

The $(g - 2)_\mu$ Saga and Supersymmetry

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Edinburgh, 10/2004

based on collaboration with
D. Stöckinger and G. Weiglein

1. Introduction
2. The saga of the anomalous magnetic moment of the Muon
3. $(g - 2)_\mu$ and Supersymmetry
4. SUSY two-loop results
5. Conclusions

1. Introduction

Q: Which Lagrangian describes the world?

Q': What describes the world better: SM or MSSM/SUSY ?

A: Two possible ways:

- Search for new SUSY particles

new SUSY particles found



SM ruled out

Problem:

SUSY particles are too heavy for today's colliders, only lower limits of $\mathcal{O}(100 \text{ GeV})$.

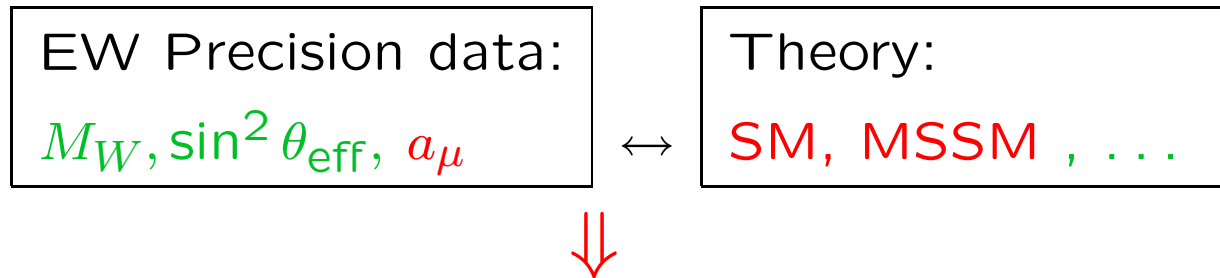
→ waiting for Tevatron (2005...?)

→ waiting for LHC (2007...?)

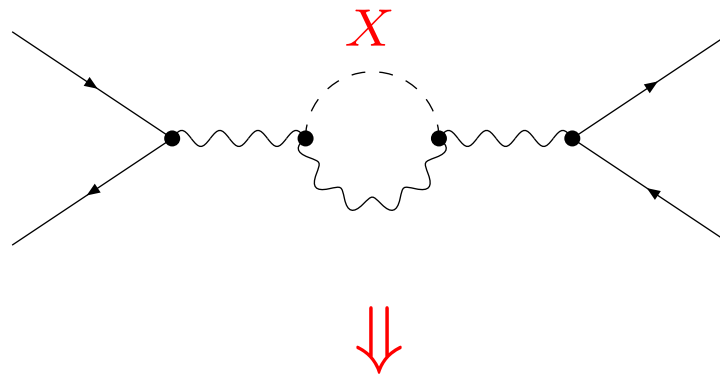
- Search for indirect effects of SUSY
via Precision Observables

Precision Observables (POs):

Comparison of electro-weak precision observables with theory:



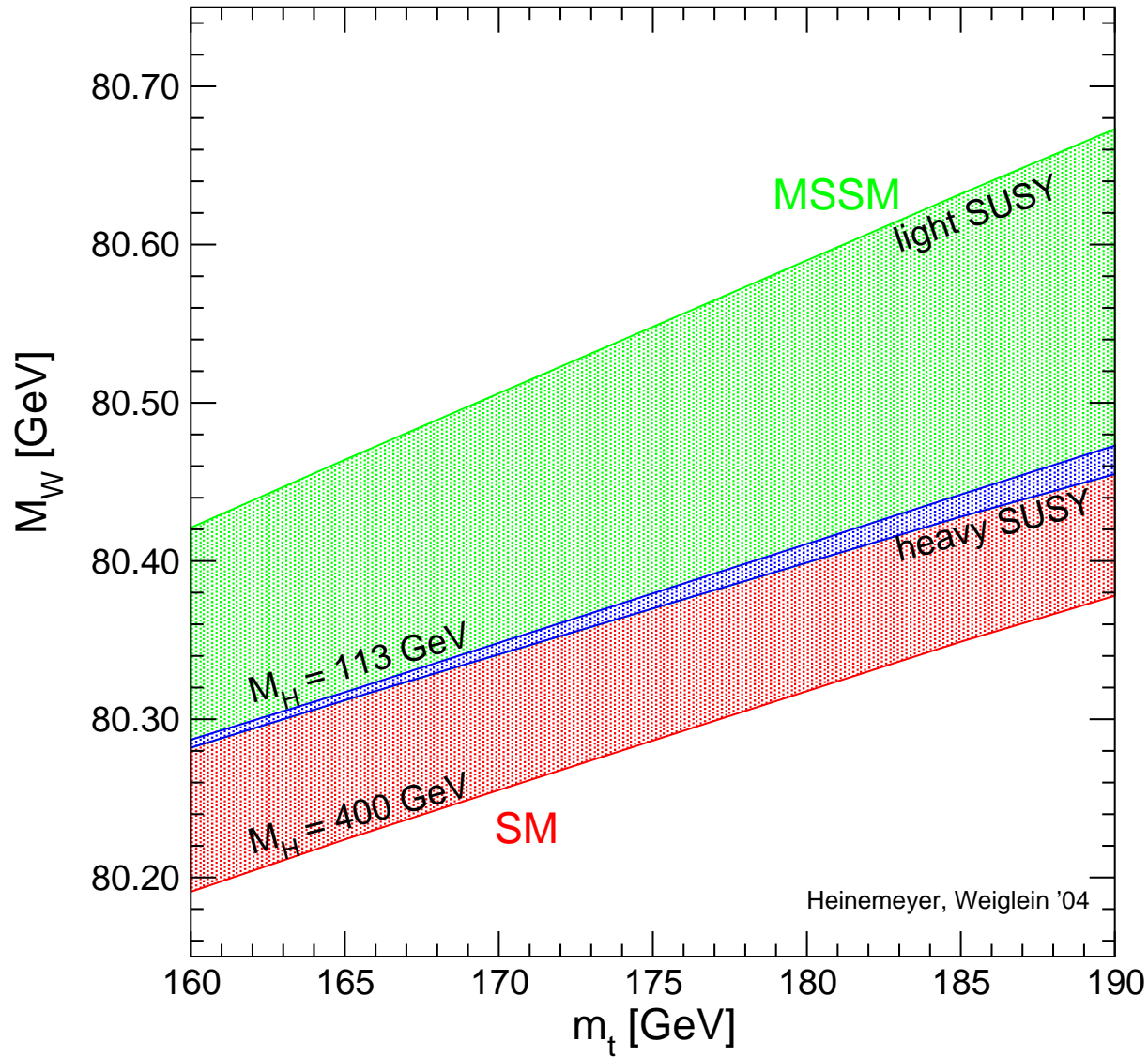
Test of theory at quantum level: Sensitivity to loop corrections



Very high accuracy of measurements and theoretical predictions needed

- Which model fits better?
- Does the prediction of a model contradict the experimental data?

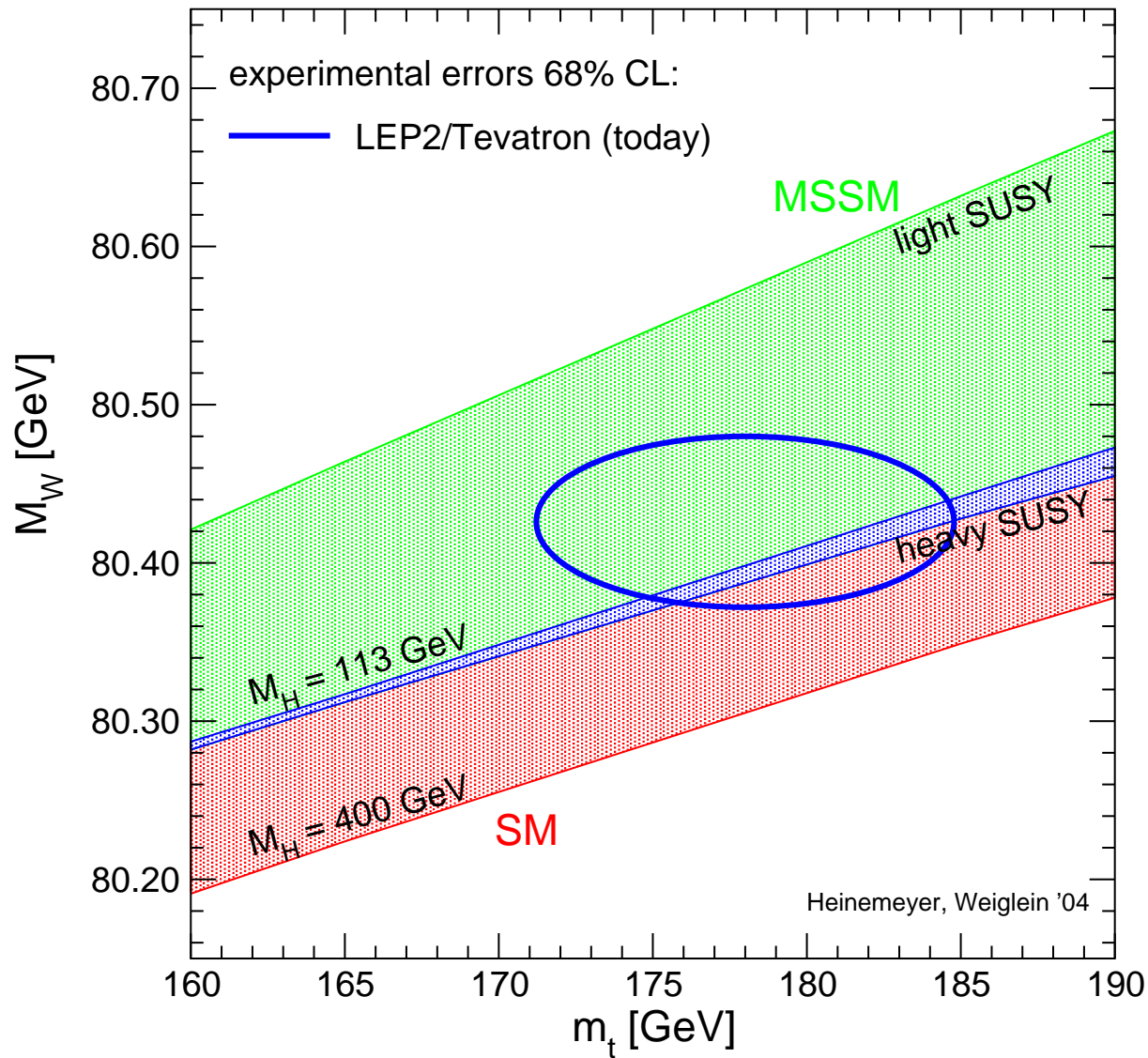
Example: Prediction for M_W in the SM and the MSSM :



