A note on frame transformations with applications to geodetic datums

Tomás Soler & John Marshall
National Geodetic Survey, NOS, NOAA
N/NGS22, #8825
1315 East-West Highway
Silver Spring, MD 20910-3282, USA
Tom.Soler@noaa.gov
Tel: +1-301-7133205 ext. 157
Fax: +1-301-7134324

Abstract. Rigorous equations in compact symbolic matrix notation are introduced to transform coordinates and velocities between ITRF frames and modern GPS-based geocentric geodetic datums. The theory is general but after neglecting higher than second-order terms it is shown that the equations revert to the formulation currently applied in most major continental datums. We discuss several examples: the North American Datum of 1983 (NAD 83), European Terrestrial Reference System of 1989 (ETRS89), Geodetic Datum of Australia of 1994 (GDA94), and the South American Geocentric Reference System (SIRGAS).

Introduction

Modern-day frame transformations have become increasingly complex to better accommodate time-dependent processes such as plate tectonics and other geophysical phenomena. In fact, many modern frame transformations extend the classical 7-parameter Helmert transformation to complex 14-parameter formulations, which augment the original 7 parameters with their time derivatives. In practice, this augmented formulation is used to transform GPS densification results from one epoch to another epoch. Although the treatment of 14-parameter transformations between geocentric terrestrial reference frames has been published in geodetic literature with various degrees of rigor (e.g. Soler 1998; Sillard et al. 1998; Boucher et al. 1999; Altamimi et al. 2002; Soler and Marshall 2002), very little attention has been devoted to the extension of this formulation, in a straight didactical manner, to the geodetic datum problem. This article complements the theory previously given in Soler and Marshall (2002) and provides a direct practical solution to the transformation between geocentric frames and geodetic datums; we focus on four major continental datums currently in use. The discussion presented here is intended to clear up the prevalent confusion about rotation of vectors (rigid body rotation) used in plate kinematics vs. rotation of coordinate frames when both operations are applied in the same transformation.

Theoretical concepts

In Soler and Marshall (2002) the general formulation to transform coordinates between two arbitrary frames was given. Nevertheless, higher than second order terms were neglected. Although the contribution of many of these terms can be ignored, some readers are interested in the most rigorous approach possible and a complete set of equations is presented here. The mapping of our transformation is denoted $ITRF00(t_D) \rightarrow ITRFyy(t_D)$ where t_D denotes an epoch associated with a datum and ITRF is an abbreviation for International Terrestrial Reference Frame. In this particular case, the equation to transform coordinates given in the ITRF00 frame to the ITRFyy frame under the condition that frame ITRF00 is not changing with time, and that the coordinates of the stations are fixed in space (no velocities are involved) may be written in compact matrix form as the well-known classical Helmert (similarity) transformation:

$$\{x(t_D)\}_{ITRFyy} = \{T_x\} + (1+s)[\delta\Re]\{x(t_D)\}_{ITRF00} \tag{1}$$

where the differential rotation matrix denoted by $[\delta\Re]$ in the above equation is given explicitly by:

$$[\delta\Re] = [I] + \underline{[\varepsilon]}^t = [I] + \begin{bmatrix} 0 & \varepsilon_z & -\varepsilon_y \\ -\varepsilon_z & 0 & \varepsilon_x \\ \varepsilon_y & -\varepsilon_x & 0 \end{bmatrix}. \tag{2}$$

The superscript *t* stands for transpose, [*I*] denotes the 3×3 unit matrix, and $\underline{[\varepsilon]}^t$ is a skew-symmetric (anti-symmetric) matrix containing the rotation parameters. To complete the description of Eq. (1), it should be mentioned that all 3×3 matrices are represented between brackets, 3×1 column vectors between braces, and scalars between parentheses. Equation (1) is consistent with counter-clockwise (anti-clockwise) rotations of the axes *x*, *y*, and *z* by angular amounts $\varepsilon_x, \varepsilon_y$, and ε_z (expressed in radians), respectively. The seven parameters involved in Eq. (1) are the standard Helmert transformation parameters (three shifts, T_x, T_y, and T_z; three differential rotations, $\varepsilon_x, \varepsilon_y$, and ε_z; and one differential scale change *s*), all of them given in the sense ITRF00 to ITRFyy at epoch t_D which is common for the two sets of coordinates.

Assume now that the coordinates on frame ITRF00 are moving at a certain rate in space with respect to this frame that remains fixed. In other words, we know the coordinates and attached linear velocities at some arbitrary epoch *t*. Clearly, in this case, the Helmert transformation takes the form:

$$\{x(t_D)\}_{ITRFyy} = \{T_x\} + (1+s)[\delta\Re]\big\{\{x(t)\}_{ITRF00} + (t_D - t)\{v_x\}_{ITRF00}\big\} \tag{3}$$

The implicit transformation in the above equation is the mapping $ITRF00(t) \rightarrow ITRFyy(t_D)$ where now *t* denotes the epoch of the initial coordinates (*t* could be the actual time of observation of a GPS survey, e.g. 2003.3254), and t_D is the epoch of the final coordinates (e.g 1995.4000). Notice that in this compact notation the vector of coordinates is abbreviated by $\{x(t)\} = \{x(t) \;\; y(t) \;\; z(t)\}^t$; velocities by $\{v_x\} = \{v_x \;\; v_y \;\; v_z\}^t$, etc. The main reason to use the subscript *x* in some parameters is to indicate that the components refer to the *x*, *y*, *z* frame and not, for example, to the local geodetic frame, e.g. $\{v_e\} = \{v_e \;\; v_n \;\; v_u\}^t$ where *e*, *n*, and *u* denote the directions toward local east, north, and up, respectively.

