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A note on the potential effects of particle noise on turbulence

G. W. Hammett, W. Dorland, et.al.
Princeton University Plasma Physics Laboratory
P.O. Box 451, Princeton, NJ 08543 USA

Abstract

1 Intro

This note is to to try answer some questions about particle noise which Curt Bolton and others have raised in cyclone and NTTP discussions.

For some time in the Cyclone and NTTP discussions, several of us have raised the issue of particle noise as a potential source of problems in particle simulations. Based on some papers by Krommes, some initial calculations I've done in the handwritten notes I've distributed indicate that the particle noise-induced diffusion in typical present simulations may be large enough to have significant impact on the turbulence. However, there are some factor-of-two uncertainties in my present calculations (I am working to improve the FLR and other parts of the calculation to reduce these uncertainties). Also, a key issue is whether a given amount of noise suppresses the turbulence by more than the noise contributes directly to the radial transport (one of the purposes of this note is to point out why we believe this has a significant possibility of being true). We are working on some numerical tests to investigate these questions.

We could be wrong, and it may be that Dimits' present particle simulations are adequately resolved with enough particles so that noise does not significantly impact the results. Perhaps the calculations we are now doing will ultimately support Dimits' results. But in this note I want to spell out why I believe the noise issue is/was at least a legitimate issue to be significantly concerned about. This note should explain why the convergence rate may be slower than 1/sqrt(N) (where N is the number of particles), and answer some other questions which have been raised. Some people have said that it seems to them that noise would only add to the transport, and not reduce it. Others have asked what the time-dependence of χ, etc., would be if noise is important. This note is to spell out my view of these things.

Again, it may turn out that we were wrong and noise is not a significant problem. I hope to have more concrete numerical simulations in the next few weeks to answer some of these questions more clearly. In combination with the "scrambled eggs" tests which are being planned with the particle codes in the near future, it seems reasonable that these issues will be resolved one way or another soon.

2 A simple noise+turbulence model

The total radial heat transport coefficient observed in a particle simulation is composed of two parts:

$$\chi = \chi_t + \chi_n \tag{1}$$

where χ_t is the component driven by the turbulence (by collective motions of the plasma), and χ_n is the component driven by "particle noise" (the diffusion caused by the random fluctuations associated with the discrete particles). The noise component χ_n increase in time as the particle weights increase, $\langle w^2 \rangle \propto \int dt \chi$. From the Eqs. on p. 35 and 14 of my handwritten notes, $\chi_n \propto \langle w^2 \rangle / N$ in a well-resolved regime where there is only a weak logarithmic dependence on the renormalized diffusion rate D_* in the g and ν_{ratio} expressions on p.35. In less well resolved regimes where ν_{ratio} is getting small, then the "strong-turbulence" limit on p.32 of my notes applies, and the noise diffusion χ_n scales only as $\langle w^2 \rangle^{1/2} / N^{1/2}$.

But in either case, the noise diffusion χ_n increases with time, and some people have assumed that this means the total χ of Eq. (1) would increase with time if noise was important. But to be sure what happens to the total χ, we need to know how the noise affects the turbulent component χ_t.

A very simple model for how the turbulence χ_t evolves in time is:

$$\frac{d}{dt} \chi_t = \left(\gamma_{lin} - c_t \chi_t k_\perp^2 - c_n \chi_n k_\perp^2 \right) \chi_t \tag{2}$$

The first term in the parenthesis, γ_{lin}, is the linear growth rate. The second and third terms are the effective damping due to nonlinear turbulent decorrelation and due to noise-induced diffusion. In this simple traditional picture (ignoring the noise) the turbulence grows until the nonlinear decorrelation rate, represented by $c_t \chi_t k_\perp^2$ balances the linear growth rate (c_t is a some coefficient which we might choose to match nonlinear simulations, or try to determine from analytic turbulence theories). This is of course an overly simple "toy" model, but it has some generic qualitative features which help explain the point I am trying to make.

