Viscous hydrodynamics in different Israel-Stewart formalisms

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References:

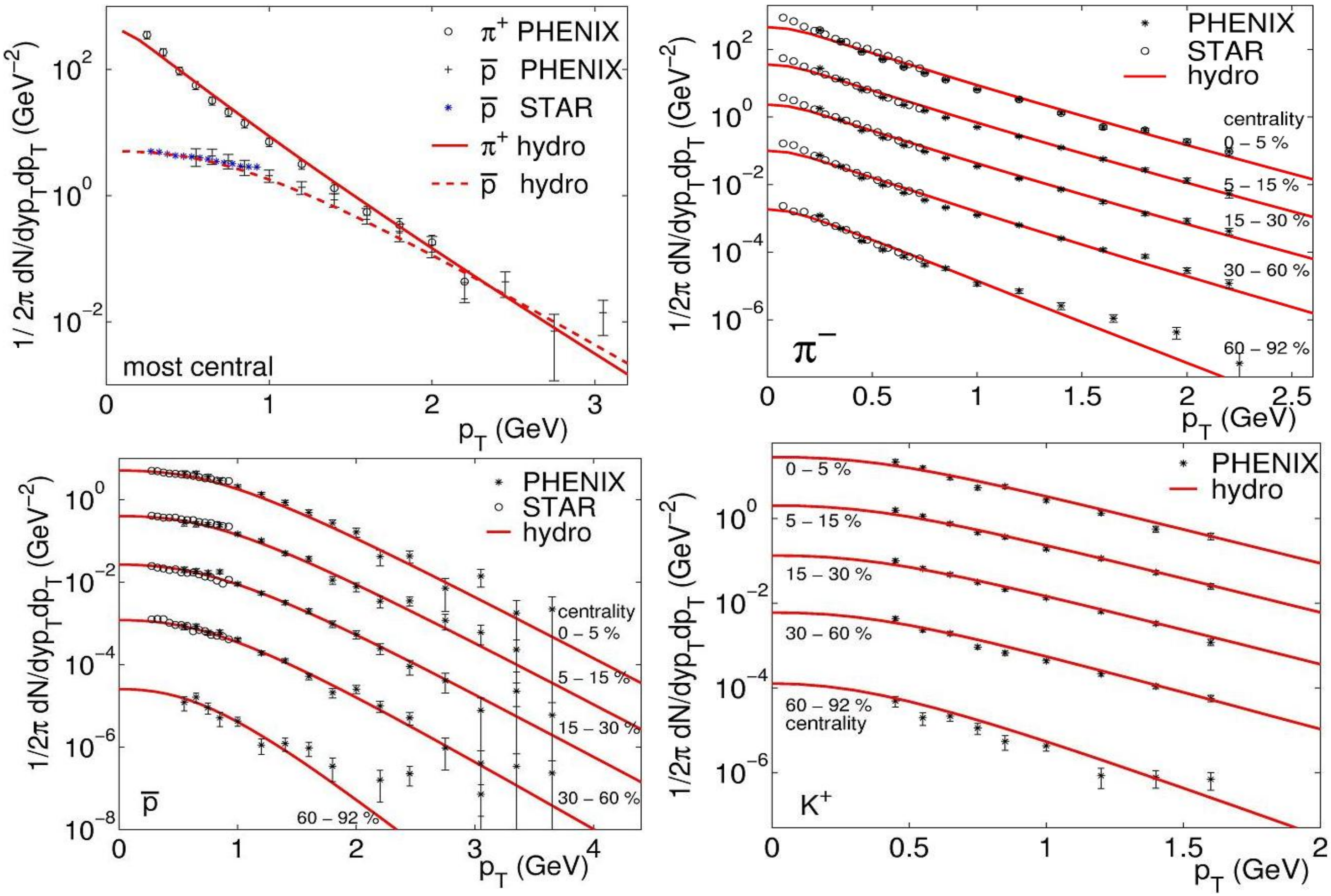
U. Heinz, H. Song and A. K. Chaudhuri, Phys. Rev. C73, 034904 (2006)
H. Song and U. Heinz, Phys. Lett. B658 279 (2008)
H. Song and U. Heinz, arXiv:0712.3715[nucl-th]

05/01/2008

The success of ideal hydrodynamics

p_T spectra for both central collision and noncentral collisions :

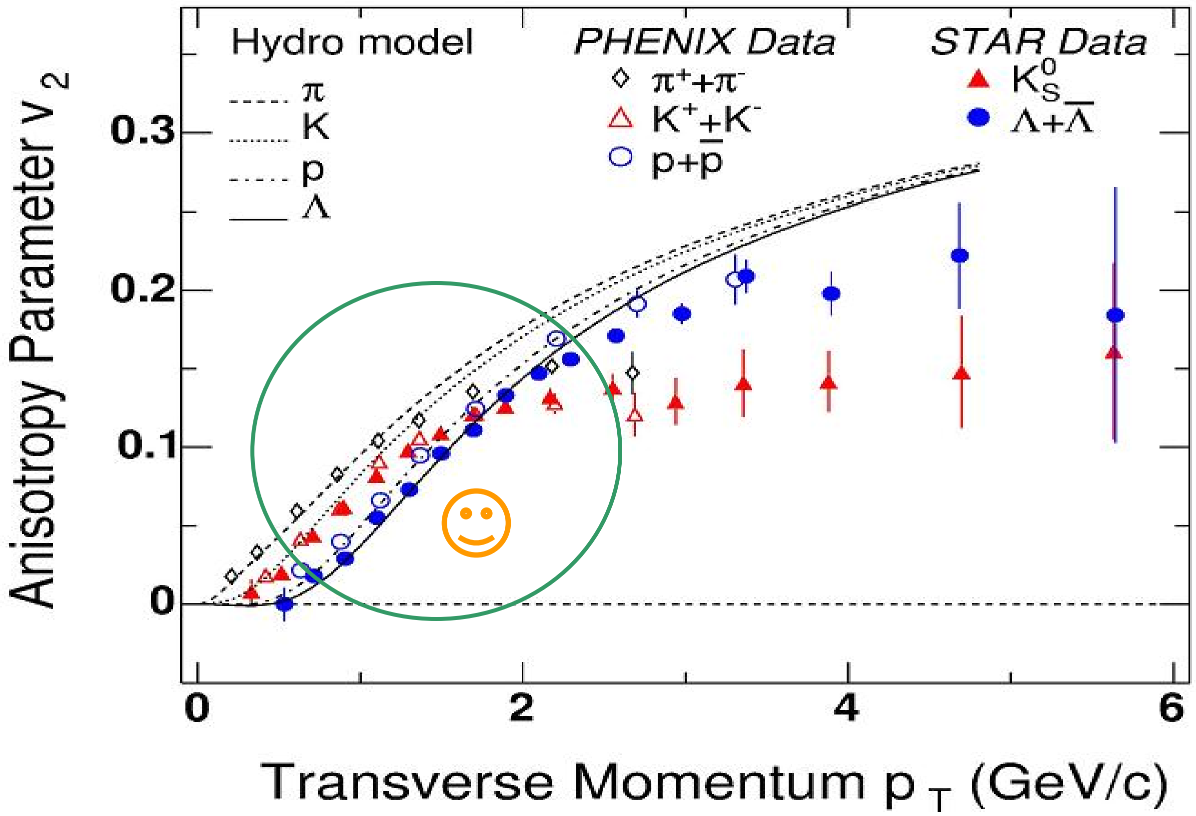
Hydro: U Heinz & P.Kolb   Data: STAR PHENIX



The success of ideal hydrodynamics

Elliptic flow coefficient v_2 in noncentral collisions:

STAR, PHENIX, PHOBOS



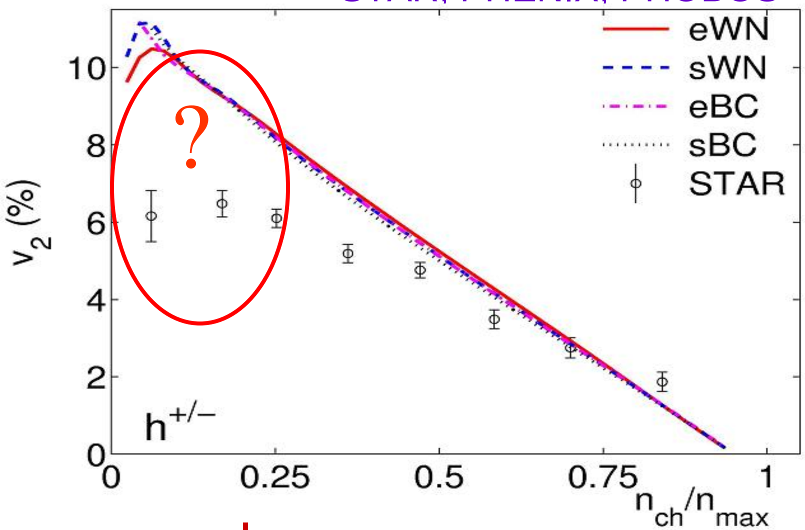
-Ideal hydro describes the data well at $b \leq 7\,fm, (n_{ch}/n_{max} \geq 0.5), \quad p_T \leq 1.5 \sim 2GeV$

-It also gives the correct mass splitting of v_2 in low p_T region

Where ideal hydro fails

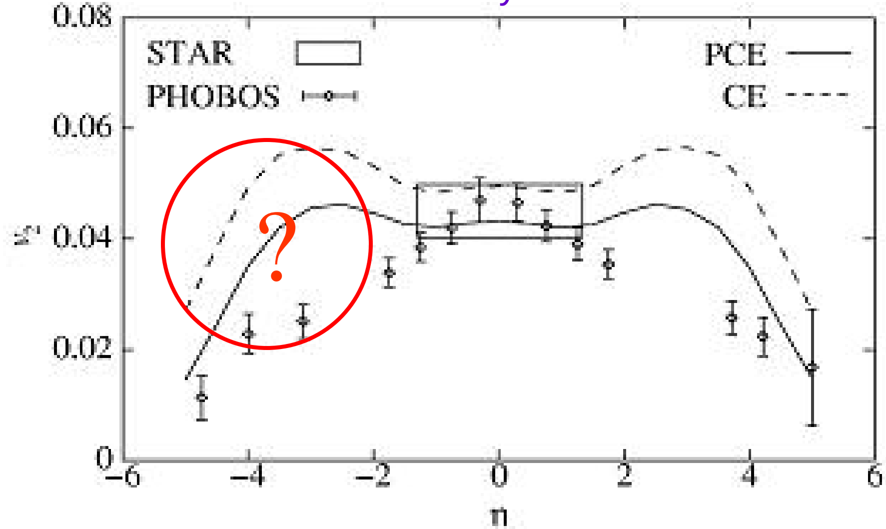
peripheral collisions:

STAR, PHENIX, PHOBOS



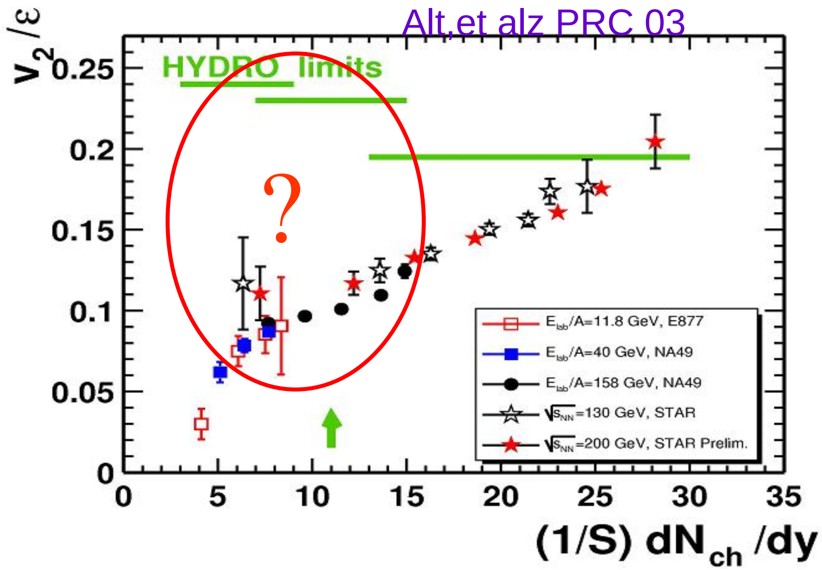
forward rapidity:

3D hydro: T.Hirano PRC02



lower energy:

Alt,et alz PRC 03



comment:

-Hadronic stage is highly viscous (Teaney 02,Hirano04)

-ideal hydro overpredicst v_2 during hadronic stage

-Glauber + ideal hydro + hadron cascade ?

-CGC + viscous hydro + hadron cascade ?

CGC initialization leaves room for QGP viscosity

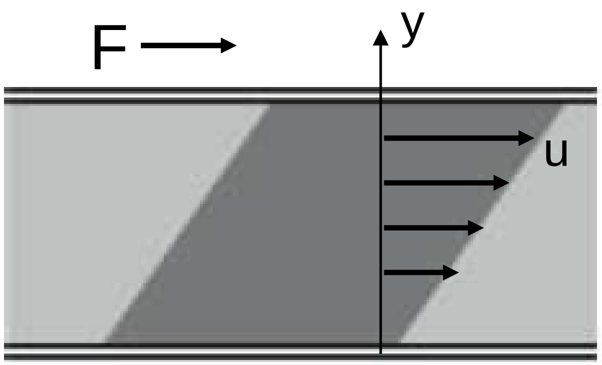
Lower bound for η/s from quantum mechanics

Quantum mechanics excludes the possibility of an absolutely ideal fluid:

Example: shear viscosity of a dilute gas

Shear viscosity η - classical definition:

$$\frac{F}{A} = \eta \frac{du}{dy}$$



Shear viscosity - microscopic view:

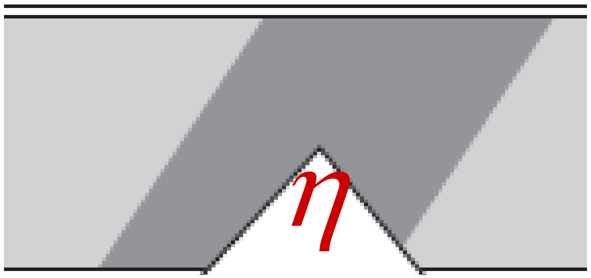
$$\eta \sim mn \bar{v} l_{mfp} , \quad s \sim k_B n$$

$$\frac{\eta}{s} \sim \frac{1}{k_B} \bar{v} m l_{mfp}$$

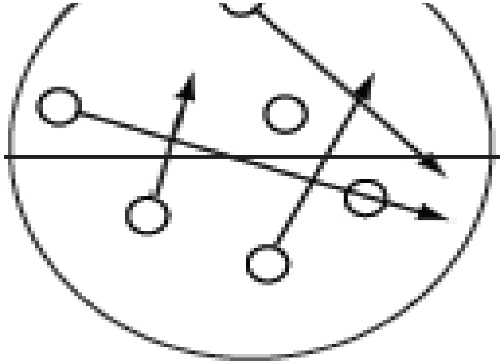
$$\sim \frac{1}{k_B} \left(\frac{1}{2} m \bar{v}^2\right)\left(\frac{l_{mfp}}{\bar{v}}\right) \sim \frac{e\tau}{k_B}$$

uncertainty principle:

$$\frac{\eta}{s} \geq \frac{h}{k_B}$$



the ability of momentum transfer



The QGP shear viscosity

Kubo formula: $\eta = \frac{1}{20}\lim_{\omega\to 0}\int d^4x e^{i\omega t} < [T^{ij}(x) T^{ij}(0)] >$

Kinetic theory: $[\frac{\partial}{\partial t} + v_k \frac{\partial}{\partial x} + F_{ext}\frac{\partial}{\partial k}] f(k,x,t) = C[f]$

- Weakly coupled QCD prediction: $T >> \Lambda_{QCD}$ P.Arnold,G.Moore & L.Yaffe '00,'03

$$\eta = \frac{T^3}{(\alpha_s)^2 \ln(1/\alpha_s)}$$

$$\eta/s = 0.15 \sim 1.5$$

However, the QGP is a strongly coupled system.