Finally, assume that the Helmert parameters change with respect to time and that they are given at epoch t_k. In Eqs. (1) and (3) it was implicitly assumed that the Helmert parameters were given at epoch t_D. In general these parameters will be defined at different epochs $t_k \neq t_D$ where t_k is the epoch at which the Helmert transformation parameters are given (e.g. t_k = 2002.5000). Consequently, it is necessary to update the so-called Helmert parameters from epoch t_k to epoch t_D using the equations:

$$\{T_x\} \equiv \{T_x(t_D)\} = \{T_x(t_k)\} + (t_D - t_k)\{\dot{T}_x\} \tag{4}$$

$$\underline{[\varepsilon]}^t \equiv \underline{[\varepsilon(t_D)]}^t = \underline{[\varepsilon(t_k)]}^t + (t_D - t_k)\underline{[\dot{\varepsilon}]}^t \tag{5}$$

$$s \equiv s(t_D) = s(t_k) + (t_D - t_k)\dot{s} \tag{6}$$

where dotted parameters indicate time derivatives. The derivative of e.g. $\underline{[\varepsilon]}^t$ with respect to time ($d\underline{[\varepsilon]}^t / dt$) takes the form:

$$\underline{[\dot{\varepsilon}]}^t = \begin{bmatrix} 0 & \dot{\varepsilon}_z & -\dot{\varepsilon}_y \\ -\dot{\varepsilon}_z & 0 & \dot{\varepsilon}_x \\ \dot{\varepsilon}_y & -\dot{\varepsilon}_x & 0 \end{bmatrix}. \tag{7}$$

Substituting Eqs. (4)-(6) into (3), and grouping terms, we arrive at:

$$\begin{aligned} \{x(t_D)\}_{ITRFyy} &= \{T_x(t_k)\} + (t_D - t_k)\{\dot{T}_x\} \\ &+ [\![(1+s(t_k))[\delta\Re] + (t_D - t_k)[\![(1+s(t_k))\underline{[\dot{\varepsilon}]}^t + \dot{s}[\delta\Re]]\!] + (t_D - t_k)^2 \dot{s}\underline{[\dot{\varepsilon}]}^t]\!] \\ &\times \left\{ \{x(t)\}_{ITRF00} + (t_D - t)\{v_x\}_{ITRF00} \right\} \end{aligned} \tag{8}$$

where, from now on, $[\delta\Re] = [I] + \underline{[\varepsilon(t_k)]}^t$.

In contrast with the formulation previously published in Soler and Marshall (2002), Eq. (8) is complete and contains second and third-order terms.

Eq. (1) is consistent with counter-clockwise rotation of frame axes. However, when applying body (vector) rotations such as in the case of plate kinematics, the skew-symmetric matrix involved will be opposite in sign (Soler 1998). In this particular instance the axes of the frame remain fixed while the position vectors (coordinates) are rotated counter-clockwise about a fixed line going through the origin of the coordinate frame. This body rotation about an arbitrary line is termed "Euler rotation" by some authors and "active rotation" by others and is the type of rotation implemented when the theory of plate tectonics is discussed.

Thus, matrices of the form:

$$\underline{[\dot{\Omega}]} = \begin{bmatrix} 0 & -\dot{\Omega}_z & \dot{\Omega}_y \\ \dot{\Omega}_z & 0 & -\dot{\Omega}_x \\ -\dot{\Omega}_y & \dot{\Omega}_x & 0 \end{bmatrix} \tag{9}$$

called the angular rotation matrix appear and are common in plate tectonic kinematics literature. More about this will be discussed later.

Notice that 14 transformation parameters relating frames ITRF00 and ITRFyy are required in Eq. (8). Seven are the standard Helmert transformation parameters, and the remaining seven parameters are their variations with respect to time.

Continuing with the same compact matrix notation, it is possible to write the transformation of velocities from frame ITRF00 to frame ITRFyy by simply taking the derivative of Eq. (8) with respect to t_D:

$$\begin{aligned} \{v_x\}_{ITRFyy} = \{\dot{T}_x\} &+ [\![(1+s(t_k))\underline{[\dot{\varepsilon}]}^t + \dot{s}[\delta\Re]]\!]\{x(t)\}_{ITRF00} \\ &+ [\![(1+s(t_k))[\delta\Re] + (2t_D-(t_k+t))[(1+s(t_k))\underline{[\dot{\varepsilon}]}^t + \dot{s}[\delta\Re]]]\!]\{v_x\}_{ITRF00} \\ &+ \dot{s}\underline{[\dot{\varepsilon}]}^t\left\{2(t_D-t_k)\{x(t)\}_{ITRF00} + (3t_D^2 + t_k^2 - 4t_Dt_k + 2t(t_k-t_D))\{v_x\}_{ITRF00}\right\} \end{aligned} \tag{10}$$

Again, Eq. (10) is complete, and no terms are neglected as previously done in Soler and Marshall (2002). Therefore, knowing the position and velocities of a point at the epoch of observation *t* on frame ITRF00, the seven Helmert parameters between frames ITRF00 and ITRFyy at epoch t_k and their seven rates, the coordinates and velocities of a point on frame ITRFyy at epoch t_D can be computed using Eqs. (8) and (10).

The datum problem

In the classical sense, a geodetic datum is a reference surface, generally an ellipsoid of revolution of adopted size and shape, with origin, orientation, and scale defined by a geocentric terrestrial frame. Once an ellipsoid is selected, coordinates of a point in space can be given in Cartesian or geodetic (curvilinear) coordinates (geodetic longitude, latitude, and ellipsoid height). Geodetic coordinates are preferred in cartographic and mapping applications.

Furthermore, the classical concept of geodetic datum implies that a datum's coordinates are fixed and do not change with time except for the effect of local tectonic motions (episodic motions, land subsidence, volcanic activity, etc.). Thus, the coordinate frame of a geodetic datum should be somewhat attached to the plate and move with it in such a way that the coordinates of the points will not change as a consequence of plate rotation. However, in actuality, the reverse process is implemented; that is, the coordinate frame is fixed to the Earth's mantle while the plates are rotated to their original position at epoch t_D (the datum epoch). This is achieved by applying the same type of correction at every point. The magnitude of this correction is determined through the angular velocity matrix associated with the continental plate where the points are located. In essence, all points are moved back to their location at epoch t_D on the frame ITRFyy which, in our example, is assumed to be the adopted datum frame and, by definition, remains fixed. In other words, the plate and the points on it are assumed frozen in space at the epoch when the datum frame was defined; all coordinates determined at epoch *t* should be taken back to epoch t_D, the datum epoch.

