Selection of modulation and codes for deep space optical communications

Jon Hamkins[a] and Bruce Moision[a]

[a]Jet Propulsion Laboratory, California Institute of Technology
4800 Oak Grove Dr., Pasadena, CA 91109, USA;

ABSTRACT

We describe several properties of deep space optical channels that lead to an appropriate selection of modulation format, pulse position modulation (PPM) order, error control code rate, and coding scheme. The selection process is motivated by capacity considerations. We compare the Shannon limit to the performance of Reed-Solomon codes and convolutional codes concatenated with PPM and show that, when iteratively decoded, concatenated convolutional codes operate approximately 0.5 dB from capacity over a wide range of signal levels, about 2.5 dB better than Reed-Solomon codes.

Keywords: Error-correcting codes, capacity, Reed-Solomon codes, convolutional codes, concatenated codes, pulse position modulation, deep space communications, optical communications

1. INTRODUCTION

The Mars Laser Communication Demonstration (MLCD) carried aboard NASA's Mars Telecommunications Orbiter, planned for launch in 2009, will be the first high-speed deep space optical communications system. While free space optical communications from Geostationary Earth Orbit (GEO) distances is nothing new, MLCD will be operating at up to 2.4 Astronomical Units, which results in a $1/r^2$ loss more than 60 dB worse than a GEO link.

NASA already has an extensive exploration program at this distances. For example, the Mars Exploration Rovers that landed on Mars in January of 2004 can communicate through Radio Frequency (RF) orbital relays at up to 128 kbps. There is an inherent frequency advantage to optical communications, and the narrower beam and smaller components make optical communications a promising technology for NASA's future missions. However, atmospheric turbulence corrupts the wavefront, making ground-based coherent reception difficult if not impossible. Also, the thinnest of cloud wisps can cause outages, even with noncoherent reception.

These challenges are compounded by some other unique properties of the optical channel. The deep space RF link is well-modeled by an additive white Gaussian noise (AWGN) channel, has performance determined by the signal-to-noise ratio (SNR), and sending a zero requires the same transmitter power as sending a one. On the other hand, the deep space optical link is often more appropriately modeled as Poisson, it's performance is generally not a function of SNR alone– generally, adding signal power is about twice as helpful as reducing background light– and sending a one, a light pulse, requires power while sending a zero does not.

What is worse, the Earth-Mars distance is maximal when the sun is between them, making sunlight the biggest problem at the time when the link is already the most challenging. Despite all these challenges, NASA's MLCD program intends to demonstrate a 1 - 30 Mbps optical link from Mars. This will require optimization in every area of the design. In this paper, we explore optimizations that involve the choice of modulation format and error control coding.

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Email: {jon.hamkins, bruce.moision}@jpl.nasa.gov

2. MODULATION FORMATS

In this section we review several types of optical modulation. We restrict our attention to single-frequency, slotted signaling, in which time is partitioned into slots of equal duration T_s during which the the laser is either on or off.

2.1. On-Off Keying (OOK)

The most general modulation of this type is On-Off Keying (OOK), which maps a sequence of input bits to transmitted laser pulses by associating each input '1' with a laser pulse in a slot, and each '0' with the absence of a laser pulse in a slot. This is illustrated in Fig. 1. For comparison with other modulation schemes which use more than one slot per bit, every other symbol is shaded gray.

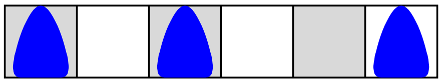

Figure 1. OOK modulation of message 101001 is shown.

OOK uses 1 bit per slot, and has a fixed peak-to-average power ratio of two. All the other orthogonal modulations considered in this paper are constrained versions of OOK, in the sense that the other modulations can be viewed as a precoding operation followed by ordinary OOK. Since those precoding operations impose constraints not present on ordinary OOK, it follows that for a fixed slot width, OOK achieves the highest possible throughput in bits/slot among all the orthogonal modulations considered here.

However, assuming equiprobable '0's and '1's at the input to the modulator, OOK will have low photon efficiency, which tops out at two bits per photon.