- Strongly coupled AdS/CFT prediction:

AdS/CFT correspondence: gauge/gravity duality

4d gauge theory at strong coupling $\longleftrightarrow$ 5d gravity at weak coupling

N=4 SYM $\longleftrightarrow$ Type IIB superstring theory on $AdS_5 \times S^5$

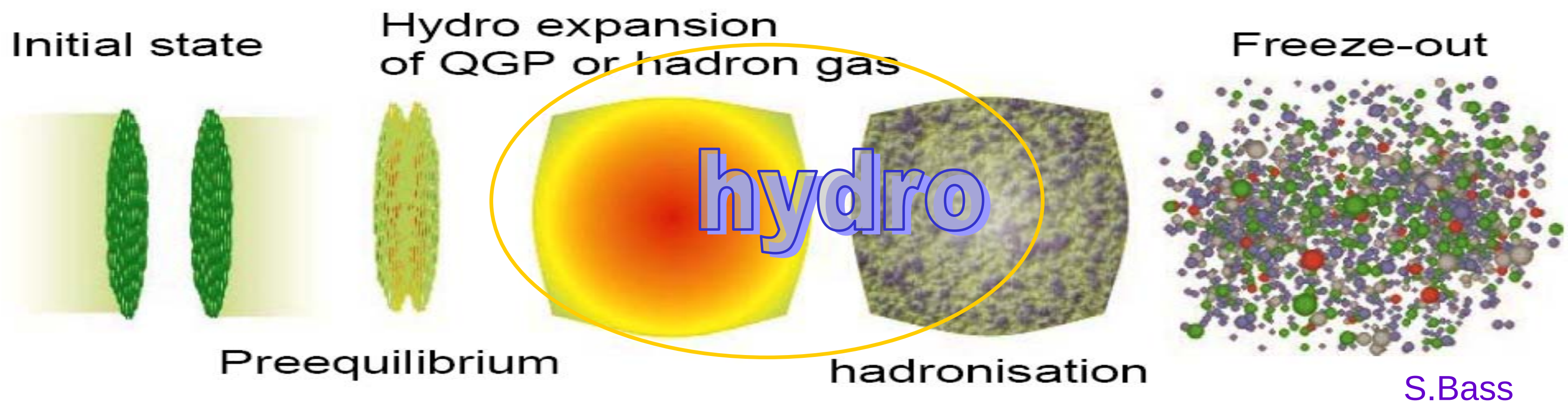
$$\eta/s \geq 1/4\pi \approx 0.08$$

D.T. Son et al. '01,'05

(not related to real QCD)

To extract the QGP viscosity from experimental data, we need viscous hydrodynamics

Ideal hydrodynamics



Hydrodynamics:

-A macroscopic tool to describe the expansion of QGP or hadronic matter

Conservation laws

$$\partial_\mu N^\mu(x) = 0$$

$$\partial_\mu T^{\mu\nu}(x) = 0$$

5 equ. 14 independent variables

- reduce # of independent variables (ideal hydro)
- or provide more equations? (viscous hydro)

Ideal hydrodynamics: $\partial_\mu S^\mu = 0$

$$N^\mu = n u^\mu$$

$$T^{\mu\nu} = (\varepsilon + p) u^\mu u^\nu - p g^{\mu\nu}$$

Input: "EOS" $\varepsilon = \varepsilon(p, n)$

$$f_{eq}(x, p)$$

Microscopically

$$N^\mu(x) = \int \frac{d^3p}{E} p^\mu (f_{eq} + \bar{f}_{eq})$$

$$T^{\mu\nu}(x) = \int \frac{d^3p}{E} p^\mu p^\nu (f_{eq} + \bar{f}_{eq})$$

$$f_{eq}(x, p) = [e^{[p.u(x)+\mu(x)]/T(x)} \pm 1]^{-1}$$

Viscous hydrodynamics - Theory

$$f(x,p) = f_{eq}(x,p)(1+\delta f(x,p))$$

$$T^{\mu\nu}(x) = \int \frac{d^3p}{E} p^\mu p^\nu (f+\bar{f}) = T^{\mu\nu}_{eq} + \delta T^{\mu\nu}$$

$$N^\mu(x) = \int \frac{d^3p}{E} p^\mu (f-\bar{f}) = N^\mu_{eq} + \delta N^\mu$$

Tensor decomposition in frame of u^μ

$$N^\mu = nu^\mu + V^\mu \qquad (\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu)$$

$$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - pg^{\mu\nu}$$

$$+(W^\mu u^\nu + W^\nu u^\mu) - \Pi\Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$V^\mu u_\mu = 0 \qquad W^\mu u_\mu = 0 \qquad W^\mu = \frac{e+P}{n} V^\mu + q^\mu$$

$$\pi^{\mu\nu} u_\nu = 0 \qquad \pi^\mu_\mu = 0 \qquad \phi^\mu u_\mu = 0$$

Landau frame: $W^\mu = 0$

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$$f(x,p) = f_{eq}(x,p)(1+\delta f(x,p))$$

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$$V^\mu u_\mu = 0 \qquad W^\mu u_\mu = 0 \qquad W^\mu = \frac{e+P}{n} V^\mu + q^\mu$$

$$\pi^{\mu\nu} u_\nu = 0 \qquad \pi^\mu_\mu = 0 \qquad \phi^\mu u_\mu = 0$$

Landau frame: $W^\mu = 0$

general conservation laws:

$$\partial_\mu N^\mu(x) = 0$$

$$\partial_\mu T^{\mu\nu}(x) = 0$$

we need more eqs. for $\Pi, \pi^{\mu\nu}, q^\mu$

-1st order theory, Navier-Stokes formalism:

$$\Pi = -\zeta\theta = \zeta X$$
$$q^\mu = \kappa\left(\nabla^\mu T - T\dot{u}^\mu\right) = \kappa X^\mu$$
$$\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} = 2\eta X^{\mu\nu}$$

-2st order theory, Israel-Stewart formalism:

a) phenomenologically: $\partial_\mu S^\mu \geq 0$

b) from kinetic theory $p_\mu d^\mu f(\vec{x},t,\vec{p}) = C(x)$

-2st order theory, Őttinger-Grmela formalism:

… …

I-S formulism: phenomenological approach

-Thermodynamics: $s = \beta p - \alpha n + \beta e$ $\quad Ts = p - \mu n + e, \;\; \alpha = \mu/T, \beta = 1/T$

-Relativistic, equilibrium: $S^{\mu} = p\beta^{\mu} - \alpha N_{eq}^{\mu} + \beta_{\nu} T_{eq}^{\mu\nu}$ $\quad \alpha = \mu/T, \beta^{\mu} = u^{\mu}/T$

-Relativistic, off equilibrium: $S^{\mu} = p\beta^{\mu} - \alpha(N_{eq}^{\mu} + \delta N^{\mu}) + \beta_{\nu}(T_{eq}^{\mu\nu} + \delta T^{\mu\nu})$

$$+ \underbrace{Q^{\mu}(\delta N^{\mu}, \delta T^{\mu\nu})}_{\text{2nd order and higher order corrections}}$$

$$\boxed{T\partial_{\mu} S^{\mu} \geq 0}$$

I-S formulism: phenomenological approach

-Thermodynamics: $s =$ … $\mu n+e, \quad \alpha=\mu/T, \beta=1/T$

-Relativistic, equilibrium: … $^{\mu\nu} \quad \alpha=\mu/T, \beta^{\mu}=u^{\mu}/T$

$$T\partial_{\mu}S^{\mu} \geq 0$$

-Relativistic, off equilibrium: $S^{\mu} = p\beta^{\mu} - \alpha(N_{eq}^{\mu} + \delta N^{\mu}) + \beta_{\nu}(T_{eq}^{\mu\nu} + \delta T^{\mu\nu}) + Q^{\mu}(\delta N^{\mu}, \delta T^{\mu\nu})$

$$T\partial_{\mu}S^{\mu} = \Pi X - q^{\mu}X_{\mu} + \pi^{\mu\nu}X_{\mu\nu} + T\partial_{\mu}Q^{\mu} \geq 0$$

$$X \equiv -\theta = -\nabla \cdot u, \quad X^{\mu} \equiv -\frac{nT}{e+p}\nabla^{\mu}\left(\frac{\mu}{T}\right), \quad X^{\mu\nu} = \nabla^{<\mu}u^{\nu>} \equiv \sigma^{\mu\nu}$$

-a simple case for second order theory (without heat flow& bulk pressure)

A. Muronga 00-04 W. Israel, J.Stewart 79

$$Q^{\mu} = -\left(\beta_2 \pi^{\lambda\nu}\pi_{\lambda\nu}\right)\frac{u^{\mu}}{2T}, \qquad (\Pi = 0, \quad q^{\mu} = 0)$$

$$T\partial_{\mu}S^{\mu} = \pi^{\alpha\beta}\left[\sigma_{\alpha\beta} - \beta_2\Delta^{\mu\alpha}\Delta^{\nu\beta}D\pi_{\mu\nu} + \frac{T}{2}\partial_{\lambda}\left(\frac{\beta_2}{T}u^{\lambda}\right)\pi_{\alpha\beta} - 2\pi^{\alpha(\mu}\omega_{\alpha}^{\nu)}\right] \qquad T\partial_{\mu}S^{\mu} = \frac{\pi_{\alpha\beta}\pi^{\alpha\beta}}{2\eta} \geq 0$$

$$\Delta^{\mu\alpha}\Delta^{\nu\beta}D\pi_{\alpha\beta} = -\frac{1}{\tau_{\pi}}\left[\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu} + \pi^{\mu\nu}\eta T\partial_{\lambda}\left(\frac{\tau_{\pi}}{2\eta T}u^{\lambda}\right)\right] - 2\pi^{\alpha(\mu}\omega_{\alpha}^{\nu)}$$

I-S formalism - Kinetic theory approach

$$f(x,p) = f_{eq}(x,p)(1+\delta f(x,p))$$

$$\delta f(x,t,p) = \varepsilon(\mathrm{x},t) + \varepsilon_{\lambda}(\mathrm{x},t)p^{\lambda} + \varepsilon_{\lambda\nu}(\mathrm{x},t)p^{\lambda}p^{\nu}$$

shear viscosity only: $\delta f(x,t,p) = \varepsilon_{\lambda\nu}(\mathrm{x},t)p^{\lambda}p^{\nu}$

$$\pi^{\mu\nu} = T^{\mu\nu} - T_0^{\mu\nu}$$
$$= \int d\omega\, p^{<\mu}p^{\nu>} f_0 \delta f$$
$$= 2T^2(e+p)\varepsilon^{\mu\nu}(x)$$

$f_0 = \exp(-p_\mu u^\mu / T)$

$$\delta f(x,t,p) = \frac{\pi_{\lambda\nu}p^{\lambda}p^{\nu}}{2T^2(e+p)}$$

$$p_\mu d^\mu f(\vec{x},t,\vec{p}) = C(x)$$

I-S formalism - Kinetic theory approach

$$f(x,p) = f_{eq}(x,p)(1+\delta f(x,p))$$

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shear viscosity only: $\delta f(x,t,p) = \varepsilon_{\lambda\nu}(\mathrm{x},t)p^{\lambda}p^{\nu}$

$$f_0 = \exp(-p_\mu u^\mu / T)$$

$$\pi^{\mu\nu} = 2T^2(e+p)\varepsilon^{\mu\nu}(x)$$

$$\delta f(x,t,p) = \frac{\pi_{\lambda\nu}p^{\lambda}p^{\nu}}{2T^2(e+p)}$$

Baier et al. ‘06, W.Israel, J.Stewart ‘79

$$p_\mu d^\mu f(\vec{x},t,\vec{p}) = C(x)$$

$$C = -p_\mu u^\mu \frac{f - f_0}{\tau_\Pi}$$

-acting $\int d\omega$, or $\int d\omega\, p^{\alpha}$ on both sides $\longrightarrow$ conservation laws: $\left(\int d\omega = \int \frac{d^3p}{(2\pi)^3 E}\right)$

$$\partial_\mu N^\mu(x) = 0, \quad \partial_\mu T^{\mu\nu}(x) = 0.$$

-acting $\int d\omega\, p^{\alpha} p^{\beta}$ on both side and expand δf to the 2nd order $\longrightarrow$ I-S formalism:

$$\Delta^{\mu\alpha}\Delta^{\nu\beta}D\pi_{\alpha\beta} = -\frac{1}{\tau_\pi}\left[\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu}\right] + \frac{1}{2}\pi^{\mu\nu}[5D\ln T - \nabla_\alpha u^\alpha] - 2\pi^{\alpha(\mu}\omega_\alpha^{\nu)}$$

massless particles: $f_0 = \exp(-p_\mu u^\mu / T)$

$$\frac{\tau_\pi}{\eta} = \frac{Tb_2}{a_2^2} = \frac{6}{sT}$$

Compare I-S eqns. from different approaches

I-S eqns. from 2nd law of thermodynamics (phenomenological)

I-S eqn. (I)

$$\Delta^{\mu\alpha}\Delta^{\nu\beta} D\pi_{\alpha\beta} = -\frac{1}{\tau_\pi}\left[\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu}\right] - \frac{1}{2}\pi^{\mu\nu}\frac{\eta T}{\tau_\pi}\partial_\lambda\left(\frac{\tau_\pi}{\eta T}u^\lambda\right) - 2\pi^{\alpha(\mu}\omega^{\nu)}_{\alpha}$$

I-S eqns. from kinetic theory:

full I-S eqn

I-S eqn. (II)

$$\Delta^{\mu\alpha}\Delta^{\nu\beta} D\pi_{\alpha\beta} = -\frac{1}{\tau_\pi}\left[\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu}\right] - \frac{1}{2}\pi^{\mu\nu}\left[5 D\ln T - \nabla_\alpha u^\alpha\right] - 2\pi^{\alpha(\mu}\omega^{\nu)}_{\alpha}$$

- η, τ_π are totally free parameters in the phenomenological approach, but could be calculated in kinetic theory approach.

-they are identical with each other for a pure massless QGP phase (conformal fluid)

$$\frac{1}{2}\pi^{\mu\nu}\frac{\eta T}{\tau_\pi}\partial_\lambda\left(\frac{\tau_\pi}{\eta T}u^\lambda\right) \longleftrightarrow \frac{1}{2}\pi^{\mu\nu}\left[5 D\ln T - \nabla_\alpha u^\alpha\right]$$

$$p = e/3, e \sim T^4, \quad De \approx -(e+p)(\nabla^\mu u_\mu) \qquad \eta \sim s, \tau_\pi \sim T^{-1}$$

-for non-conformal fluid (with a phase transition), it is better to use I-S eqn. (I)

-Numerically, the differences between I-S eqn. (I) and I-S eqn. (II) are small if the system size and initial energy density are not too small

Conformal 2nd order formulism

-For a conformal fluid, conformal symmetry constrains the form of the 2nd order viscous eqns.:

Baier, Romatschke, Son, Starinets & Stephanov, arXiv:0712.2451[hep-th]

$$\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \tau_\pi \left[\Delta^{\mu\alpha}\Delta^{\nu\beta} D\pi_{\alpha\beta} + \frac{4}{3}\pi^{\mu\nu}\nabla_\alpha u^\alpha \right]$$

full I-S eqn

$$- \frac{\kappa}{2}\left[R^{<\mu\nu>} + 2u_\alpha R^{\alpha<\mu\nu>\beta} u_\beta \right]$$

$$- \frac{\lambda_1}{2\eta^2}\pi^{\alpha<\mu}\pi^{\nu>}_{\alpha} - \frac{\lambda_2}{2\eta}\pi^{\alpha(\mu}\omega^{\nu)}_{\alpha} - \frac{\lambda_3}{2}\omega^{\alpha<\mu}\omega^{\nu>}_{\alpha}$$

-to preserve the conformal symmetry, $\Delta^{\mu\alpha}\Delta^{\nu\beta} D\pi_{\alpha\beta}$ & $\frac{4}{3}\pi^{\mu\nu}(\nabla_\alpha u^\alpha)$ must appear simultaneously

-flat space: $\kappa = 0$

-fluid vorticity, $\omega_{\mu\nu} = -\nabla_{[\mu} u_{\nu]}$ for irrotational fluid, $\lambda_2, \lambda_3 = 0$

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full I-S eqn

$$\cancel{-\frac{\kappa}{2}\left[R^{<\mu\nu>} + 2u_\alpha R^{\alpha<\mu\nu>\beta}u_\beta\right]}$$

$$-\frac{\lambda_1}{2\eta^2}\pi^{\alpha<\mu}\pi^{\nu>}_\alpha \cancel{- \frac{\lambda_2}{2\eta}\pi^{\alpha(\mu}\omega^{\nu)}_\alpha - \frac{\lambda_3}{2}\omega^{\alpha<\mu}\omega^{\nu>}_\alpha}$$

-to preserve the conformal symmetry, $\Delta^{\mu\alpha}\Delta^{\nu\beta}D\pi_{\alpha\beta}$ & $\frac{4}{3}\pi^{\mu\nu}(\nabla_\alpha u^\alpha)$ must appear simultaneously

-flat space: $\kappa = 0$

-fluid vorticity, $\omega_{\mu\nu} = -\nabla_{[\mu}u_{\nu]}$ for irrotational fluid, $\lambda_2, \lambda_3 = 0$

-for strongly coupled N=4 plasma,

$$\eta/s = 1/(4\pi), \quad \tau_\pi = (2-\ln 2)/(2\pi T), \quad \lambda_1 = \eta/(2\pi T)$$

-for weakly coupled plasma,

$$\tau_\pi = 6\eta/sT, \qquad \lambda_1 = 0$$

$$\partial_\mu T^{\mu\nu}(x) = 0 \qquad T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - pg^{\mu\nu} + \pi^{\mu\nu}$$

$$\Delta^{\mu\alpha}\Delta^{\nu\beta}D\pi_{\alpha\beta} = -\frac{1}{\tau_\pi}\left[\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu}\right] + \frac{1}{2}\pi^{\mu\nu}[5D\ln T - \nabla_\alpha u^\alpha]$$

(2+1)-d viscous hydrodynamics

-Bjorken approximation: $v_z = z/t$

(τ, x, y, η) coordinates 3+1 ➡ 2+1

-For simplicity, assuming: $N^\mu = 0, \quad \Pi = 0$

(2+1)-d viscous hydrodynamics

-Romatschke & Romatschke: full I-S eqn. EOS I EOS L*

PRL'07 **Au+Au,** T_{dec} **=150MeV** (EOS L* here is the quasi-particle one based on lattice QCD)

-Song & Heinz: simplified I-S eqn. & full I-S eqn. EOS I SM-EOS Q EOS L

PLB'08 & arXiv:0712.3715[nucl-th] **Cu+Cu,** simplified I-S eqn., T_{dec} **= 130MeV**

(**Au+Au**, **Cu+Cu**, system size effects, full I-S eqn. vs. simplified I-S eqn., EOS L etc, in preparation)

-Dusling & Teaney: Öttinger-Grmela (O-G) eqn. EOS I

PRC'08 **Au+Au,** decoupling by scattering rate, arXiv:0803.1262 [nucl-th], (dilepton production)

-Huovinen & Molnar: full I-S eqn. EOS I

QM08 talk: comparing the results from viscous hydro and from transport model

-Chaudhuri: simplified I-S eqn. EOS I EOS Q

arXiv:0708.1252 [nucl-th], arXiv:0801.3180 [nucl-th], arXiv:0803.0643 [nucl-th] **Au+Au**

Issues:

- verification of the codes individually developed by different groups
- effects from different 2nd order formalisms

  simplified I-S eqn. vs. full I-S eqn., I-S eqn. vs. O-G eqn.
- effects from different EoS, systems sizes and freeze-out procedures

Will show numerical results for:

- ideal hydro vs. viscous hydro

Cu+Cu, simplified I-S eqn., Song & Heinz, PLB07 & arXiv:0712.3715[nucl-th]

- viscous hydro in different I-S formalisms

Simplified Israel-Stewart equation vs. full Israel-Stewart equation:

As $\tau_\pi \rightarrow \mathbf{0}$, simplified I-S eqn. and full I-S eqn. approach the same Navier-Stokes limit