One way to apply plate motions in Eq. (8) is to replace the individual velocity of each point --which generally is unknown-- by the velocity generated by the rotation of the plate. It is known that the velocity of a point on a plate can be computed by:

$$\{v_x\}_{ITRF00} = \underline{[\dot{\Omega}]}\{x(t)\}_{ITRF00} \tag{11}$$

where the matrix $\underline{[\dot{\Omega}]}$ is given explicitly in Eq. (9) and contains the components of the counter-clockwise rotation of magnitude $\dot{\Omega}$ about a rotation axis, defined by a line with spherical longitude λ and spherical latitude ϕ:

$$\lambda = \arctan\frac{\dot{\Omega}_y}{\dot{\Omega}_x}; \quad 0 \le \lambda \le 2\pi \tag{12}$$

$$\phi = \arctan\frac{\dot{\Omega}_z}{\sqrt{\dot{\Omega}_x^2 + \dot{\Omega}_y^2}}; \quad -\frac{\pi}{2} \le \phi \le \frac{\pi}{2} \tag{13}$$

Values of $\dot{\Omega}_x, \dot{\Omega}_y$, and $\dot{\Omega}_z$ for the major tectonic plates are given in literature by various authors. For example, McCarthy (1996, p. 14) tabulates the components of the angular rotation vector $\dot{\Omega}$ corresponding to the global geologic model (no net rotation) NNR-NUVEL1A. More recently, a new model called REVEL based primarily on GPS observations has been published (Sella et al. 2002). Alternatively, new values of $\dot{\Omega}$, λ, and ϕ for the Pacific, North American, and Australian plates determined through a combination of geodetic space techniques were published by Beavan et al. (2003). In this latter case the components of $\dot{\Omega}$ required in Eq. (9) are easily computed from

$$\{\dot{\Omega}_x\} = \dot{\Omega}\{\ell_x\} = \dot{\Omega}\begin{Bmatrix}\cos\phi\cos\lambda \\ \cos\phi\sin\lambda \\ \sin\phi\end{Bmatrix} \tag{14}$$

where $\{\dot{\Omega}_x\} = \{\dot{\Omega}_x \;\; \dot{\Omega}_y \;\; \dot{\Omega}_z\}^t$. The column vector $\{\ell_x\}$ contains the direction cosines of the so-called "Euler rotation axis." See Appendix A for a clarification between rotation of vectors (active rotation) and rotation of frames (passive rotation).

In the datum problem we need to rotate the plate backwards to the datum epoch, thus, we should insert Eq. (11) with $\underline{[\dot{\Omega}]}$ replaced by $\underline{[\dot{\Omega}]}^t$ into Eq. (8). The transformation of coordinates between the frame ITRF00 and a datum frame based on the ITRFyy frame (denoted DITRFyy, where the D stands for "datum") considering the plate angular velocity matrix takes the form:

$$
\begin{aligned}
\{x(t_D)\}_{DITRFyy} &= \{T_x(t_k)\} + (t_D - t_k)\{\dot{T}_x\} \\
&\quad + [\![(1+s(t_k))[\delta\Re] + (t_D - t_k)[(1+s(t_k))\underline{[\dot{\varepsilon}]}^t + \dot{s}[\delta\Re] + (t_D - t_k)^2\, \dot{s}\,\underline{[\dot{\varepsilon}]}^t]]\!] \\
&\quad \times [\![[I] + (t_D - t)\underline{[\dot{\Omega}]}^t]\!] \{x(t)\}_{ITRF00}
\end{aligned} \tag{15}
$$

The above equation permits the transformation of coordinates from epoch t (the epoch of the GPS observations) referred to the GPS orbit frame used during processing (currently ITRF00$\equiv$IGS 2000) to coordinates referred to a predefined geodetic datum frame denoted as DITRFyy. This notation implies that the adopted terrestrial reference frame datum at epoch t_D is ITRFyy. Also needed are the 14 parameters of the transformation ITRF00$\rightarrow$ITRFyy assumed to be known at epoch t_k and the matrix $\underline{[\dot{\Omega}]}^t$ containing the components of the angular velocity $\dot{\Omega}$ of the tectonic plate spanning the datum in question.

The value of the velocities on the datum frame are obtained by differentiating Eq. (15) with respect to t_D:

$$
\begin{aligned}
\{v_x\}_{DITRFyy} &= \{\dot{T}_x\} + [\![(1+s(t_k))\underline{[\dot{\varepsilon}]}^t + \dot{s}[\delta\Re] + 2(t_D - t_k)\, \dot{s}\,\underline{[\dot{\varepsilon}]}^t]\!] \{x(t)\}_{ITRF00} \\
&\quad + [\![(1+s(t_k))[\delta\Re] + (2t_D - t - t_k)[(1+s(t_k))\underline{[\dot{\varepsilon}]}^t + \dot{s}[\delta\Re]]]\!] \underline{[\dot{\Omega}]}^t \{x(t)\}_{ITRF00} \\
&\quad + [\![(3t_D^2 + t_k^2 - 4t_D t_k - 2t_D t + t_k t)\dot{s}\,\underline{[\dot{\varepsilon}]}^t]\!] \underline{[\dot{\Omega}]}^t \{x(t)\}_{ITRF00}
\end{aligned} \tag{16}
$$

Keep in mind that for geodetic datums the plate is assumed fixed to the frame ITRFyy at epoch t_D. As a result, we substitute $\underline{[\dot{\Omega}]}^t = [0]$ and the velocities on the DITRFyy datum take the simplified form:

$$
\{v_x\}_{DITRFyy} = \{\dot{T}_x\} + [\![(1+s(t_k))\underline{[\dot{\varepsilon}]}^t + \dot{s}[\delta\Re] + 2(t_D - t_k)\, \dot{s}\,\underline{[\dot{\varepsilon}]}^t]\!] \{x(t)\}_{ITRF00} \tag{17}
$$

Neglecting second order terms we arrive at:

$$\{v_x\}_{DITRFyy} = \{\dot{T}_x\} + \llbracket \underline{[\dot{\varepsilon}]}^t + \dot{s}[I] \rrbracket \{x(t)\}_{ITRF\,00} \tag{18}$$

Note that the velocity of a point on the datum frame DITRFyy in Eq. (18) is independent of t_k and the seven standard Helmert transformation parameters; however, the velocity of a point is not independent of the rates. Thus, in the present GPS technological era, there are small datum point velocities due to the changes with time of the Helmert parameters. Because in the old classical datums these rates were ignored, the velocities referred to the datum were exactly zero and the points did not physically move on the datum from epoch to epoch. Ignoring these rates is not totally rigorous for accurate modern GPS-determined datums.

