Mathematical Modeling and Analysis

Error Minimization Based Rezone Strategy for ALE Methods

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The philosophy of the Arbitrary Lagrangian-Eulerian (ALE) methodology for solving multidimensional fluid flow problems is to move the computational grid, using the flow as a guide, to improve the robustness, accuracy and efficiency of the simulation. The main elements in the ALE simulation are an explicit Lagrangian phase, a rezone phase in which a new grid is defined, and a remap (conservative interpolation) phase in which a Lagrangian solution is transferred to the new grid. In most multidimensional ALE codes, the main goal of the rezone phase is to maintain geometrical quality of the mesh, that is, to prevent it from tangling. The example of such a strategy is the Reference Jacobian Matrix (RJM) method described in [1].

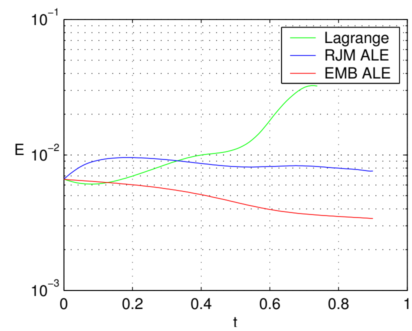
The RJM rezone strategy does not directly address the question of improving accuracy and relies on assumption that if the mesh has good geometrical quality and is close to the Lagrangian mesh then it improves accuracy. This is clearly true in comparison with pure Lagrangian methods and some rezone strategies which were available to authors of [1], but the question still remains: Can we improve accuracy even more by using a strategy based directly on reducing some measure of error?

Developing a rezone strategy for reducing error is a challenging task for full gas dynamics equations in 2D and 3D. Therefore we have started with a simpler problem, the 1D viscous Burgers' equation:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \varepsilon\frac{\partial^2 u}{\partial x^2}.$$

This equation has many important features of gas-dynamics equations: it expresses conservation law, it can develop shock-like structures, and its Lagrangian form resembles one for a full system of gas dynamics equations. In [2], we developed a new Error-Minimization-Based (EMB) rezone strategy which minimizes the L_2 norm of a global error and maintains smoothness of the grid.

The L_2 norm of the global error on new time level $t = t^{n+1}$ is a superposition of an interpolation error, an error due to the time advancing method, and the space discretization error. It is clear that if the mesh is not rezoned, then the interpolation error is zero; however, the grid at time level $t = t^n$ may not be the best grid to represent features of the solution at time $t = t^{n+1}$. The goal of the EMB rezone strategy is to achieve balance (if possible) between the errors which will minimize the global error at time $t = t^{n+1}$.



The accuracy of the pure Lagrangian, RJM ALE and EMB ALE simulations for viscous Burgers' equation with $\varepsilon = 0.005$ *on the mesh with 32 cells.*

In the figure, we demonstrate the superiority of the new method over the ALE method with the RJM rezone strategy and the pure Lagrangian method. The soluion at $t = 0$ consists of two shock-like structures, which move with different speeds and merge at $t \approx 0.5$. The pure Lagrangian method is accurate up to about $t = 0.1$. After this moment, dynamics of mesh cells do not correspond to dynamics of features and accuracy starts to degrade. It confirms the well-know statement that the method of characteristics is not well-suited for general hyperbolic partial differential equations and in particular for Burgers' equation. For the RJM ALE method, the error first starts to grow because the RJM rezone strategy tends to make the mesh smoother and by doing this, it reduces the resolution of features. Then, the error stays approximately constant. After $t = 0.3$ the accuracy of the RJM ALE is significantly better then accuracy of the pure Lagrangian method. For the EMB ALE method, the error is about the same as for the pure Lagrangian method for $t \leq 0.15$, but after this moment, it is significantly less than errors for the pure Lagrangian and the RJM ALE methods.

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References

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