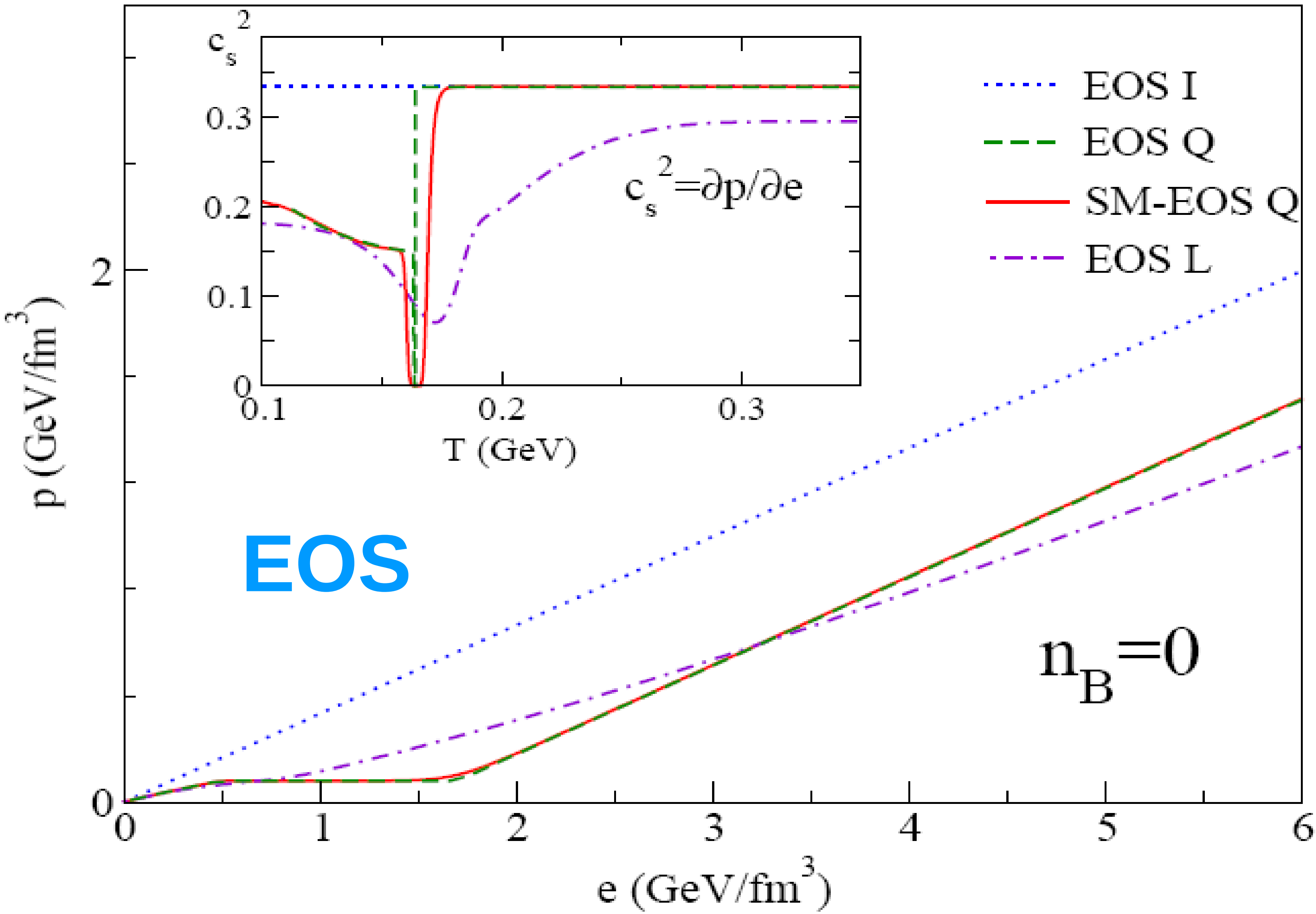
For EOS with a phase transition, the differences between simplified I-S eqn. and full I-S eqn. are small (but not for EOS I!)

- system size effects:

v_2 / ε multiplicity scaling: ideal vs. viscous, simplified vs. full I-S

- entropy production

EOS



c_s^2
0.3
0.2
0.1
0
0.1
0.2
0.3
T (GeV)
$c_s^2 = \partial p / \partial e$
EOS I
EOS Q
SM-EOS Q
EOS L
2
0
p (GeV/fm^3)
EOS
$n_B = 0$
0
1
2
3
4
5
6
e (GeV/fm^3)

Causal viscous hydro in (2+1)-d

Heinz, Song & Chaudhuri, PRC06

(τ, x, y, η) coordinates 3+1 ⇨ 2+1

$$T^{mn}{}_{,m} = 0 \qquad T^{mn} = (\varepsilon + p) u^m u^n - p g^{mn} + \pi^{mn}$$

$$\Delta^{ml} \Delta^{nk} D \pi_{lk} = -\frac{1}{\tau_\pi} \left[\pi^{mn} - 2\eta \sigma^{mn} \right]$$

simplified I-S eqn

-also the eqns used in Song & Heinz, PLB07 & arXiv:0712.3715[nucl-th]& the one in preparation

$$T^{mn}{}_{,m} = 0 \qquad T^{mn} = (\varepsilon + p) u^m u^n - p g^{mn} + \pi^{mn}$$

$$\Delta^{ml} \Delta^{nk} D \pi_{lk} = -\frac{1}{\tau_\pi} \left[\pi^{mn} - 2\eta \sigma^{mn} \right] + \frac{1}{2} \pi^{mn} [5 D \ln T - \nabla_\alpha u^\alpha]$$

full I-S eqn

-also the eqns used in P&U Romatschke PRL07, Song & Heinz, in preparation

Vortices terms are neglected, $\nabla^{<m} u^{n>} = \frac{1}{2}[\nabla^m u^n + \nabla^m u^n] - \frac{1}{3} \Delta^{mn} \partial_\sigma u^\sigma$

numerical results

-Viscous effects: Ideal hydro vs. viscous hydro, Cu+Cu, simplified I-S eqn

Song & Heinz, PLB07 & arXiv:0712.3715[nucl-th]

-Comparisons between simplified I-S eqn full I-S eqn

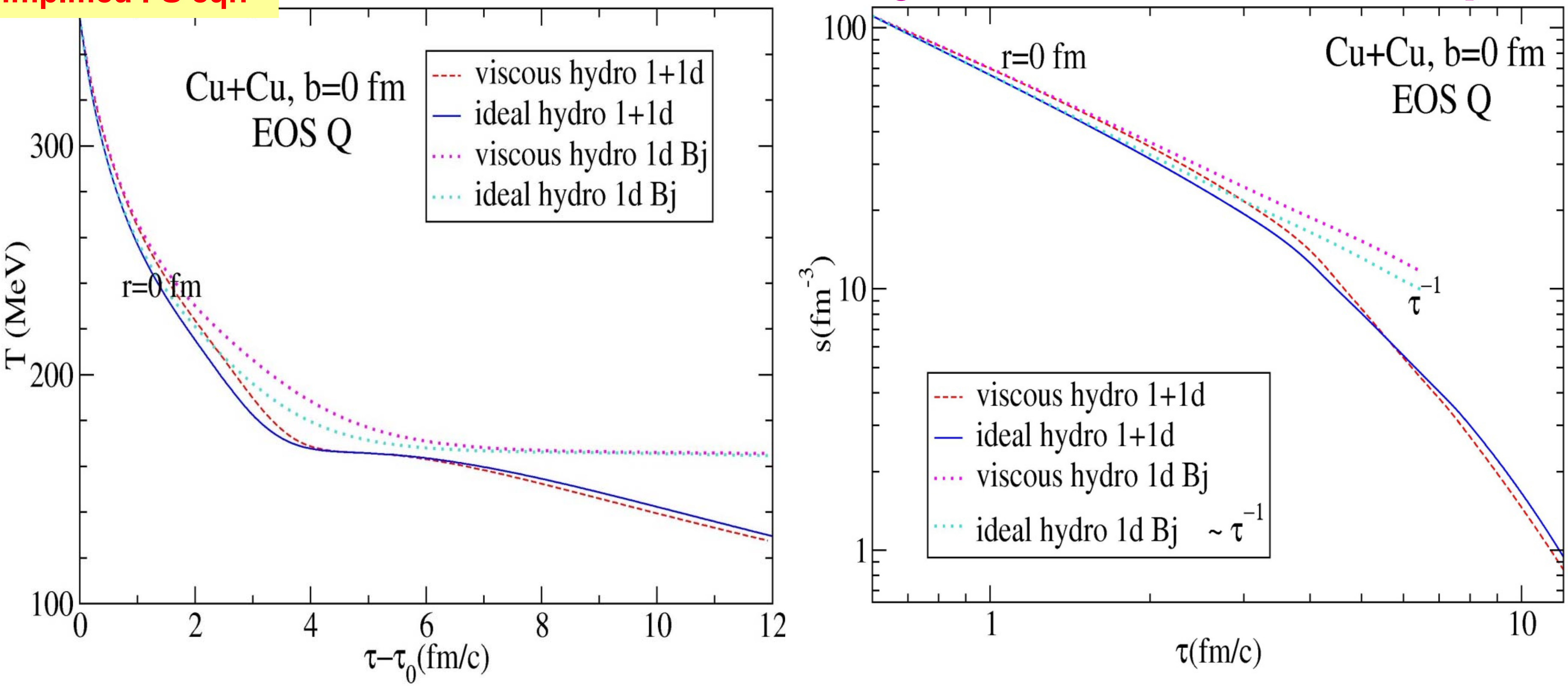
Viscous vs. ideal hydro: temperature & entropy

Initial & final conditions: (viscous & ideal) $e_0 = 30\mathrm{GeV/fm^3}, \tau_0 = 0.6\mathrm{fm/c}, T_{dec} = 130\mathrm{MeV}$

Other viscous hydro parameter: Shear viscosity& relaxation time $\eta / s = 1/4\pi, \quad \tau_\pi = 3\eta / sT$

Simplified I-S eqn

Song & Heinz, PLB07 & arXiv:0712.3715[nucl-th]



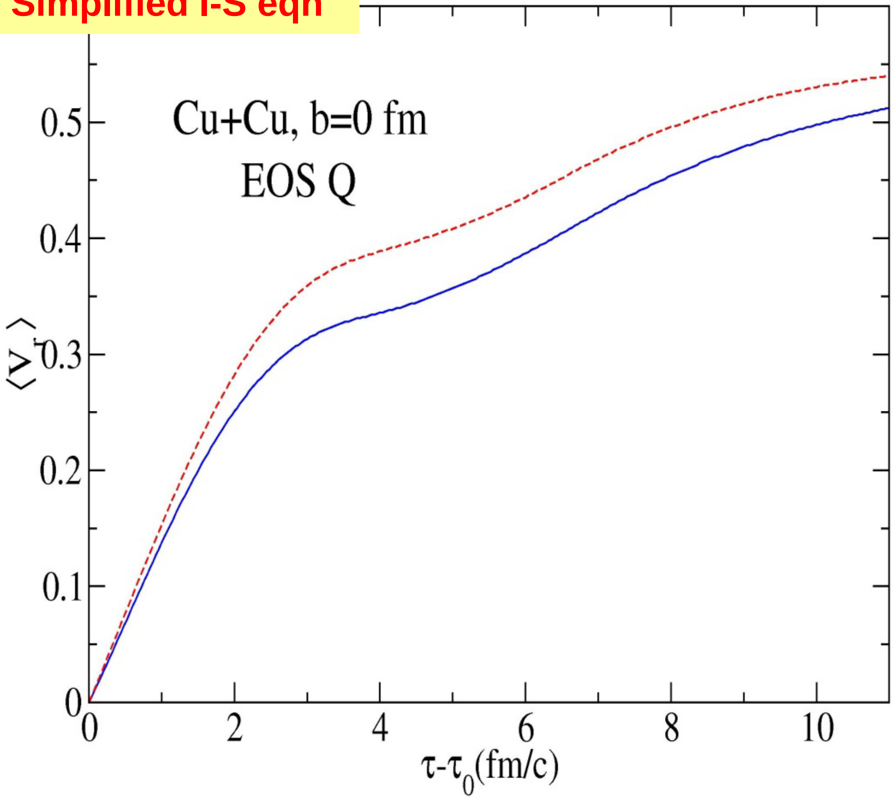
-slowing down of cooling process due to decelerated longitudinal expansion initially , but faster cooling in middle and late stages due to stronger transverse expansion

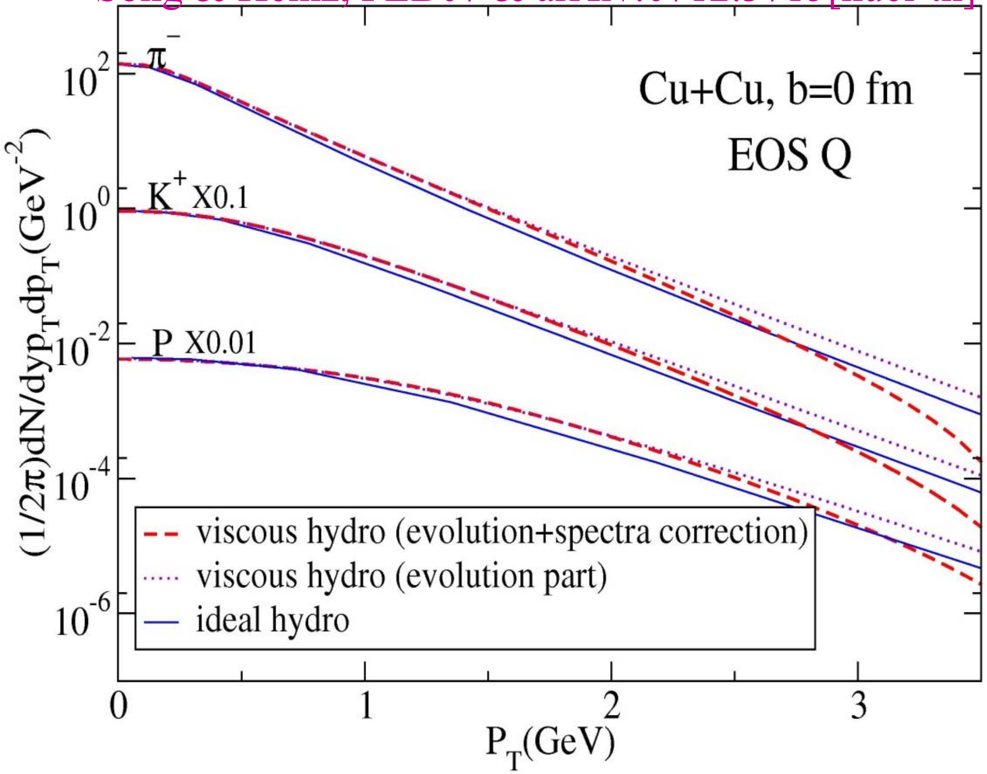
-viscous effects are larger in early and middle stages, but neglectable in late stage

Viscous vs. ideal hydro: radial flow & spectra

Simplified I-S eqn

Song & Heinz, PLB07 & arXiv:0712.3715[nucl-th]





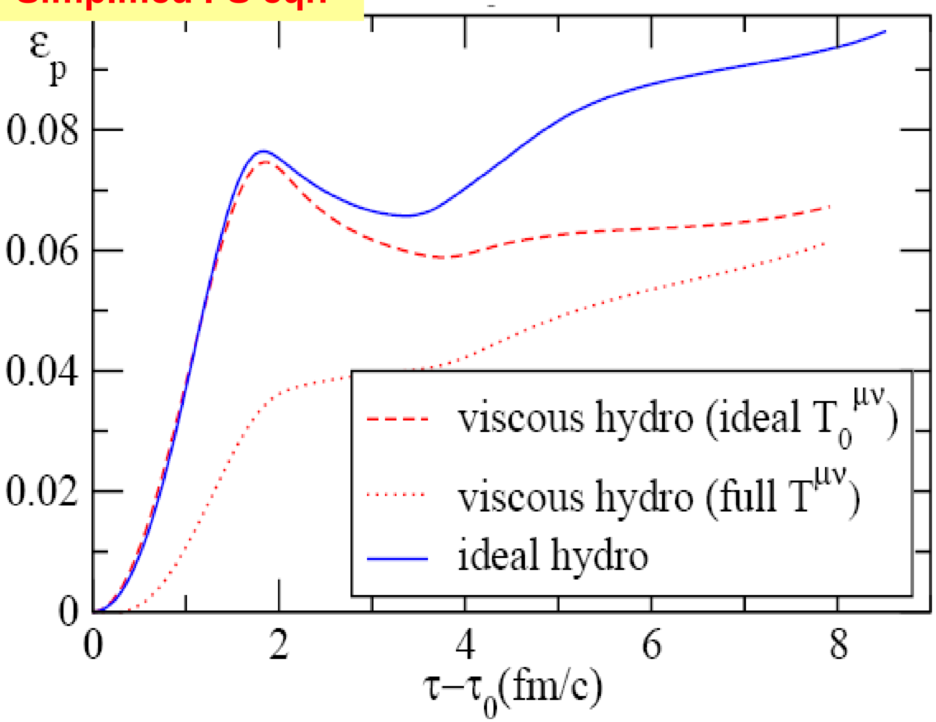
$$E\frac{dN}{d^3p}=\int_\Sigma \frac{p\cdot d^3\sigma(x)}{2\pi^3}\left[f_{eq}(x,p)+\delta f(x,p)\right]=\int_\Sigma \frac{p\cdot d^3\sigma(x)}{(2\pi)^3}f_{eq}(x,p)\left(1+\frac{1}{2}\frac{p^\alpha p^\beta}{T^2(x)}\frac{\pi_{\alpha\beta}(x)}{(e+p)(x)}\right)$$