Assuming that the Helmert parameters are given at epoch $t_k = t$, Eq. (15) takes the form:

$$\begin{aligned}
\{x(t_D)\}_{DITRFyy} &= \{T_x(t_k)\} + (1+s(t_k))[\delta\Re]\{x(t)\}_{ITRF\,00} \\
&+ (t_D - t)\Big\{\{\dot{T}_x\} + \llbracket (1+s(t_k))\underline{[\dot{\varepsilon}]}^t + \dot{s}[\delta\Re] + (t_D - t)\dot{s}\underline{[\dot{\varepsilon}]}^t \rrbracket \{x(t)\}_{ITRF\,00}\Big\} \\
&+ (t_D - t)\llbracket (1+s(t_k))[\delta\Re] + (t_D - t)\llbracket (1+s(t_k))\underline{[\dot{\varepsilon}]}^t + \dot{s}[\delta\Re] \rrbracket + (t_D - t)^2\dot{s}\underline{[\dot{\varepsilon}]}^t \rrbracket \underline{[\dot{\Omega}]}^t \{x(t)\}_{ITRF\,00}
\end{aligned}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots(19)$$

Finally, neglecting second and higher order terms we arrive at:

$$\begin{aligned}
\{x(t_D)\}_{DITRFyy} &= \{T_x(t_k)\} + (1+s(t_k))[\delta\Re]\{x(t)\}_{ITRF\,00} \\
&+ (t_D - t)\Big\{\{\dot{T}_x\} + \llbracket \underline{[\dot{\Omega}]}^t + \underline{[\dot{\varepsilon}]}^t + \dot{s}[I] \rrbracket \{x(t)\}_{ITRF\,00}\Big\}
\end{aligned} \tag{20}$$

This is the simplified equation currently used in most datum transformations.

In principle, the elements of the matrix $\underline{[\dot{\Omega}]}^t$ could be arbitrarily selected; however, it makes sense to use values which are closely related to the motion of the plate spanning the area on which the geodetic datum is defined. Consequently, every datum definition involves the adoption of a plate model.

Our unified convention assumes positive counter-clockwise rotation of coordinate axes and vectors. We can write explicitly:

$$
\begin{aligned}
\underline{[\dot{\Omega}]}^t + \underline{[\dot{\varepsilon}]}^t &= \begin{bmatrix} 0 & \dot{\Omega}_z & -\dot{\Omega}_y \\ -\dot{\Omega}_z & 0 & \dot{\Omega}_x \\ \dot{\Omega}_y & -\dot{\Omega}_x & 0 \end{bmatrix} + \begin{bmatrix} 0 & \dot{\varepsilon}_z & -\dot{\varepsilon}_y \\ -\dot{\varepsilon}_z & 0 & \dot{\varepsilon}_x \\ \dot{\varepsilon}_y & -\dot{\varepsilon}_x & 0 \end{bmatrix} \\
&= \begin{bmatrix} 0 & (\dot{\Omega}_z + \dot{\varepsilon}_z) & -(\dot{\Omega}_y + \dot{\varepsilon}_y) \\ -(\dot{\Omega}_z + \dot{\varepsilon}_z) & 0 & (\dot{\Omega}_x + \dot{\varepsilon}_x) \\ (\dot{\Omega}_y + \dot{\varepsilon}_y) & -(\dot{\Omega}_x + \dot{\varepsilon}_x) & 0 \end{bmatrix} = \underline{[\dot{\omega}]}^t
\end{aligned} \tag{21}
$$

Another simplified form of Eq. (15) is:

$$
\begin{aligned}
\{x(t_D)\}_{DITRFyy} &= \{T_x(t_k)\} + (1+s(t_k))[\delta\Re]\{x(t)\}_{ITRF\,00} \\
&\quad + (t_D - t_k)\big\{\{\dot{T}_x\} + [\![\underline{[\dot{\varepsilon}]}^t + \dot{s}[I]]\!]\{x(t)\}_{ITRF\,00}\big\} + (t_D - t)\underline{[\dot{\Omega}]}^t\{x(t)\}_{ITRF\,00}
\end{aligned} \tag{22}
$$

where now the Helmert transformation parameters are given at arbitrary epoch t_k.

The above equation is equivalent to:

$$
\begin{aligned}
\{x(t_D)\}_{DITRFyy} &= \{T_x(t_k)\} + (1+s(t_k))[\delta\Re]\{x(t)\}_{ITRF\,00} \\
&\quad + (t_D - t_k)\{v_x(t_D)\}_{DITRFyy} + (t_D - t)\underline{[\dot{\Omega}]}^t\{x(t)\}_{ITRF\,00}
\end{aligned} \tag{23}
$$

which is now written in the form adopted for transforming GPS positions to the European Terrestrial Reference System of 1989 (Boucher and Altamimi 2001).

Equations (22) and (17) could be implemented for transforming coordinates and velocities between the ITRF00 frame at the epoch of observation t and any geodetic datum. The required input includes the standard Helmert parameters of the transformation ($T_x, T_y, T_z,\ \varepsilon_x, \varepsilon_y, \varepsilon_z$, s) given at epoch t_k, and their rates ($\dot{T}_x, \dot{T}_y, \dot{T}_z,\ \dot{\varepsilon}_x, \dot{\varepsilon}_y, \dot{\varepsilon}_z,\ \dot{s}$), and the angular velocity components ($\dot{\Omega}_x, \dot{\Omega}_y, \dot{\Omega}_z$) of a pre-specified plate model or an equivalent set of input.

Brief review of continental datums

An important consequence of the recent GPS technological revolution is the establishment, at the highest level of accuracy, of a number of geocentric geodetic datums of continental extent. Next, we will briefly discuss the four major geodetic datums currently in place.

The North American Datum of 1983 (NAD 83) is used everywhere in North America except Mexico. This datum is realized in the conterminous United States and Alaska (North American Plate) through the National CORS (Continuously Operating Reference Stations) which provides the basis for obtaining rigorous transformations between the ITRF series and NAD 83 as well as a myriad of scientific applications (Snay et al. 2002).

At this writing there are more than 330 National CORS sites participating in the network, and this number continuously grows with the addition of several new stations each month. The latest realization of NAD 83 is technically called NAD 83 (CORS96), epoch 2002.00, constituting the framework for the definition of the National Spatial Reference System (NSRS). In Canada NAD 83 is also monitored through the Canadian Active Control System. Thus, the two organizations responsible for monitoring and making changes to the NAD 83 are the National Geodetic Survey (NGS) and Natural Resources of Canada (NRCan). Mexico's Instituto Nacional de Estadística, Geografía, e Informática (INEGI), the Federal agency responsible for geodesy and mapping in the country, adopted the geocentric frame ITRF92, epoch 1988.0, as the basis for their datum definition. The realization of the datum is achieved through the Red Nacional Activa (RNA) a 14 station network of permanent GPS receivers (Hernández-Navarro 1992).