Assuming that a quasi-equilibrium in Eq. (2) is maintained throughout the nonlinear phase, so the expression in parentheses vanishes, we can solve for χ_t to find:

$$\chi_t = \frac{\gamma_{lin} - c_n \chi_n k_\perp^2}{c_t k_\perp^2} \tag{3}$$

If the $\chi_n = 0$ so the noise is ignored, this is just the usual $\gamma / k_\perp^2$ mixing-length scaling. One notes that the noise-induced diffusion will reduce the effective growth rate which drives the instabilities and will thus suppress the turbulence. Inserting this expression for χ_t in Eq. (1), yields

$$\chi = \frac{1}{c_t} \frac{\gamma}{k_\perp^2} + \left(1 - \frac{c_n}{c_t} \right) \chi_n$$

How strongly the noise suppresses the turbulence depends on the ratio c_n / c_t. There could be regimes where this ratio is large, and the measured χ will actually drop as the noise increases. There might be other regimes where this ratio is of order unity, so that as one increases the number of particles the drop in the noise χ_n is offset by the rise in the turbulent χ_t, and the convergence rate could be slow. [In the limit that the noise is very large, then

Eq. (3) breaks down because it predicts a negative χ_t. In this limit where the total χ is all due to noise, χ would again grow in time.]

The real nonlinear system is very complicated, but this example serves to show that various types of behaviour are possible. One of the interesting points is that it would seem that the ratio c_n/c_t is often greater than one, so that one would often expect χ to actually drop in time. This is because χ_t in nonlinear simulations (and apparently in experiments) is often 2-8 times larger than the mixing length level, i.e., $c_t \sim 0.15 - 0.5$. This is usually attributed to several effects. One is the downshift of the nonlinear spectrum from the fastest growing linear instability, to modes with larger $\gamma/k_\perp^2$. But another is that the $\chi_t k_\perp^2$ term in Eq. (2) is a renormalized approximation to a more complicated $E \times B$ nonlinearity in the full equations. This nonlinearity has the form $\vec{v}_{ExB} \cdot \nabla \propto \hat{z} \cdot \vec{k}'' \times \vec{k}'$. The strength of this nonlinearity tends to be reduced for anisotropic turbulence where the eddies tend to be elongated in one particular direction. I.e., the nonlinearity completely vanishes if $\vec{k}''$ and $\vec{k}'$ both point in the same direction. Another way to look at this is in real space, where the nonlinearity has the Poisson-bracket form:

$$\vec{v}_{ExB} \cdot \nabla f \propto \frac{\partial \phi}{\partial x}\frac{\partial f}{\partial y} - \frac{\partial \phi}{\partial y}\frac{\partial f}{\partial x}$$

In the limit that the eddies for ϕ (which depends on the distribution function f) are infinitely elongated in the radial direction, then $\partial\phi/\partial x = 0$ in the first term and $\partial f/\partial x = 0$ in the second term, so the nonlinearity completely vanishes.

Yet another way to think of this is that we are looking at turbulence driven by bad-curvature instabilities, and these instabilities preferentially cause transport in the radial direction, and less transport in the poloidal direction. [This is related to a point which Bryan Taylor has made to me, that though the primary instability is driven by bad curvature, the “secondary instability”, which causes flow in the poloidal direction and breaks up the radial streamers, can not be driven by bad curvature and must depend on some relatively weaker mechanisms like a variant of the slab ITG mode.] Thus the damping due to χ_t should perhaps be split up into two terms related to radial and poloidal transport:

$$c_t \chi_t k_\perp^2 \approx \chi_{tx} k_x^2 + \chi_{ty} k_y^2$$

The right hand side can be significantly smaller than $\chi_t k_\perp^2$ (so c_t is significantly smaller than one), because the typical k_x^2 is small compared to the typical k_y^2 for radially elongated eddies, and the poloidal turbulent diffusion χ_{ty} is probably small compared to the radial $\chi_{tx} = \chi_t$, which is the component usually reported.

