# Three-Dimensional Track Finding in the PHENIX Drift Chamber by a Combinatorial Hough Transform Method[1]

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**Abstract.** We describe a pattern recognition method based on Combinatorial Hough Transforms (CHT) for 3-D track finding in the large drift chambers of the PHENIX experiment at RHIC/BNL. The CHT algorithm first identifies tracks in the bending plane from hits in the two separated sets of wires parallel to the beam axis; each track is then analyzed to obtain $z$-coordinate information from the two sets of UV stereo wires. The method is coded in C++, using appropriate objects to describe various geometric entities inherent to the problem. Track finding efficiency evaluated using realistic simulated hits from central Au+Au collisions is >98% in the bend plane, and >90% in three dimensions. Our CHT method has been incorporated into the standard PHENIX reconstruction code package, where its execution time is a small fraction of the overall event reconstruction time. The algorithm is quite robust and performs well at twice the highest expected track density. It has also recently been applied, with minor modifications, to a second PHENIX sub-system, the time-expansion chamber (TEC).

## INTRODUCTION

The Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory (BNL) will be commissioned and begin colliding nuclei from protons to Au at $\sqrt{s} = 200\text{GeV/A}$ beginning in the spring of 1999. The RHIC physics program is to probe nuclear matter at extreme temperatures and create and study a deconfined and/or chirally restored state of matter. The expected multiplicity from central

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collisions of Au nuclei at these energies is approximately 10,000 primary particles. As simple road following algorithms fail at large track densities [1] due to frequent track crossings, in particular in projective measurements, new, more robust and time efficient pattern recognition algorithms must be developed.

We report here on a novel and highly effective method based on Combinatorial Hough Transforms (CHT) [2] for recognizing straight tracks in the drift chambers of the PHENIX detector at RHIC. The same CHT algorithm has also recently been successfully applied, with only minor modifications, to a second PHENIX detector, the time-expansion chamber (TEC) [3].

## DETECTOR AND MAGNET DETAILS

The PHENIX experiment is a multi-subsystem detector designed to make simultaneous measurements of $e^{\pm}$, $\gamma$ and hadrons in the central arms and $\mu^{\pm}$ in the muon arms [4]. The two opposed PHENIX central arms cover the mid-rapidity range $|\eta| < 0.35$ with multiple detectors for effective track reconstruction and particle identification.

PHENIX differs from most colliding beam detectors in that the central arm tracking elements are situated predominantly outside the axial magnetic field [5]. The field bends particles in the $x$-$y$ plane, perpendicular to the beam, and is essentially limited to $R < 2$ m. The fringe field for $R > 2$ m is sufficiently small that the residual deflection in the $x$-$y$ or bend plane is at most 5% of the primary deflection, i.e. no more than one degree. In the polar angle ($\theta$) the tracks are to first order not bent, except for tracks very close to the magnet pole edges or at low momentum [6].

The drift-chamber (DC) is the innermost tracking element. The two independent, cylindrically-shaped DC's straddle the beam line and occupy a volume defined by $2.0 < R < 2.4$ m, $-.8 < z < +.8$ m and $\Delta\phi = 90°$. The two symmetric detector volumes each contain 40 layers of wires at increasing radii; each layer has 80 sense or anode wires separated by $1.25°$ in azimuth. Most of these sense wires are parallel to the beam axis and measure in the main bend direction ('x-wires'). A subset is tilted by a small stereo angle ($\sim \pm 5°$) to allow full three-dimensional track reconstruction. The wires are arranged, in order of increasing radius from the origin, as 12X, 4U, 4V, 12X, 4U, 4V. The drift cells are designed to remove left-right ambiguities by making anode wires uni-directionally sensitive. As a result, the number of wires sensing each particle is nominally 20. Furthermore, the wire nets are divided electrically at the $z = 0$ midplane. Even so, the average occupancy for a central Au+Au collision is still about 2 hits per wire.

## THE COMBINATORIAL HOUGH TRANSFORM

The modest bend of the tracks in the drift chamber reduces the task of pattern recognition to finding straight-line trajectories. The conventional or simple Hough Transform (SHT) is a well-known technique for this [7] that could in principle be used. In practice, however, the SHT fails; because of the high hit density there are too many actual or potential lines relative to data points in the 'image.' This situation results in 'ridges' in the Hough feature space that create many false peaks and, therefore, ghost tracks.