A multinational European scientific organization, EUREF (European Reference Frame), has developed an ambitious program in order to preserve, to the highest level of accuracy, the European datum. Currently, it is called ETRS89 (European Terrestrial Reference System of 1989) based on ITRF89, epoch 1989.0 and monitored by a network of about 150 permanent GPS trackers known as EPN (EUREF Permanent Network). More information and a bibliography about ETRS89 and EPN may be found on the web at http://epncb.oma.be.

The Geocentric Datum of Australia of 1994 (GDA94) is referred to the frame ITRF92, at epoch 1994.0 (Featherstone, 1996; Steed and Luton, 2000). GDA94 is controlled by the Australian Regional GPS Network (ARGN) which is presently comprised of a network of 15 GPS stations permanently tracking in Australia and its Territories, with the 10 stations in Australia known as the Australian Fiducial Network (AFN). The organization responsible for monitoring GDA94 is Geoscience Australia. More information related to GDA94 is available at: http://www.auslig.gov.au/geodesy/datums/gda.htm.

The South American Geocentric Reference System (SIRGAS) was established to support a unified geodetic and mapping frame for the South American continent. Most South American and Caribbean countries participated in this enterprise which was later extended to Central and North America. The adopted reference frame was ITRF94, epoch 1995.4. At this writing there is not an integrated continent-wide network of permanent GPS receivers that monitor and provide transformations from ITRF to SIRGAS. Two countries (Brazil and Argentina) have their own independent national CORS network established. However, periodic continental adjustments of the GPS observations on a continental scale are not performed at this time. There are several web pages with abundant information about SIRGAS, such as:
http://www.ibge.gov.br/home/geografia/geodesico/sirgas/principal.htm

The datum transformation equations developed in this paper (Eqs. (22) and (17)) could be specifically adapted to any of the above datums. In all equations that follow, as mentioned before, t is the actual time of the GPS observation. This epoch is, generally, the mean point of data time interval during which the GPS observations were collected. Recall that the coordinates $\{x(t)\}$ refer to the frame of the GPS orbit (ephemeris) used in the reduction of the observations. Currently, it is the IGS00 (International GPS Service 2000) frame. It should be stressed that the IGS00 ephemeris frame is not exactly equal to the ITRF00 frame. However, on a practical basis, both frames can be assumed equivalent. The present difference is one of the questions that the international organizations involved with generating GPS products should resolve as soon as possible to establish rigorous consistency of information required in GPS applications. For example, in current GPS differential positioning the orbit selected at the processing stage refers to the IGS frame; however, the position of the fixed (reference) station is generally updated to the epoch of observation using ITRF coordinates and velocities. In the present article we are not concerned with other possible satellite orbits available to the GPS user such as WGS84 (G1150) and GLONASS.

Transforming from ITRF00 to various geodetic datums

In this section, various forms of Eqs. (22) and (17) will be applied to each of the four specific geodetic datums discussed above. Table 2 contains all 14-parameters given at epoch t_k required for transforming GPS data from the currently available ephemeris frame (ITRF00$\approx$IGS00) at observation epoch t to each specific datum frame. Except for the case of NAD83, the values contained in the Table were adapted from Boucher and Altamimi (2000). As noted in Table 2, the tabulated values assume anti-clockwise rotations positive, which is the notation followed through this paper.

NAD 83(CORS96)

For NAD 83 at t_D =2002.0 (see Table 1), Eq. (22) takes the form:

$$\{x(2002.0)\}_{NAD83} = \{T_x(t_k)\} + (1+s(t_k))[\delta\mathfrak{R}]\{x(t)\}_{ITRF00}$$
$$+(2002.0-t_k)\Big\{\{\dot{T}_x\} + [\![\underline{[\dot{\varepsilon}]}^t + \dot{s}[I]]\!]\{x(t)\}_{ITRF00}\Big\} + (2002.0-t)\underline{[\dot{\Omega}]}^t\{x(t)\}_{ITRF00} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(24)$$

A simplified version of the above equation was adopted by NGS and NRCan and is currently used for transforming coordinates between ITRF00 and NAD 83. The explicit form of the equations is given in Soler and Snay (2003). Table 2 presents the values of the 14-transformation parameters as adopted by NGS and NRCan for the transformation between ITRF00 and the latest realization of NAD 83 known as NAD 83 (CORS96). As indicated in the table, t_k =1997.0.

The velocity on the NAD 83 can be written using the simplified form:

$$\{v_x\}_{NAD83} = \{\dot{T}_x\} + [\![\underline{[\dot{\varepsilon}]}^t + \dot{s}[I]]\!]\{x(t)\}_{ITRF00} \qquad (25)$$

If necessary, the coordinates on the NAD 83 at some other time, $t_c \neq 2002.0$, could be determined using the equation:

$$\{x(t_c)\}_{NAD83} = \{x(2002.0)\}_{NAD83} + (t_c - 2002.0)\{v_x\}_{NAD83} \tag{26}$$

Rigorously speaking, NAD 83 is not a geocentric datum because, as Table 2 shows, there is a shift of about 2 meters in the *y* component of $\{T_x\}$. This non-geocentricity results from the TRANSIT Doppler System observations used in the definition of the original NAD 83 datum (Schwarz 1989). The components of $\underline{[\dot{\Omega}]}^t$ for the North American plate required in Eq. (24) were extracted from the NNR-NUVEL1A global geologic model as given in McCarthy (1996, p. 14) (See Table 3).

ETRS89

Using the compact matrix notation introduced here, the equations to transform positions and velocities from the GPS derived ITRF00 frame to the European Terrestrial Reference System of 1989 (ETRS89) could be written for t_D =1989.0 as

$$\begin{aligned}\{x(1989.0)\}_{ETRS89} &= \{T_x(t_k)\} + (1+s(t_k))[\delta\mathfrak{R}]\{x(t)\}_{ITRF00} \\ &\quad + (1989.0-t_k)\big\{\{\dot{T}_x\} + [\![\underline{[\dot{\varepsilon}]}^t + \dot{s}[I]]\!]\{x(t)\}_{ITRF00}\big\} + (1989.0-t)\underline{[\dot{\Omega}]}^t\{x(t)\}_{ITRF00}\end{aligned}$$

……………………………(27)

$$\{v_x\}_{ETRS89} = \{\dot{T}_x\} + [\![\underline{[\dot{\varepsilon}]}^t + \dot{s}[I]]\!]\{x(t)\}_{ITRF00} \tag{28}$$

The 14 transformation parameters in the above equations are, by definition, those of the mapping $ITRF00 \rightarrow ITRF89$. They are given at epoch t_k =1988.0 in Table 2. The values of the elements of the matrix $\underline{[\dot{\Omega}]}^t$ for the Eurasian plate were originally extracted from the NNR-NUVEL1A model. If desired, equations similar to (26) could be written for datum ETRS89.