MSSM uncertainty:
unknown masses
of SUSY particles

SM uncertainty:
unknown Higgs mass

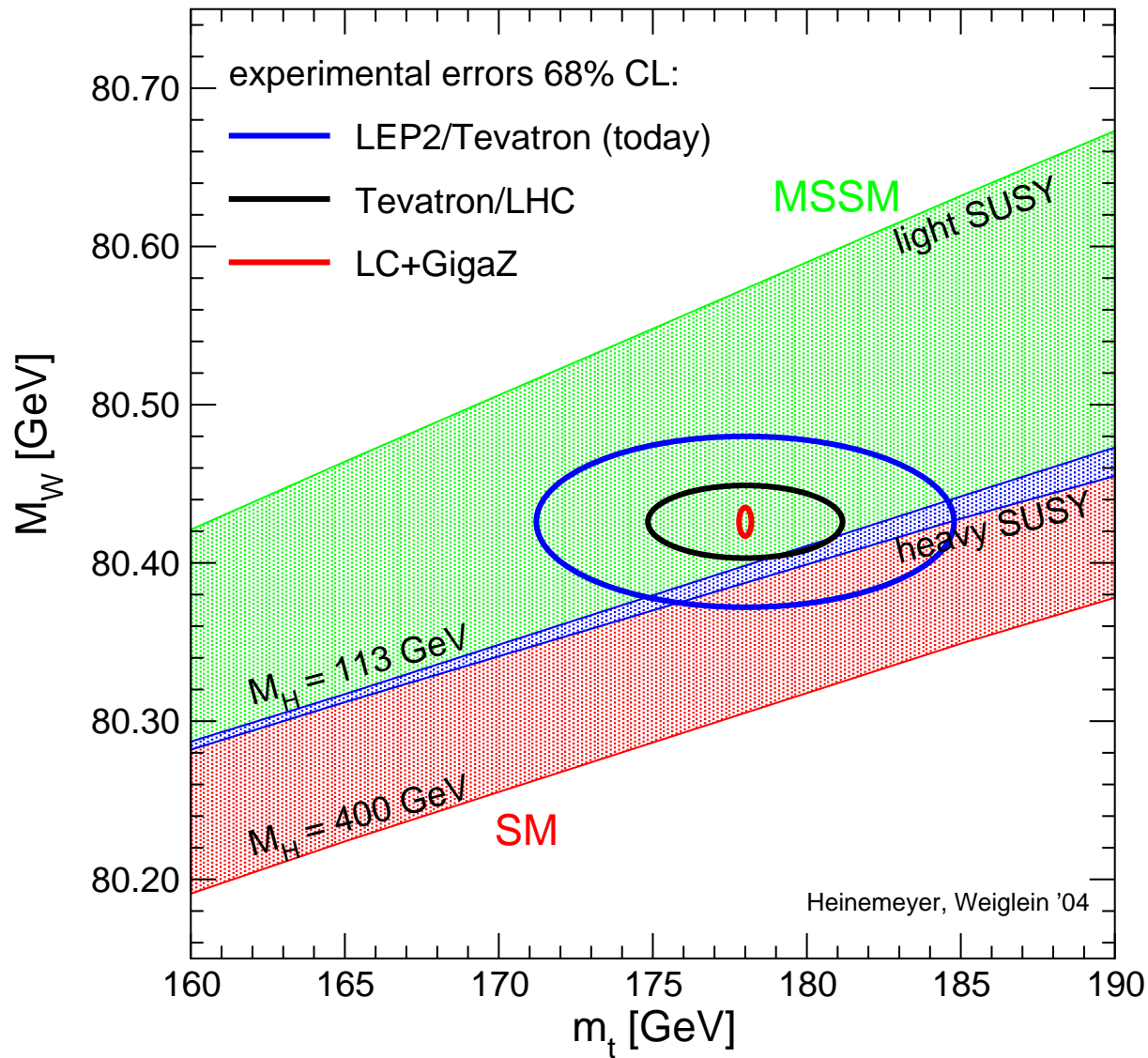
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The Minimal Supersymmetric Standard Model (MSSM)

Superpartners for Standard Model particles

$$\begin{array}{llll} [u, d, c, s, t, b]_{L,R} & [e, \mu, \tau]_{L,R} & [\nu_{e,\mu,\tau}]_L & \text{Spin } \frac{1}{2} \\ [\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}]_{L,R} & [\tilde{e}, \tilde{\mu}, \tilde{\tau}]_{L,R} & [\tilde{\nu}_{e,\mu,\tau}]_L & \text{Spin } 0 \\ g & \underbrace{W^\pm, H^\pm}_{\text{Spin } 1} & \underbrace{\gamma, Z, H_1^0, H_2^0}_{\text{Spin } 0} & \text{Spin } 1 / \text{Spin } 0 \\ \tilde{g} & \tilde{\chi}_{1,2}^\pm & \tilde{\chi}_{1,2,3,4}^0 & \text{Spin } \frac{1}{2} \end{array}$$

Enlarged Higgs sector: Two Higgs doublets

Problem in the MSSM: many scales

Enlarged Higgs sector: Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$
$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$
$$+ \underbrace{\frac{g'^2 + g^2}{8}}_{\text{gauge couplings, in contrast to SM}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}}_{\text{gauge couplings, in contrast to SM}} |H_1 \bar{H}_2|^2$$

physical states: h^0, H^0, A^0, H^\pm

Goldstone bosons: G^0, G^\pm

Input parameters:

$$\tan \beta = \frac{v_2}{v_1}, \quad M_A^2 = -m_{12}^2 (\tan \beta + \cot \beta)$$

⇒ Prediction for m_h :

MSSM tree-level bound: $m_h < M_Z$, excluded by LEP Higgs searches

Large radiative corrections:

Dominant one-loop corrections: $\sim G_\mu m_t^4 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$

Measurement of m_h , Higgs couplings ⇒ test of the theory

LHC: $\Delta m_h \approx 0.2$ GeV, LC: $\Delta m_h \approx 0.05$ GeV

⇒ m_h will be (the best?) electroweak precision observable

Present status of m_h prediction in the MSSM:

Feynman-diagrammatic result:

complete one-loop, leading + subleading two-loop

FeynHiggs2.1 (www.feynhiggs.de)

[S. H., W. Hollik, G. Weiglein '98, '00, '02]

[T. Hahn, S. H., W. Hollik, G. Weiglein '04]

⇒ used for later Higgs evaluation

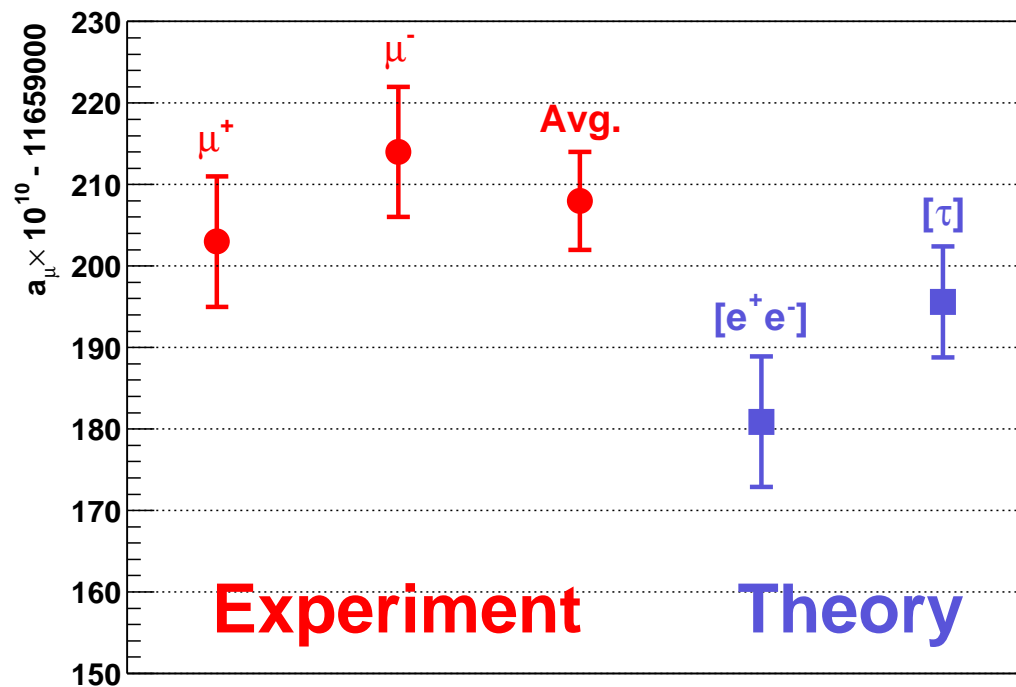
2. The Saga of the anomalous magnetic moment of the muon

$$a_\mu \equiv (g - 2)_\mu / 2$$

Overview about the current **experimental** and **SM (theory)** result:

[*g-2 Collaboration, hep-ex/0401008*]

→ T

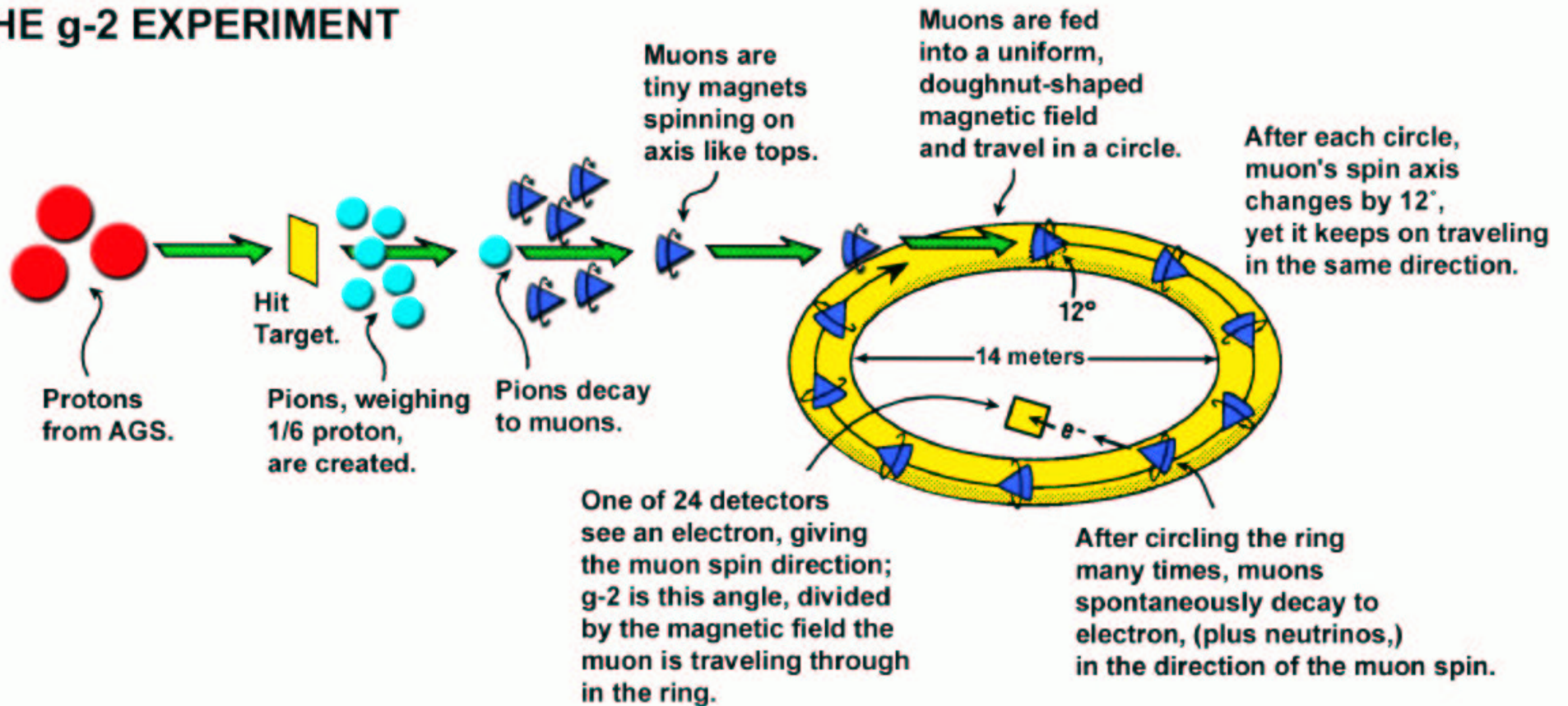


$$a_\mu^{\text{exp}} - a_\mu^{\text{theo,SM}} \approx (25.2 \pm 9.2) \times 10^{-10}$$

long history ...

The $(g - 2)_\mu$ experiment:

LIFE OF A MUON: THE g-2 EXPERIMENT

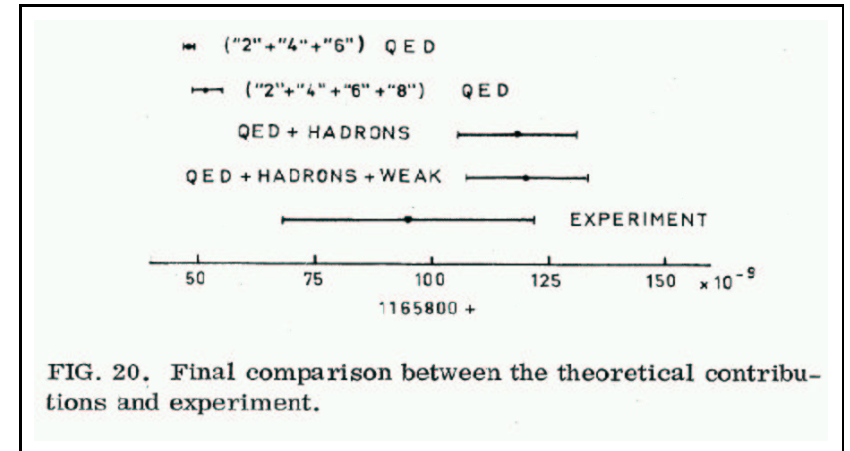


Coupling of muon to magnetic field : $\mu - \mu - \gamma$ coupling

$$\bar{u}(p') \left[\gamma^\mu F_1(q^2) + \frac{i}{2m_\mu} \sigma^{\mu\nu} q_\nu F_2(q^2) \right] u(p) A_\mu \quad F_2(0) = (g - 2)_\mu$$

The early years:

'49 Schwinger: QED 1L	$\frac{\alpha}{2\pi}$
'57 QED 2L terms	-1.77×10^{-6}
'68: CERN measurement	$\pm 2.7 \times 10^{-7}$
'72 QED 2L $\log \frac{m_\mu}{m_e}$	$+5.9 \times 10^{-6}$
'75: CERN measurement	$\pm 2.7 \times 10^{-8}$
'76 QED 3L numerically	$+3 \times 10^{-7}$
'76 QED 4L leading	$(4 \pm 2) \times 10^{-9}$
'76 hadronic contr.	$(6.7 \pm .9) \times 10^{-8}$
'78: CERN measurement	$\pm 1.0 \times 10^{-7}$