A variation on the SHT proposed by Ben-Tzvi and Sandler [2] for general image-processing about ten years ago has proven to be much better suited to the DC tracking problem, even though particle tracking was not the original intent. In this 'combinatorial Hough transform' (CHT) the image points (drift chamber hits) are mapped *pair-wise* into the feature space. This results in a 'gain' effect that greatly enhances the peak-to-background ratio in feature space. In the SHT the nominal peak height is equal to the number of points $n$ on the track; in the CHT on the other hand the peak height is $n(n-1)/2$. The combinatorial approach is limited by the $\mathcal{O}(n^2)$ growth in the number of possible combinations, but with current computers $n = 10^3$ or more is quite practical.

Essentially the same combinatorial method has also been proposed at almost the same time by Ohlsson, Peterson and Yuille [8] specifically for particle tracking. They use the term 'local Hough transform' (LHT) to denote their technique for limiting the number of combinations considered when searching for circular tracks in a time projection chamber (TPC). It must be emphasized that up to now these visual methods [8] have been intended only for 'seeding' a powerful but potentially very time-consuming [9] iterative algorithm, such as elastic tracking (ET), rather than as a *stand-alone* method as in the present work. The one exception that we are aware of is a study on the possible use of LHT in the STAR TPC experiment at RHIC [10].

## TRACKING IN THE BEND PLANE

Fig. 1 sketches the path of a sample track in the $xy$ or bending plane of the drift chamber. The coordinates we chose to describe tracks in the drift chamber are $\phi$, the polar angle at the intersection of the track with a 'reference radius' near the mid-point of the drift chamber, and $\alpha$, the inclination of the track at that point. These variables are also shown schematically in Fig.1. In principle $\phi$ and $\alpha$ are equivalent to a slope and

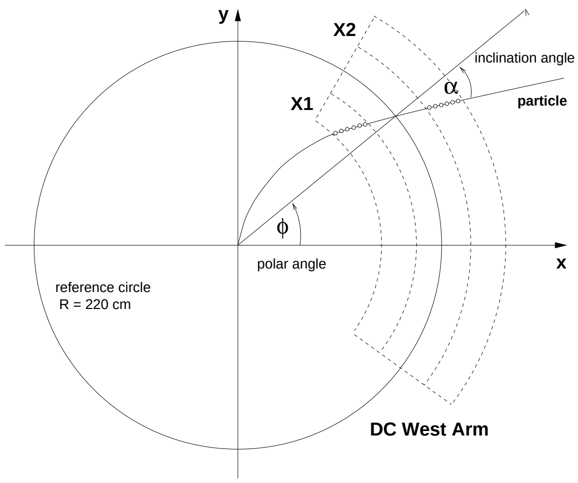


**FIGURE 1.** A sample track in the central arm shown in an xy-plane cut-away. The X1 and X2 hits in the drift chamber are shown as small circles with an outline of the drift chamber. $\phi$ and $\alpha$ are the feature space variables.

intercept; however, the latter are not bound and therefore the feature space is not finite. An added benefit of choosing $\phi$ and $\alpha$ is that $\alpha \sim 1/p_t$, making limited searches for, for example, high momentum tracks, trivial.

Fig. 2 shows an example of a cluster of points in the $\alpha$–$\phi$ feature space corresponding to one typical track passing through the drift chamber. Feature space points from the X1–X2 combinations are tightly clustered while those from X1 or X2 only combinations are both smeared out and displaced. The displacements are due to the non-collinearity of the X1 and X2 track segments in the drift chamber, a result of the residual magnetic fields in the drift chamber; the smearing reflects the increased uncertainties for angular measurements over a shorter distance. In order to have the sharpest possible peaks in feature space, and hence the best peak to background separation, only the 36 X1–X2 combinations were used in the final algorithm. As a result, the image has a width of about 2.75 mrad in $\phi$ by 50 mrad in $\alpha$. To minimize computation time $\alpha$ and $\phi$ were only calculated for hit pairs separated in azimuth angle by physically reasonable amounts corresponding to $p_\perp \simeq 150$ MeV/c, comparable to the natural cut-off of the spectrometer.