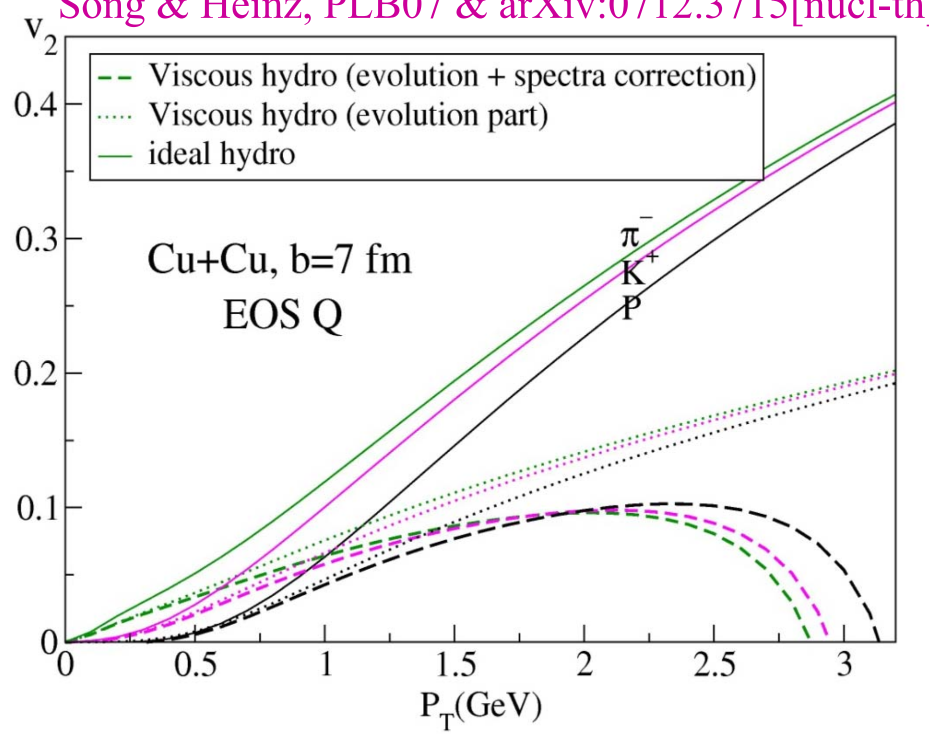
- -More radial flow, flatter spectra
- -the viscous effects to the hadron spectra could be absorbed by starting viscous hydro later with lower initial energy density

Viscous vs. ideal hydro: momentum anisotropy & v_2

Simplified I-S eqn

Song & Heinz, PLB07 & arXiv:0712.3715[nucl-th]





-momentum anisotropy: $\varepsilon_p = \dfrac{< T_0^{xx} - T_0^{yy} >}{< T_0^{xx} + T_0^{yy} >}$ (ideal) $\varepsilon'_p = \dfrac{< T_0^{xx} - T_0^{yy} + \pi^{xx} - \pi^{yy} >}{< T_0^{xx} + T_0^{yy} + \pi^{xx} + \pi^{yy} >}$ (full)

-viscous effects are small at later stage (fixed η/s)

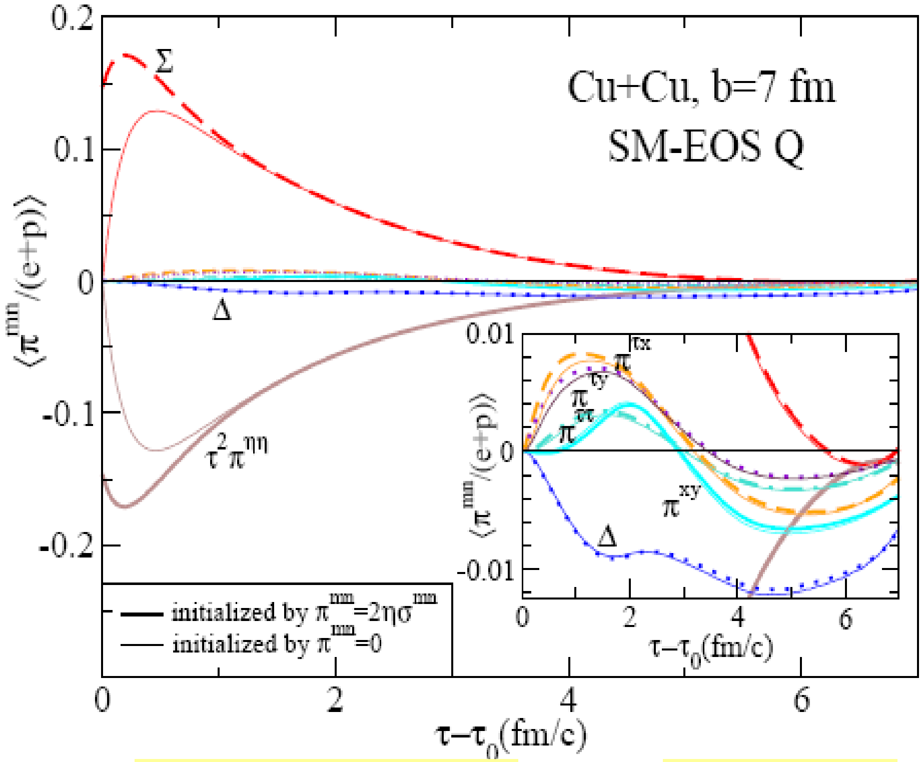
-Elliptic flow is very sensitive to even minimal shear viscosity.

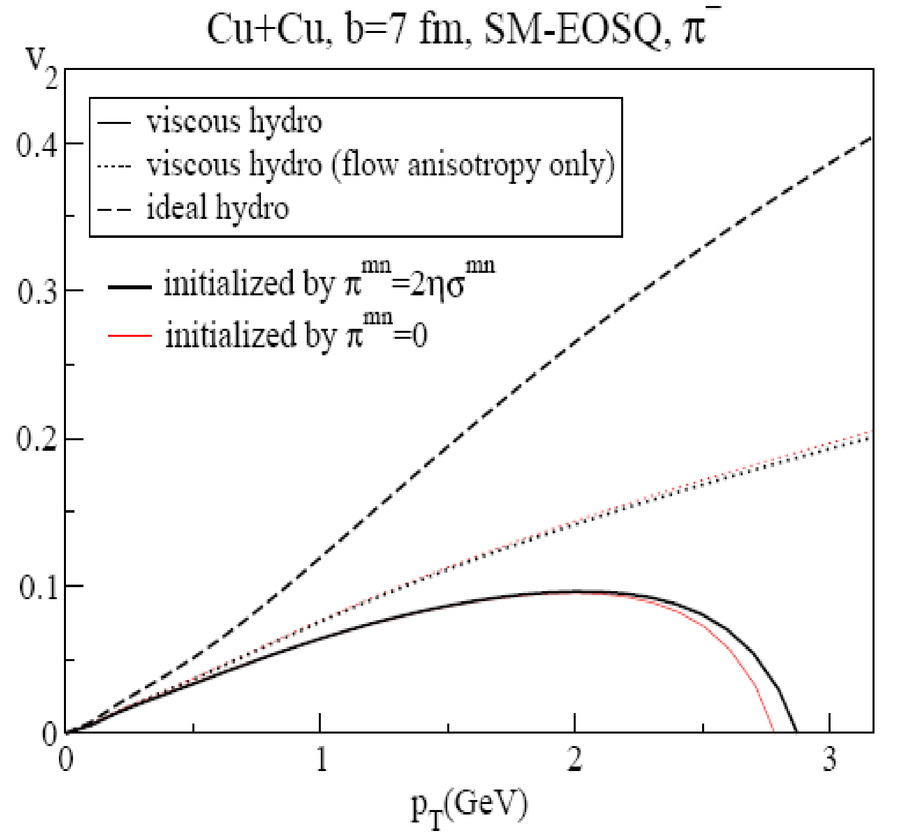
-Both the evolution corrections (viscous corrections to f_0) and spectra corrections (viscous corrections to δf) have significant effects to v_2, for low p_T region evolution correction dominant.

Sensitivity to initialization of π^{mn}

$\Sigma = \pi^{xx} + \pi^{yy}$, $\Delta = \pi^{xx} - \pi^{yy}$

Song & Heinz, PLB07 & arXiv:0712.3715[nucl-th]





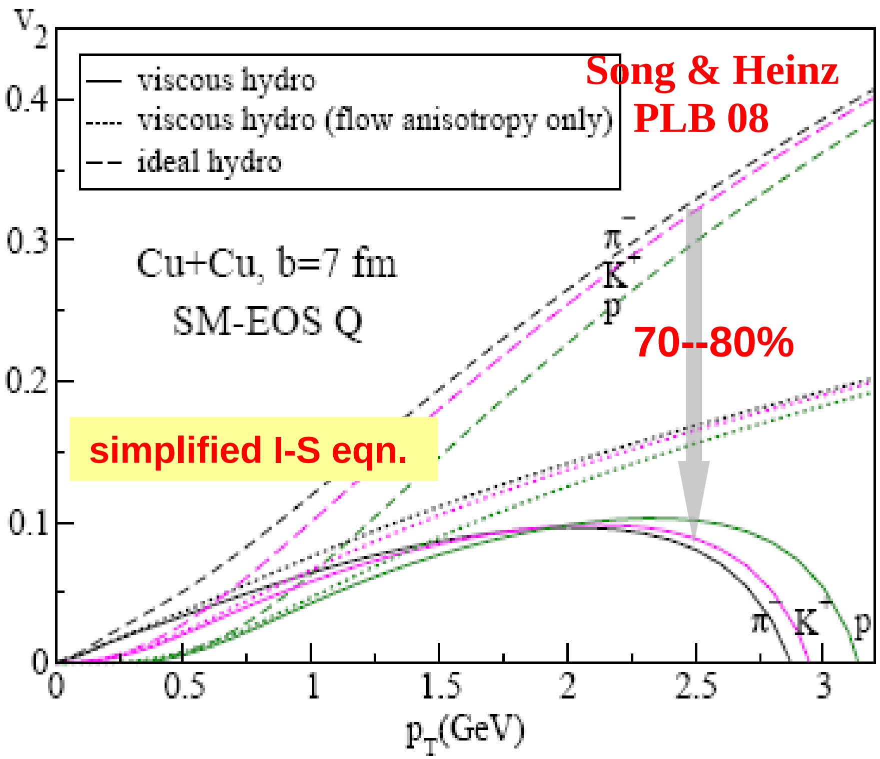
$\pi^{mn} = 2\eta\sigma^{mn}$ **vs.** $\pi^{mn} = 0$

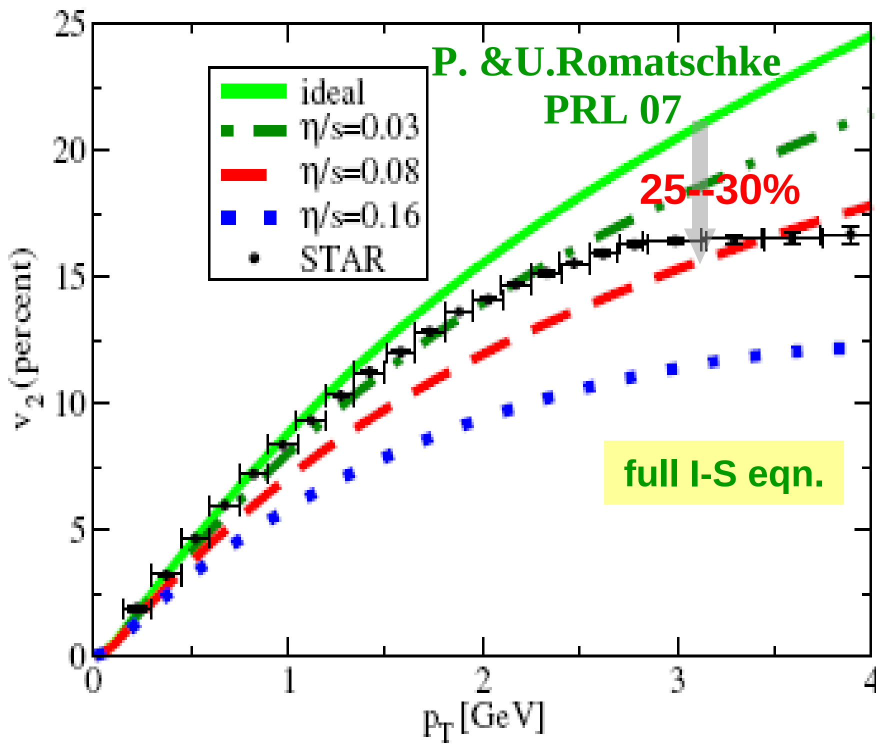
-viscous effects are smaller (or even negligible) in late stage (fixed η/s)

-after ~1fm/c (several relaxation times), viscous pressure loses memory of initial cond

-v_2 is insensitive to different initializations of π^{mn}

Comparison with Romatschke 07 results





- different systems & EOS: CuCu, b=7, SM-EOS Q vs. Au+Au, min bias, EOS Lattice

- different Isreal-Stewart eqns. used: simplified I-S eqn. vs. full I-S eqn.

Effect of using different I-S eqns.?

Simplified I-S eqn. vs. full I-S eqn.:

simplified I-S eqn.: $$\Delta^{\mu\alpha}\Delta^{\nu\beta}D\pi_{\alpha\beta} = -\frac{1}{\tau_\pi}\left[\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu}\right]$$

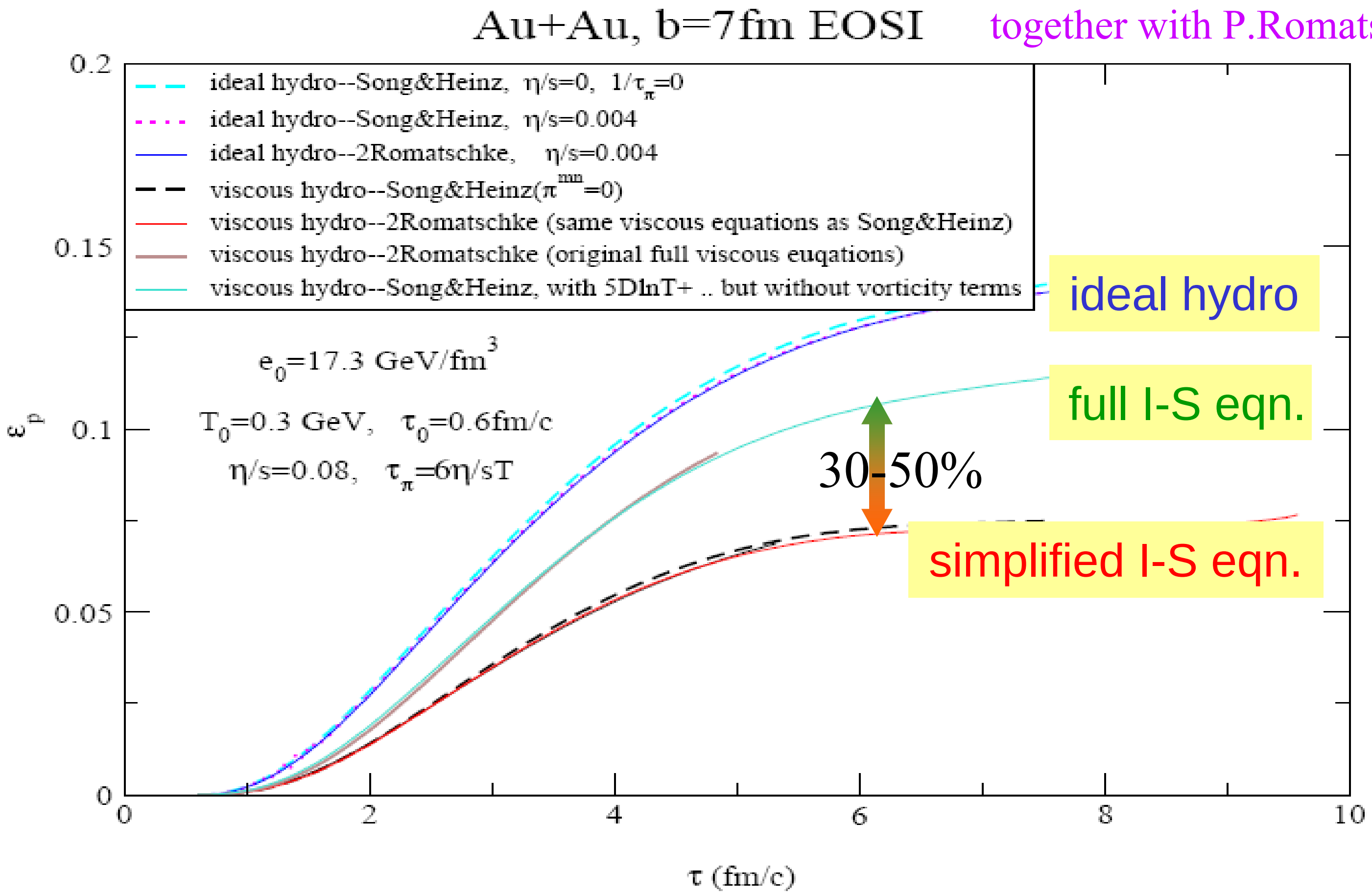
full I-S eqn.: $$\Delta^{\mu\alpha}\Delta^{\nu\beta}D\pi_{\alpha\beta} = -\frac{1}{\tau_\pi}\left[\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu}\right] + \frac{1}{2}\pi^{\mu\nu}[5D\ln T - \nabla_\alpha u^\alpha] - 2\pi^{\alpha(\mu}\omega_\alpha^{\nu)}$$

important for preserving the conformal symmetry

(Baier et al. ‘07)