2.2. Pulse Position Modulation

In Pulse Position Modulation (PPM), a k-bit source $\mathbf{U} = (U_1, \ldots, U_k) \in \{0,1\}^k$ is modulated with M-ary PPM, $M = 2^k$, to yield a signal $\mathbf{X} = (0, \ldots, 0, 1, 0, \ldots, 0) \in \{0,1\}^M$, which contains a single one in the position indicated by the binary representation of $\mathbf{U}$. The transmission channel is a binary-input unconstrained-output memoryless channel ($X_i = 0$ or 1 in Fig. 2). One use of the overall PPM-symbol channel consists of M serial uses of the binary-input channel, and produces the received vector $\mathbf{Y} = (Y_1, \ldots Y_M) \in \mathbb{R}^M$. An example of a PPM mapping for 101001 is illustrated in Fig. 1. Again, every other symbol is shaded gray.

Since PPM uses one pulse per M slots, it has a duty cycle of $1/M$ and a peak-to-average power ratio of M. It uses $\log_2(M)/M$ bits per slot, which for varying M results in a flexible trade-off between photon efficiency and bandwidth efficiency.

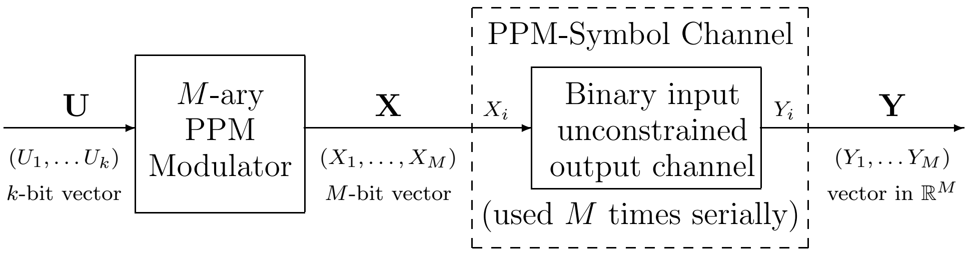

Figure 2. PPM signaling.

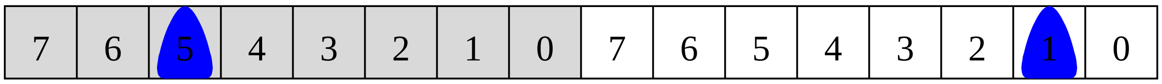

Figure 3. PPM modulation of message 101001 is shown.

2.3. Multipulse PPM (MPPM)

In Multipulse PPM (MPPM), a group of $\left\lfloor \log_2 \left[\binom{M}{n} \right] \right\rfloor$ bits is mapped to n pulses in M slot positions. This is a generalization of conventional PPM, in which $n = 1$. The encoding of 101001 with $n = 2$, $M = 4$ (2-pulse 4-PPM) is shown in Fig. 4, where each group of three bits determines the location of the two pulses in a symbol. MPPM uses $\frac{\left\lfloor \log_2 \left[\binom{M}{n} \right] \right\rfloor}{M}$ bits per slot, which results in a peak to average power ratio of M/n, and as in PPM,

Figure 4. Multipulse PPM modulation of message 101001 is shown.

allows for a flexible trade-off between photon efficiency bandwidth efficiency.

2.4. Overlapping PPM (OPPM)

In Overlapping PPM (OPPM), a group of bits is mapped to one pulse in $N(M-1)+1$ positions. This is the only modulation we consider that is not orthogonal. The encoding of 101001 with $N = 3$ (overlap index), $M = 2$ is shown in Fig. 5. OPPM uses $\frac{\log(N(M-1)+1)}{N(M-1)+1}$ bits per slot, which results in a peak to average power ratio of

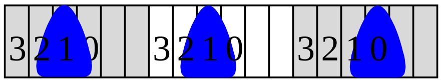

Figure 5. Overlapping PPM modulation of message 101001 is shown.

M/N, and as in PPM, allows for a flexible trade-off between photon efficiency bandwidth efficiency.

2.5. Differential PPM (DPPM)

Differential PPM (DPPM), also called truncated PPM, is similar to conventional PPM, except that the symbol is flushed after a pulse. That is, after the slot containing the pulse, the remaining slots of the PPM symbol are not transmitted, and instead a new symbol is begun. In this way, information is conveyed by the time between pulses. The encoding of 101001 with $M = 8$ is shown in Fig. 5.

