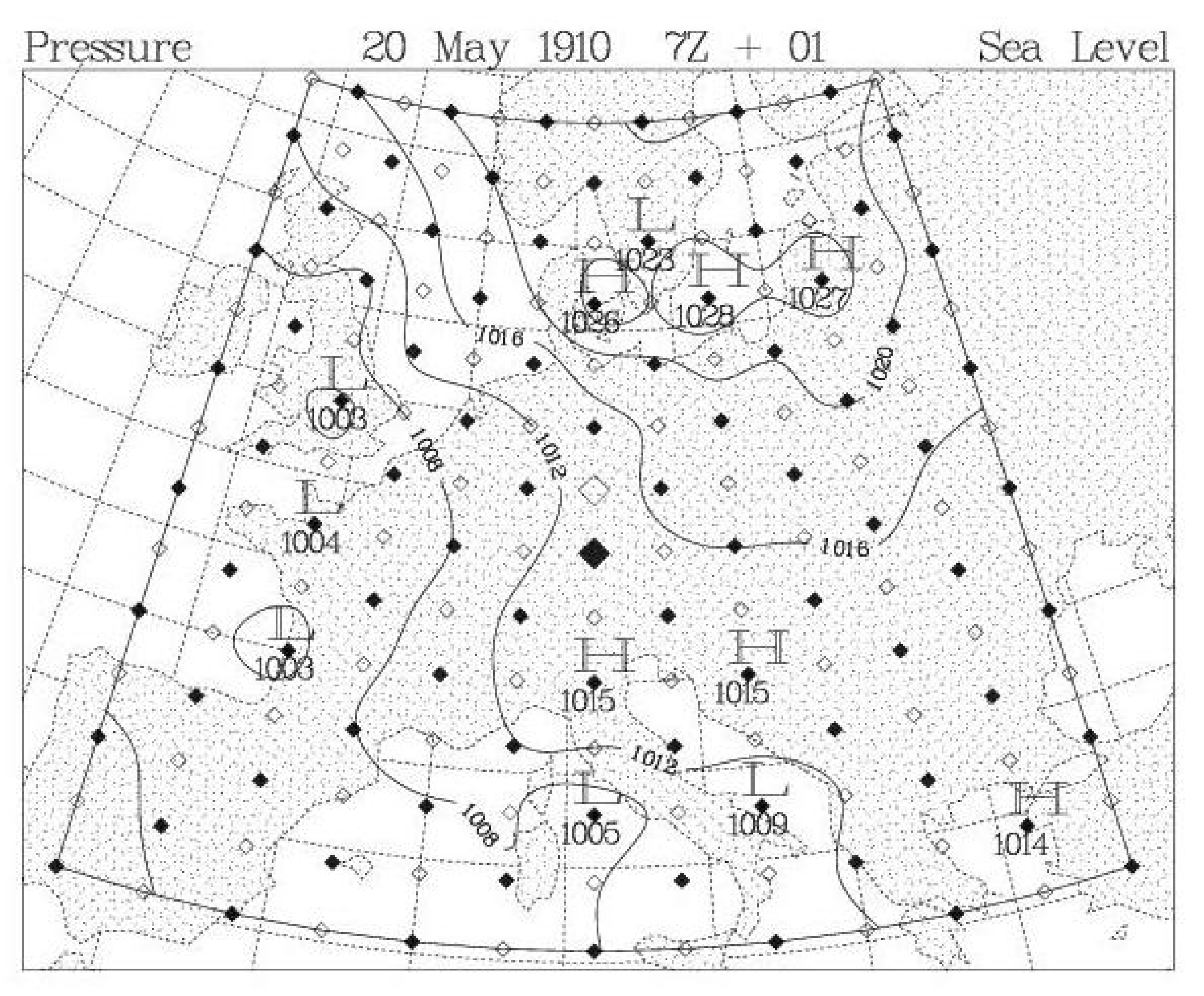
Richardson's Method B is a close cousin of Digital Filtering Initialization, which has some current popularity.

Forecast without Filtering



Short-range forecast of sea-level pressure, from *uninitialized data.* The contour interval is 4 hPa. Single forward time step of size $\Delta t = 3600\,\mathrm{s}$.

Forecast with Filtering



Short-range forecast of sea-level pressure, from *filtered data.* The contour interval is 4 hPa. Single forward time step of size $\Delta t = 3600\,\text{s}$.