Code testing: VISH2+1 vs. Romatschke code

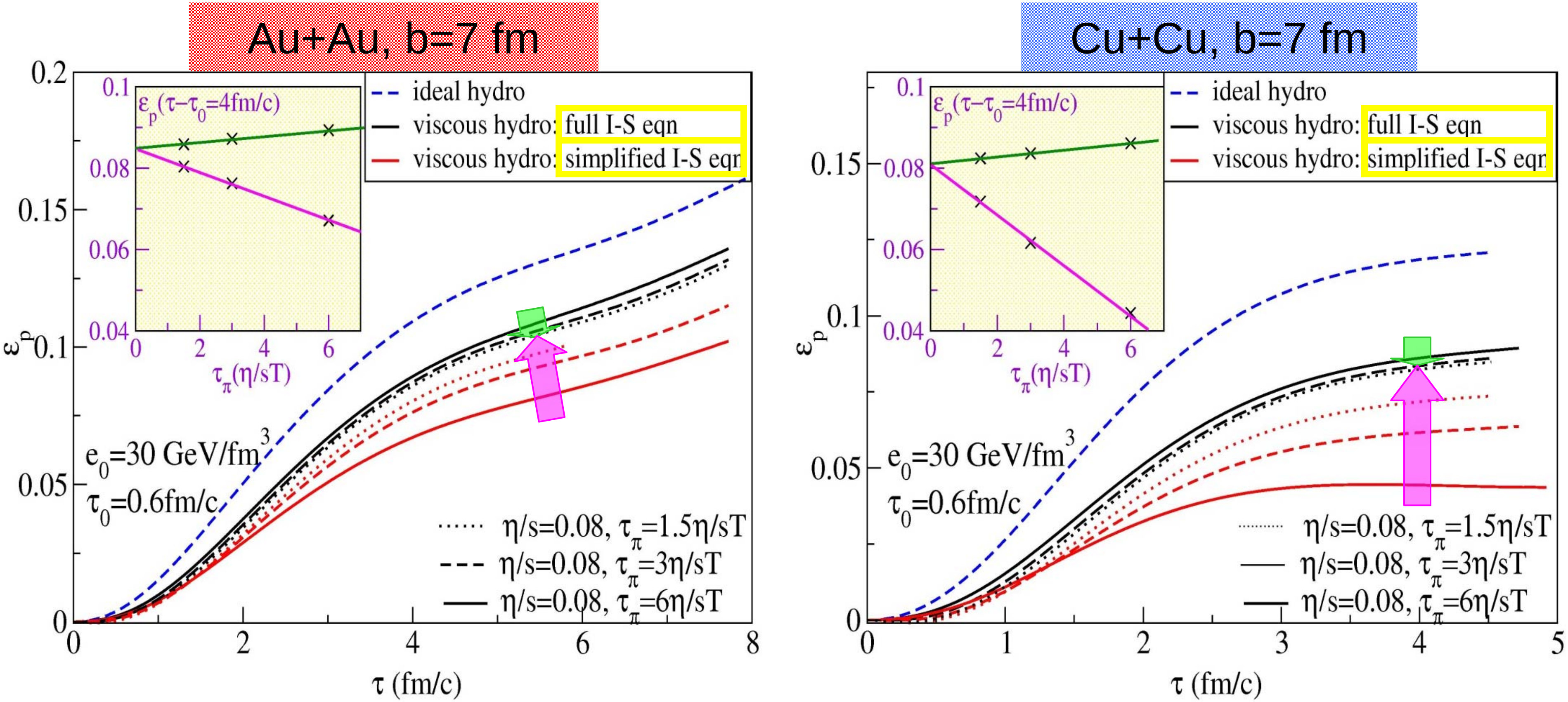
together with P.Romatschke



- for identical initial conditions, EOS (EOS I here), transport eqns. (full I-S eqn. or simplified I-S eqn.), the two codes agree well with each other

- But: the additional terms in the full I-S eqn. bring big differences for the late-stage momentum anisotropy and final elliptic flow ?!

simplified I-S eqn. vs. full I-S eqn. EOS I



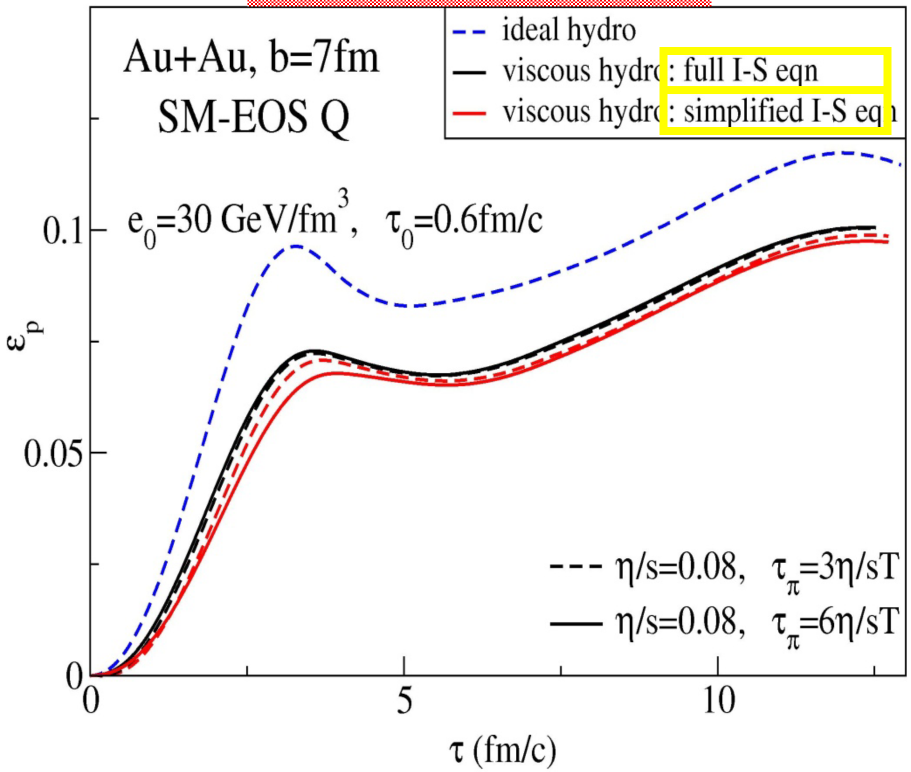
Momentum anisotropy evolution: simplified I-S eqn. vs. full I-S eqn. with different τ_π :

- for EOS I, the additional terms in full I-S eqn. bring 30-50% difference in the late-time momentum anisotropy and final v_2 suppression

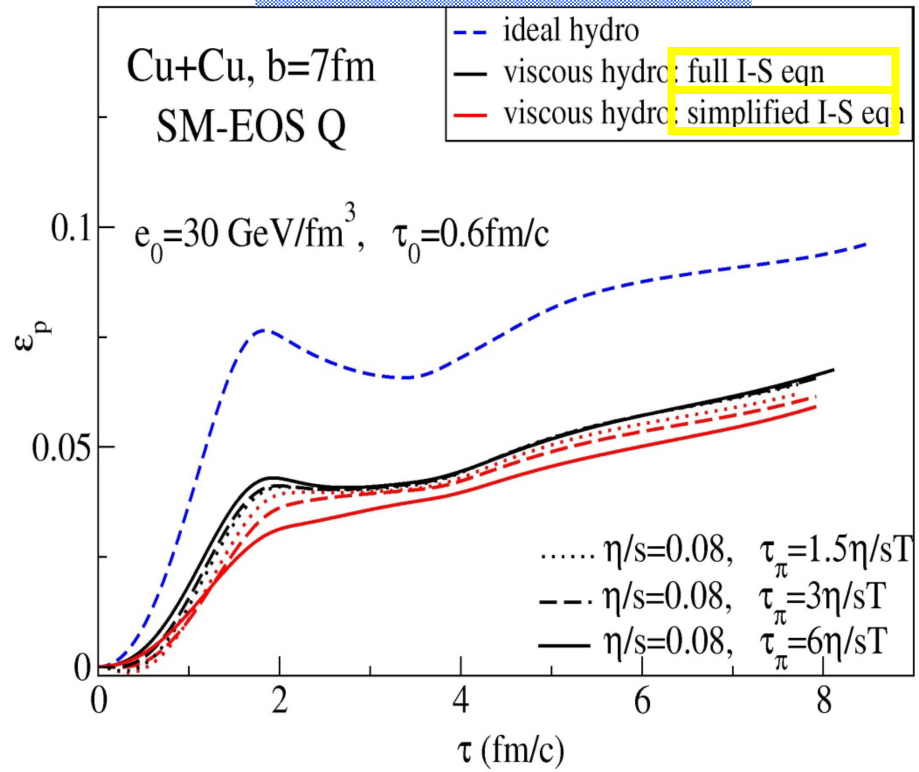
- numerical simulations also show that simplified I-S eqn. and full I-S eqn. approach the same Navier-Stokes limit as $\tau_\pi \rightarrow 0$, but the full I-S eqn. shows much weaker sensitivity to τ_π

simplified I-S eqn. vs. full I-S eqn.: EOS Q

Au+Au, b=7 fm



Cu+Cu, b=7 fm

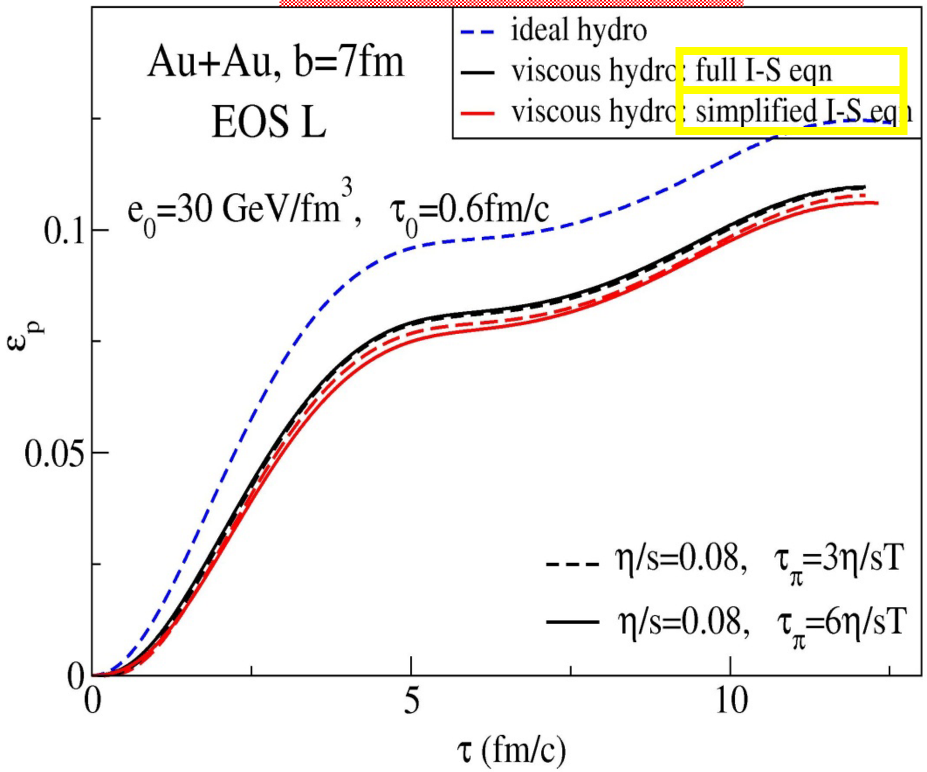


Momentum anisotropy evolution: simplified I-S eqn. vs. full I-S eqn. with different τ_π :

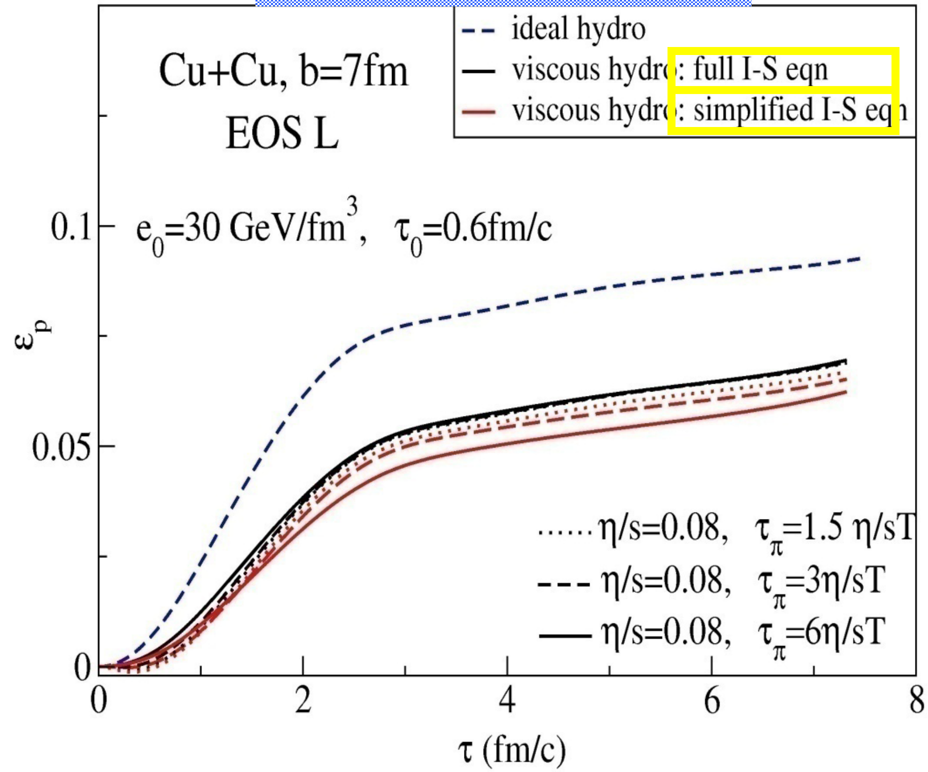
- simplified I-S eqn. vs. full I-S eqn. only: 5-10% differences (for SM-EOS Q)
- for smaller systems, the difference between full and simplified I-S eqns. increases
- for simplified I-S eqn. the sensitivity to the value of the relaxation time is stronger

simplified I-S eqn. vs. full I-S eqn.: EOS L

Au+Au, b=7 fm



Cu+Cu, b=7 fm

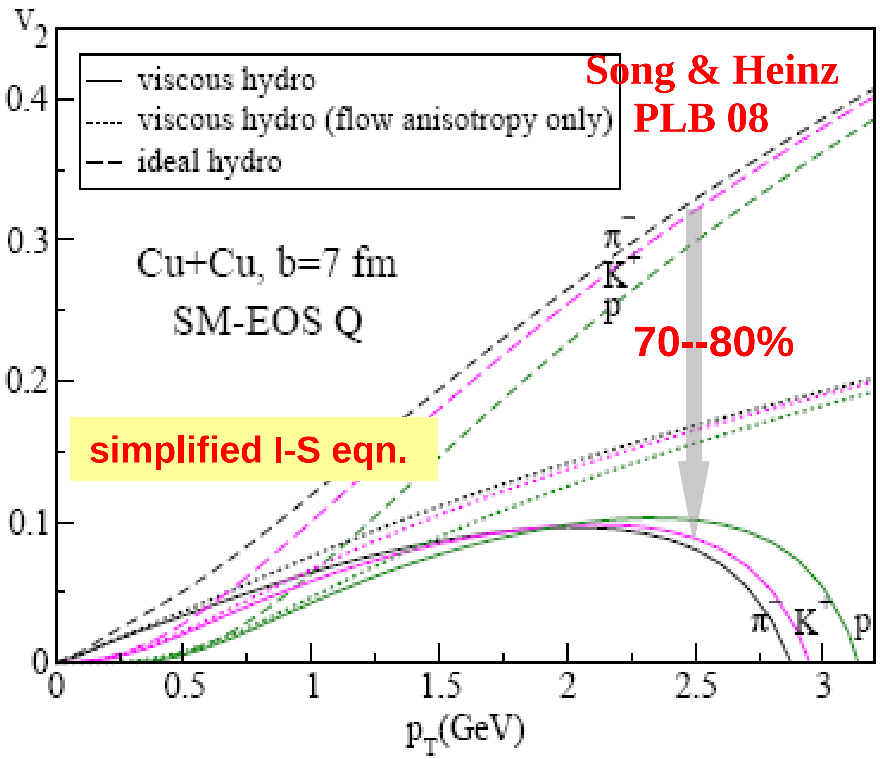


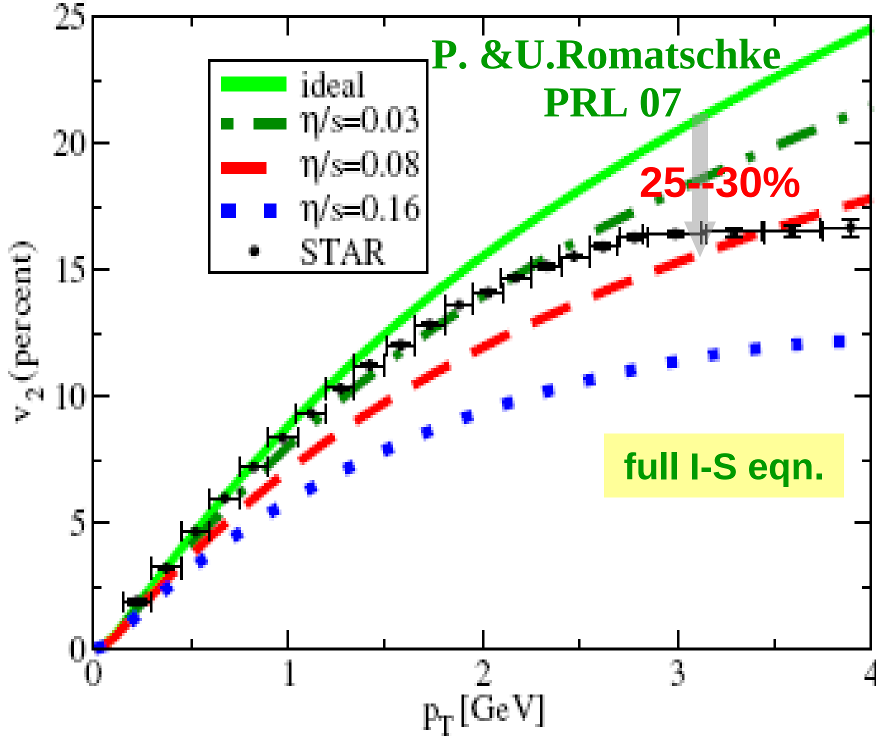
Momentum anisotropy evolution: simplified I-S eqn. vs. full I-S eqn. with different τ_π :

- for **EOS Q** and **EOS L**, viscous effects are largely similar

- for realistic EOS with a phase transition, the difference between simplified and full I-S eqns. for the viscous suppression of v_2 are small if the systems are not too small and the initial energy density is not too low

Comparison with Romatschke 07 results



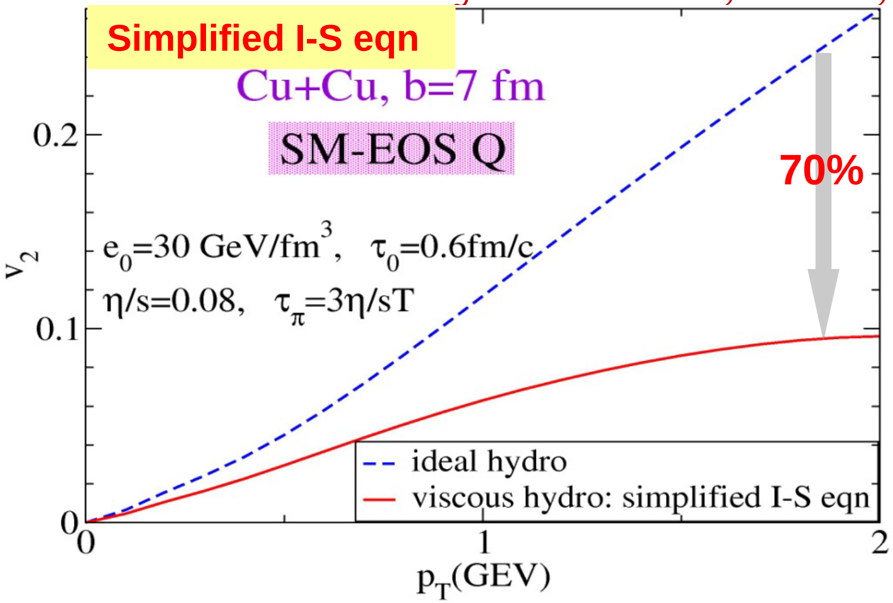


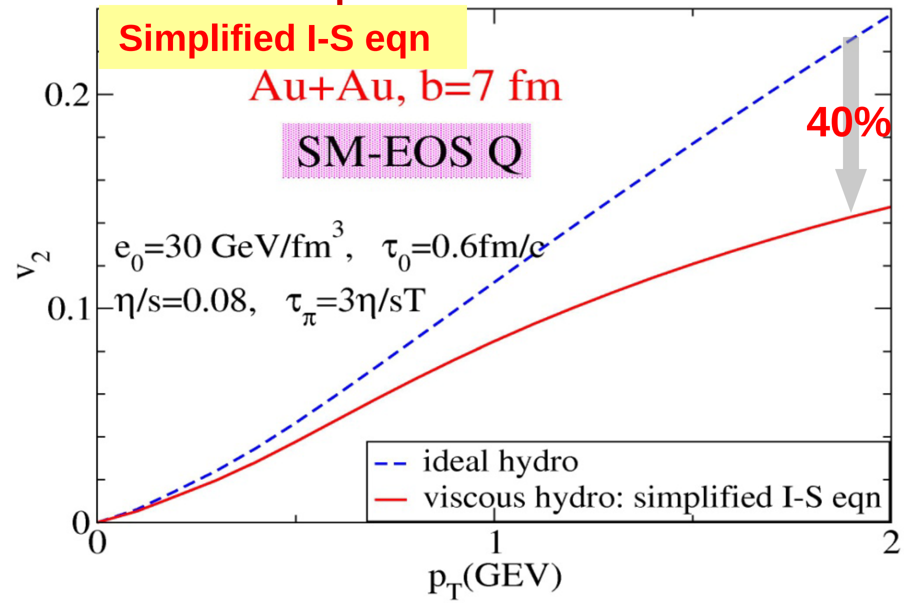
- different Isreal-Stewart eqns. used: simplified I-S eqn. vs. full I-S eqn.
- different systems & EoS: CuCu, b=7, SM-EOS Q vs. Au+Au, min bias, EOS Lattice

System size effects and EOS ?