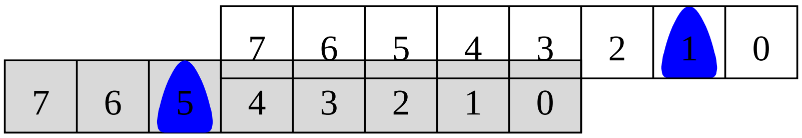

Figure 6. Differential PPM modulation of message 101001 is shown.

DPPM has moderate to high peak-to-average power ratio of $M/2$. Its variable symbol length averages 1/2 that of PPM, resulting in twice the throughput as PPM. However, it also has twice the average power as M-PPM, and handling a variable symbol length at the receiver may be more difficult.

3. UNCODED PERFORMANCE OF PPM

3.1. For Continuous-Valued Detector Output

At the receiver, the detected photons in each slot are counted. If hard decisions are used, the slot with the greatest photon-count is declared to be the signal-slot. This has been shown to be the ML decision for a variety of channel models.[1] On a continuous-output channel, the PPM symbol error probability SER is the well-known performance[2] of an ML detector for M-ary orthogonal signaling

$$\begin{aligned}\mathrm{SER} &= 1 - \Pr(Y_1 = \max\{Y_1, \dots, Y_M\} | X_1 = 1) \\ &= 1 - \int_{-\infty}^{\infty} f_{Y|X}(y|1) \left[\int_{-\infty}^{y} f_{Y|X}(y'|0) dy' \right]^{M-1} dy,\end{aligned}$$

Observations in the $M-1$ nonsignal slots are assumed to be independent. This may be evaluated numerically by first producing a table lookup for the fenced term, and then computing the outer integral numerically in the usual way.

3.2. For Discrete-Valued Detector Output

When the detector outputs take on discrete values, there is a possibility of a tie for the maximum count. Suppose k photons are detected in the slot containing the pulse, l nonsignal slots also have count k, and the remaining nonsignal slots have count strictly less than k. Then the correct decision is made with probability $1/(l+1)$. Otherwise, an error is made. By summing over all possible values of k and l, it follows that

$$SER = 1 - \sum_{k=0}^{\infty} \sum_{l=0}^{M-1} \Pr \begin{bmatrix} \text{correct decision when } l \\ \text{nonsignal slots tie the sig-} \\ \text{nal slot for the maximum} \\ \text{count} \end{bmatrix} \times \Pr \begin{bmatrix} \text{exactly } l \text{ of } M-1 \\ \text{nonsignal slots have} \\ \text{value } k\text{, all others} \\ \text{smaller} \end{bmatrix} \times \Pr \begin{bmatrix} \text{signal slot} \\ \text{has value } k \end{bmatrix} \quad (1)$$

$$= 1 - \sum_{k=0}^{\infty} \sum_{l=0}^{M-1} \frac{1}{l+1} \times \binom{M-1}{l} f(k|0)^l F(k-1|0)^{M-l-1} \times f(k|1) \quad (2)$$

where $f(k|1)$ and $f(k|0)$ denote the conditional probabilities that a received slot $Y_i = k$ when a '1' (pulse) or '0' (no pulse), respectively, is transmitted in the slot, and $F(k|1)$ ($F(k|0)$) denote the cumulative distributions. After some algebraic manipulation, this can be rewritten in a single summation as[3]

$$SER = 1 - \sum_{k=0}^{\infty} \frac{f(k|1)}{f(k|0)M} \left(F(k|0)^M - F(k-1|0)^M \right). \quad (3)$$

An extension of (2) to n-pulse PPM, $n \geq 2$, is straightforward, and involves a triple summation in place of the double summation.

4. CHANNEL CAPACITY

PPM, MPPM, and DPPM are each a constrained version of OOK, i.e., they are slotted signaling schemes with on and off slots, but with extra constraints (e.g., exactly 1 pulse per M slots, for PPM) that define the modulation. Hence the capacity of unconstrained OOK will be an upper bound on the capacity of these other modulations.

