AN APPLICATION OF TRAMO-SEATS: CHANGES IN SEASONALITY AND CURRENT TREND-CYCLE ASSESSMENT¹

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Abstract

The paper details an application of programs TRAMO and SEATS to seasonal adjustment and trend-cycle estimation of the German Retail Trade Turnover series. When adjusting with X12-ARIMA, the Bundesbank identified two problems: heteroscedasticity in the seasonal component, associated with different moving patterns for some of the months, and unstability of the trend-cycle at the end of the series. It is seen how, starting with the fully automatic procedure and adding some simple modifications, TRAMO and SEATS deal properly with both problems, and provide good, stable, and robust results.

Keywords: Time-series; ARIMA models; Seasonal adjustment; Trend estimation

1. Introduction: Description of the Problem

We present an application of the programs TRAMO, "Time Series Regression with ARIMA Noise, Missing Observations and Outliers", and SEATS, "Signal Extraction in ARIMA Time Series", (Gómez and Maravall, 1996). TRAMO is a program for estimation and forecasting of regression models with ARIMA errors and missing values. The program interpolates these values, identifies and corrects for several types of outliers, and estimates special effects such as Trading Day and Easter and, in general, intervention-variable type effects. SEATS is a program for estimation of unobserved components in time series following the so-called ARIMA-model-based (AMB) method; the basic components are the trend-cycle, seasonal, and irregular components, which are estimated and forecast with signal extraction techniques applied to ARIMA models. The two programs are structured so as to be used together, both for in-depth analysis of few series or for routine applications to a large number of them, and can be run in an entirely automatic manner. When used for seasonal adjustment, TRAMO preadjust the series to be adjusted by SEATS. The two programs are intensively used at present by data producing and economic agencies, including Eurostat and the European Central Bank.

The AMB methodology for seasonal adjustment was originally proposed by Burman (1980) and Hillmer and Tiao (1982). A description of the methodology behind TRAMO and SEATS can be found in Gómez and Maravall (2000 a,b). In essence, given the vector of observations $y = (y_{ti}, ..., y_{tm})$ where 0 < t1 < ..., < tm, TRAMO fits the regression model $y_t = z'_t \beta + x_t$, where β is a vector of regression coefficients, z'_t denotes a matrix of regression variables, and x_t follows the ARIMA process $\phi(B)\delta(B)x_t = \theta(B)a_t$, where B is the backshift operator, a_t is a n.i.i.d. $(0, V_a)$ white-noise variable, and $\phi(B)$, $\delta(B)$, $\theta(B)$ are finite polynomials in B that often have a multiplicative form. SEATS decomposes x_t as in $x_t = n_t + s_t$, $n_t = p_t + u_t$, where n_t , p_t , s_t , u_t , are seasonally adjusted (SA) series, the trend-cycle, seasonal, and irregular components, which also follow ARIMA-type models, possibly with deterministic effects added.

This paper illustrated application of the programs to the monthly German Retail Trade (RT) Turnover series (a more complete discussion is contained in Kaiser and Maravall, 2000a). We consider first the 24-year period 1/1975 - 12/1998, comprising 288 observations. The series, displayed in Figure 1, was made available by the Bundesbank for its October 1999 workshop on seasonal adjustment. The series had been already corrected for several effects (working days in the month, holidays, Easter, and German shopping hours).

Some additional information was also provided: (a) in July 90 the D-Mark was made the legal tender in easter Germany, (b) and (c) in January 93 and April 94 there were VAT increases, (d) in January 94 part of the reporting sample was new, (e) in January 95 there was a legal change concerning the declaration of acquired firms by companies, (f) in November 98 there was a special sales campaign by a large company, (g) special emphasis was

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given to the evolution of the Christmas Bonus (CB) during the period. The bonus, usually paid in November, had been gradually decreasing, and eventually frozen in November 94.



Figure 1: The RT series

Besides general interest in the TRAMO-SEATS results, two points of special concern were made: 1) The treatment of different seasonal factor variability for different months (related to the CB effect). 2) The assessment of the current situation by means of the trend-cycle component (related to the effect of treating, or not, as an outlier an observation at the end of the series, namely, the November 98 sales campaign).

2. An X11-X12ARIMA-Type Approach

The two points of concern mentioned emerge from the X11-type treatment of the RT series by the Bundesbank (now X12ARIMA, Findley et al, 1998). In brief, the problem can be detected from the plot of the preliminary seasonalirregular (SI) factors versus the seasonal factors (S). Figure 2a presents the plot for the month of November when the Bundesbank standard (3×9) X11-seasonal filter is used.