Different effects contributing to v_2 suppression

system size, EOS, different I-S equations:

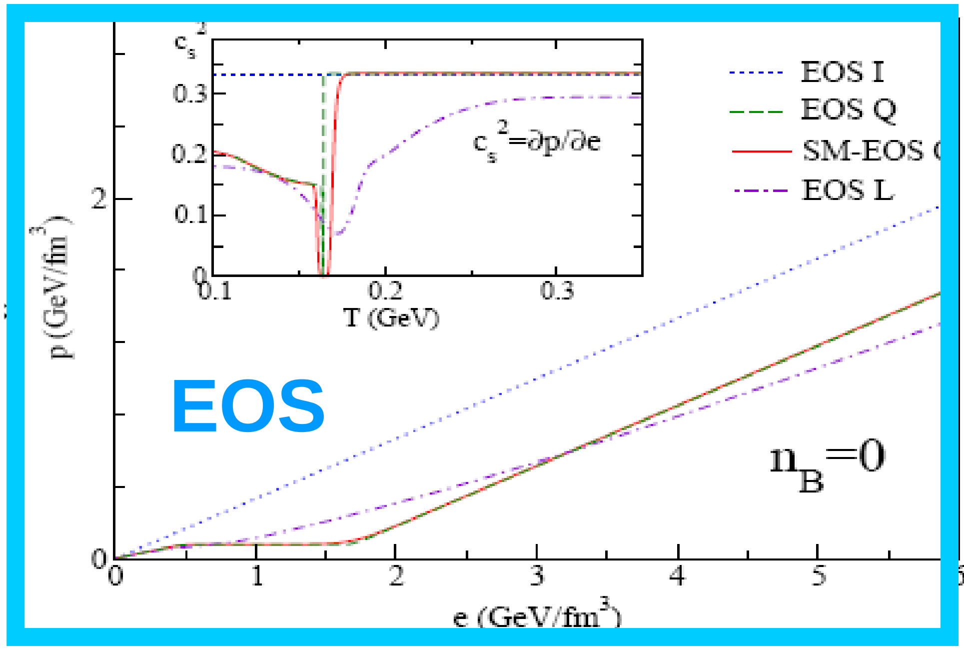


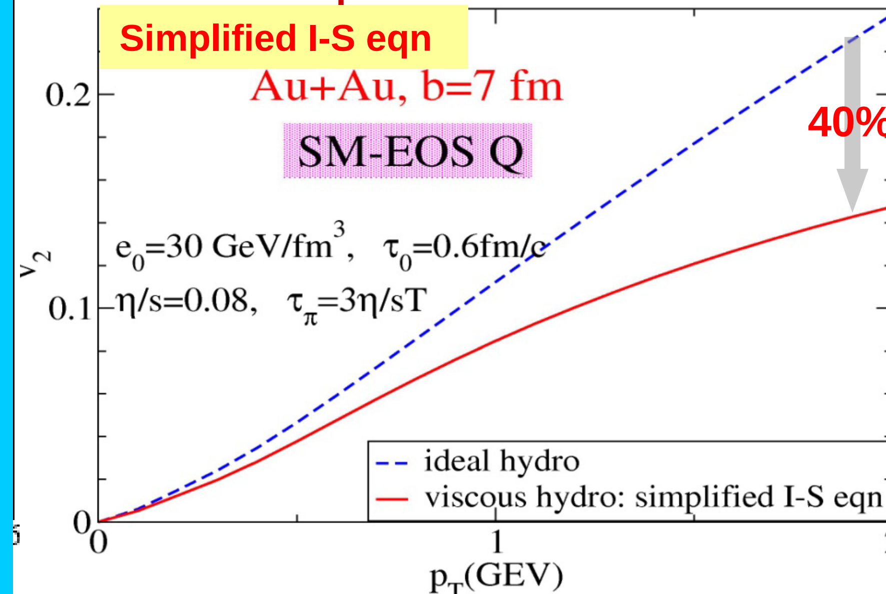


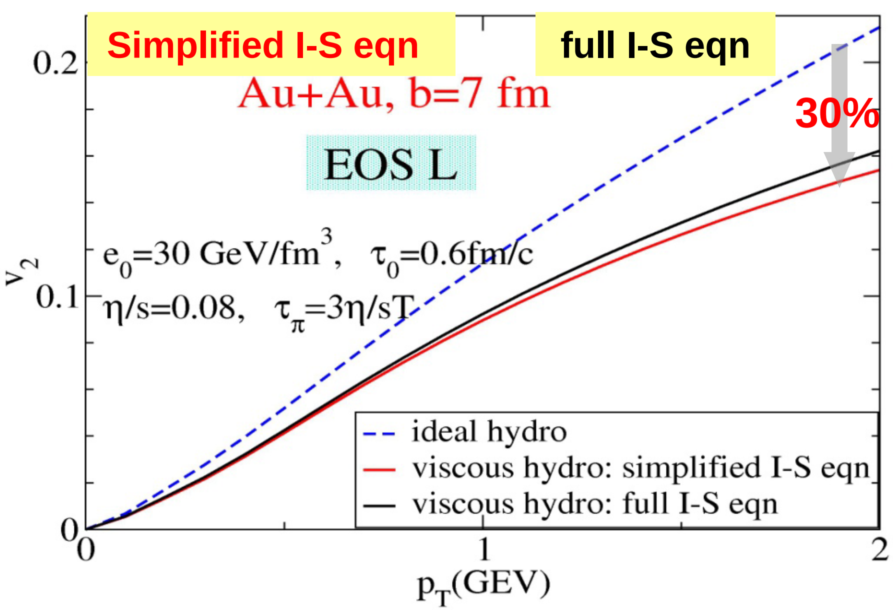
-system size: CuCu b=7fm vs. AuAu b=7fm: 20-30% effect

Different effects contributing to v_2 suppression

ifferent I-S equations:







-system size: CuCu b=7fm vs. AuAu b=7fm: 20-30% effect

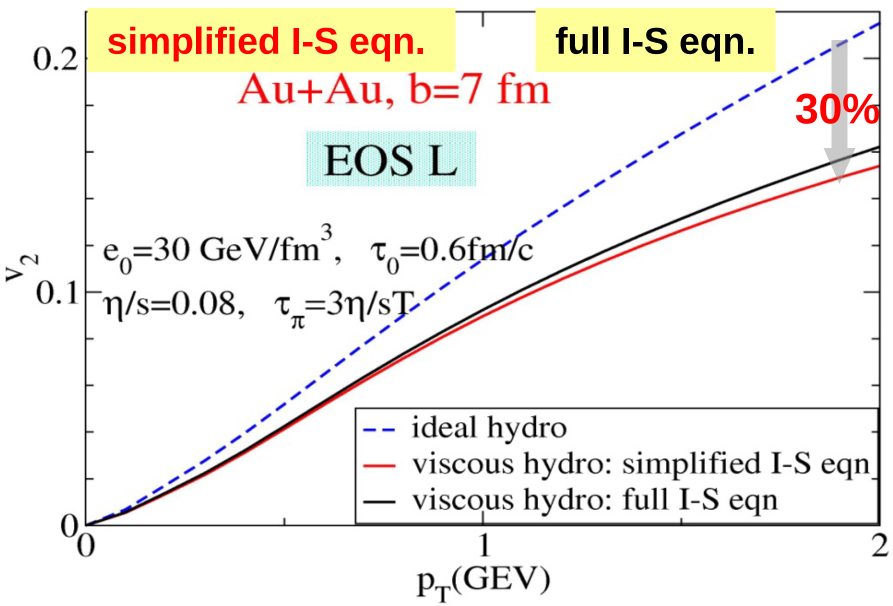
-EOS: SM-EOS Q vs. EOS L: ~10% effect

-different I-S eqns: simplified I-S eqn. vs. full I-S eqn.: ~5% effects (EOS Q and EOS L only)

Different contributions to the suppression of v_2

System size, EOS, different I-S equations:

Considering all of these effects, the final suppression of v_2 for Au+Au with EOS L and the full I-S eqn., for minimal shear viscosity $\eta/s=0.08$, is ~25%, approaching the results of P. & U. Romatschke (PRL 99, 172301 (2007)).

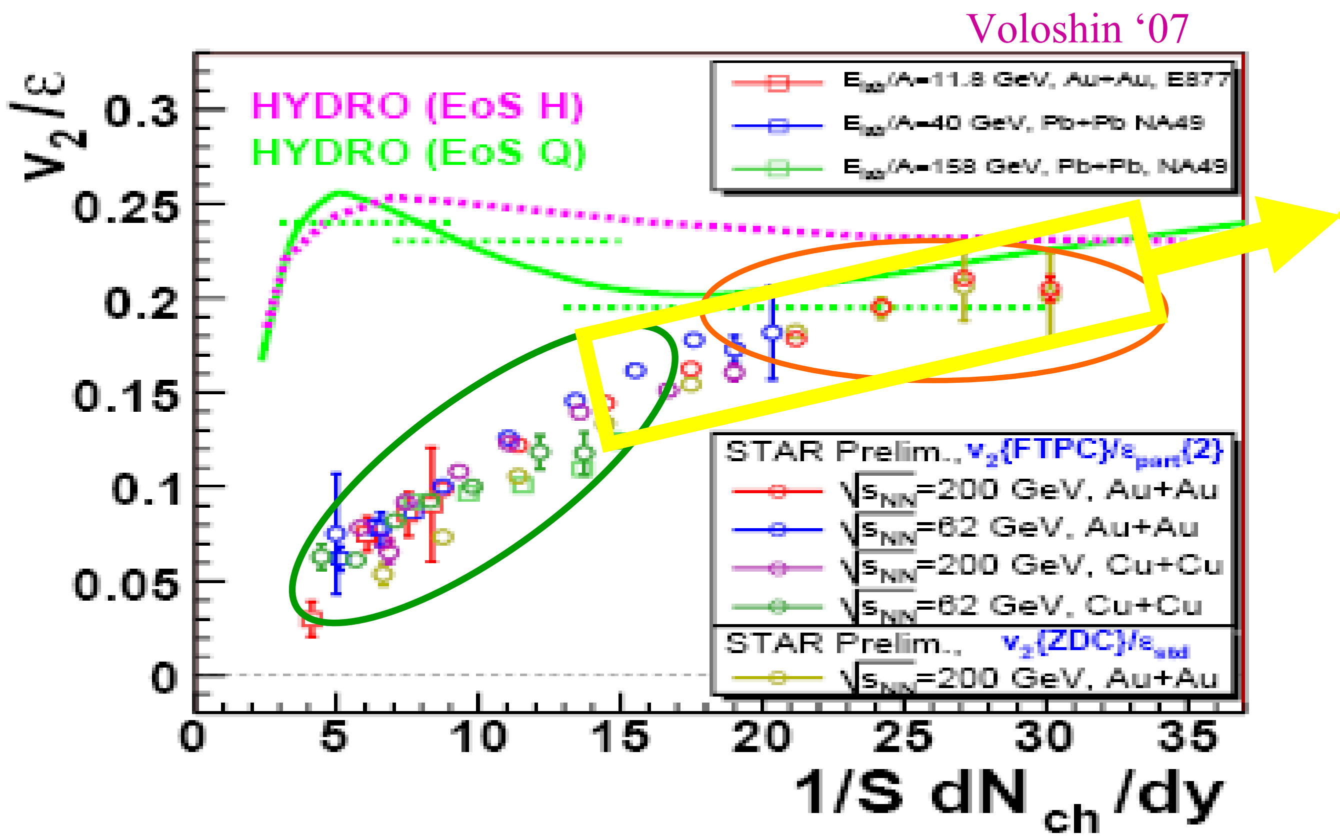


- system size: CuCu b=7fm vs. AuAu b=7fm: 20-30% effect
- EoS: SM-EOS Q vs. EOS L: ~10% effect
- different I-S eqn.: simplified vs. full I-S eqn.: ~5% effect (EOS Q and EOS L only)

Comment: To extract QGP viscosity from exp. data by using viscous hydro, one needs a better description of EoS (Lattice EoS + chemical non-equil. HRG EoS)

System size effects

Multiplicity scaling of v_2: Experiment



Central & semi-central collisions:

200 GeV Au+Au
62.5 GeV Au+Au
200 GeV Cu+Cu

Compare these above data with viscous hydro

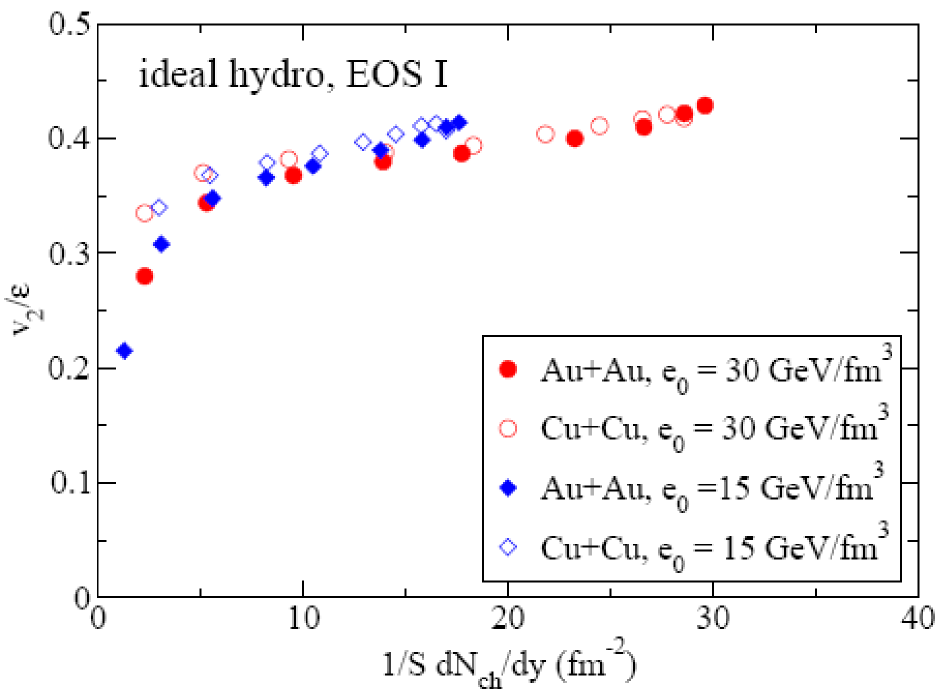
low energy limit: $v_2/\varepsilon \propto (1/S)(dN_{ch}/dy)$ Voloshin PLB'00

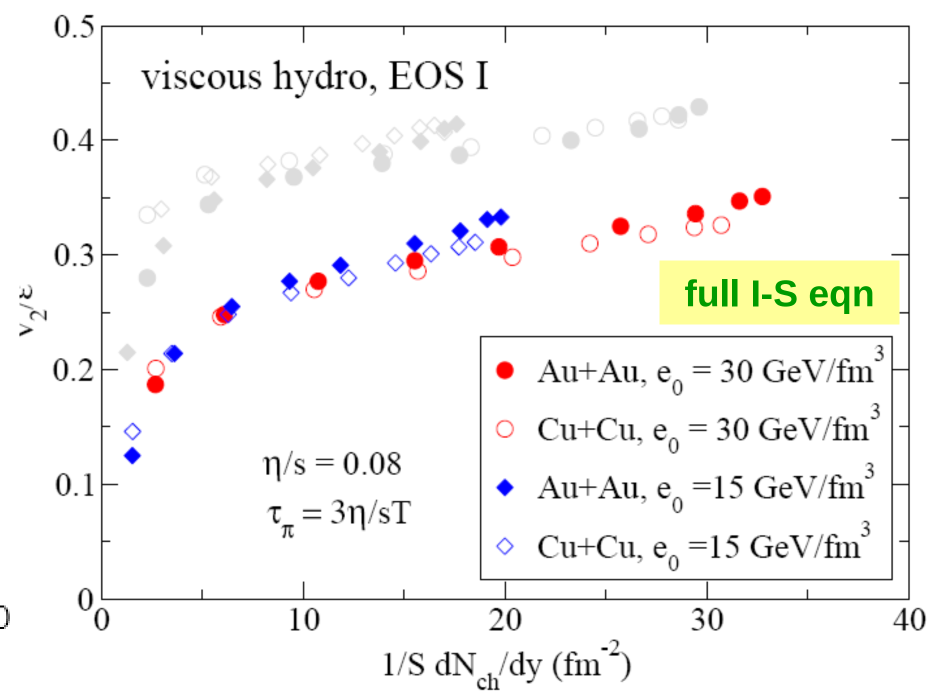
ideal hydro limit: v_2/ε → a saturation value around 0.21-0.23 (EOS Q; depends on c_s)