Fig. 3 shows an example of a portion of the drift chamber hits and the associated feature space for the expected particle multiplicity in a central Au+Au collision. The tracks are clearly visible in the feature space and a simple algorithm can find these peaks. At this point in the tracking algorithm each of the peaks corresponds to an actual track in the drift chamber. At the expected multiplicity of a Au+Au collision at RHIC, over 98% of the tracks from the origin and passing completely through the drift chamber are found from the combinatorial Hough transform with no further tracking. The ghost track rate is less than 1%.

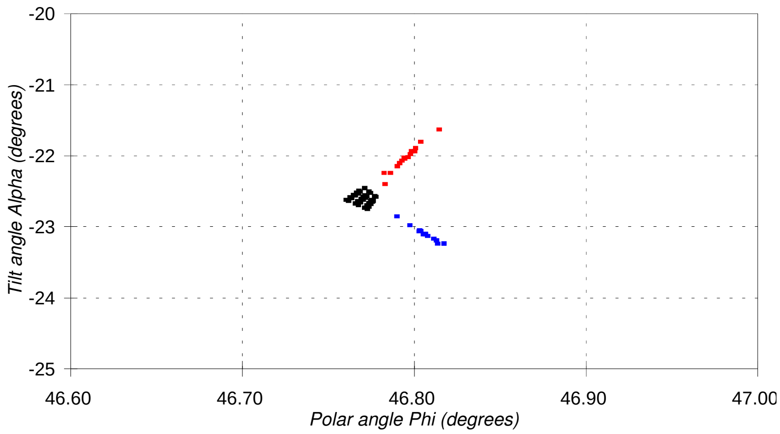


**FIGURE 2.** Bending-plane Hough space distribution for one sample track. Black points (tight cluster) correspond to X1 *versus* X2 hit combinations while red (upper right streak) and blue (lower right streak) points come from X1–only and X2–only combinations respectively.

# TRACKING WITH THE STEREO WIRES

After reconstructing the track in the bend plane, the direction of the track is specified by $\phi$ and $\alpha$. However, there is as yet no $z$ information; therefore, the track is constrained to a plane defined by $\phi$ and $\alpha$, extending in $z$, as shown in Fig. 4. The path of the trajectory through this plane (the '$x$-plane') is a straight line and is determined by exploiting the information measured by the stereo wires.

Drift chamber hits correspond to lines in physical space parallel to the anode wire and perpendicular to the drift direction. To determine the trajectory through the $x$-plane, we intersect each of the drift chamber hits corresponding to stereo wires with the $x$-plane for each track. In the $x$-plane of a track, the intersection of UV hits results in a number of points. Those UV hits corresponding to the track will align. A schematic of this procedure is shown in Fig. 4. At this point a second combinatorial Hough transform is implemented; here the coordinates of the feature space are $zed$, the $z$ point at which the track crosses the reference radius, and $\beta$, the polar angle at that point. Since the $x$-plane is defined by one track a unique solution in this space is expected.

# IMPLEMENTATION AND PERFORMANCE

The drift chamber tracking algorithm described above has been implemented in the PHENIX tracking package. It was coded in C++ using the standard template and CLHEP libraries. Natural objects for the drift chamber, including lines, planes and their