If we apply only a duty-cycle constraint on OOK, we can compute the capacity of the resulting memoryless channel as

$$C_{\mathrm{OOK}} = \frac{1}{M} E_Y \log \frac{f_{Y|X}(Y|0)}{f_Y(Y)} + \frac{1}{1-1/M} E_Y \log \frac{f_{Y|X}(Y|1)}{f_Y(Y)}. \quad (4)$$

where

$$f_Y(y) = \frac{1}{M} f_{Y|X}(y|0) + \frac{M-1}{M} f_{Y|X}(y|1)$$

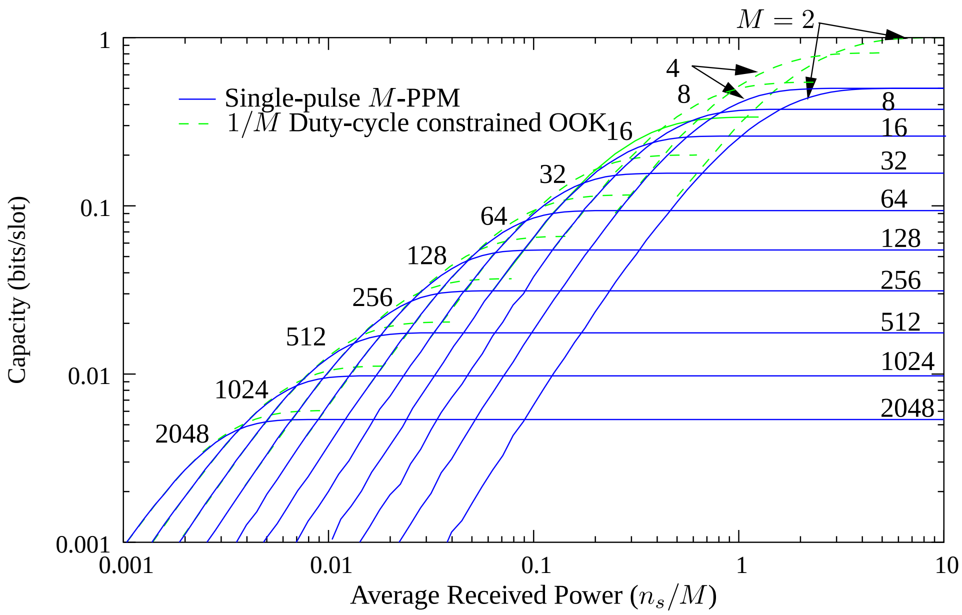

Figure 7. Capacity of duty-cycle constrained OOK and PPM on a Poisson channel with background $n_b = 1$.

is the probability mass function for a randomly chosen slot. This is shown for a Poisson channel in the dashed green line of Fig. 7. On the other hand, the capacity of PPM is[4]:

$$C_{\text{PPM}} = E_{\mathbf{Y}} \log_2 \left[\frac{M L(Y_1)}{\sum_{j=1}^{M} L(Y_j)} \right] \text{ bits per PPM symbol,} \tag{5}$$

an expectation over $\mathbf{Y}$, where $L(y) = f_{Y|X}(y|1)/f_{Y|X}(y|0)$ is the channel likelihood ratio, the Y_j have density $f_{Y|X}(y|1)$ for $j = 1$ and density $f_{Y|X}(y|0)$ otherwise.

As can be seen in Fig. 7, for low average signal power, the capacity of PPM is near that of OOK, and thus, no other constrained form of OOK (MPPM, DPPM, etc.) can offer substantial improvement over PPM. The optimal PPM order is high, since a higher order creates the higher peak power needed to overcome the weak average power

For high average signal power, modulations more general than PPM can offer up to twice the data rate, as can be seen by the factor of two difference in the horizontal asymptotes of OOK and PPM in Fig. 7. The optimal PPM order is low, since high peak power not needed and low orders offer higher throughput per slot.

Based on the capacity, PPM should be preferred in the region of average power less than approximately $n_s/M \leq 1$. For higher powers, MPPM or another signaling scheme could offer gains.

5. OPTIMIZING CODE PARAMETERS

If PPM is the chosen modulation format, we may optimize the PPM order, data rate, and code rate as follows:

1. Identify target n_s/M and background rate n_b.
2. Choose optimal PPM size M^* from upper shell of capacity curves.
3. Identify achievable data rate R_d from capacity curve.
4. Identify code rate: $R_c = R_d M^* T_s / \log_2 M^*$

On the Poisson channel, this results in the PPM orders shown in Fig. 8, and the outer code rates shown in Fig. 9.