- viscous hydro works for near equilibrium systems, which lie between the low energy and ideal hydro limits (how does shear viscosity change the scaling of ideal hydro?)
- v_2/ε is insensitive to the type of initialization used (CGC or Glauber model)

Multiplicity scaling of v_2/ε EOS I

Preliminary





Ideal hydrodynamics: multiplicity scaling of v_2/ε is weakly broken:

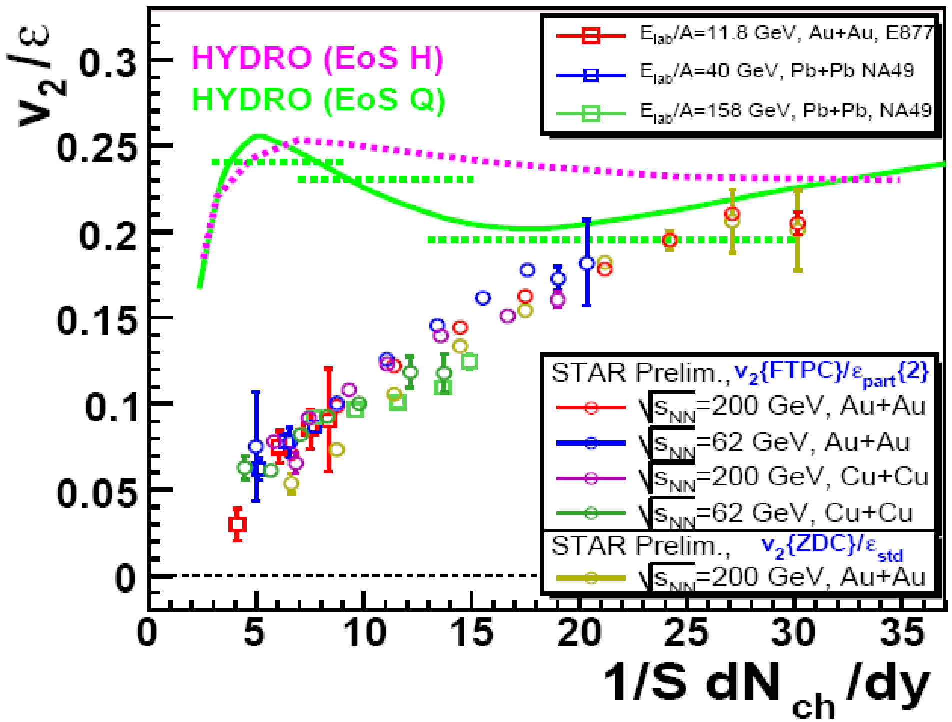
- freeze-out condition introduces time scale, breaking scale invariance of id. hydro eqns.
- Cu+Cu and Au+Au systems are not identical after a rescaling

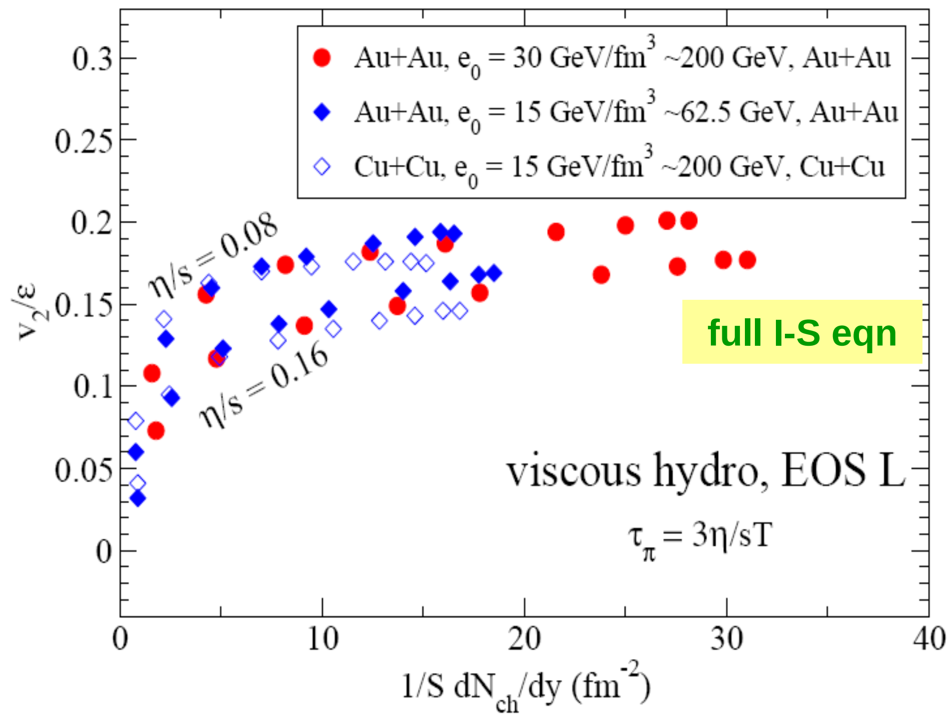
Viscous hydrodynamics: additional scale breaking by shear viscosity, resulting in fine structure of v_2/ε :

- for similar initial energy density, Cu+Cu curves are slightly below the Au+Au curves
- at fixed $\frac{1}{S}\frac{dN_{ch}}{dy}$, the $e_0 = 15\mathrm{GeV/fm}^3$ curves are slightly above the $e_0 = 30\mathrm{GeV/fm}^3$ ones

Viscous effects are larger for smaller systems and lower collision energies

Multiplicity scaling of v_2/ε EOS L *Preliminary*

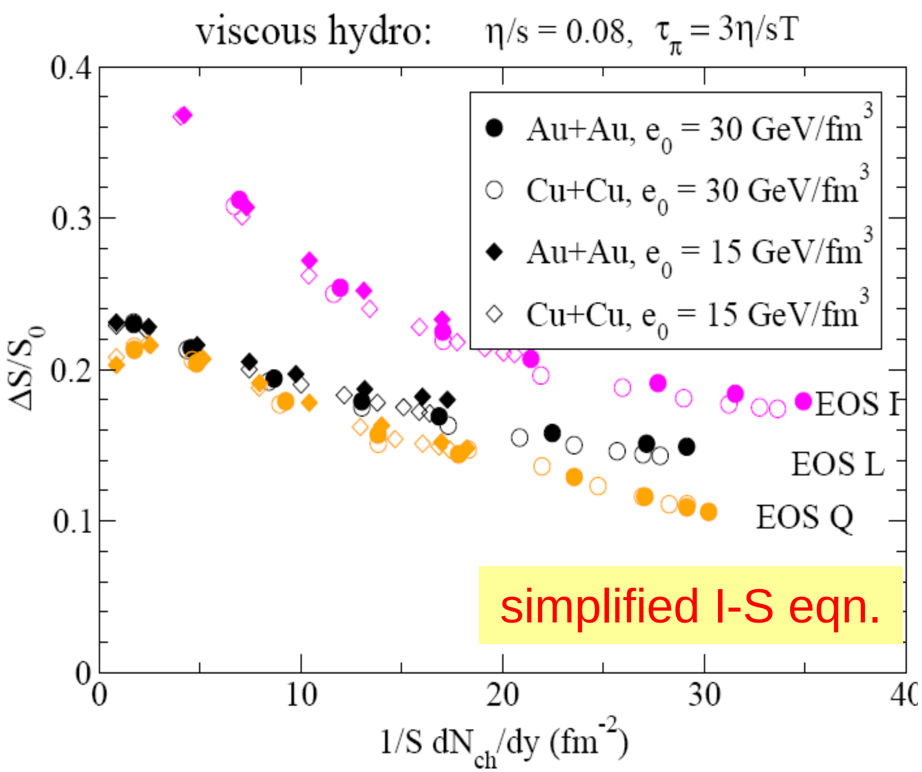


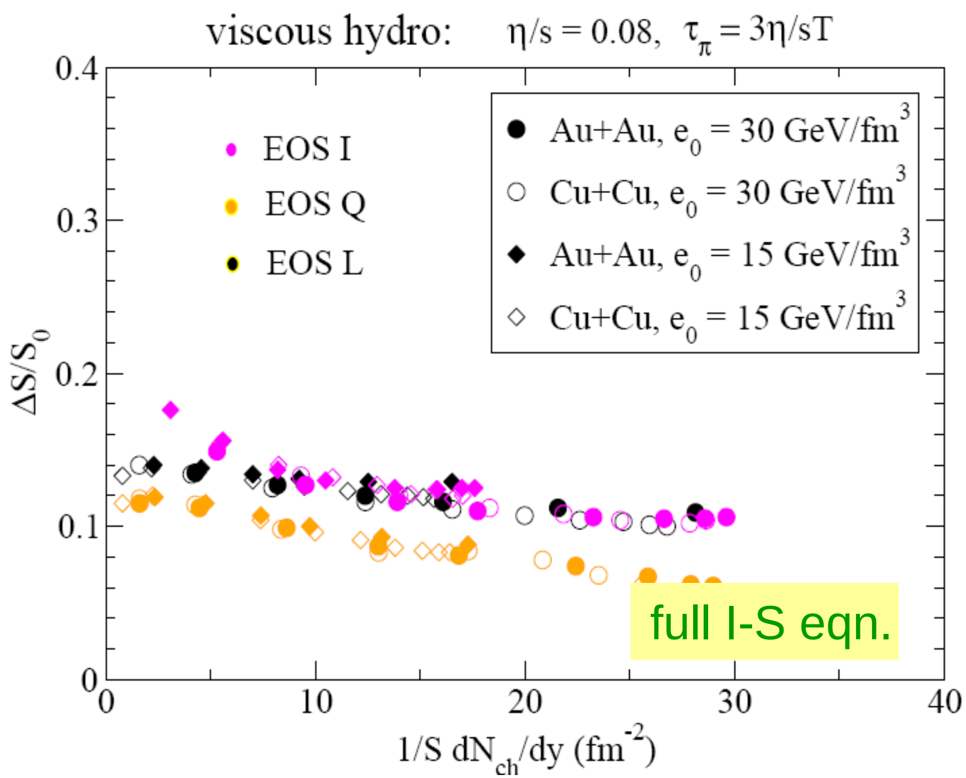


- experimental data show qualitatively similar fine ordering as viscous hydro prediction
- to reproduce slope of v_2/ε vs. (1/S)dN/dy, a better description of the highly viscous hadronic stage is needed: viscous hydro + hadron cascade
- the experimental v_2/ε vs. (1/S)dN/dy scaling (slope and fine structure) is another good candidate to constrain η/s (insensitive to Glauber-type vs. CGC initialization)
- this requires, however, experimental and theoretical improvements: reduced error bars, accounting for *T*-dependence of η/s, ζ/s near T_c, modeling hadronic phase with realistic cascade

Entropy production

Entropy production (I)

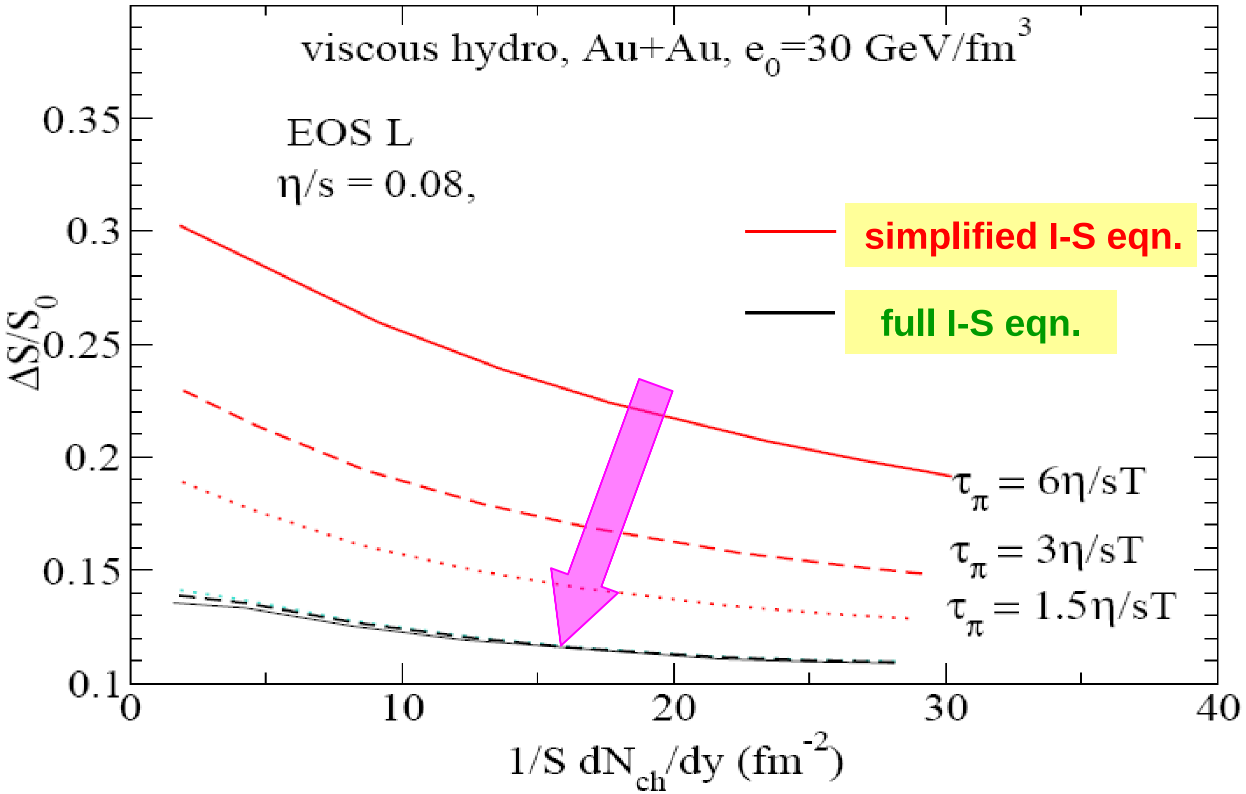




- for each EoS, see good scaling of with (1/S)dN/dy (for both simplified and full I-S eqns)
- larger viscous effects in smaller systems and at lower collision energies:
  entropy production increases as $(1/S)dN_{ch}/dy$ decreases)

- for EOS L, the simplified I-S eqn. gives 5-10% more entropy production than the full I-S

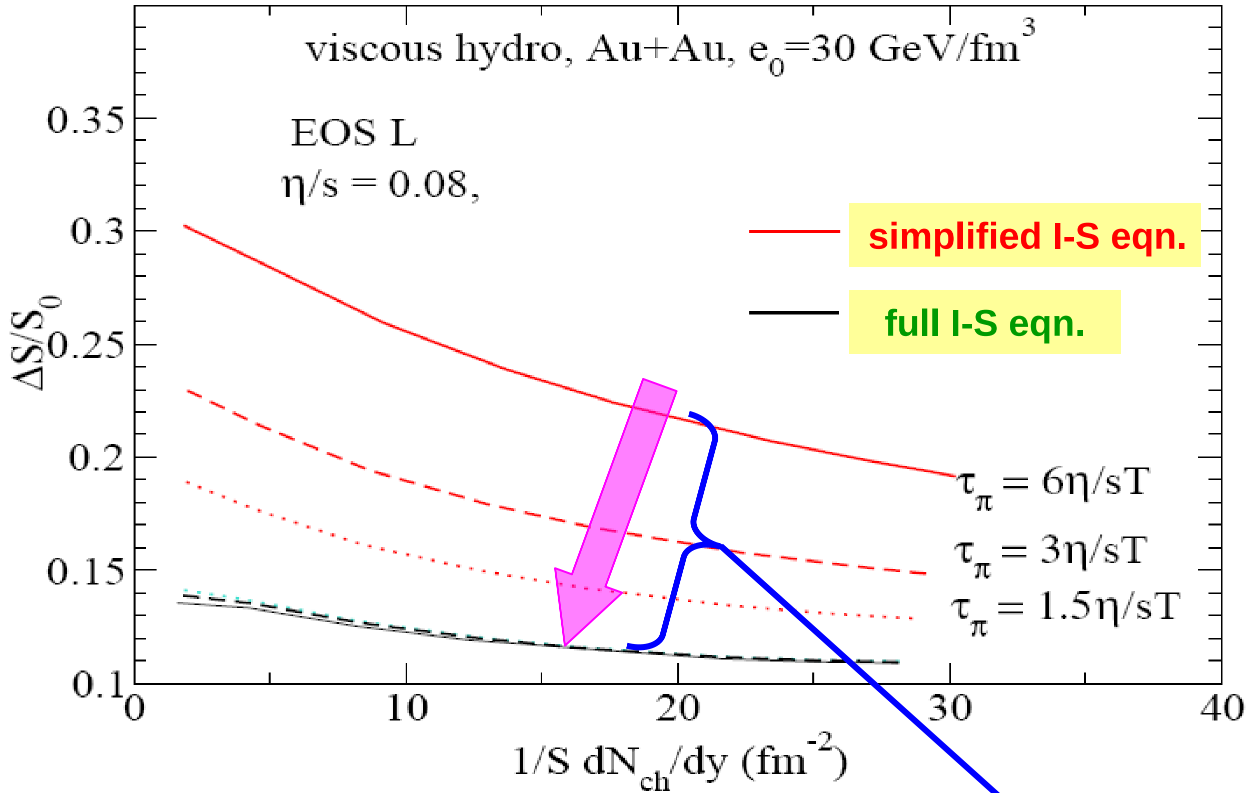
Entropy production (II)



- simplified vs. full I-S eqn.: entropy production approaches the same Navier-Stokes limit as we let $\tau_\pi \to 0$

- for the full I-S eqn., entropy production is very insensitive to τ_π

Entropy production (III)



$$\Delta^{\mu\alpha}\Delta^{\nu\beta}D\pi_{\alpha\beta} = -\frac{1}{\tau_\pi}\left[\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu}\right] + \frac{1}{2}\pi^{\mu\nu}\left[5D\ln T - \nabla_\alpha u^\alpha\right] - 2\pi^{\alpha(\mu}\omega_\alpha^{\nu)}$$