[Calmet, Narison, Perrottet, de Rafael '78]

Towards the first discrepancy:

QED calculation down to $\pm 0.1 \times 10^{-10}$

'75 – '96 3L analytically [*Remiddi*]

'83 – today 4L, 5L numerically [*Kinoshita*]

Electroweak 2L down to $\pm 0.5 \times 10^{-10}$

'96 [*Czarnecki, Krause, Marciano*]

'04 [*S.H., Stöckinger, Weiglein*]

Hadronic light-by-light contributions

'85 [*Kinoshita et al.*] $(+4.9 \pm 0.5) \times 10^{-10}$

'95 [*Kinoshita et al.*] $(-5.2 \pm 1.8) \times 10^{-10}$

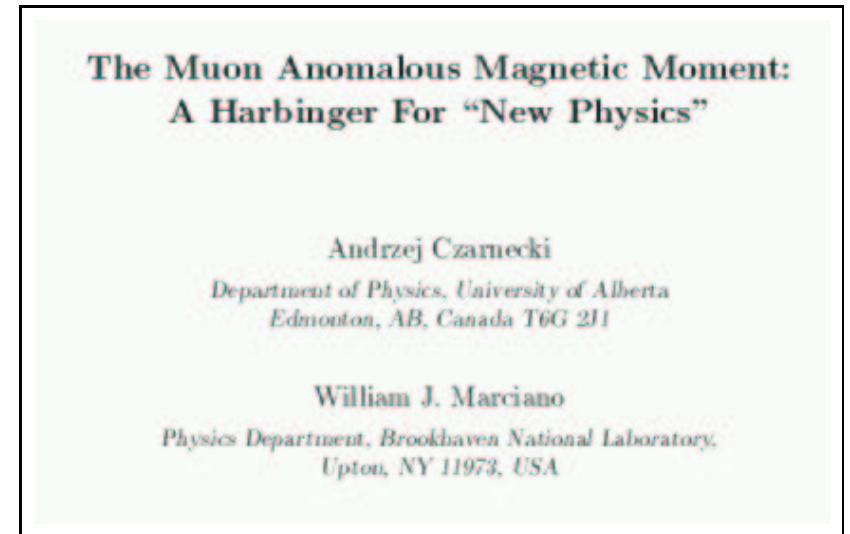
'96 [*Bijnens et al.*] $(-9.2 \pm 3.2) \times 10^{-10}$

'98 [*Kinoshita et al.*] $(-7.9 \pm 1.5) \times 10^{-10}$

'01: BNL measurement $\pm 10 \times 10^{-10}$

$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{theo}} = (43 \pm 16) \times 10^{-10}$$

⇒ A Harbinger For “New Physics”?



[*A. Czarnecki, W. Marciano '01*]

The full light-by-light history:

'85 [<i>Kinoshita et al.</i>]	$(+4.9 \pm 0.5) \times 10^{-10}$
'95 [<i>Kinoshita et al.</i>]	$(-5.2 \pm 1.8) \times 10^{-10}$
'96 [<i>Bijnens et al.</i>]	$(-9.2 \pm 3.2) \times 10^{-10}$
'98 [<i>Kinoshita et al.</i>]	$(-7.9 \pm 1.5) \times 10^{-10}$
'01: BNL measurement	2.6 σ discrepancy
'02 [<i>Knecht, Nyffeler</i>]	$(+8 \pm 4) \times 10^{-10}$
'02 [<i>Kinoshita et al.</i>]	$(+8 \pm 3) \times 10^{-10}$
'02 [<i>Bijens et al.</i>]	$(+8 \pm 3) \times 10^{-10}$
'02 [<i>other groups</i>]	$(+8 \pm 3) \times 10^{-10}$
'03 [<i>Melnikov, Vainshtein</i>]	$(13 \pm 2.5) \times 10^{-10}$
	(TBC!)

seems to get settled, **improvement still necessary**

The hadronic vacuum polarization history:

$$a_{\mu}^{\text{had}} \times 10^{10} - 11659000$$

720 \Rightarrow SM ok

'95 – '01 [*many groups*] ~ 700 \pm 12

'02 new CMD-2 $\pi^+\pi^-$ data ~ -15

'03 discrepancy between e^+e^- and τ data

[Davier et al.]

'03 error in luminosity determination ~ +10
at CMD-2 discovered

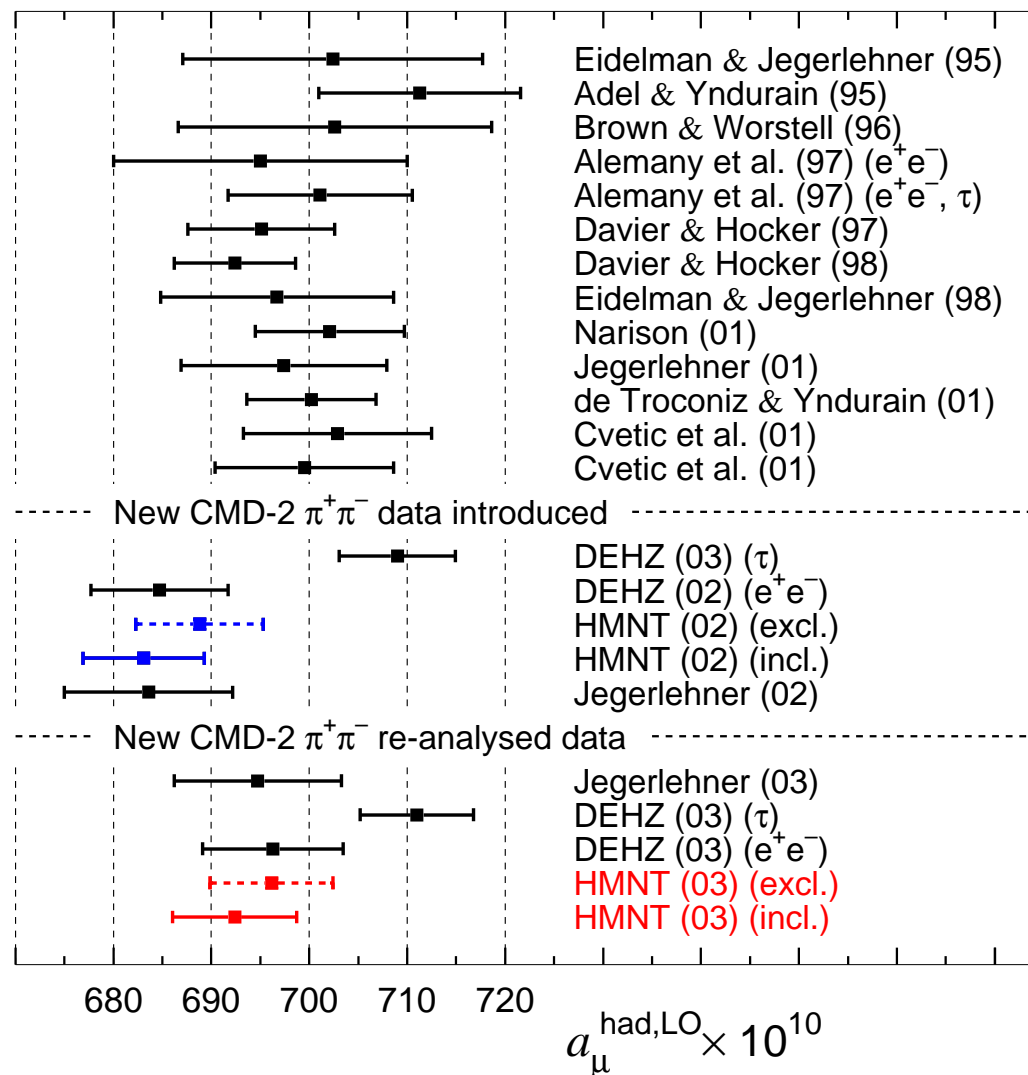
'04 KLOE data (rad. return) \Rightarrow agrees with e^+e^-

'04 isospin breaking effects \Rightarrow τ discarded ~ 695 \pm 7

[Ghozzi, Jegerlehner]

seems to get settled, **improvement still necessary**

The $a_\mu^{\text{had,LO}}$ history (graphically):



Overview of the SM theory evaluation:

Source	contr. to $a_\mu [10^{-10}]$	
LO hadr.	$\sim 695 \pm 7 (e^+e^-)$	[Davier et al, Hagiwara et al. '03]
		[Ghozzi, Jegerlehner '03]
	$711.0 \pm 6 (\tau)$	[Davier, Eidelman, Höcker, Zhang '03]
LBL	8 ± 4	[Knecht, Nyffeler '02]
	13.6 ± 2.5 tbc	[Melnikov, Vainshtein '03]
EW 1L	19	
EW 2L	-4	[Czarnecki, Krause, Marciano '98]
exp. res.	6	[BNL E821 '04]

“Isospin breaking effects” in τ based evaluation of LO hadr. not properly under control ? [Ghozzi, Jegerlehner '03]

general agreement at ICHEP'04 Beijing: discard τ data

$$a_\mu^{\text{exp}} - a_\mu^{\text{theo,SM}} \approx (25.2 \pm 9.2) \times 10^{-10}$$

3. $(g - 2)_\mu$ and Supersymmetry

Despite of . . .

→ many discovered errors

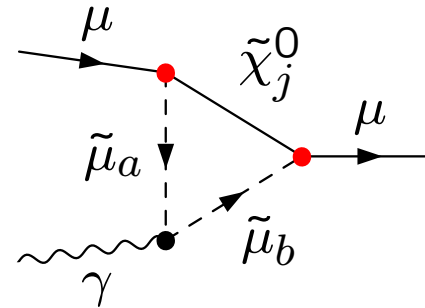
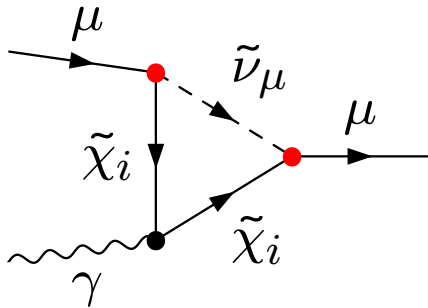
→ many new evaluations

. . . discrepancy persists

$$a_\mu^{\text{exp}} - a_\mu^{\text{theo,SM}} \approx (25.2 \pm 9.2) \times 10^{-10}$$

Could **SUSY** be the explanation?

Feynman diagrams for MSSM 1L corrections:



- Diagrams with chargino/sneutrino exchange
- Diagrams with neutralino/smuon exchange

Enhancement factor as compared to SM:

$$\mu - \tilde{\chi}_i^\pm - \tilde{\nu}_\mu : \sim m_\mu \tan \beta$$

$$\mu - \tilde{\chi}_j^0 - \tilde{\mu}_a : \sim m_\mu \tan \beta$$

$$\text{SM, EW 1L: } \frac{\alpha}{\pi} \frac{m_\mu^2}{M_W^2}$$

$$\text{MSSM, 1L: } \frac{\alpha}{\pi} \frac{m_\mu^2}{M_{\text{SUSY}}^2} \times \tan \beta$$

SUSY corrections at 1L:

$$a_{\mu}^{\text{SUSY,1L}} \approx 13 \times 10^{-10} \left(\frac{100 \text{ GeV}}{M_{\text{SUSY}}} \right)^2 \tan \beta \text{ sign}(\mu)$$