GDA94

The Geodetic Datum of Australia was defined at epoch t_D =1994.0 based on the terrestrial frame ITRF92. Positions and velocities on the GDA94 could be transformed according to the equations:

$$\begin{aligned}\{x(1994.0)\}_{GDA94} &= \{T_x(t_k)\} + (1+s(t_k))[\delta\mathfrak{R}]\{x(t)\}_{ITRF00} \\ &\quad + (1994.0-t_k)\big\{\{\dot{T}_x\} + [\![\underline{[\dot{\varepsilon}]}^t + \dot{s}[I]]\!]\{x(t)\}_{ITRF00}\big\} + (1994.0-t)\underline{[\dot{\Omega}]}^t\{x(t)\}_{ITRF00}\end{aligned}$$

………..……………..(29)

$$\{v_x\}_{GDA94} = \{\dot{T}_x\} + [\![\underline{[\dot{\varepsilon}]}^t + \dot{s}[I]]\!]\{x(t)\}_{ITRF00} \tag{30}$$

The 14 parameters of the transformation used in Eqs. (29) and (30) are given by the mapping $ITRF00 \rightarrow ITRF92$ and are known at epoch t_k =1988.0 (see Table 2). As far as the authors were able to determine, no values for the elements of $\underline{[\dot{\Omega}]}^t$ have been selected yet for GDA94, although it is assumed that they will follow those of the Australian plate according to some kinematic plate model.

SIRGAS

Transformations of positions and velocities from the GPS determined ITRF00 frame to the South American Geocentric Datum of epoch t_D =1995.4 could be determined by the equations:

$$\{x(1995.4)\}_{SIRGAS} = \{T_x(t_k)\} + (1+s(t_k))[\delta\mathfrak{R}]\{x(t)\}_{ITRF00}$$
$$+(1995.4-t_k)\Big\{\{\dot{T}_x\} + [\![\underline{[\dot{\varepsilon}]}^t + \dot{s}[I]]\!]\{x(t)\}_{ITRF00}\Big\} + (1995.4-t)\underline{[\dot{\Omega}]}^t\{x(t)\}_{ITRF00} \quad \text{……...…………………..(31)}$$

$$\{v_x\}_{SIRGAS} = \{\dot{T}_x\} + [\![\underline{[\dot{\varepsilon}]}^t + \dot{s}[I]]\!]\{x(t)\}_{ITRF00} \quad (32)$$

Table 2 tabulates the 14 parameters of the transformation to be used given by the mapping $ITRF00 \rightarrow ITRF94$, which are known at epoch t_k =1997.0. The values of the South American plate angular rotation matrix $\underline{[\dot{\Omega}]}^t$ were originally extracted from the NNR-NUVEL1A model, although more recent determinations of $\underline{[\dot{\Omega}]}^t$ are also suggested as possible alternatives.

Conclusions

It should be clear by the theory described herein that the realization of a geodetic datum using GPS observations involves two basic problems:

1) Accurate determination of ITRF coordinates using GPS methodology

2) Accurate transformation of these coordinates to the adopted datum frame at some specified epoch t_D

However, the problem is slightly more complicated due to the possible influence on the coordinates of local tectonic motions (earthquakes, volcanic uplift, subsidence, etc.) that were totally neglected in this general discussion. If one wants to provide accurate positions on both the ITRF and datum frames, episodic motions and other types of geophysical disturbances should be taken into consideration and corrected for. Every time an earthquake occurs, the GPS antennas and the ground marks near the epicenter move. It is important to model and/or correct for these displacements to account for the change in coordinates around the affected region. This situation is common along the San Andreas Fault and other areas spanning the western states of the United States. Consequently, NGS has developed a software package called Horizontal Time Dependent Positioning (HTDP) to transform ITRF positions to NAD 83(CORS96); it incorporates major earthquakes and other known local motions of geophysical origin. For more information about HTDP consult Snay (1999). At this time, it appears that NAD 83 is the only continental datum where local tectonic motions are modeled and compensated for during the transformation of positions from ITRFxx to the datum frame.

In view of inconvenient physical disturbances, it could be appropriate to ask if we should extrapolate the old concept of geodetic datum of continental extent into the present era of advanced GPS technology. Obviously, the definition of a continental datum is the only available alternative for large land masses such as the United States, Australia, the European Union, etc. However, thanks to GPS, in the near future each individual country of sizable extent (Mexico was the first example) will be able to independently establish their own CORS network and, as a byproduct, their own national geodetic datum. Using the available IGS ephemeris, accurate geocentric coordinates referred to the latest ITRFxx frame could be easily determined. Nevertheless, it is important to emphasize that individual National agencies responsible for archiving the GPS data should make these RINEX files available to the international scientific community, preferably through the Internet, to facilitate a large array of global investigations currently under way (sea level studies, ionospheric modeling, etc.). Using transformation equations such as the ones discussed in this paper, the coordinates from any GPS densification network referred to ITRFxx could be transformed to the regional reference datum and tailored to the particular plate and geophysical conditions of the country at hand. Finally, ITRFxx and datum coordinates should be disseminated for applications in all types of surveying, mapping, GIS, and cadastral applications. Datum coordinates at epoch t_D should then be used for producing maps. With the type of accuracies currently obtainable through GPS methodology, discontinuities along national borders of countries belonging to the same tectonic plate will not be larger that 1-2 cm.

One added difficulty is introduced in the case of large continents, such as South America, which contains political boundaries spanning several plates (Caribbean, South American, Pacific, Nazca, Antarctica, etc.). This situation requires, by necessity, the adoption of different values of $\underline{[\dot{\Omega}]}$, and thus, in essence, the definition of several datums. This is the case for the NAD 83 which implements the velocities of the Pacific plate for stations in Hawaii and the Mariana plate for stations in Guam. This problem is not so critical for countries whose borders are located within a single plate. However, as discussed above, even minor earthquakes can produce changes at the centimeter level to the coordinates of an established datum, and procedures must be available to correct for these changes if an accurate set of coordinates assigned to ground marks is intended at all times.