- simplified vs. full I-S eqn.: entropy production approaches the same Navier-Stokes limit as we let $\tau_\pi \to 0$

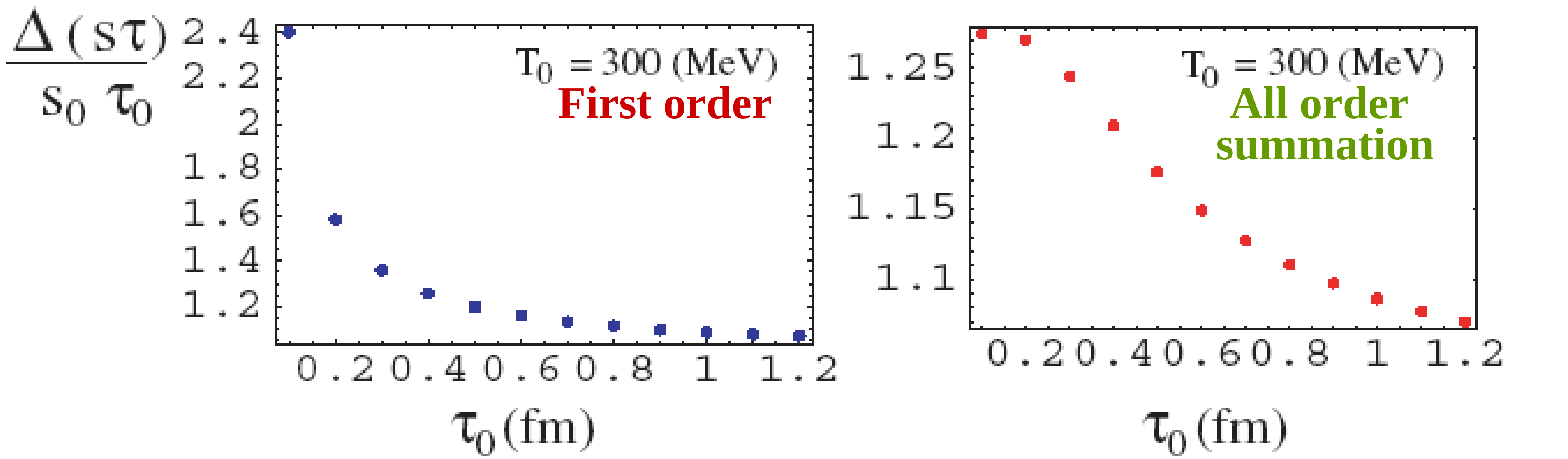
- for the full I-S eqn., entropy production is very insensitive to τ_π

Entropy production (IV)

- effects of higher order terms on entropy production

We argue that estimates of dissipative effects based on first-order hydrodynamics with shear viscosity are potentially misleading because higher order terms in the gradient expansion of the dissipative part of the stress tensor tend to reduce them. Using recently obtained sound dispersion relations in thermal $\mathcal{N} = 4$ supersymmetric plasma, we calculate the *resummed* effect of these high-order terms for Bjorken expansion appropriate to heavy ion collisions such as those performed at the BNL Relativistic Heavy Ion Collider (RHIC) and the CERN Large Hadron Collider (LHC). A reduction of entropy production is found to be substantial, up to an order of magnitude.

--Lublinsky & Shuryak PRC2007

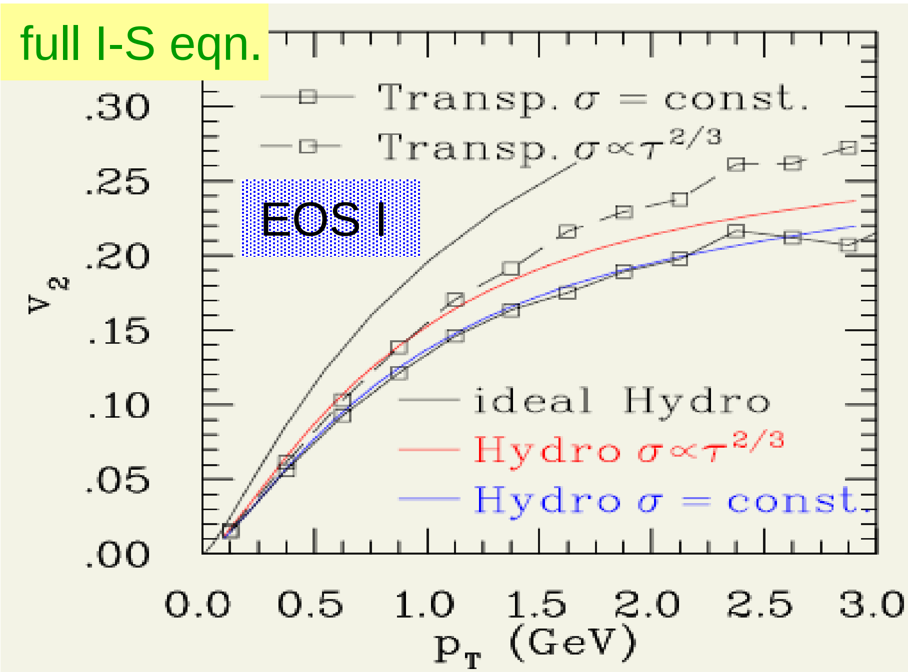


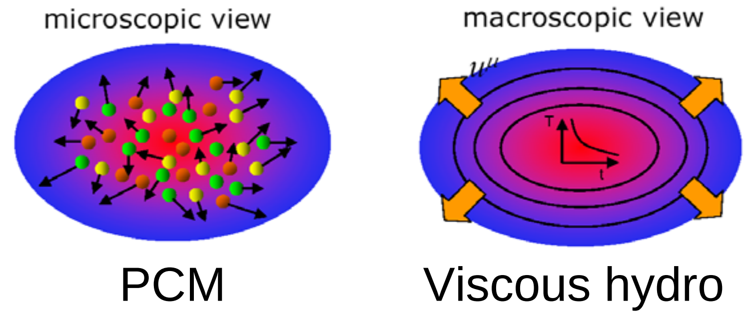
- Is the the 2nd order expansion formalism enough to describe the more viscous fluids created in smaller collision systems or at lower energies?

full I-S eqn. vs. parton cascade model

Important insight from comparing viscous hydro with parton cascade model:

Molnar & Huovinen QM08





massless Boltzmann particles $2 \leftrightarrow 2$ processes

$\sigma \propto \tau^{2/3} \Rightarrow \eta/s \approx 1/(4\pi)$ $\eta = 4T/(5\sigma_{\mathrm{tr}})$

- $\tau_0 = 0.6$ fm$/c$
- $b = 8$ fm
- $T_0 = 385$ MeV and $dN/d\eta|_{b=0} = 1000$
- freeze-out at constant $n = 0.365$ fm^{-3}

- parton cascade model: evolves directly $f(\vec{x}, t, \vec{p})$ with Boltzmann eqn.
- 2nd order viscous hydro: I-S eqn. is obtained from Boltzman eqn. by expanding to 2nd order in δf
- amazingly, the two approaches give results that agree well with each other

It appears that the 2nd order I-S equation is still sufficient for semi-central collisions starting with high enough initial energy density

Summary and discussion (I)

- Elliptic flow is very sensitive to even minimal shear viscosity

- Viscous effects are larger in smaller systems and at lower collision energies:

  - 20-30% more v_2 suppression in Cu+Cu b=7 fm than Au+Au b=7fm (similar initial ecc.)

  - more entropy production in smaller systems and at lower collision energies

  - fine structure in the multiplicity scaling curves for v_2/ε predicted by viscous hydro agrees qualitatively with experimental data and can be used to constrain the QGP shear viscosity

- Simplified I-S equation vs. full I-S equation:

  - approach same Navier-Stokes limit for $\tau_\pi \to 0$ (entropy prod., momentum anisotropy etc.)

  - for non-conformal fluids (EOS Q, EOS L) both eqns. are OK (~5-10% difference) if the system size and initial energy density are not too small (the approx. behind the simplified I-S eqn. are "OK" (~90-95% reliable) for EOS L and EOS Q)

  - for conformal fluids (EOS I) the full I-S eqn. (which preserves the conserve conformal symmetry) works but the simplified I-S eqn. fails

  - entropy production: the full I-S eqn. is much less sensitive to τ_π, leaving less ambiguity in the extraction of η/s due to the choice of τ_π

Summary and discussion (II)

- Extracting the QGP shear viscosity requires a better understanding of all sources of uncertainty from both sides, theory and experiment:

- For causal viscous hydro, one needs to consider at least the following aspects:

  - resolve the ambiguities among different 2nd order formalisms used by different groups when simulating causal viscous hydrodynamics

    a) simplified I-S (Song & Heinz 07-08) vs. full I-S eqn. (P.&U.Romatschke) ~5-10% (EoS L & EOS Q)

    b) I-S formalism vs. O-G formalism (Dusling & Teaney) ~ ? %

  - a realistic EOS: EOS L vs. SM-EOS Q ~10% (for v_2 and v_2/ε)

  - realistic initial conditions: CGC vs. Glauber-type initialization ~15-30% (for v_2)

  - bulk viscosity: with vs. without bulk viscosity ~?%

  - T-dependent η/s : ~?%

... ...

Thank You

I-S formalism - Kinetic theory approach

$$f(x,p) = f_{eq}(x,p)(1+\delta f(x,p))$$

$$\delta f(x,t,p) = \varepsilon(\mathrm{x},t) + \varepsilon_{\lambda}(\mathrm{x},t)p^{\lambda} + \varepsilon_{\lambda\nu}(\mathrm{x},t)p^{\lambda}p^{\nu}$$

shear viscosity only: $\delta f(x,t,p) = \varepsilon_{\lambda\nu}(\mathrm{x},t)p^{\lambda}p^{\nu}$

$$f_0 = \exp(-p_{\mu}u^{\mu}/T)$$

$$\pi^{\mu\nu} = 2T^2(e+p)\varepsilon^{\mu\nu}(x)$$

$$\delta f(x,t,p) = \frac{\pi_{\lambda\nu}p^{\lambda}p^{\nu}}{2T^2(e+p)}$$

Baier et al. '06, W.Israel, J.Stewart '79

$$p_{\mu}d^{\mu}f(\vec{x},t,\vec{p}) = C(x)$$

$$C = -p_{\mu}u^{\mu}\frac{f-f_0}{\tau_{\Pi}}$$

-acting $\int d\omega$, or $\int d\omega\, p^{\alpha}$ on both sides $\longrightarrow$ conservation laws: $\left(\int d\omega = \int \frac{d^3p}{(2\pi)^3 E}\right)$

$$\partial_{\mu}N^{\mu}(x) = 0, \quad \partial_{\mu}T^{\mu\nu}(x) = 0.$$

-acting $\int d\omega\, p^{\alpha}p^{\beta}$ on both side and expand δf to the 2nd order $\longrightarrow$ I-S formalism:

$$\frac{-a_2}{T} < \nabla^{\alpha}u^{\beta} > + \frac{b_2}{a_2}(\Delta^{\gamma\alpha}\Delta^{\beta\delta}D\pi_{\gamma\delta} + 2\pi^{\delta(\alpha}\omega_{\delta}^{\beta)}) + b_2\pi^{\alpha\beta}Da_2^{-1} = -\frac{1}{\tau_{\pi}}\frac{b_2}{a_2}\pi^{\alpha\beta}$$

massless particles: $f_0 = \exp(-p_{\mu}u^{\mu}/T)$ $\quad a_2 = 4T^6/\pi^2 \quad b_2 = 24T^7/\pi^2$

Causal viscous hydro in (2+1)-d

Heinz, Song & Chaudhuri, PRC06

(τ, x, y, η) coordinates 3+1 $\Rightarrow$ 2+1

$$T^{mn}{}_{,m} = 0 \qquad T^{mn} = (\varepsilon + p)u^m u^n - p g^{mn} + \pi^{mn}$$

$$\Delta^{ml}\Delta^{nk} D\pi_{lk} = -\frac{1}{\tau_\pi}\left[\pi^{mn} - 2\eta\sigma^{mn}\right]$$

simplified I-S eqn

-also the eqns used in Song & Heinz, PLB07 & arXiv:0712.3715[nucl-th]& the one in preparation

$$T^{mn}{}_{,m} = 0 \qquad T^{mn} = (\varepsilon + p)u^m u^n - p g^{mn} + \pi^{mn}$$

$$\Delta^{ml}\Delta^{nk} D\pi_{lk} = -\frac{1}{\tau_\pi}\left[\pi^{mn} - 2\eta\sigma^{mn}\right] + \frac{1}{2}\pi^{mn}[5D\ln T - \nabla_\alpha u^\alpha]$$

full I-S eqn

-also the eqns used in P&U Romatschke PRL07, Song & Heinz, in preparation

Vortices terms are neglected, $\nabla^{<m}u^{n>} = \frac{1}{2}[\nabla^m u^n + \nabla^m u^n] - \frac{1}{3}\Delta^{mn}\partial_\sigma u^\sigma$

the transport equations for energy momentum tensor are explicit written as:

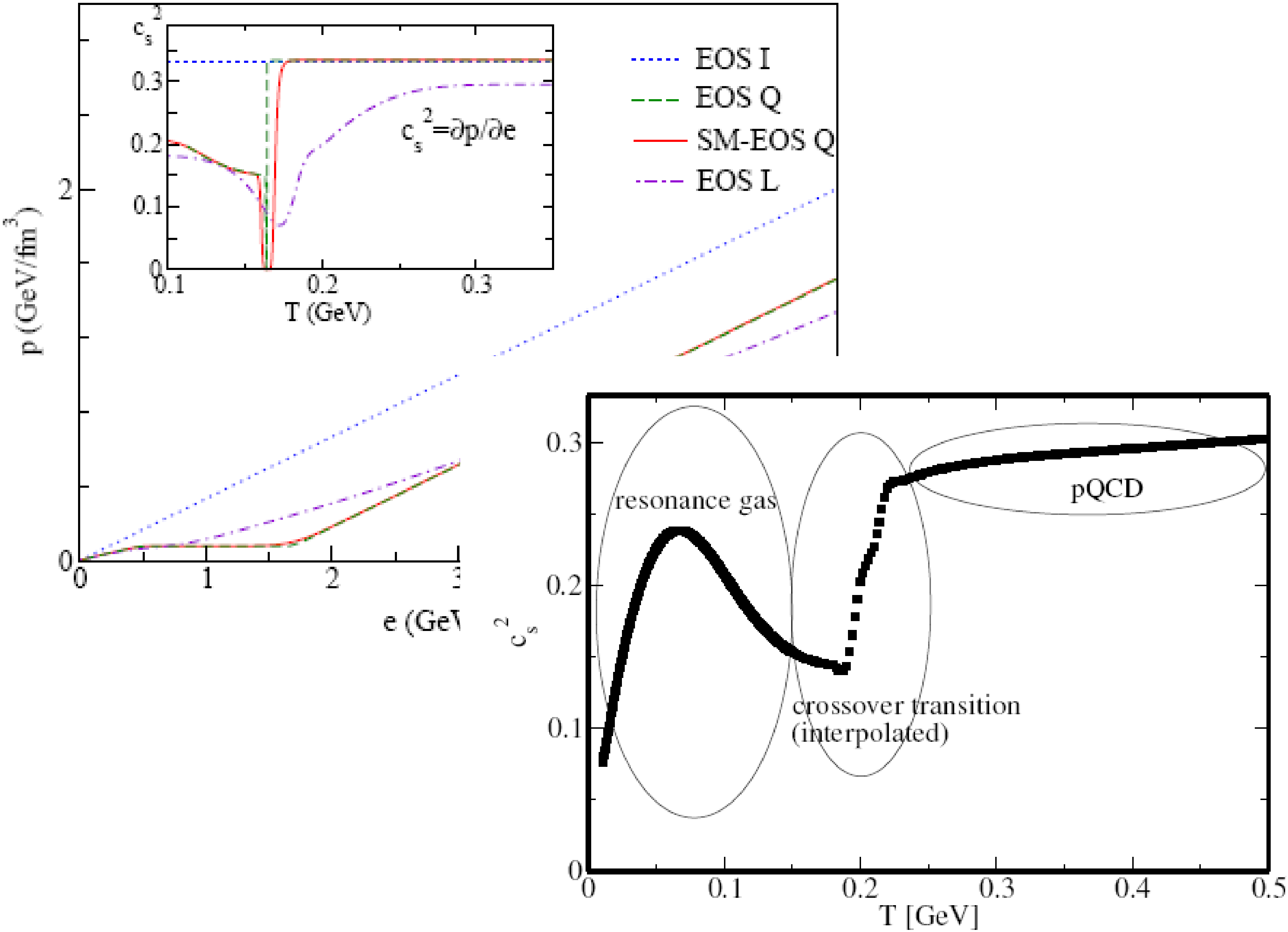
$$\frac{1}{\tau}\partial_\tau(\tau T^{\tau\tau}) + \partial_x(T^{\tau x}) + \partial_y(T^{\tau y}) = -\frac{p + \tau^2\pi^{\eta\eta}}{\tau}$$

$$\frac{1}{\tau}\partial_\tau(\tau T^{\tau x}) + \partial_x((T^{\tau x} - \pi^{\tau x})v_x) + \partial_y((T^{\tau x} - \pi^{\tau x})v_y) = -\partial_x(p + \pi^{xx}) - \partial_y \pi^{xy}$$

$$\frac{1}{\tau}\partial_\tau(\tau T^{\tau y}) + \partial_x((T^{\tau y} - \pi^{\tau y})v_x) + \partial_y((T^{\tau y} - \pi^{\tau y})v_y) = -\partial_y(p + \pi^{yy}) - \partial_x \pi^{xy}$$

$$v_x = \frac{T^{\tau x} - \pi^{\tau x}}{T^{\tau\tau} + p - \pi^{\tau\tau}}$$

$$v_y = \frac{T^{\tau y} - \pi^{\tau y}}{T^{\tau\tau} + p - \pi^{\tau\tau}}$$


EOS I
EOS Q
SM-EOS Q
EOS L
$c_s^2=\partial p/\partial e$
T (GeV)
p (GeV/fm^3)
resonance gas
crossover transition
(interpolated)
pQCD
T [GeV]
c_s^2