However, the situation is different for the noise-induced diffusion term, since the noise spectrum is isotropic in k (ignoring the possible enhancement of the $m = n = 0$ modes) and tends to cause diffusion in all directions equally. In other words, a given level of noise χ_n can be much more efficient at providing the poloidal motion needed to break up a radially elongated streamer than the same level of turbulent χ_t. Thus while the coefficient c_t for the turbulence is often less than one, one would usually expect the coefficient c_n for the noise to be closer to one.

Also, because the noise tends to be at high k (the noise ϕ spectrum is relatively flat in k until FLR and particle size cuts it off at high k, but the noise spectrum of velocity fluctuations $k\phi$ is weighted to high k), there is a better separation of scales between the noise and the turbulence and one can more accurately treat the effect of the short-wavelength noise on the longer wavelength turbulence with a diffusive approximation, so $c_n \approx 1$. Our experience with linear calculations is that adding diffusion D to all of the equations causes the net linear growth rate to just drop by approximately $D\langle k_\perp^2 \rangle$ (the eigenmode structure can adjust a bit so this isn't exact), suggesting that $c_n \approx 1$. [Similar results occur when D is added to all equations except the density equation, to preserve the adiabatic electron constraint of no particle transport.]

[For simplicity, I've been ignoring the possible enhancement of the $m = n = 0$ component of the noise due to adiabatic electron constraints. This would only further increase the poloidal transport caused by the noise, and more strongly suppress the turbulence, without adding to the radial transport in Eq. (1) at all.]

In the cyclone conference call last week, I believe I understood Andris to say that there are some low N cases where he believe there is noise and it appears to be causing the net transport to drop at the end of a run, consistent with the above picture. He also said that this appears to depend on the regime, as there are other cases where noise causes the net transport to rise. I agree that the net result may vary depending on the parameter regime.

In a recent message, Wei-li Lee said that noise acts as enhanced collisions and, consequently, drives up (but not down) the diffusion. There can be some regimes where collisional de-trapping indeed raises turbulence levels, particularly in weak turbulence regimes, but I don't think this is a universal feature. Furthermore, while it is true that noise in a regular particle simulation acts as enhanced collisions in the traditional sense, where collisions between discrete particles causes diffusive scattering of the particles in velocity space, gyrokinetic particle simulations are quite unusual in this regard. A curious feature of noise in a gyrokinetic particle simulation is that since the adiabatic invariant μ of each particle is conserved, collisions between two gyrokinetic ring particles causes no scattering in velocity space. Instead, the two gyrokinetic guiding-center particles scatter off of each other in configuration space as they $E \times B$ drift due to the field from each other. For the reasons outlined above, I would expect this noise-induced configuration-space diffusion to usually reduce the turbulence level, not enhance it.

3 Other items

One should note that there are two types of errors from the discrete set of simulation particles. One is the "particle noise" which I have been discussing here. But that noise is proportional to $\langle w^2 \rangle$, and so in the early linear phase the noise can be made arbitrarily small in a δf simulation by scaling δf to be arbitrarily small. (Another way of thinking of this is that "particle noise" is an intrinsically nonlinear effect of two discrete particles scattering off of each other.) But this does not mean that one can accurately reproduce the linear eigenmode structure and eigenfrequency with only 1 particle. There must be a second criterion to have enough particles to adequately sample phase space to reproduce the linear dynamics. It would seem reasonable (as I recall having heard Bruce or some of the other particle experts state), that one generally needs more particles near marginal stability in order to adequately sample the resonant part of phase space (which shrinks near marginal stability). The criterions for reducing these two types of errors (linear sampling errors and particle noise) are somewhatat related in the nonlinear gyrokinetic ordering, so that one converges with respect to both of them at similar levels of N. But it could be that, in some cases, as the number of particles is increased one converges first with respect to one of these criterion, and only later with even more particles does one converge with respect to the other criterion.

4 Conclusions

See Section 1.