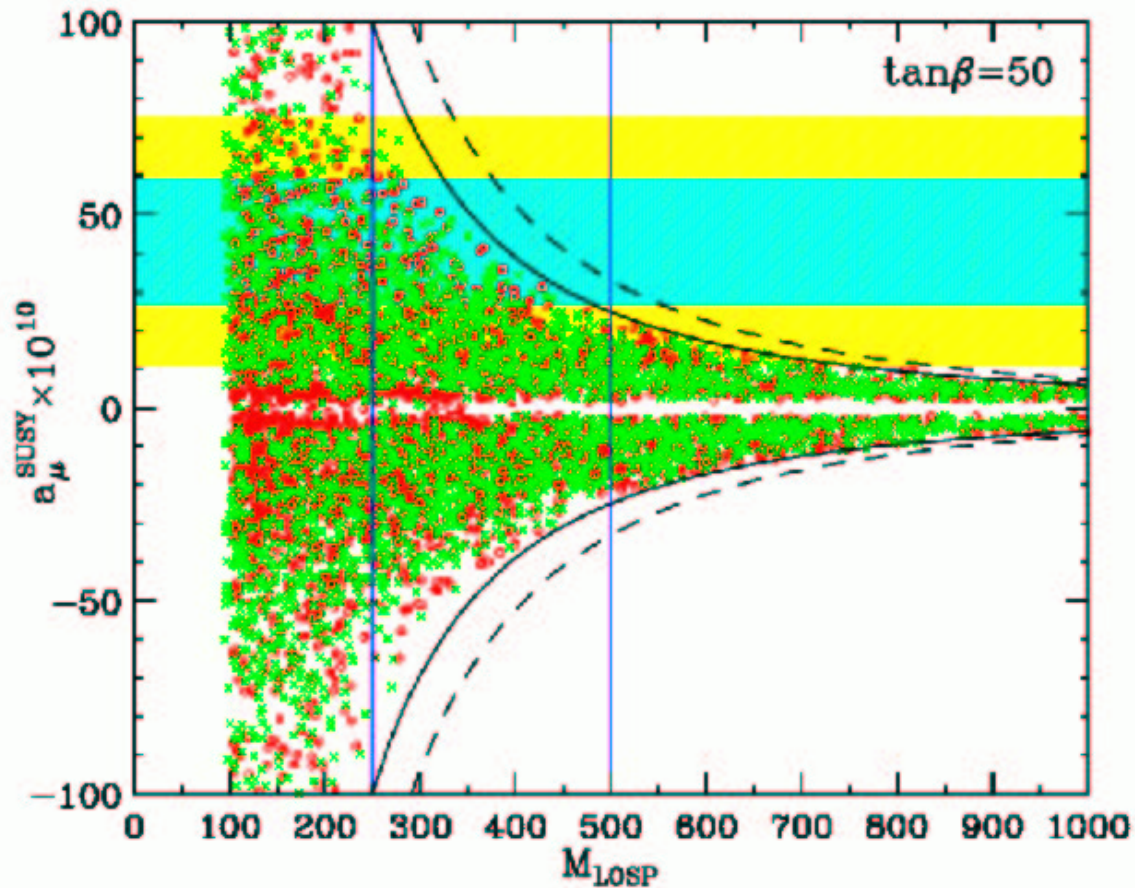
$M_{\text{SUSY}} (= m_{\tilde{\mu}} = m_{\tilde{\nu}} = m_{\tilde{\chi}})$: generic SUSY mass scale

$$a_{\mu}^{\text{SUSY,1L}} = (-100 \dots + 100) \times 10^{-10}$$
$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{theo,SM}} \approx (25.2 \pm 9.2) \times 10^{-10}$$

⇒ SUSY could easily explain the “discrepancy”

⇒ a_{μ} can provide bounds on SUSY parameter space
(by requiring agreement at the 95% C.L.)

Example I: Scan over SUSY parameter space



Scan over

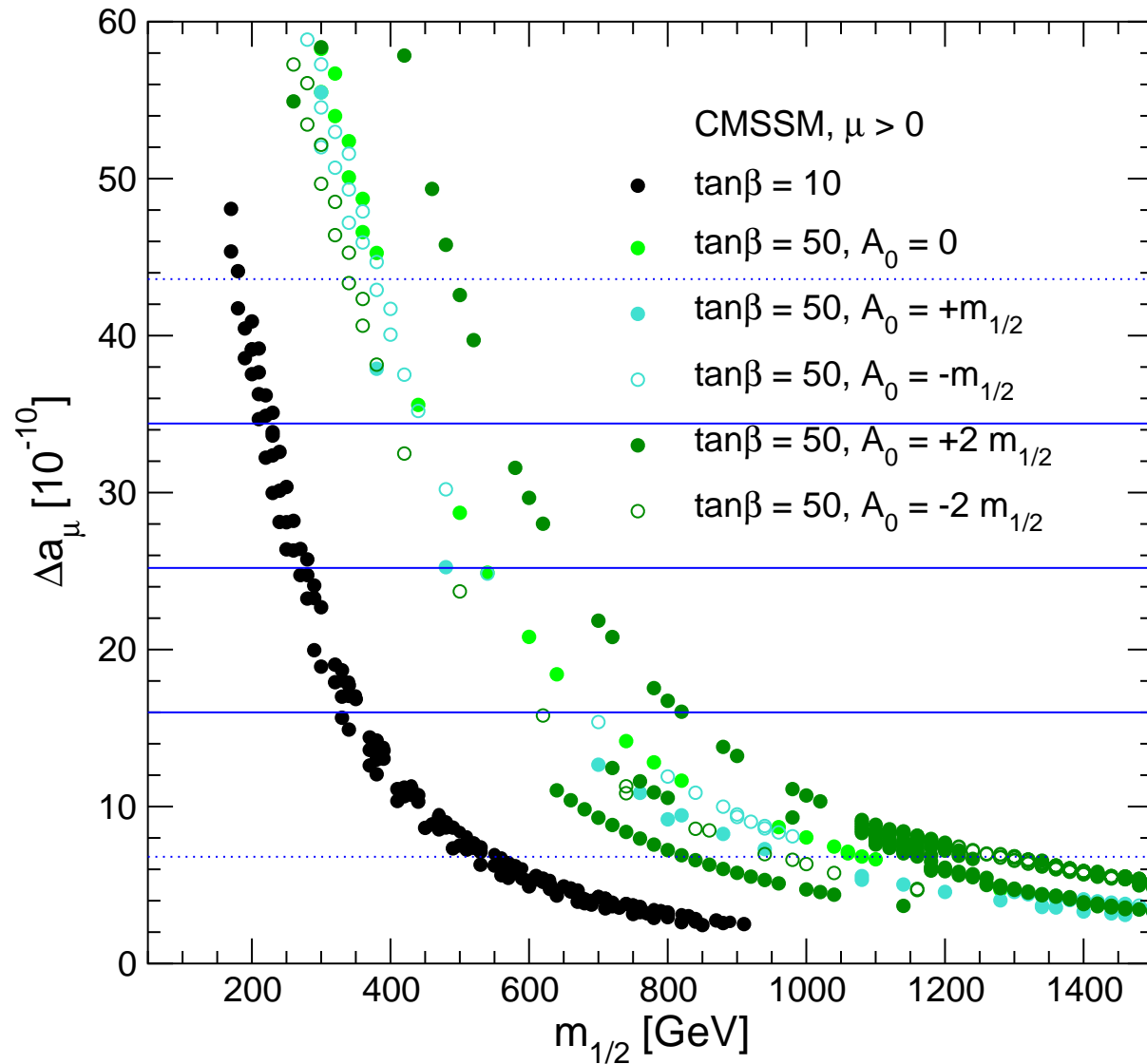
$\mu, M_2, m_{\tilde{\mu}}, A_\mu$

LOSP = lightest observable
SUSY particle

[J. Feng, K. Matchev '01]

SUSY could easily explain
discrepancy

Example II: Investigation of mSUGRA with cold dark matter constraint



Scan over $m_{1/2}, m_0, A_0$
 $\tan\beta = 10, 50$
selected points give correct
amount of cold dark matter

[Ellis, S.H., Olive, Weiglein '04]

Severe bounds on e.g. $m_{1/2}$

⇒ SUSY 1-loop calculation gives interesting results

Motivation for 2-loop calculation:

- precise knowledge of a_{μ}^{SUSY} to $\sim 1 \times 10^{-10}$ necessary to derive **precise bounds** on SUSY parameter space
- $a_{\mu}^{\text{SUSY},2\text{L}}$ can be large even if 1-loop is small
(e.g. due to heavy slepton/neutralino/chargino masses)
- $a_{\mu}^{\text{SM},2\text{L},\text{EW}} \approx -23\%$ of $a_{\mu}^{\text{SM},1\text{L},\text{EW}}$
⇒ 2L can be large in principle

Classification of MSSM one-loop contributions

- A) diagrams with a throughgoing μ/ν_μ line \rightarrow THDM diagrams
- B) diagrams with a throughgoing $\tilde{\mu}/\tilde{\nu}_\mu$ line \rightarrow SUSY diagrams

Classification of MSSM two-loop contributions

1. diagrams with a closed SM fermion/scalar fermion loop (attached to class A)
2. diagrams with a closed chargino/neutralino (attached to A)
3. pure THDM diagrams (class A)
4. diagrams with a closed mixed f/\tilde{f} loop (attached to B)
5. THDM loop correction to class B

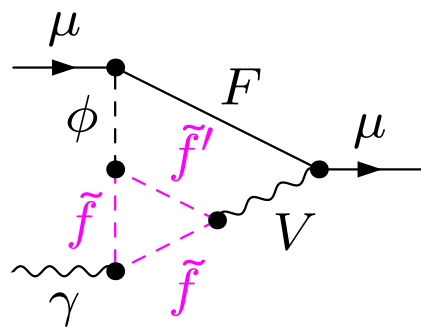
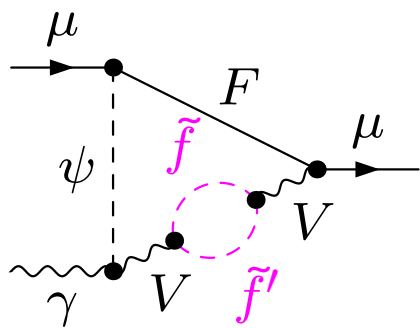
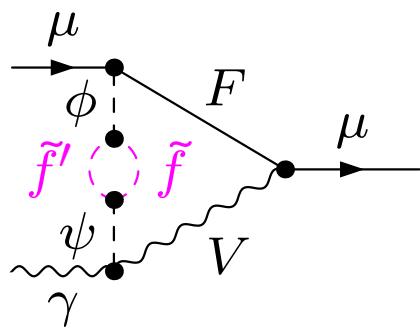
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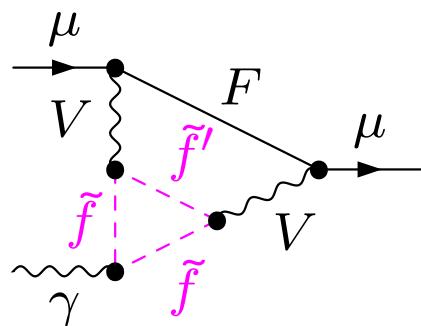
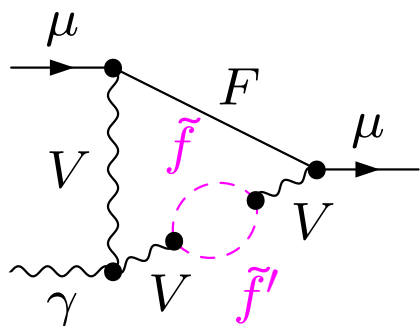
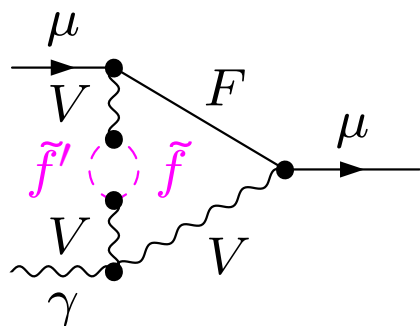
Classification of MSSM two-loop contributions

1. diagrams with a closed SM fermion/scalar fermion loop (attached to class A) **done**
2. diagrams with a closed chargino/neutralino (attached to A) **done**
3. pure THDM diagrams (class A) **done**
4. diagrams with a closed mixed f/\tilde{f} loop (attached to B) **not yet**
5. THDM loop correction to class B **not yet**