Equations of the type (22) also could be adapted to web-based utilities that supply the GPS user with positional coordinates through the Internet. For example, NGS developed OPUS (Online Positioning User Service) to provide interested GPS users positional coordinates via email in a timely fashion, usually within a few minutes. The output of the GPS processing includes ITRF as well as NAD 83 (CORS96) coordinates. Similar utilities were independently developed by other institutions such as the Australian Online GPS Processing Service (AUSPOS) (http://www.auslig.gov.au/geodesy/sgc/wwwgps/); Auto Gipsy (AG) developed at JPL (http://www.unavco.ucar.edu/data_support/processing /gipsy/auto_gipsy_info.html); and SCOUT (Scripps Coordinate Update Tool) developed by

Scripps Orbit and Permanent Array Center (SOPAC) (http://sopac.ucsd.edu/cgi-bin/SCOUT.cgi). Presently none of these latter utilities provide national datum coordinates.

In conclusion, rigorous equations to transform ITRF coordinates to current or future geodetic datums are introduced in this paper. The theory is general and applies to existing datums. Several datums are investigated including NAD 83, ETRS89, GDA94, and SIRGAS.

It should be understood that the main application of the datum concept belongs to the areas of surveying, cartography, and cadastral work. Accurate geodesy and geophysics should be primarily concerned with the instantaneous position of points in space and their time series behavior referred to geocentric frames of the IGS/ITRF type. The definition of a datum, on the other hand, requires coordinates of points whose positions remain constant in time, except when local tectonic motions disturb them. The datum definition allows surveyors, cartographers, and GIS experts to avoid dealing with changing spatial coordinates due to the ever present motion of the plates.

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APPENDIX A

A counter-clockwise active rotation of vectors (body rotation) by an angle $\dot{\Omega}$ about a line of direction cosines $\{\ell_x\}$ can be written in compact matrix notation as:

$$\mathbb{R}_\ell(\dot{\Omega}) = [I] + \sin\dot{\Omega}[\underline{\ell}] + (1-\cos\dot{\Omega})[\underline{\ell}]^2 \tag{A1}$$

This equation can be reduced by assuming a differential rotation $\dot{\Omega}$, substituting $\sin\dot{\Omega} \approx \dot{\Omega}$, $\cos\dot{\Omega} \approx 1$, and taking into consideration Eq. (14). The result is:

$$\delta\mathbb{R}_\ell(\dot{\Omega}) = [I] + [\underline{\dot{\Omega}}] \tag{A2}$$

It can be easily proved by simple geometric arguments that an active counter-clockwise rotation of vector coordinates is equivalent to a passive clockwise rotation of frames. However, if we assume that all rotations (active and passive) are positive when rotated in the counter-clockwise sense, a passive counter-clockwise rotation by an angle θ around the three frames *i* = 1,2,3, using the transpose of Eq. (A1) we can write:

$$R_i(\theta) = [I] + \sin\theta[\underline{\ell_i}]^t + (1-\cos\theta)[\underline{\ell_i}]^2 \tag{A3}$$

The fact that $[\underline{\ell_i}]^2$ is a symmetric matrix was taken into consideration to write Eq. (A3). The symbols $\ell_i; i = 1,2,3$ correspond to the three direction cosines of the three Cartesian axes, i.e., $\ell_1 = \{1 \quad 0 \quad 0\}^t$; $\ell_2 = \{0 \quad 1 \quad 0\}^t$; and $\ell_3 = \{0 \quad 0 \quad 1\}^t$. For example, if we want to obtain the standard counter-clockwise rotation about the first axis denoted as $R_1(\theta)$, it follows from Eq. (A3):

$$R_1(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \sin\theta \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} + (1-\cos\theta) \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

…………………(A4)

which reduces to the well-known definitions of counter-clockwise rotation around coordinate axes $R_i(\theta),\ i = 1,2,3$ prevalent on the geodetic literature (Kaula 1966, p. 13; Mueller 1969, p. 43). The reader should be reminded that the counter-clockwise positive sense for rotations and the basic rotation matrices described above are adopted by most scientists, and it is even assumed in the formulation presented in the set of IERS conventions when discussing rotation of frames due to precession, nutation, and polar motion.

APPENDIX B

The following partial derivatives based on Eqs. (8) and (10) are rigorous (no high order terms are neglected). This new set of partial derivatives should replace the original ones previously given in Soler and Marshall (2002) in order to have a consistent set of transformation equations, and accompanying partial derivatives, totally independent of any approximations. In our notation $\Im$ is associated with the functional relationship of Eq. (8); similarly, $\mathcal{A}$ is associated with Eq. (10).

$$[\delta\Re] = [I] + [\underline{\varepsilon(t_k)}]^t$$

$$\partial\Im / \partial\{x\} = (1+s(t_k))[\delta\Re] + (t_D - t_k)[\![(1+s(t_k))[\underline{\dot{\varepsilon}}]^t + \dot{s}[\delta\Re]]\!] + (t_D - t)^2 \dot{s}[\underline{\dot{\varepsilon}}]^t = [\partial x]$$

$$\partial\Im / \partial\{v_x\} = (t_D - t)[\![(1+s(t_k))[\delta\Re] + (t_D - t_k)[\![(1+s(t_k))[\underline{\dot{\varepsilon}}]^t + \dot{s}[\delta\Re]]\!] + (t_D - t)^2 \dot{s}[\underline{\dot{\varepsilon}}]^t]\!] = [\partial v_x]$$

$$\partial\Im / \partial\{T_x(t_k)\} = [I]$$

$$\partial\Im / \partial\{\varepsilon(t_k)\} = \big((1+s(t_k)) + (t_D - t_k)\dot{s}\big)[\![[\underline{x}] + (t_D - t)[\underline{v_x}]]\!] = [\partial\varepsilon]$$

$$\partial\Im / \partial s(t_k) = [\![[\delta\Re] + (t_D - t_k)[\underline{\dot{\varepsilon}}]^t]\!]\big\{\{x\} + (t_D - t)\{v_x\}\big\} = \{\partial s\}$$

$$\partial\Im / \partial\{\dot{T}_x\} = (t_D - t)[I]$$

$$\partial\Im / \partial\{\dot{\varepsilon}\} = \big((t_D - t_k)(1+s(t_k)) + (t_D - t_k)^2 \dot{s}\big)[\![[\underline{x}] + (t_D - t)[\underline{v_x}]]\!] = [\partial\dot{\varepsilon}]$$

$$\partial\Im / \partial\dot{s} = [\![(t_D - t_k)[\delta\Re] + (t_D - t)^2[\underline{\dot{\varepsilon}}]^t]\!]\big\{\{x\} + (t_D - t)\{v_x\}\big\} = \{\partial\dot{s}\}$$