Figure 2: SI versus S factors

Two things can be noticed: 1. Starting in 94, there seems to be a break in the pattern of the (SI)-(S) difference. 2. November 98 clearly departs from the new pattern, and could be considered an outlier. Consultation with experts provided an explanation for the new November pattern: the CB evolution mentioned above. According to this information, the change in 94 was of a permanent nature, leading to a more stable seasonal factor. Further, the November 98 sales campaign could justify treating the month as an outlier. The need to accommodate a break in the seasonal pattern, lead to the use of a more flexible (3×3) filter for the months of November and December. Figure 2b presents the (SI)-(S) plot, with the modification implemented: the systematic difference after 1994 has disappeared , and the November 98 factor appears to be, as before, an outlier, the estimated seasonal factors are shown in Figure 3a.

The November 98 correction was important because it had an impact on the trend-cycle at the end of the series. As Figure 4a shows, in one case, the series is experiencing explosive growth, in the other it seems to be approaching a minimum. Short-term extrapolation of the trend-cycle would lead to different cyclical implications (the trend-cycle we use is the one selected automatically by the program).



3. The Tramo-Seats Results

As a starting point, we use the automatic procedure which yields the "Airline model", in the logs and with no mean. The July 90 outlier, associated with German monetary reunification, is automatically detected as a Level Shift, with a t-value of 4.5. The model provides a good fit, and the first column of Table 1 summarizes the results.

Model	Default	Model 1	Model 2	Model 3
Parameter estimates:				
θ_1	689	694	692	690
θ_{12}	636	686	687	675
Outliers:				
LS 7/90	.053	.053	.053	.053
t-value	(4.47)	(4.58)	(4.57)	(4.54)
SLS 11-12/94	-	032	033	030
t-value		(-3.52)	(-3.58)	(-3.23)
LS 1/93	-	-	-	022
t-value				(1.86)
Residual statistics:				
BIC	-7.989	-8.014	-8.019	-8.010
SE (a_t) *100	1.796	1.759	1.754	1.747
$N(a_t)$	2.62	1.57	1.44	1.76
$Q_{24}(a_t)$	25.3	25.8	25.3	25.6
$Q_{\rm S}(a_{\rm t})$.78	.72	.76	.68
$Q_{24}(a_t^2)$	27.5	25.6	26.3	24.1

Table 1. Summary of TRAMO results. BIC denotes the Bayesian information criterion, and SE(a_t) the residual standard error, both should be as small as possible. N denotes the Bowman-Shenton test for normality, and is asymptotically distributed as a x_2^2 , it should be smaller than 6. $Q_{24}(a_t)$ denotes the Ljung-Box test for residual autocorrelation using the first 24 autocorrelations, and is asymptotically distributed as a χ_2 with (24-# of parameter estimates) degrees of freedom; for the Airline model it should be smaller than 34. $Q_{24}(a_t^2)$ is the McLeod-Li test for linearity, equal to the previous test, but computed on the squared residuals; it has the same asymptotic distribution as the Ljung-Box one. The N, $Q(a_t)$, and $Q(a_t^2)$ test are described in, for example, Harvey (1993). $Q_s(a_t)$ is a test for residual seasonal autocorrelation described in Pierce (1978); it is distributed approximately as a x_2^2 , and should be smaller than 6.

The estimated seasonal factors (Figure 3b) are seen to be close to the ones obtained with the modified X12A, but, as figure 2c shows, the (SI)-(S) plot still suggests a mild change in the pattern for the last years. It seems sensible to test for whether the CB evolution has produced a change in the level of the November-December seasonal factors. A way to do this is by means of the Seasonal Level Shift (SLS) outlier of Kaiser and Maravall (1999). Several specifications are possible and the BIC criterion lead to the one whereby the outlier (in the one-month case) is modelled as $\omega \nabla_{12}^{-1} d_{it}$, where $d_{it} = 1$ for the month when the outlier effect starts, and $d_{it} = 0$ otherwise. The SLS produces a correction in the level of the seasonal factors for the months that correspond to the outlier, the correction also has a small effect on the mean. Given that the effect affects both months, two SLS outliers were introduced for November and December 94. The parameter estimates were similar, setting them equal, a t-value of -3.52 is obtained, and overall results are improved. The results are presented in Figure 3c: they clearly reflect the gradual decrease that stabilizes in November 94. (The November and December seasonal factor correction could have been enforced in TRAMO-SEATS simply from the heteroscedasticity in the factor correction could have been enforced in 1999). The SLS effect on the seasonal component for the RT series is displayed in Figure 5.