Crucial Advances, 1920–1950

- ***Dynamic Meteorology***
  - **Rossby Waves**
  - **Quasi-geostrophic Theory**
  - **Baroclinic Instability**
- ***Numerical Analysis***
  - **CFL Criterion**
- ***Atmopsheric Observations***
  - **Radiosonde**
- ***Electronic Computing***
  - **ENIAC**

Electronic Computer Project, 1946 (under direction of John von Neumann)

Von Neumann's idea:

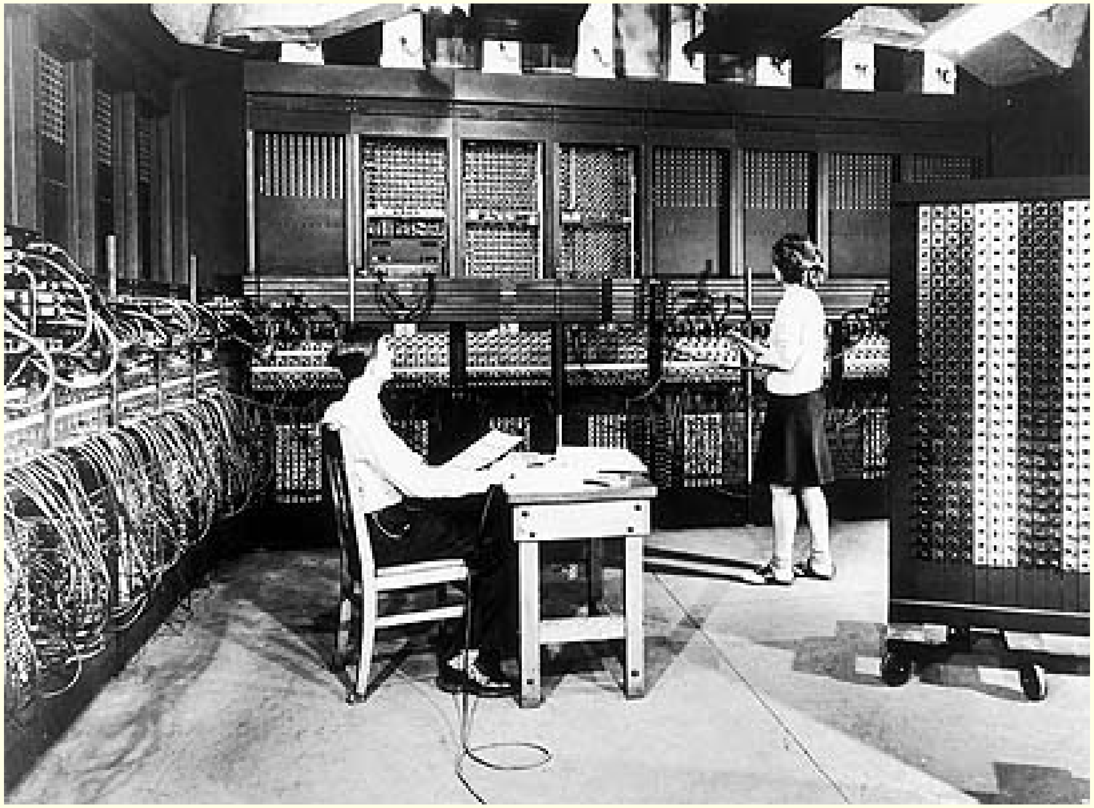
Weather forecasting was, *par excellence*, a scientific problem suitable for solution using a large computer.

The objective of the project was to study the problem of predicting the weather by simulating the dynamics of the atmosphere using a digital electronic computer.

A Proposal for funding listed three "possibilities":

1. **Entirely new methods of weather prediction by calculation will have been made possible;**
2. **A new rational basis will have been secured for the planning of physical measurements and field observations;**
3. **The first step towards influencing the weather by rational human intervention will have been made.**

The ENIAC



The ENIAC was the first multi-purpose programmable electronic digital computer.