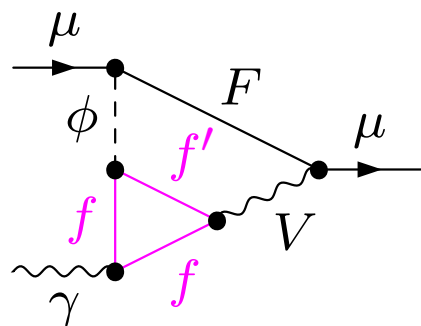
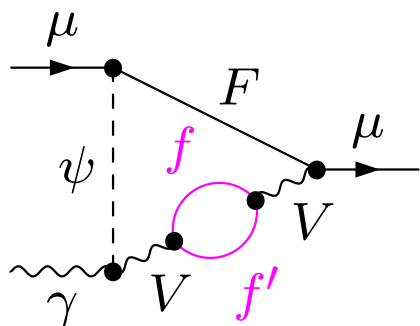
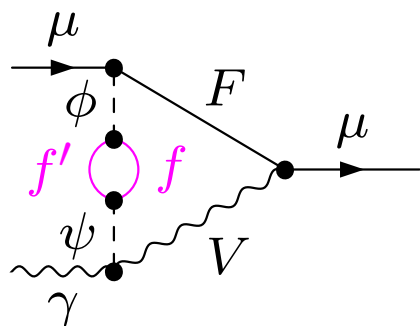
1. diagrams with a closed SM fermion/scalar fermion loop (class A)



type: $(\tilde{f}V\phi)$

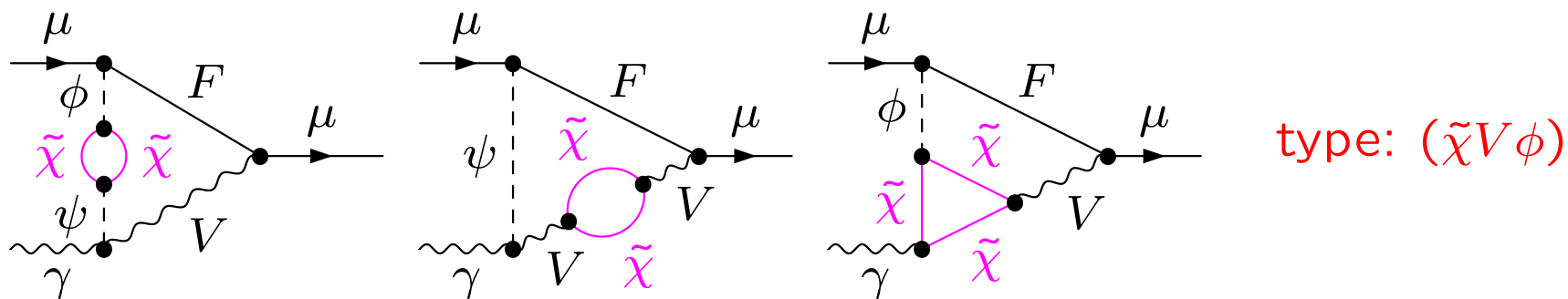


type: $(\tilde{f}VV)$



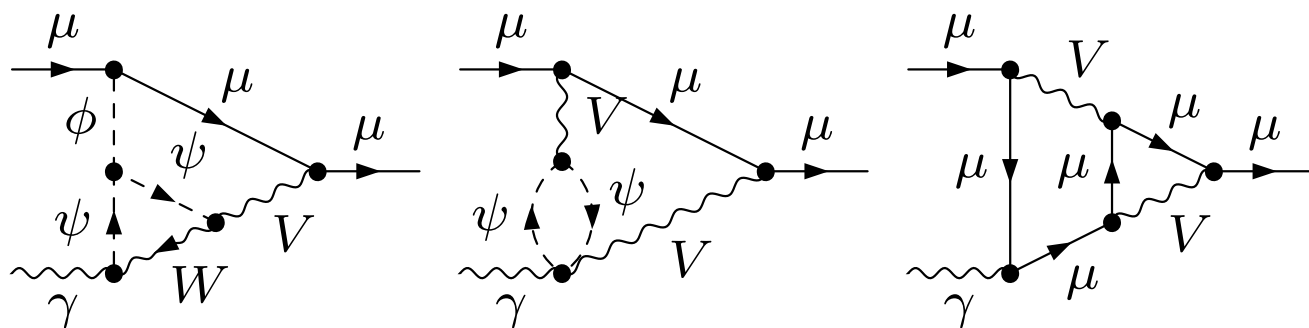
type: $(fV\phi)$

2. diagrams with a closed chargino/neutralino loop (class A)



Can be large, even if one-loop diagrams are suppressed by heavy smuon/sneutrino mass

3. pure THDM diagrams (class A)



→ check of corresponding SM part

4. diagrams with a mixed f/\tilde{f} loop (class B)

- same enhancement factors as diagrams with a closed f/\tilde{f} loop attached to class A
- ⇒ possibly of same size

5. THDM corrections to class B

THDM corrections to SUSY one-loop diagrams

Already known:

QED corrections to $a_\mu^{\text{SUSY}}(1\text{L})$: $\sim -8\%$ for $M_{\text{SUSY}} = 500 \text{ GeV}$

[*G. Degrandi, G. Giudice '98*]

- only evaluated for a common SUSY mass scale
- ⇒ non-negligible corrections possible

Overview about technical details of the MSSM 2-loop calculation:

1. Generate Feynman diagrams for $\mu\mu\gamma$ at two-loop
→ *FeynArts* [Küblbeck, Böhm, Denner '90] [Hahn '00 - '03]
2. Extract contribution of $(g - 2)_\mu$ from 2-loop amplitude
→ Dirac algebra, traces, ...
→ reduction to a “basic” set of 2-loop integrals
⇒ no analytic expression available
3. Expand integrals in $m_\mu \ll M_{\text{weak}}, M_{\text{SUSY}}$ (Taylor, Large Mass exp.)
→ all integrals are reduced to vacuum integrals: T_{134}, A_0, B_0
4. ⇒ analytic expression of a_μ in $M_{\text{weak}}, M_\phi, M_{\text{SUSY}}$
→ implementation in *FeynHiggs2.1* (www.feynhiggs.de)

Performed checks of our result:

- Cancellation of UV divergences
- Cancellation of terms $\sim m_\mu^{-4}, m_\mu^{-2}, m_\mu^0$
- Cancellation of field renormalization constants
- Reevaluation of SM 2L diagrams from [Czarnecki, Krause, Marciano '98] \Rightarrow perfect analytical agreement (after going to the appropriate limit)
- f/\tilde{f} loop corrections:
Going to the limit of [A. Arhrib, S. Baek '01] \Rightarrow agreement
(i.e. [C. Chen, C. Geng '01] is too large by a factor of 4)

4. SUSY two-loop results

We have obtained analytical/numerical results for all two-loop corrections to class A diagrams:

3. pure THDM diagrams (class A)

1. diagrams with a closed SM fermion/scalar fermion loop (class A)

2. diagrams with a closed chargino/neutralino loop (class A)

Work in progress:

4. diagrams with a mixed f/\tilde{f} loop (class B)

5. THDM corrections to class B

3. pure THDM diagrams (class A)

$$a_{\mu}^{\text{bos},2\text{L}} = \frac{5 G_{\mu} m_{\mu}^2}{38\pi^2 \sqrt{2}} \frac{\alpha}{\pi} \left(c_L^{\text{bos},2\text{L}} \log \frac{m_{\mu}^2}{M_W^2} + c_0^{\text{bos},2\text{L}} \right)$$

with

$$c_L^{\text{bos},2\text{L}} = \frac{1}{30} \left[107 + 23(1 - 4s_W^2)^2 \right]$$

as in the SM!

⇒ perfect agreement with existing calculation

[Czarnecki, Krause, Marciano '98]

$\log \frac{m_{\mu}^2}{M_W^2}$ constitutes the leading part in the THDM corrections

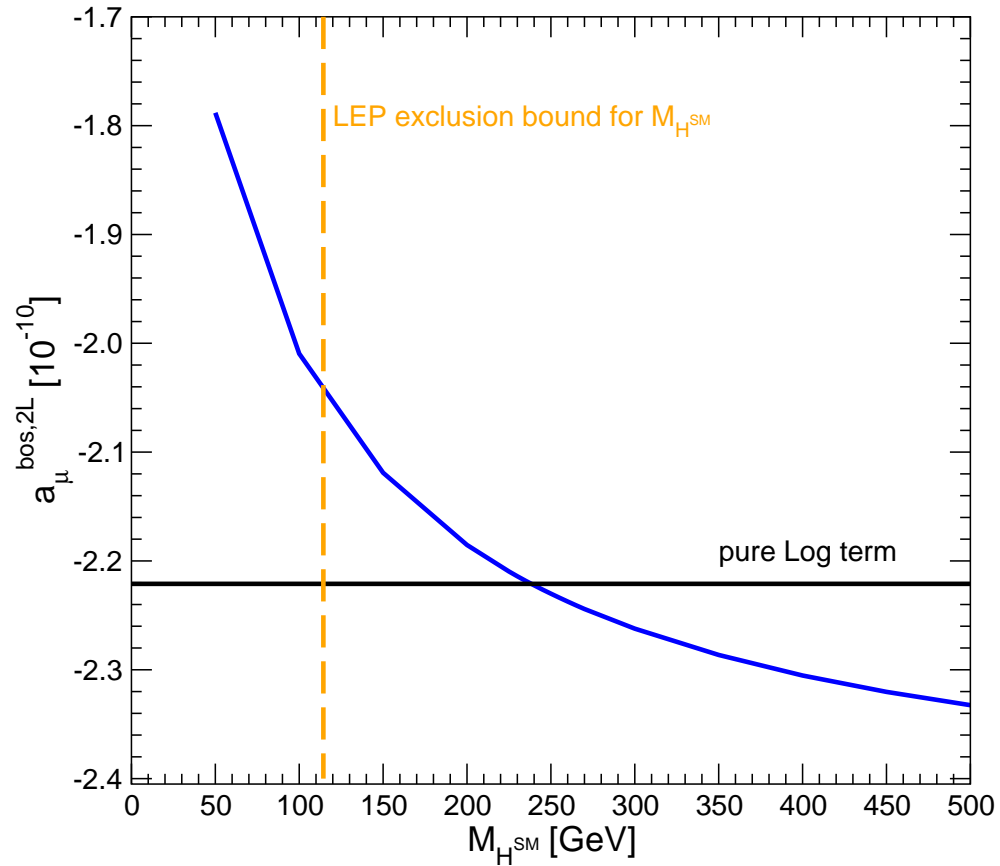
$c_0^{\text{bos},2\text{L}}$: complicated function of MSSM parameters

→ T

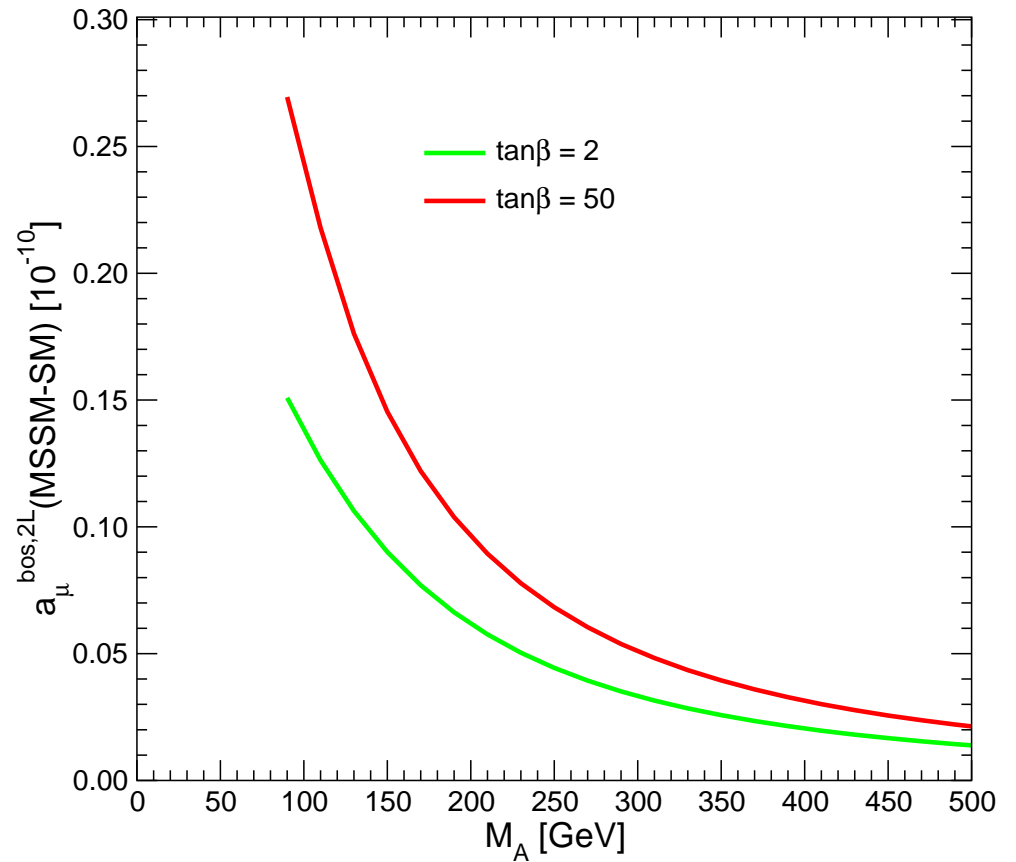
⇒ THDM corrections well under control, $\lesssim 2 \times 10^{-10}$

pure THDM two-loop contributions:

full SM result:



MSSM - SM:



1. diagrams with a closed SM fermion/scalar fermion loop (class A)

Old evaluations of presumably leading terms: corrections up to $\sim 20 \times 10^{-10}$

Experimental constraints?