Using the previous model, which already incorporates effects (a) and (g), we proceed to test for the significance of effects (b) to (f), introducing them as regression variables in TRAMO. Each variable is specified as an additive outlier (AO), a transitory change (TC), and a level shift (LS), and the most significant specification is chosen, the results are in Table 2 (the regression variables are basically orthogonal). In summary, with the (borderline)

exception of the January VAT increase, the other effects are clearly not significant. The fact that an event does not seem to produce a significant effect does not imply that there was no effect, but that its magnitude is not large enough to merit correction. Based on the sample evidence, to correct for the November 98 AO would be hard to justify. However, there is a presumably very precise independent expert estimation of the effect, equal to 1% of the level of the series for that month. Considering that the SD of the monthly innovation is 1.8%, it is understandable that a 1% effect is not significant. Be that as it may, given the reliability of the expert's estimate, the 1% November 98 effect can be directly applied to the series avoiding parameter estimation.

Event	(b)	(c)	(d)	(e)	(f)
t-value	-1.9	-0.1	-1.3	1.2	1.4
Specification	LS	LS	LS	LS	AO

Table 2. Special event effects.

In summary, three models seem worth comparing. All are obtained with the automatic TRAMO procedure, with the 2-month seasonal level shift of November 94 incorporated. This yields, in fact, Model 1. Adding the "ad-hoc" 1% November 98 correction, Model 2 is obtained. Model 3 also includes the January 93 VAT increase effect. In all three cases the Airline model was obtained, with the LS outlier for January 90 (monetary reunification). No additional outlier was detected. Writing the general model as

$$\begin{split} y_t &= \omega_1 \frac{1}{\nabla} d_{1t} + \omega_2 \frac{1}{\nabla_{12}} d_{2t}^{(2)} + \omega_3 d_{3t} + \omega_4 \frac{1}{\nabla} d_{4t} + x_t ,\\ \nabla \nabla_{12} x_t &= (1 + \theta_1 B) (1 + \theta_{12} B^{12}) a_t . \end{split}$$

The first equation specifies the outlier-intervention variables, that is, the deterministic part of the series, the second equation specifies the ARIMA model, that is, the stochastic part. The d-variables are such that $d_{1t} = 1$ for January 90 (monetary reunification), $d_{2t}^{(2)} = 1$ for November and December 94 (CB effect), $d_{3t} = 1$ for November 98 (sales campaign), $d_{4t} = 1$ for January 93 (VAT increase), and 0 otherwise. Model 1 sets $\omega_3 = \omega_4 = 0$, Model 2 sets $\omega_3 = .01$, $\omega_4 = 0$, and Model 3 sets $\omega_3 = .01$. The last 3 columns of Table 1 summarize the TRAMO results. Very marginally, the "ad-hoc" November 98 modification does more good than damage, while the VAT January 93 correction seems neutral. The closeness of the models is appreciated in Figure 6, which displays the 2-year ahead forecast function of the 3 models. (The purely automatic result, with no SLS, is also included: missing the CB correction has little effect on forecasts.)



Fitting criteria are not enough to clearly select a model. Given that the purpose of the application is seasonal adjustment perhaps differences in the way the series are decomposed can be of help. Table 3 presents some results from SEATS. First, the variances of the component innovations are displayed. Interest centers on a more stable seasonal component, thus we seek to minimize its innovation variance. Second, the percent reduction in the revision variance of the concurrent estimator, after one more year of data is presented. The next rows present the variances

of the concurrent estimator. Naturally, we would like fast convergence, small estimation error, and small revisions. The table shows how not including the CB correction produces a more unstable seasonal component, and SA series that are estimated with larger error and subject to larger revisions. Among the 3 models that include the CB correction, Model 3 performs marginally worse on practically all accounts. Adding the fact that it is less parsimonious, models 1 and 2 seem preferable.

The differences between these two models are, for all practical purposes, negligeable. Figure 7 compares the complete X12A (modified) and TRAMO- SEATS (Model 2) decomposition of the series. The two SA series are close, the SEATS trend-cycle is less noisy, the seasonal component more stable, and the irregular component more homocedastic. Figure 8 compares the trend-cycle of X12A (with all corrections enforced) and of Model 2 in SEATS. The short-term oscillations of the X12A trend-cycle around the SEATS trend are clearly discernible.