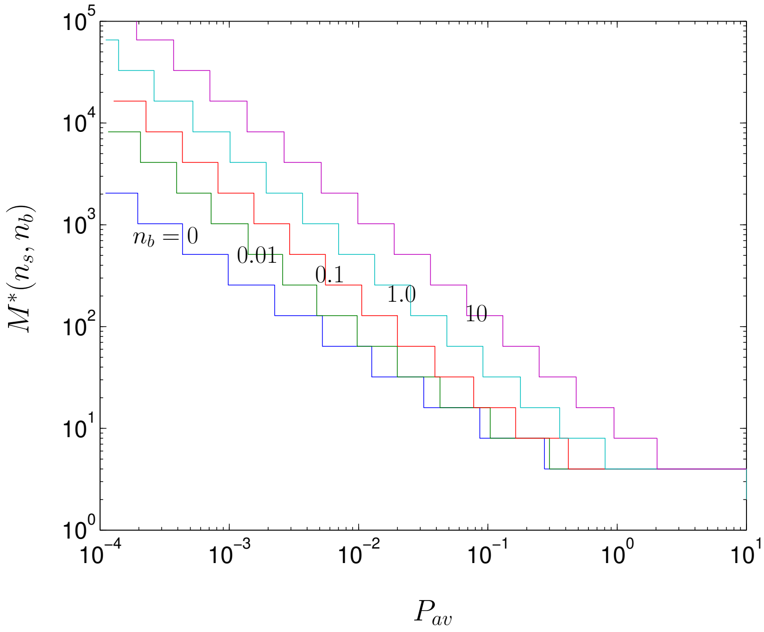

Figure 8. Optimal PPM orders on a Poisson channel.

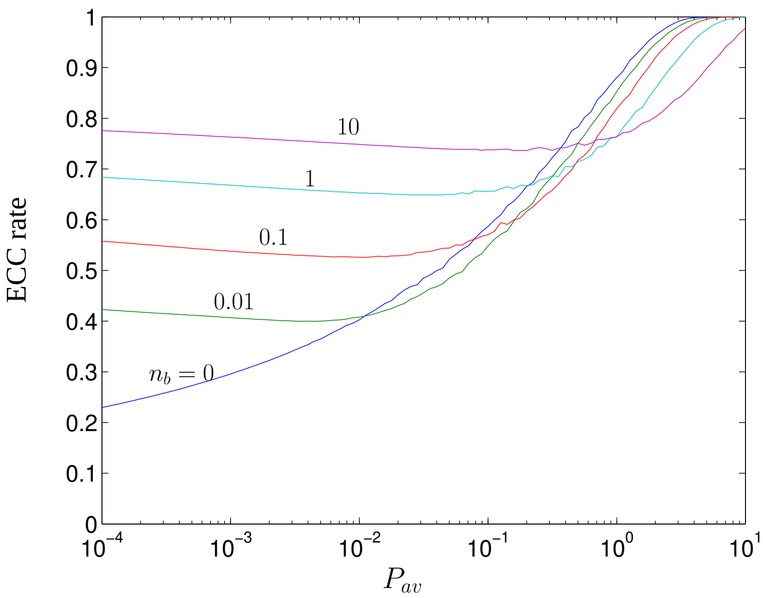

Figure 9. Optimal outer code rates.

6. CODES

It remains to design an outer code with rate R_c for use with M^*-PPM that achieves near-capacity performance. Here we compare two coding methods, one old and one new.

6.1. Reed-Solomon codes

Reed-Solomon (RS) codes have long been proposed for use with PPM, because a PPM symbol can naturally be matched to a $\mathrm{RS}(n,k)$ code symbol in $\mathrm{GF}(n+1)$. The convention is to take $n = M-1$, and define an RS code over $\mathrm{GF}(n+1)$, but performance can be poor when when M is small. Since small M is optimal for moderate values of average power, an alternative approach is needed, such as grouping two or more PPM symbols per RS code symbol to obtain a RS code with a longer blocklength.

Unfortunately, even the trick of grouping PPM symbols does not result in performance closer than 2-3 dB from capacity. This is illustrated in Fig. 10. Part of this gap is due to the fact that conventional decoding of RS codes is performed with hard decisions. Recent work on soft decision RS decoding may provide some improvement.