⇒ Scan over the MSSM parameter space

$$\begin{aligned} -3 \text{ TeV} &\leq \mu \leq 3 \text{ TeV} \\ -3 \text{ TeV} &\leq A_{t,b} \leq 3 \text{ TeV} \\ 150 \text{ GeV} &\leq M_A \leq 1 \text{ TeV} \\ 0 &\leq M_{\text{SUSY}} \leq 1 \text{ TeV} \end{aligned}$$

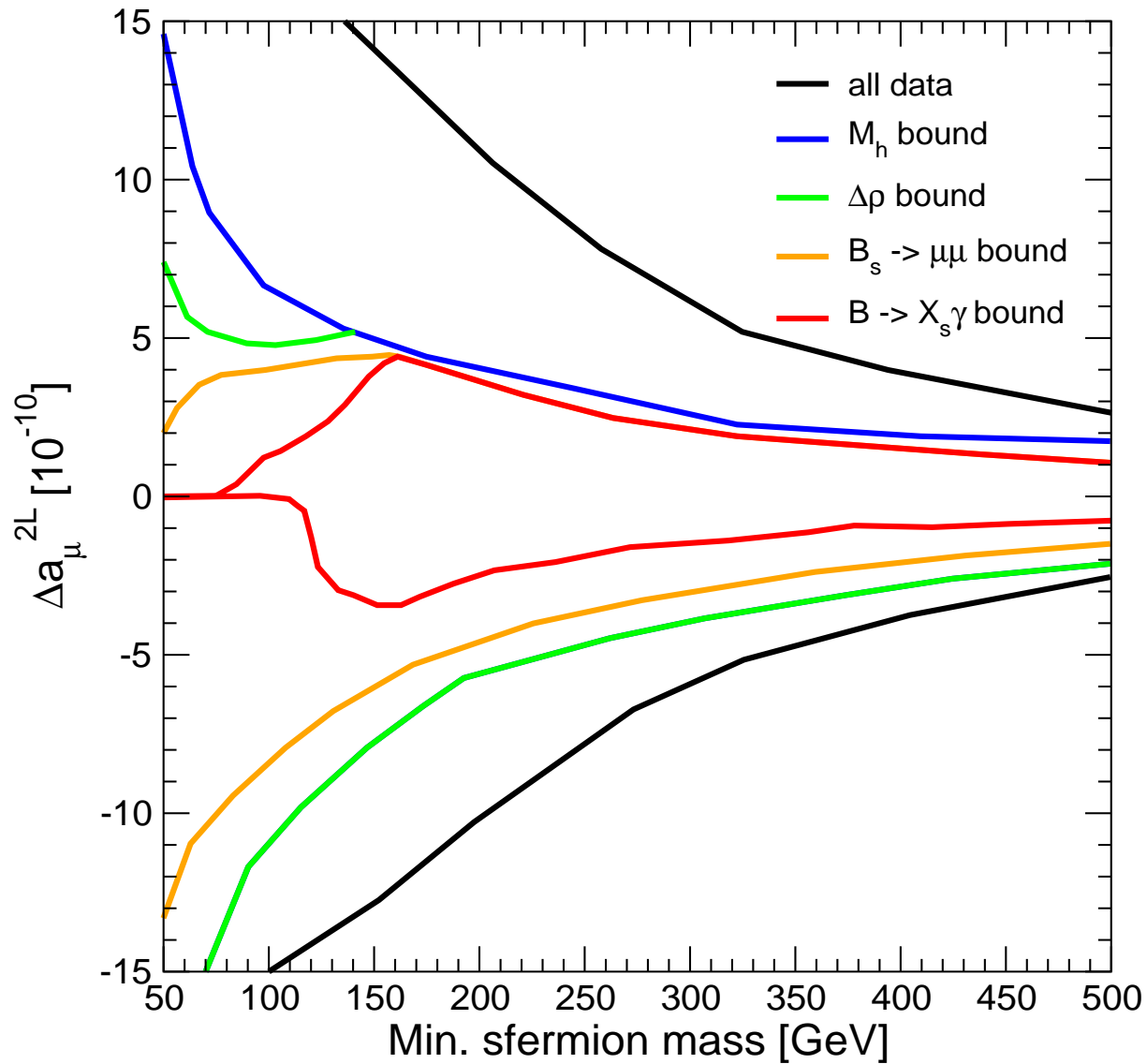
$\tan \beta = 50$

$M_{\text{SUSY}} = M_Q = M_L = M_U = M_D = M_E$ (later relaxed ...)

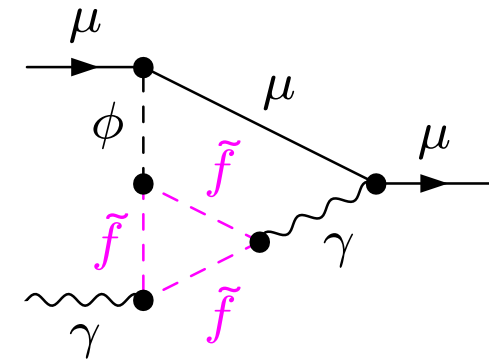
$A_\tau = A_b$

Constraints: M_h , $\Delta\rho^{\text{SUSY}}$, $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$, $\text{BR}(B \rightarrow X_s \gamma)$

Successive application of constraints:

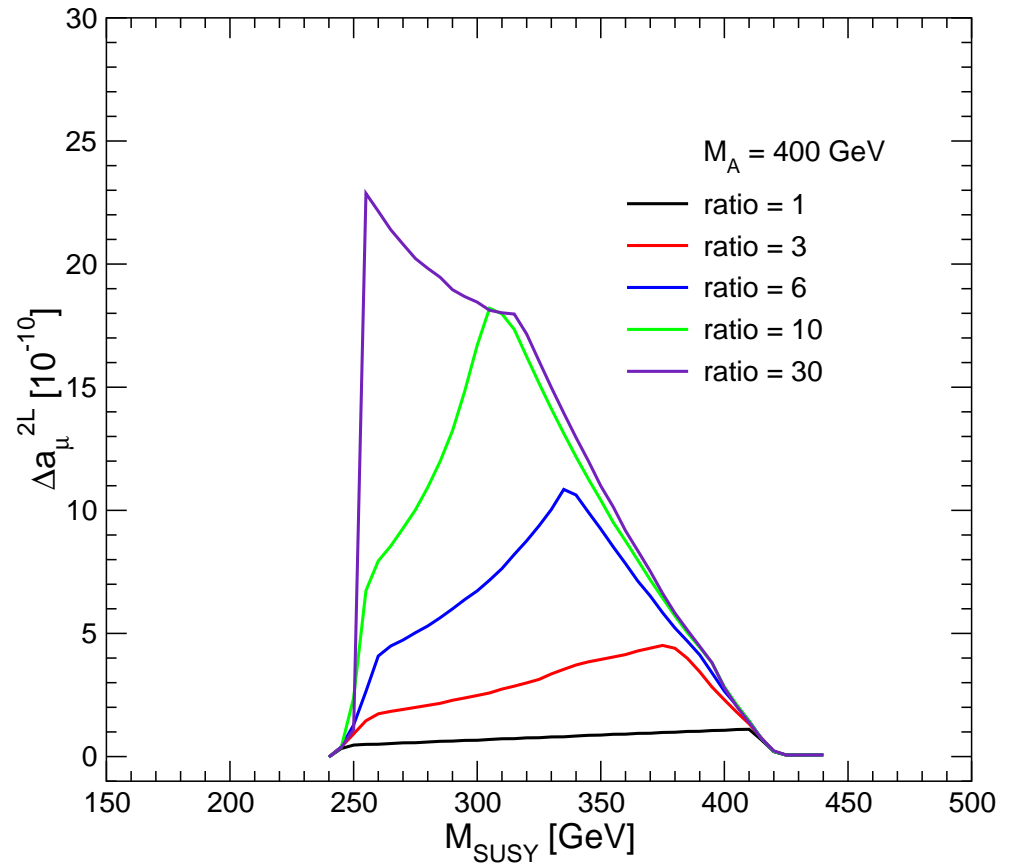
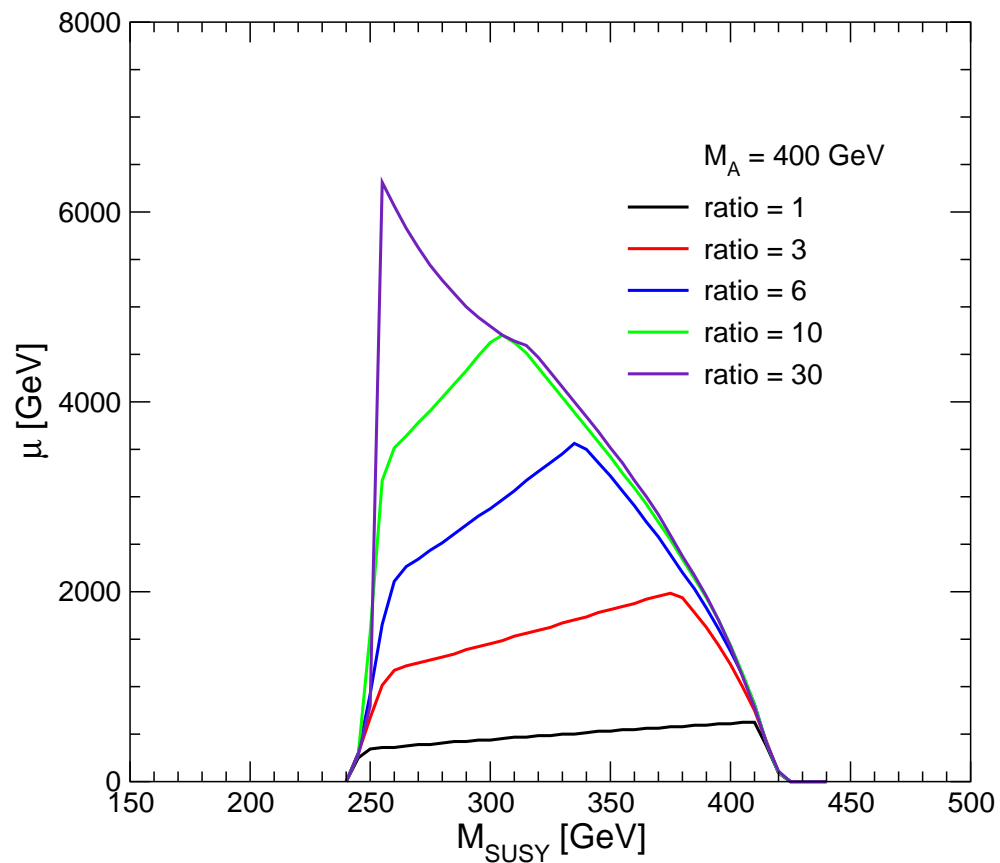


- largest contributions from $(\tilde{f}V\phi)$ (Barr-Zee diag.)



- no constraints
 \Rightarrow corrections up to 20×10^{-10} possible
- constraints
 $\Rightarrow |\Delta a_\mu^{2L}| \lesssim 5 \times 10^{-10}$

Results for relaxed universality condition:



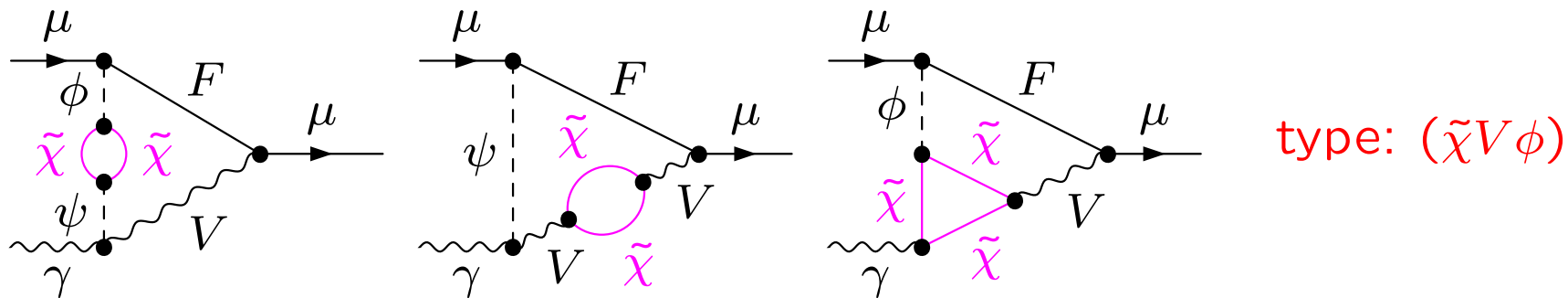
$M_D/M_U \gg 1 \Rightarrow$ very large μ possible

$\Rightarrow \Delta a_\mu^{2L} > 20 \times 10^{-10}$ possible

(this shows how “difficult” it is to obtain large corrections)

2. diagrams with a closed chargino/neutralino loop (class A)

All diagrams with a closed chargino/neutralino loop:



Approximation formula:

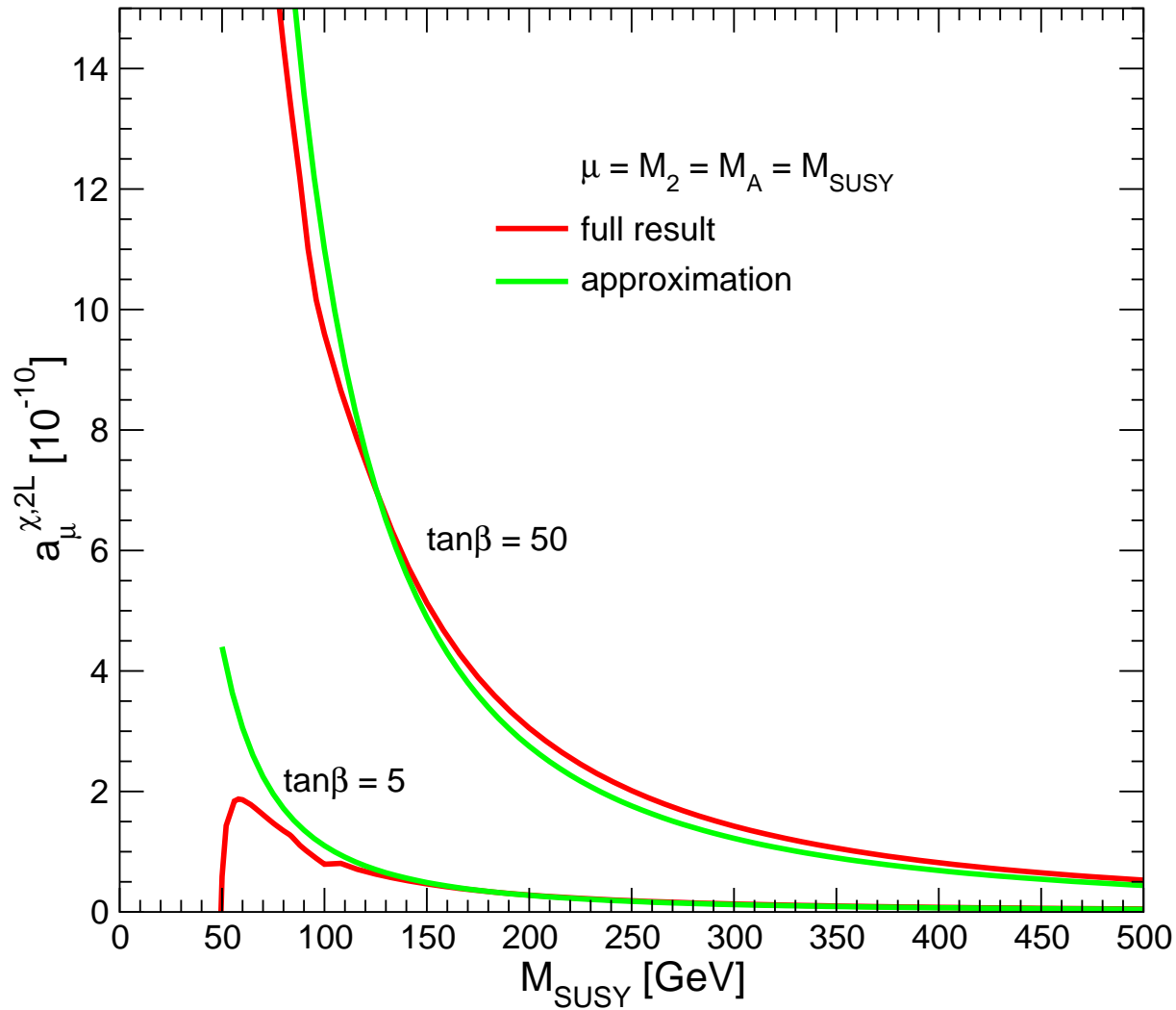
$$\Delta a_{\mu}^{2L\tilde{\chi}} \approx 11 \times 10^{-10} \times \left(\frac{\tan \beta}{50} \right) \times \left(\frac{100 \text{ GeV}}{M_{\text{SUSY}}} \right)^2 \times \text{sgn}(\mu)$$

⇒ suppressed by a factor of ~ 50 as compared to the 1L result provided that $m_{\tilde{\mu}} \approx m_{\tilde{\nu}_{\mu}} \approx \mu \approx M_2 \approx M_A$

– Quality of approximation ?

– Size of 2L corrections for $m_{\tilde{\mu}} \approx m_{\tilde{\nu}_{\mu}} > \mu \approx M_2 \approx M_A$

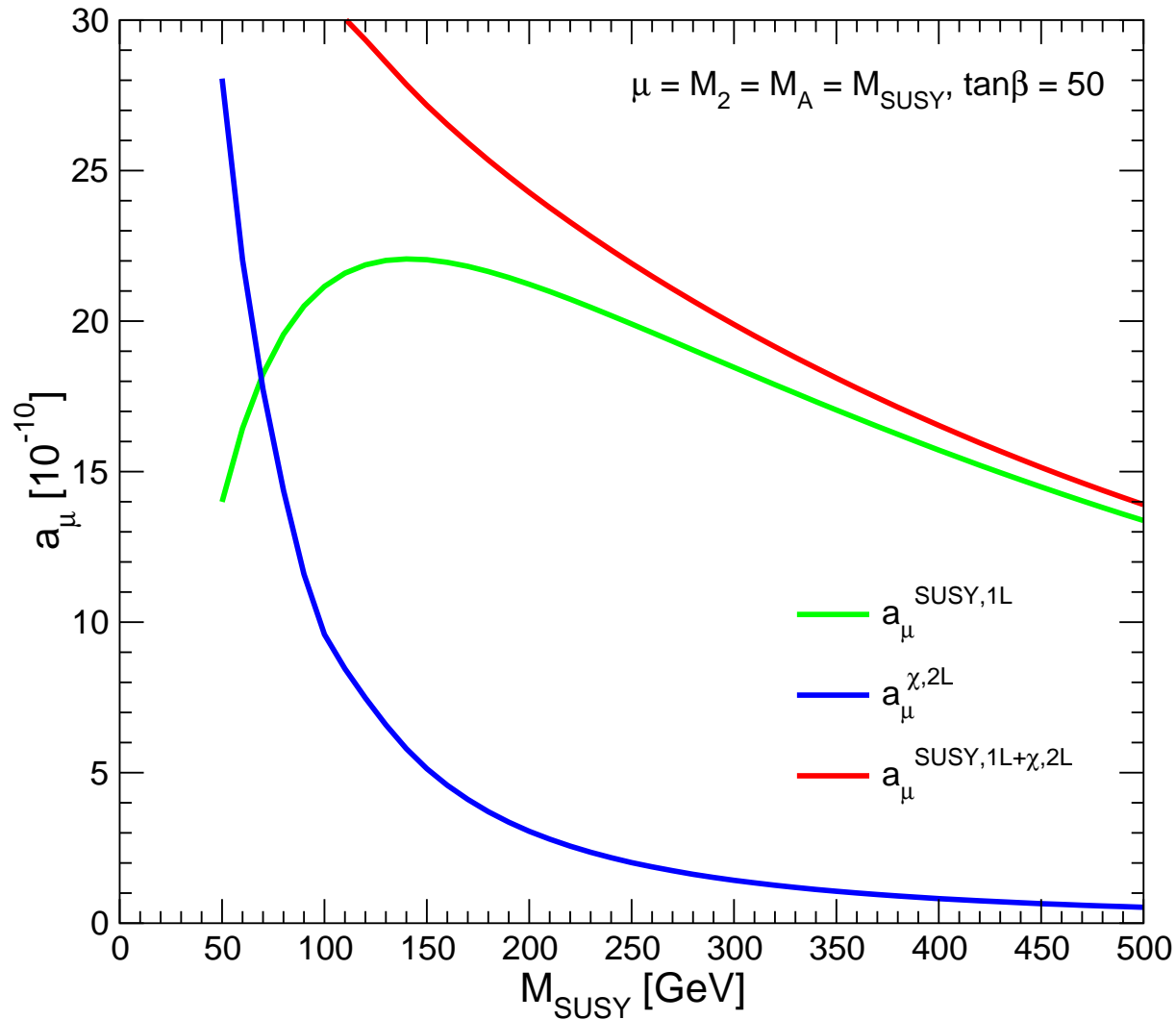
Quality of approximation:



⇒ excellent approximation

2L corrections can be sizeable

Comparison of 1L and 2L results:

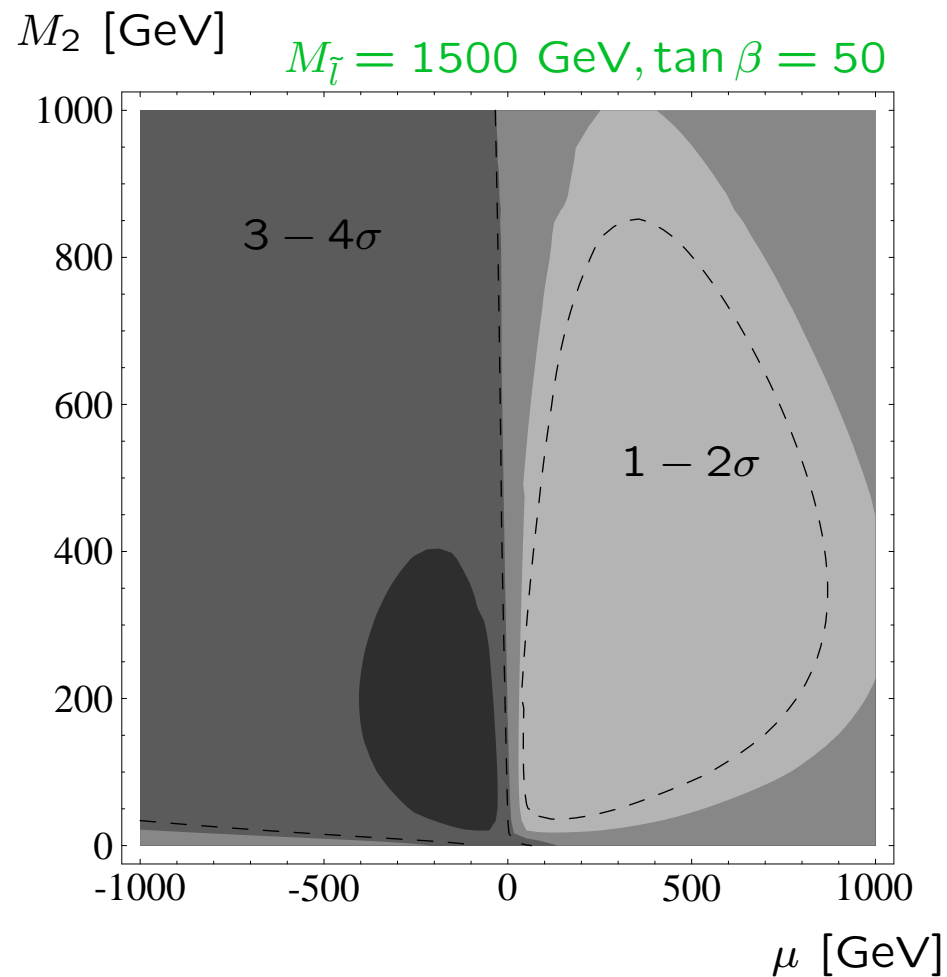
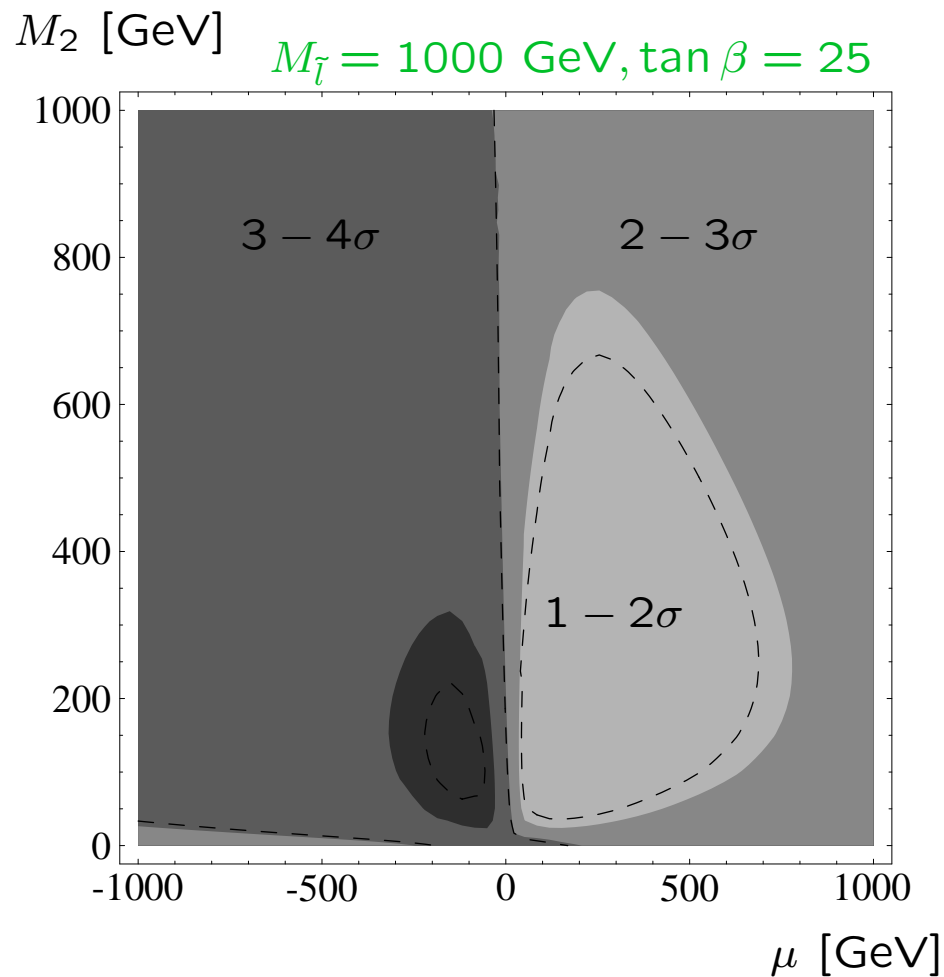