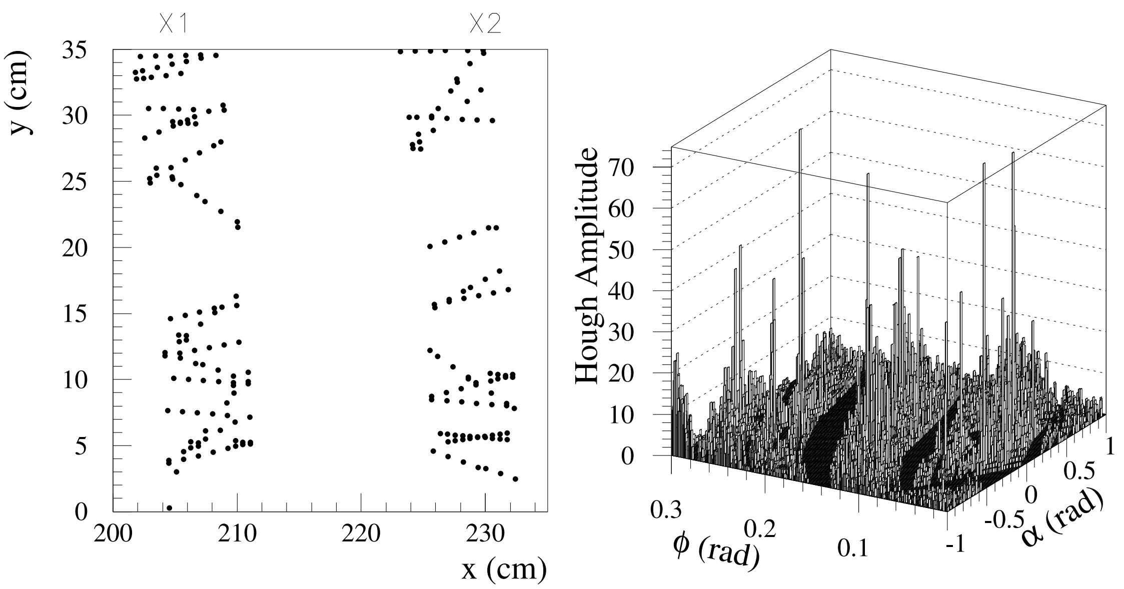


**FIGURE 3.** Simulated hits in a portion of the drift chamber and the corresponding Hough transform for X1 and X2 wires

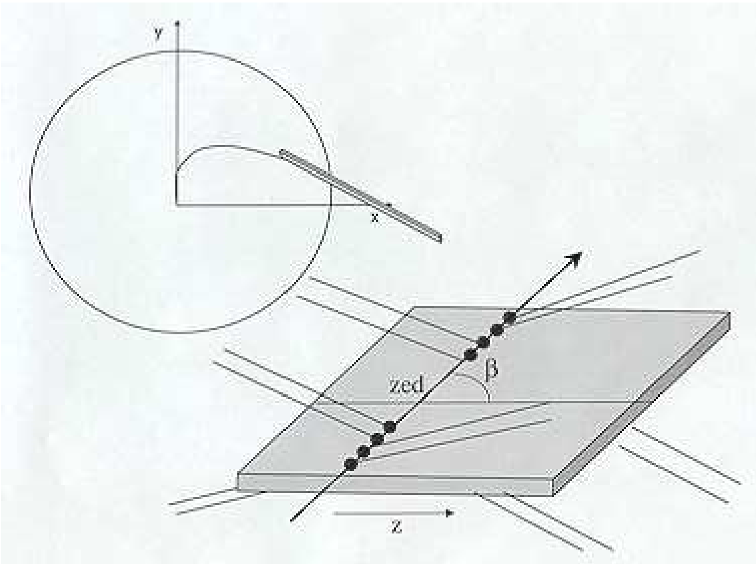


**FIGURE 4.** A schematic of the procedure for tracking with the UV wires. The first Hough transform defines a track in the $x$-$y$ plane with no $z$ information, effectively a plane extending in $z$ as shown in the upper-left figure. By intersecting the UV wire hits with this plane, the direction of the trajectory through this plane can be determined.

points of intersection, were incorporated.

Track-finding efficiency for the complete three dimensional algorithm was determined with the help of Monte Carlo simulations that included realistic detector response and position resolution. More than 92% of all tracks which pass fully through the drift chamber are reconstructed with fewer than 1% spurious (ghost) tracks. The tracking efficiency is constant for high momentum and drops precipitously at about $p_t = 200$MeV while the resulting resolution of the algorithm is comparable to the resolution of the detector.

The CPU time to process one central Au+Au event is currently ~120 SPECINT95 seconds, which is about 10% of the total central arm reconstruction time. Recent studies of the robustness of the algorithm at double the expected central Au+Au track density show that the tracking efficiency is still > 80% for this extreme situation. As a result of this postive experience with a combinatorial Hough transform, another tracking subsystem in PHENIX, the time-expansion chamber (TEC), has chosen to adopt it [3].

## CONCLUSIONS

In conclusion, we have developed a reasonably fast and highly effective pattern recognition algorithm based on combinatorial Hough transforms for tracking in three dimensions using a drift chamber with stereo wires.

## ACKNOWLEDGMENTS

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