Model	Default	Model 1	Model 2	Model 3
SD of component innovation variance:				
• Trend	.230	.229	.230	.228
Seasonal	.387	.329	.326	.337
• Irregular	1.230	1.249	1.274	1.229
Convergence of concurrent estimator in				
1 year (% decrease in revision variance):				
• Trend	90	90	89	89
• SA series	36	31	31	32
SD of concurrent estimation error				
• Trend	.753	.742	.742	.739
• SA series	.756	.710	.706	.712
SD of revision in concurrent estimator				
• Trend	.568	.533	.552	.553
• SA series	.517	.488	.487	.488

Table 3. Summary of SEATS results. SD: Standard deviation, all are expressed in 10^{-2} (i.e., in percent points), and are obtained from the standarized variances provided by SEATS. To express them in the series units, they have been multiplied by the residual SD.

4. Assessment of the Current Trend-Cycle

As shown in Figure 4a, although the AO correction is small, the current evolution of the X12A trend-cycle component is markedly affected by the treatment of the November 98 observation. Figure 4b displays the SEATS trend-cycles produced by Models 1 and 2. The difference reflects the effect of the November 98 AO correction. The correction has a small effect, and the two trend-cycles show similar behavior at the end.

	Original Series	SA Series	Trend-Cycle
a) <u>Current measures</u> Month-to-month rate of growth	15.3	-1.9 (0.6)	0.0 (0.1)
Rate of growth for last 12 months	1.5	1.6	0.5
	-	(0.7)	(0.5)
Current rate of annual growth (centered in December 98 and using forecasts)	2.0	2.0	0.4
	(2.11)	(2.0)	(1.4)
b) <u>Forecasts</u> Monthly rate of growth for January 1999	-25.9 (1.7)	0.6 (2.2)	0.2 (1.7)
Rate of growth for the next 12 months	0.9	0.9	0.5
	(2.5)	(2.9)	(2.4)

Table 4. Rates of growth (in percent points), last observation: 12/98

To help analysis of the present evolution of the trend-cycle, SEATS offers two additional tools of interest: the standard error (SE) of the component estimator (as well as of its rates of growth) and its optimal forecast, with the associated SE. Table 4 presents, for the December 98 observation, the rates of growth of the original and SA series and of the trend-cycle. Three rates are considered: a) the (annualized) monthly rate-of-growth, b) the rate of growth for the last 12 months (December 98 versus December 97), c) the present rate of annual growth, centered in December 98 and measured using forecasts (i.e., June 99 versus June 98), SE of the rates-of-growth are given in parenthesis. First, the trend-cycle is seen to provide a more stable and more precise signal than the SA series. Second, the underlying current rate-of-growth of the series, measured with the trend-cycle, can be comfortably accepted as zero.

The table also contains forecasts of the rate-of-growth for the next month and for the next year (with SE), they indicate that the series may experience very mild growth is influenced by seasonality. Comparing the SA series with the trend-cycle, it is seen that, although non-trivial, the noise plays a second-order role.



Figure 7: X12 ARIMA and TRAMO-SEATS

If interest goes beyond present evolution of the trend-cycle, it is possible to apply SEATS to get an estimator of the business cycle along the lines of the Modified Hodrick-Prescott (MHP) filter of Kaiser and Maravall (1999b, 2000b). This is done, in essence, by extending the trend-cycle component with forecasts and backcasts, and using

the extended trend-cycle series as input to SEATS, run in the fixed model-based Hodrick-Prescott (HP) format. The MHP filter depends on a paramater, λ , that for quarterly series usually takes the value 1600 (see Prescott, 1986). We use $\lambda = 129000$, which (as shown in Del Rio and Maravall, 2000) is the monthly equivalent of the previous quarterly value. Compared to the standard HP filter applied to X11-SA series, the modified procedure provides a cleaner cyclical signal (Figure 9), improves end-point estimation and reduces revisions, and can be given a sensible model-based interpretation, based on which confidence intervals and forecasts can be computed. Figure 9a indicates that, in December 98, the series seemed to be slowly recovering from a mild recession.



Figure 8: Trend–Cycle comparison

A Comment on Ex-Post Corrections. The X11-Bundesbank approach relies heavily on careful analysis of the data using tools supplied by X11-X12A, that are unquestionably of help. Yet, the practice of ex-post corrections to the data can be dangerous. Every year many special events happen. Surely God could explain the world in a deterministic manner, we certainly cannot. In the limit we could possibly find ex-post explanations for any unexpected shock. In practice this is unfeasible, and that is why stochastic models were invented. Their basic assumption is that there are many unexpected shocks, that are better treated as random inputs. Having a proper model, we can then test for the significance of some specific event. Allowing for ex-post correction of non-significant effects cannot be universally recommended. First, ex-post, ad-hoc modifications will increase revisions in the series. Second, these modifications difficult transparency of the procedure. Further, they introduce an element of arbitrariness that could, in theory, foster data manipulation (i.e., correcting only the ones that are convenient).