1L suppressed:

$$m_{\tilde{\mu}} = m_{\tilde{\nu}_\mu} = 1 \text{ TeV}$$

\Rightarrow 2L corrections can be sizeable

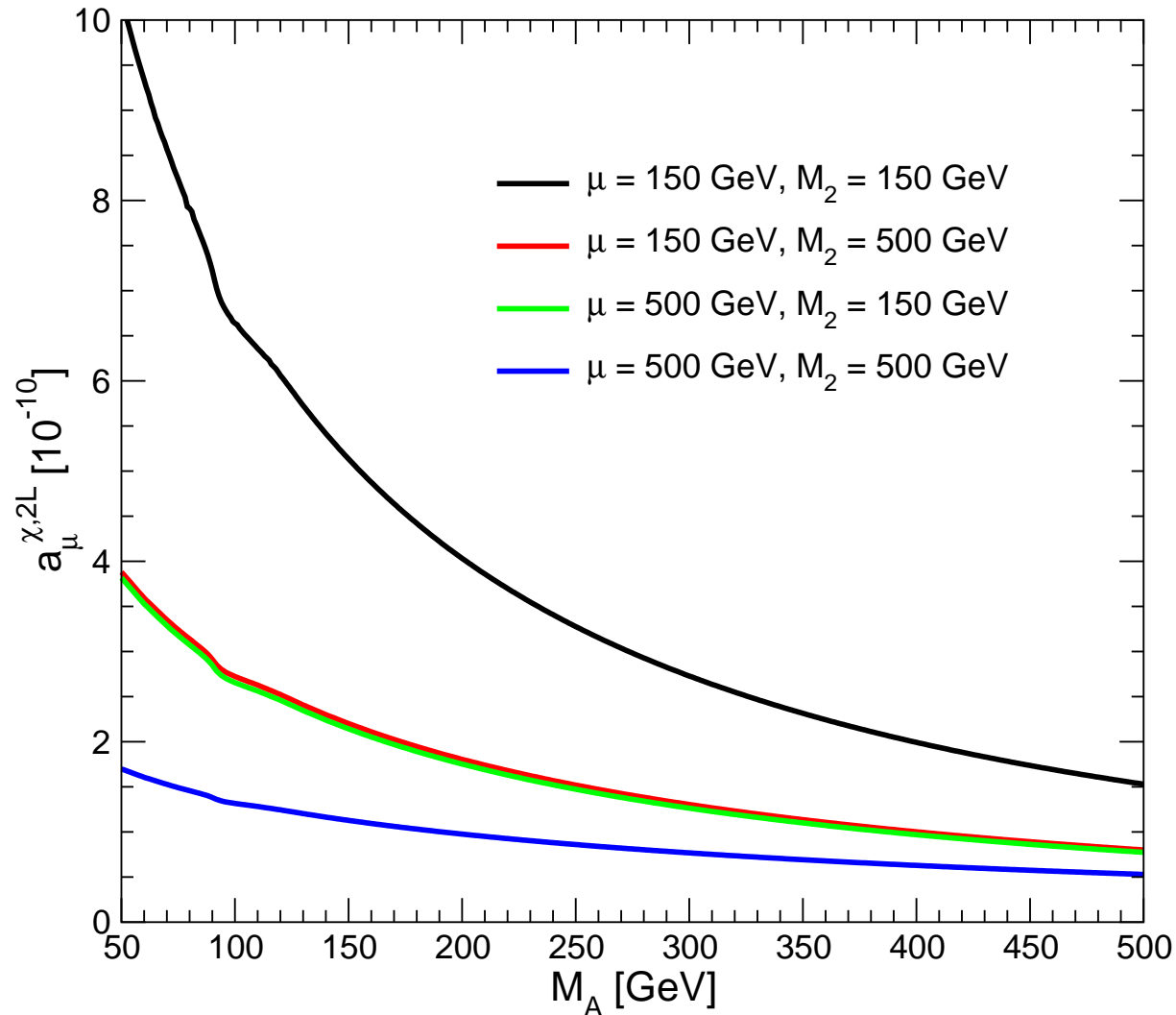
Shift in 2σ contours:



dashed: pure 1L

solid border: $1L + 2L \Rightarrow$ shift by $\gtrsim 100$ GeV possible

Results in Split SUSY:



Split SUSY:

[Arkani-Hamed, Dimopoulos '04]

- scalar masses large
($\gtrsim 10^9 \text{ GeV}$)
- fermion masses “normal”

only non-vanishing contribution to $(g - 2)_\mu$: $(\tilde{\chi} V \phi)$

\Rightarrow Split SUSY excluded at 95% C.L.

5. Conclusinos

- Precision observables can give valuable information about the “true” Lagrangian
- new experimental result for a_μ :
 $a_\mu^{\text{exp}} - a_\mu^{\text{theo}} \approx (25.2 \pm 9.2) \times 10^{-10} : 2.7 \sigma$ (e^+e^- data)
- SUSY could easily explain “discrepancy”
 a_μ can provide bounds on SUSY parameters
goal: $\Delta a_\mu^{\text{theo, MSSM}} \lesssim 1 \times 10^{-10}$
- \Rightarrow Evaluation of MSSM two-loop contributions
- Expansion \Rightarrow analytic result in vacuum integrals
- pure THDM 2L corrections: $\Delta a_\mu^{2L} \lesssim 2 \times 10^{-10}$
- closed f/\tilde{f} loop corrections: $\Delta a_\mu^{2L} \lesssim 3 - 5 \times 10^{-10}$
can be large, even if 1L is suppressed (crucial: experimental constraints)
- closed chargino/neutralino loop corrections: $\Delta a_\mu^{2L} \lesssim 5 \times 10^{-10}$
can be large, even if 1L and/or 2L(f/\tilde{f}) is suppressed

Outlook: evaluation of remaining two-loop corrections

6. Calculation of MSSM two-loop corrections

Overview:

1. Generate Feynman diagrams for $\mu\mu\gamma$ at two-loop
2. Extract contribution of $(g - 2)_\mu$ (given in terms of two-loop integrals)
3. Expand integrals in $m_\mu \ll M_{\text{weak}}, M_{\text{SUSY}}$
4. \Rightarrow analytic expression in $M_{\text{weak}}, M_\phi, M_{\text{SUSY}}$

1. Generate Feynman diagrams for $\mu\mu\gamma$ at two-loop

– use *FeynArts* (www.feynarts.de)

[*J. Küblbeck, M. Böhm, A. Denner '90*]

[*T. Hahn '00 - '03*]

– use *MSSM model file*

[*T. Hahn, C. Schappacher '01*]

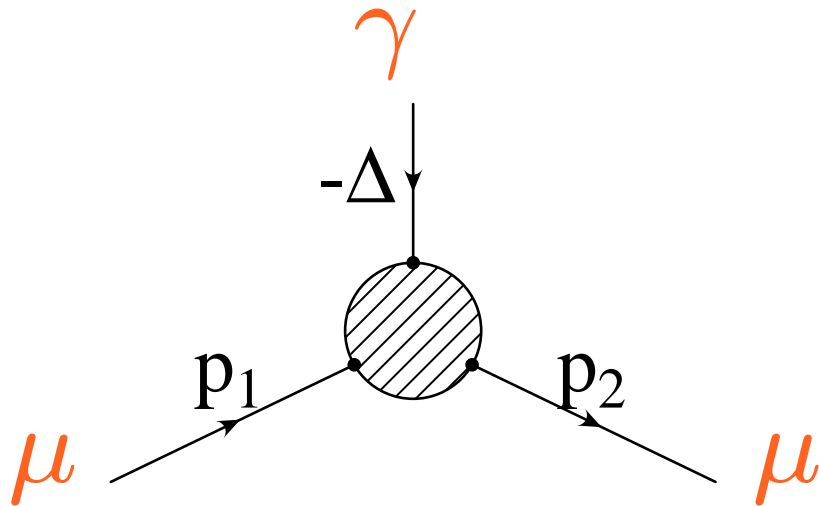
⇒ obtain all *two-loop* diagrams

one-loop diagrams with counter term insertion

one-loop diagrams for renormalization

⇒ transform *diagrams to amplitudes*

2. Extract contribution of $(g - 2)_\mu$



Amplitude: $M_\mu(p, \Delta)$

Expansion in γ momentum:

$$M_\mu(p, \Delta) \approx V_\mu(p) + \Delta^\nu T_{\nu\mu}(p)$$

($V_\mu, T_{\nu\mu}$ given in terms
of 2L **self-energies**)

next step: project out $(g - 2)_\mu$:

$$a \sim \frac{1}{m_\mu^2} \text{Tr} \left[P^\mu V_\mu + Q^{\nu\mu} T_{\nu\mu} \right]$$

Next step:

- Dirac algebra
- traces, ...
- reduction to a “basic” set of 2L integrals

⇒ $a = (g - 2)/2$ is given in terms of 2L integrals:

$$a = (g - 2)/2 = C_1 \times \text{Integral}_1 + C_2 \times \text{Integral}_2 + \dots$$

Integrals have non-zero external momentum $p^2 = m_\mu^2$
multiple propagators
complicated numerator
masses: $0, m_\mu, M_W, \dots, M_{\text{SUSY}}$

⇒ no analytic expressions available

3. Expand integrals in $m_\mu \ll M_{\text{weak}}, M_{\text{SUSY}}$

Leading term in a_μ : $\sim \frac{m_\mu^2}{M_{W,\text{SUSY}}^2} \approx 10^{-6}$

$\Rightarrow \frac{m_\mu}{M_{W,\text{SUSY}}}$ is a good expansion parameter

\Rightarrow perform Taylor expansion or “Large mass expansion” of the integrals

\Rightarrow all integrals are reduced to vacuum integrals: T_{134}, A_0, B_0

– Further simplification of coefficients

– Insertion of analytical expressions for T_{134}, A_0, B_0

\Rightarrow Analytical result for $a = (g - 2)_\mu$

\Rightarrow numerical evaluation possible

(implementation in *FeynHiggs2.1*)

Performed checks of our result:

- Cancellation of UV divergences
- Cancellation of terms $\sim m_\mu^{-4}, m_\mu^{-2}, m_\mu^0$
- Cancellation of field renormalization constants
- Reevaluation of SM 2L diagrams from [Czarnecki, Krause, Marciano '98] \Rightarrow perfect analytical agreement (after going to the appropriate limit)
- f/\tilde{f} loop corrections:
Going to the limit of [A. Arhrib, S. Baek '01] \Rightarrow agreement
(i.e. [C. Chen, C. Geng '01] is too large by a factor of 4)

Experimental constraints:

Quantity	M_h	$\Delta\rho^{\text{SUSY}}$	$\text{BR}(B_s \rightarrow \mu^+ \mu^-)$	$\Delta_{B \rightarrow X_s \gamma}$
strong bound	$> 111.4 \text{ GeV}$	$< 3 \times 10^{-3}$	$< 0.97 \times 10^{-6}$	$< 1.0 \times 10^{-4}$
weak bound	$> 106.4 \text{ GeV}$	$< 4 \times 10^{-3}$	$< 1.2 \times 10^{-6}$	$< 1.5 \times 10^{-4}$

M_h : strong: exp. bound - 3 GeV theory uncertainty

weak: effect of $\delta m_t^{\text{exp}} = +5 \text{ GeV}$

$\Delta\rho^{\text{SUSY}}$: strong: 2σ , weak: 3σ

$\text{BR}(B_s \rightarrow \mu^+ \mu^-)$: strong: 90% CL, weak: 95% CL

$\Delta_{B \rightarrow X_s \gamma} = |\text{BR}(B \rightarrow X_s \gamma) - 3.34 \times 10^{-4}|$

strong: 90% CL, weak: 95% CL