It had:

- **18,000 vacuum tubes**
- **70,000 resistors**
- **10,000 capacitors**
- **6,000 switches**
- **Power: 140 kWatts**

Evolution of the Meteorology Project:

- **Plan A: Integrate the Primitive Equations**

  Problems similar to Richardson's would arise

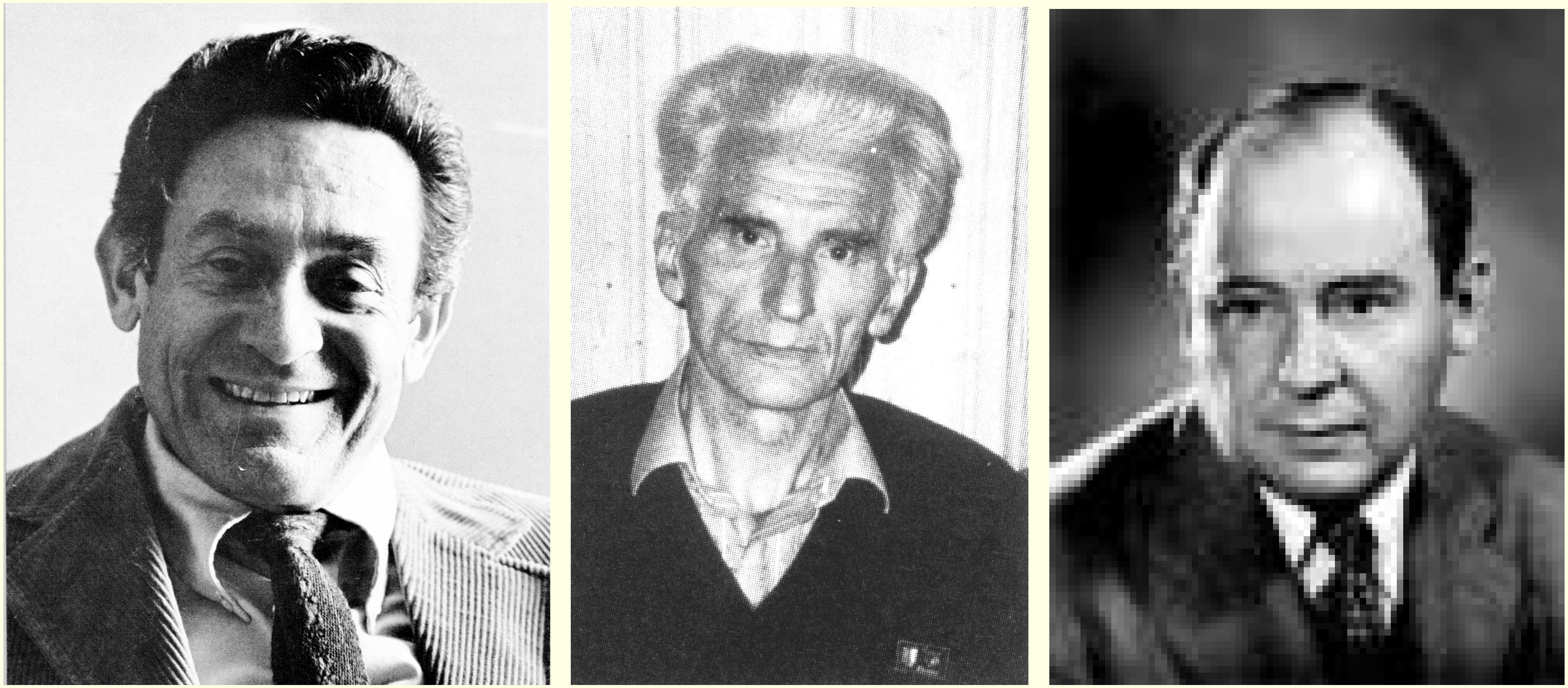
- **Plan B: Integrate baroclinic Q-G System**