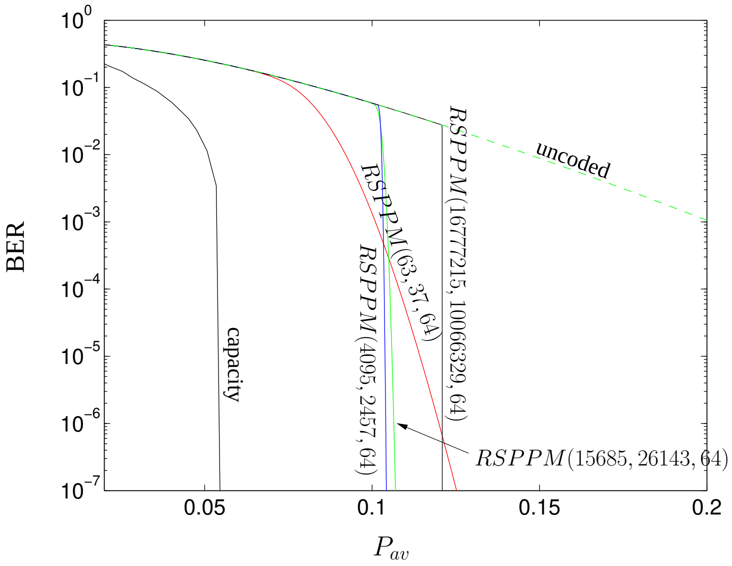

Figure 10. Performance of RS codes with PPM.

6.2. Serially Concatenated PPM (SCPPM)

As an alternative to RSPPM, we may serially concatenate a convolutional code with the inner modulation, via an interleaver. This is shown in Fig. 11. We place an accumulator prior to the modulator, which results in improved performance. A comparison of RSPPM and SCPPM codes are shown in Fig. 12. The upper envelope of capacity from Fig. 7 is included for reference. SCPPM is low complexity, suitable for high speed (100+ Mbps)

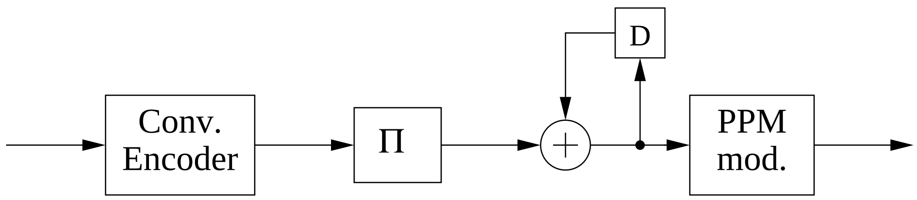

Figure 11. Encoder for SCPPM.

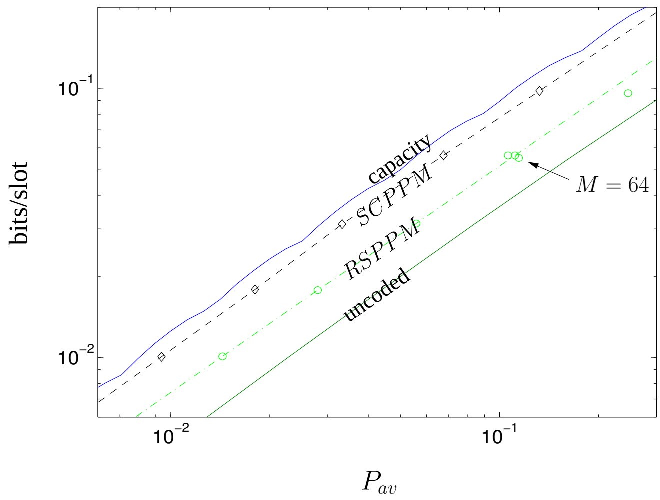

Figure 12. Comparison of capacity, SCPPM, RSPPM, and uncoded PPM.

decoding. It also outperforms RSPPM by about 2.5 dB at $n_b = 1$, and has a gap of about 0.5 dB over the range of operating points.

REFERENCES

1. V. Vilnrotter, M. Simon, and M. Srinivasan, "Maximum likelihood detection of PPM signals governed by an arbitrary point process plus additive Gaussian noise," JPL Publication 98-7, JPL, April 1998.
2. R. Gallager, *Information Theory and Reliable Communication*, Wiley, New York, NY, 1968.
3. J. Hamkins, "Accurate computation of the performance of M-ary orthogonal signaling on a discrete memoryless channel," *IEEE Trans. Commun.* , 2003, submitted.
4. B. Moision and J. Hamkins, "Deep-space optical communications downlink budget: Modulation and coding," in *IPN Progress Report*, **42-154**, Aug. 2003.