4. Out-of-Sample Analysis

The series for the Bundesbank workshop covered the period January 1975 - December 1998. In order to look at the out-of-sample behavior of TRAMO-SEATS, we asked the Bundesbank for the more recent observations, but were informed that the series had been revised, and were provided with the new revised series, for the period January

1975 – February 2000. The revision is small though, for some periods it is not negligeable, this is particularly noticeable towards the end of the series. Two questions of interest are: a) would the results from TRAMO-SEATS have been different for the revised series? b) would the TRAMO-SEATS procedure have been stable over the 14 additional months?

For the new series, the purely automatic procedure provided again the Airline model and the LS outlier associated with monetary reunification. The results are very similar to those obtained for the old series, perhaps marginally better. Heteroscedasticity in the seasonal component is still noticeable, and introduction of the SLS for November and December, associated with the CB freezing, yields a highly significant effect, and improves results. The BIC and residual SE decrease, seasonality becomes more stable, is subject to smaller revisions, and estimated with more precision. Adding the November 1998 effect, as the 1% ad-hoc correction, has a very small (positive) effect. Testing for the other special effects yields similar results and, as before, the specification of Model 1 or 2 for the 14 additional periods, with the parameters fixed at their December 1998 value, and the implied forecast errors. The figure evidences the good out-of-sample behavior of the forecast. In summary, the selected TRAMO-SEATS procedure is unaffected by the series revision.





Using Model 2 on the new series, we apply the routine procedure recommended in Gómez and Maravall (1998). This implies fixing the (0,1,1) $(0,1,1)_{12}$ –in the logs and with no mean-specification, maintaining the LS outlier (associated with the monetary reunification), the SLS outlier (associated with the freezing of the CB), and the "adhoc" AO (associated with the November 1998 campaign). Every month, the model parameters are reestimated and, after the additional 14 months, the model is reidentified. The 1-period-ahead forecasts are very similar to those of Figure 10, and none of the forecast errors is cause for alarm. The estimates of the ARIMA parameters and of the two regression variable coefficients remain practically unchanged, and no new outliers are detected. Reidentification of the model after the 14 months have became available replicates the arguments of Section 3.

Tables 5 and 6 summarize the TRAMO-SEATS results, and suggest again Model 2 or Model 1 as the best choice.

Model	Default	Model 1	Model 2	Model 3
Parameter estimates:				
θ_1	685	682	681	686
θ_{12}	633	685	686	676
Outliers:				
LS 7/90	.053	0.53	.053	.052
t-value	(4.52)	(4.55)	(4.55)	(4.59)
SLS 11-12/94	-	033	033	030
t-value		(-3.63)	(-3.63)	(-3.34)
LS 1/93	-	-	-	022
t-value				(-1.94)
Residual statistics:				
BIC	-8.031	-8.058	-8.063	-8.055
SE (a_t) *100	1.760	1.723	1.719	1.711
$N(a_t)$	2.86	1.43	1.28	1.63
$Q_{24}(a_t)$	26.3	26.6	25.6	25.5
$Q_{s}(a_{t})$	1.06	1.24	1.30	.67
$\tilde{Q}_{24}(a_t^2)$	28.0	29.7	30.5	28.4

Model	Default	Model 1	Model 2	Model 3
SD of component innovation variance:				
• Trend	.228	.233	.233	.227
Seasonal	.380	.319	.317	.327
• Irregular	1.201	1.214	1.197	1.202
Convergence of concurrent estimator in				
1 year (% decrease in revision variance):				
• Trend	89	89	89	89
• SA series	36	31	31	32
SD of concurrent estimation error				
• Trend	.741	.737	.735	.728
• SA series	.743	.696	.692	.697
SD of revision in concurrent estimator				
• Trend	.559	.548	.546	.541
• SA series	.507	.478	.477	.478

 Table 5. Summary of TRAMO results. Extended period (see Table 1).

 Table 6. Summary of SEATS results. Extended period. (see Table 3)

Comparison of these two tables with Tables 1 and 3 shows that the TRAMO-SEATS procedure is robust with respect to moderate revisions in the series and that it is stable over the out-of-sample period considered. It is of interest to notice that, although the differences are small, the relative ranking of the models remains unchanged. One important point related to the stability of the TRAMO-SEATS procedure concerns the convergence of the concurrent estimator to the historical one. We started with the series ending in 1993, and compared the sequence of estimators for the year 1992 and 1993, as more years of data are added. In agreement with the SEATS message, the revision is small, although it takes about 5 years for the SA series to converge.

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