The partials of the functional relationship (10) with respect to the 14 transformation parameters are:

$$\partial\mathcal{A} / \partial\{x\} = \partial[\partial x] / \partial t_D = (1+s(t_k))[\underline{\dot{\varepsilon}}]^t + \dot{s}[\delta\Re] + 2(t_D - t_k)\dot{s}[\underline{\dot{\varepsilon}}]^t = [\bar{\partial} x]$$

$$\begin{aligned}\partial\mathcal{A} / \partial\{v_x\} = \partial[\partial v_x] / \partial t_D = (1+s(t_k))[\delta\Re] + (2t_D - (t_k + t))[\![(1+s(t_k))[\underline{\dot{\varepsilon}}]^t + \dot{s}[\delta\Re]]\!] \\ + (3(t_D^2 + t_k^2) - 4t_D t_k + 2t(t_k - t_D))\dot{s}[\underline{\dot{\varepsilon}}]^t \qquad = [\bar{\partial} v_x]\end{aligned}$$

$$\partial\mathcal{A} / \partial\{T_x(t_k)\} = [0]$$

$$\partial\mathcal{A} / \partial\{\varepsilon(t_k)\} = \partial[\partial\varepsilon] / \partial t_D = (1+s(t_k))[v_x] + \dot{s}[\![[\underline{x}] + (2t_D - (t_k + t))[\underline{v_x}]]\!] = [\bar{\partial}\varepsilon]$$

$$\partial\mathcal{A} / \partial s(t_k) = \partial\{\partial s\} / \partial t_D = [\delta\Re]\{v_x\} + [\underline{\dot{\varepsilon}}]^t\big\{\{x\} + (2t_D - (t_k + t))\{v_x\}\big\} = \{\bar{\partial} s\}$$

$$\partial\mathcal{A} / \partial\{\dot{T}_x\} = [I]$$

$$\partial\mathcal{A}/\partial\{\dot{\varepsilon}\} = \partial[\partial\dot{\varepsilon}]/\partial t_D = (1+s(t_k))[\![\underline{[x]} + (2t_D - (t_k - t))\underline{[v_x]}]\!]$$
$$+\dot{s}[\![2(t_D - t_k)\underline{[x]} + (3t_D^2 + t_k^2 - 4t_D t_k + 2t(t_k - t_D))\underline{[v_x]}]\!] = [\bar{\partial}\dot{\varepsilon}]$$

$$\partial\mathcal{A}/\partial\dot{s} = \partial\{\partial\dot{s}\}/\partial t_D = [\delta\mathfrak{R}]\{\{x\} + (2t_D - (t_k + t))\{v_x\}\}$$
$$+\underline{[\dot{\varepsilon}]}^t\{2(t_D - t_k)\{x\} + (3t_D^2 + t_k^2 - 4t_D t_k + 2t(t_k - t_D))\{v_x\}\} = \{\bar{\partial}\dot{s}\}$$

Table 1. Transformations from ITRF00 to other major datum frames and datum epochs.

NAD83 (CORS96)	**ETRS89**	**GDA94**	**SIRGAS**
ITRF00→ NAD83 t_D =2002.00	ITRF00→ITRF89 t_D =1989.0	ITRF00→ITRF92 t_D =1994.0	ITRF00→ITRF94 t_D =1995.4

Table 2. Transformation parameters and their rates from ITRF00 to other frames.

ITRF00→		NAD83	ITRF89	ITRF92	ITRF94
		t_k =1997.0	t_k =1988.0	t_k =1988.0	t_k =1997.0
T_x	cm	99.56	2.97	1.47	0.67
T_y		-190.13	4.75	1.35	0.61
T_z		-52.15	-7.39	-1.39	-1.85
ε_x	mas	25.915	0.00	0.00	0.00
ε_y		9.426	0.00	0.00	0.00
ε_z		11.599	0.18	0.18	0.00
s	ppb	0.62	5.85	0.75	1.55
$\dot{T}_x$	cm/y	0.07	0.00	0.00	0.00
$\dot{T}_y$		-0.07	-0.06	-0.06	-0.06
$\dot{T}_z$		0.05	-0.14	-0.14	-0.14
$\dot{\varepsilon}_x$	mas/y	0.013	0.00	0.00	0.00
$\dot{\varepsilon}_y$		-0.015	0.00	0.00	0.00
$\dot{\varepsilon}_z$		-0.020	-0.02	-0.02	-0.02
$\dot{s}$	ppb/y	-0.18	0.01	0.01	0.01

Note: All rotations are given counterclockwise positive; mas = milliarc second; ppb = parts per billion ≡ 10^{-6} ppm.

Table 3. Angular velocity components according to two models for the four plates spanning the major continental datums.

PLATE		NORTH AMERICA	EURASIA	AUSTRALIA	SOUTH AMERICA
ANGULAR VELOCITY	MODEL	mas/y	mas/y	mas/y	mas/y
$\dot{\Omega}_x$	NNR-NUVEL1A	0.0532	-0.2023	1.6169	-0.2141
	REVEL	0.1358	-0.1030	1.4534	-0.2454
$\dot{\Omega}_y$	NNR-NUVEL1A	-0.7423	-0.4940	1.0569	-0.3125
	REVEL	-0.7036	-0.4763	1.1463	-0.2422
$\dot{\Omega}_z$	NNR-NUVEL1A	-0.0316	0.6503	1.2957	-0.1794
	REVEL	-0.0299	0.7882	1.2893	-0.1669

Note: The signs of the rotations are consistent with counter-clockwise positive rotation of vectors. Some of the differences noted above maybe due to the fact that REVEL gives rotations relative to ITRF97 whereas NNR-NUVEL1A the rotations are given with respect to a "no-net-rotation" frame.