  Too computationally demanding

- **Plan C: Solve barotropic vorticity equation**

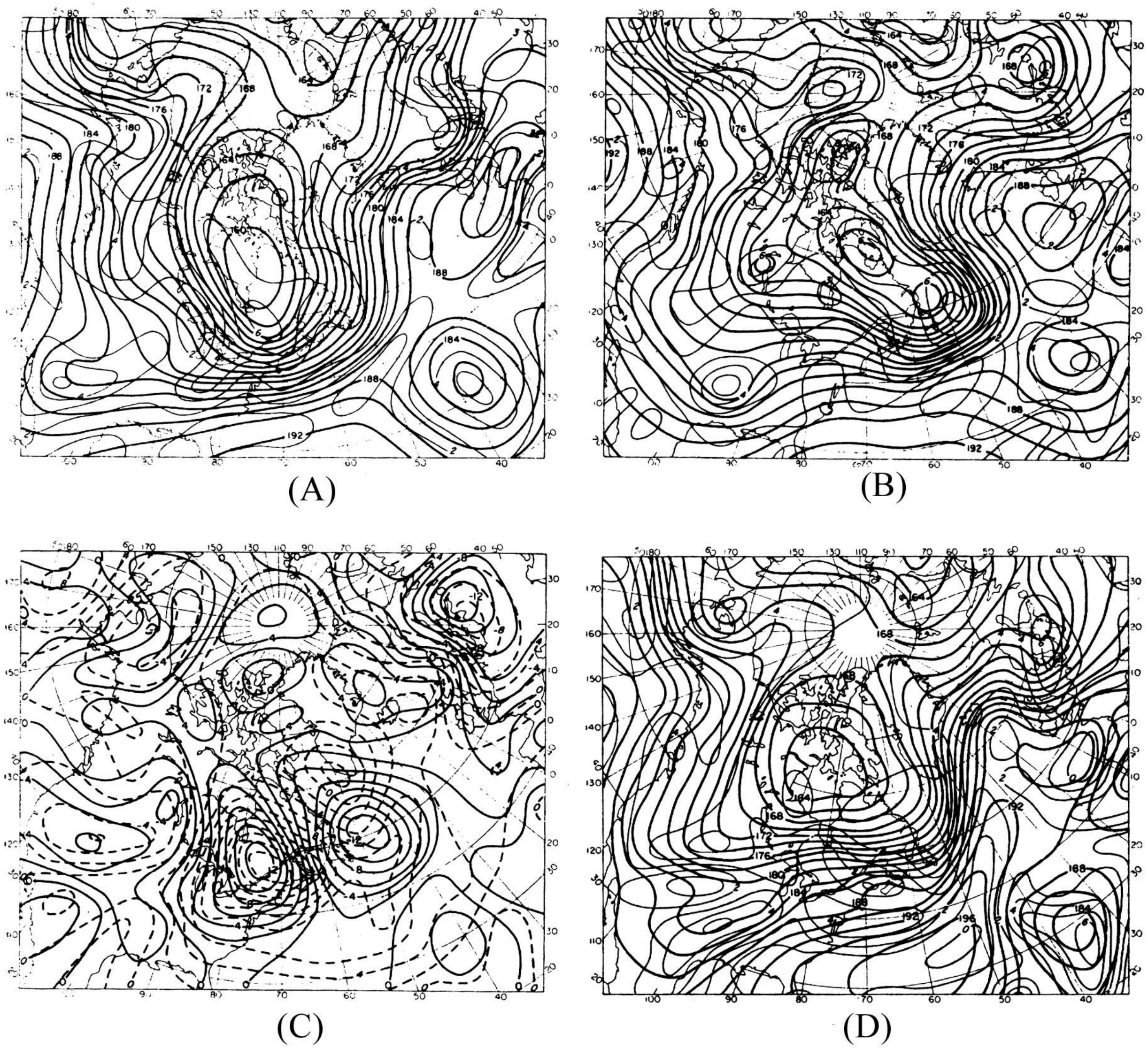
  Very satisfactory initial results

Charney, Fjørtoft, von Neumann



Charney, J.G., R. Fjørtoft and J. von Neumann, 1950: Numerical integration of the barotropic vorticity equation. *Tellus*, 2, 237–254.

ENIAC: First Computer Forecast



(A) (B)

(C) (D)

Richardson’s reaction

- ***“Allow me to congratulate you and your collaborators on the remarkable progress which has been made in Princeton.***
- ***“This is . . . an enormous scientific advance on the single, and quite wrong, result in which Richardson (1922) ended.***

Karl-Heinz Hinkelmann.

At an early stage, Karl-Heinz Hinkelmann proposed that if geostrophic winds were assumed initially, the high frequency noise would remain under control.

Moreover, the process of geostrophic adjustment, elucidated by Carl Gustav Rossby a decade earlier, implied that the flow would soon adjust to harmony with the pressure field.

Hinkelmann, K., 1951: Der Mechanismus des meteorologischen Lärmes. *Tellus*, 3, 285–296.

NWP Operations

The Joint Numerical Weather Prediction Unit was established on July 1, 1954:

- ***Air Weather Service of US Air Force***
- ***The US Weather Bureau***
- ***The Naval Weather Service.***

Operational numerical forecasting began in May, 1955, with a three-level quasi-geostrophic model.

You will hear a great deal more about this during the rest of this Symposium.

Thank you for listening.

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