Multilevel Accelerated Grid Optimization

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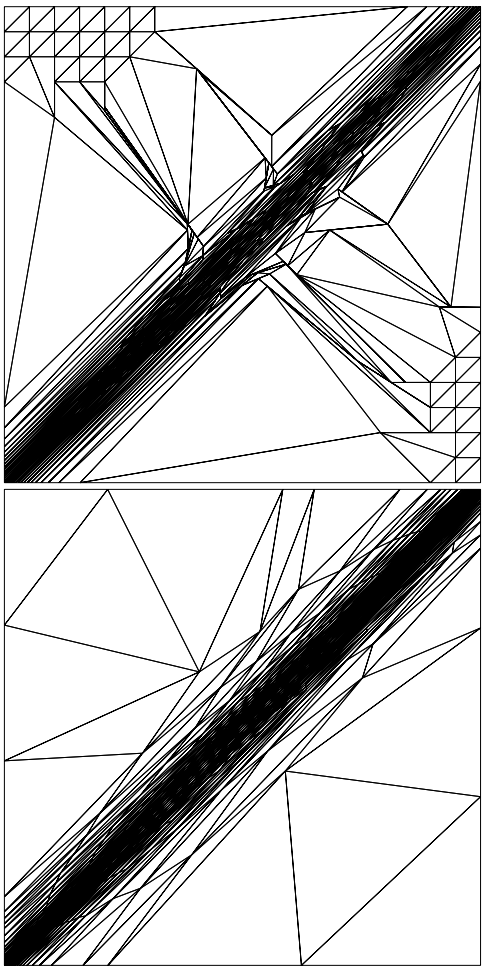
The quality of numerical simulations of processes that are modeled by partial differential equations strongly depends on the quality of the mesh that is used for their discretization. This quality is affected, for example, by mesh smoothness, or discretization error. To improve the mesh, a functional that is in general nonlinear must be minimized. This minimization is constrained by the validity of the mesh, since no mesh folding is allowed. Classical optimization techniques, such as nonlinear CG, or Gauss-Seidel steepest descent, perform very poorly on this class of minimization problems. We introduce a new minimization technique that utilizes the underlying geometry of the problem. By coarsening the mesh successively, in a multilevel-like fashion, minimizing appropriate coarse grid quality measures, and interpolating finer meshes from coarser ones, a more rapid movement of fine mesh points results, and the overall convergence of the minimization procedure is accelerated (see [1]).

We first describe the coarsening strategy. In an initial step in Delaunay coarsening [2], the list of vertices is reordered in such a way that all boundary vertices come first. In a loop over this list of vertices, the current vertex is added to the list of coarse vertices and its neighbors are deleted from the list of vertices. Hence, initially the boundary is coarsened, and then the interior. As a slight modification of this algorithm, we consider such boundary vertices first that are necessary to properly resolve the shape of the domain. An example for such vertices are the four corner vertices of a square. The resulting list of coarse vertices is then triangulated using a Delaunay algorithm (e.g. [3]).

Interpolating a grid from a coarser one is achieved by injection in the case of coarse vertices that are also fine vertices. All other fine vertices are interpolated using their barycentric coordinates with respect to their underlying coarse triangle as interpolation weights. In this procedure, the resulting grid might be tangled in some places. We handle this situation by using the untangling procedure described in [4].

For an approximate minimization procedure, we will call it relaxation, we use a few iterations standard gradient based optimizer, such as Gauss-Seidel steepest descent.



Grid after 500 iterations of CG (top) and after two cycles of multilevel optimization procedure (bottom)

The three components restriction, interpolation, and relaxation are combined to yield a multigrid-style V-cycle iteration (see [5]). In the figure, we show the final grid after 500 itera-

tions of Gauss-Seidel steepest descent (top), and the final grid after three iterations of our new V-cycle optimization procedure (bottom). The objective was to minimize the approximation error of a given function with a steep gradient by moving the underlying grid. In both cases the initial grid was a regular triangular grid. The V-cycle procedure completed in 25.9 seconds, while 500 iterations of the fine level Gauss-Seidel steepest descent procedure took more than 1000 seconds.

It is essential for this approach that a coarse grid representation of the objective function that is to be minimized can be derived. In other words, the change in the initial grid that is required to obtain the optimal grid must be expressible as small changes of vertex positions relative to the positions of neighbor vertices, plus larger changes of positions of groups of vertices. This is possible for applications where the objective is to find a grid that is optimal for the approximation of a function. We plan to extend this work to include the case where an error estimate, and not the actual error, is to me minimized.

Acknowledgements

Los Alamos Report LA-UR-03-3474. Funded by the Department of Energy under contracts W-7405-ENG-36 and the DOE Office of Science Advanced Computing Research (ASCR) program in Applied Mathematical Sciences.

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