Validation of Roundtrip Charged-Particle Calibrations Derived From S- and X-Band Doppler Via DRVID Measurements

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In this article it is shown that roundtrip signal path calibrations for charged-particle effects can be computed from S- and X-band doppler data. The method by which this is done is described, and possible error sources are examined. It is shown that roundtrip calibrations computed from S- and X-band doppler compare to direct measurements using DRVID to within the DRVID data noise.

I. Introduction

The Tracking Systems Analytic Calibration (TSAC) function provides calibrations for spacecraft radio metric data based on dual frequency (S- and X-band) doppler data. These calibrations are based on modeling the charged-particle effects on the uplink portion of the roundtrip path using the downlink only information in the S/X data and certain assumptions about the spatial and temporal distribution of the contributing charged-particle environment.

The presence of a low-density plasma along the propagation path of a radio signal causes the group propagation velocity to decrease and the phase velocity to increase. The DSN ranging system uses measurement of the roundtrip signal propagation time and the assumption of a constant value for the group velocity to imply the roundtrip distance to the spacecraft. The presence of charged particles along the path causes an apparent increase in this range value. The doppler measurement system measures the increment in the roundtrip path length by counting cycles of the returning signal and assuming a constant wavelength. A positive change in the charged-particle content along the signal path will result in a decrease in the measured range change from counted (integrated) doppler. The apparent increase in range inferred from group measurements is equal in magnitude to the apparent decrease inferred from phase measurements taken over the same time period. The magnitude of this effect is proportional to the plasma density and inversely proportional to the square of the frequency of the signal involved. A comparison of the apparent increase in range from differencing two consecutive range measurements with the apparent decrease in range as measured by doppler data counted over the same interval gives a measurement of the change in the charged particles along the signal path. An implementation of this technique has been mechanized in the PLanetary OPerational (PLOP) ranging system and the resulting data type is referred to as Differenced Range Versus Integrated Doppler (DRVID) data (Ref. 1). The equivalent range change measured by DRVID is output in PLOP Range Units (RU). An RU is approximately one light nanosecond.

On December 1, 1978, a test was run at Deep Space Station 14 to determine the effects of moving the antenna subreflector. For this pass of data, range, doppler and DRVID data were taken from the Voyager 1 spacecraft at both S- and X-band. DRVID data provide a measurement of the charged-particle environment along the entire signal path. Thus, simultaneous acquisition of dual frequency doppler data and DRVID allows a comparison to be made between a complete roundtrip calibration reconstructed from the doppler data using a model and an actual measurement of the effect by DRVID. This in turn allows the modeling technique used to be validated.

A similar pass of data during which both dual frequency doppler and DRVID data were taken occurred on April 13, 1977 for the Viking 2 orbiter.

In the following sections, the technique used to provide roundtrip signal calibrations from S/X doppler data are described, and several possible error sources are identified. For the passes of data processed we find that the roundtrip calibrations derived from S/X doppler data are equivalent to the same data derived from DRVID measurements to within the DRVID measurement noise.

II. The TSAC S/X Doppler Model

The model which TSAC uses to compute roundtrip calibrations from downlink S/X data is pictured in Fig. 1. Working backward from the reception time, the signal is assumed to pass through Earth's ionosphere at a time negligibly different from the reception time. The solar plasma is assumed to affect the signal as if it were concentrated at the point on the ray path closest to the Sun. Depending on the Sun-Earth-probe geometry, this can occur anywhere from zero to approximately 500 seconds (light time to Sun) prior to the reception time. Prior to this, the signal encounters roughly the same plasma configuration at a time earlier by a roundtrip light time (RTLT) less twice the Earth-to-plasma time. Initially, the signal is affected by the Earth's ionosphere at a time negligibly different from the transmission time, i.e., the reception time minus one RTLT. The reliability of the assumption that the plasma acts as though it were concentrated at the point on the signal path closest to the Sun is examined in Ref. 2.

The equation for the total effect due to charged particles on a range point received at time t is

$$R(t) = I(t) + P(t) + \lambda P(t - \text{TPLAS}) + \lambda I(t - \text{RTLT}) \quad (1)$$

where

$R(t)$ = total charged particle effect on range at time t

$I(t)$ = ionosphere contribution computed from Faraday rotation data

$P(t)$ = plasma contribution computed by removing ionosphere effect from total downlink effect as measured by S/X doppler data

$P(t - \text{TPLAS})$ = plasma effect evaluated at time corresponding to uplink passage of signal through space where plasma is assumed to be located.

$I(t - \text{RTLT})$ = ionosphere contribution at transmission time

$\lambda = (240/221)^2$ = correction factor to account for increased effect on uplink due to lower uplink transmitted frequency

The increment in the S-band cycle count over a particular interval is given by

$$\Delta S \Big|_{t-T}^{t} = fo \int_{t-T}^{t} \left(\frac{\dot{r}_{s/c}}{c} - \left(\frac{240}{221}\right)^2 \frac{\dot{r}_{up}}{c} - \frac{\dot{r}_{dp}}{c} \right) dt \quad (2)$$

and the corresponding X-band cycle count is

$$\Delta X \Big|_{t-T}^{t} = \frac{11}{3} fo \int_{t-T}^{t} \left(\frac{\dot{r}_{s/c}}{c} - \left(\frac{240}{221}\right)^2 \frac{\dot{r}_{up}}{c} - \left(\frac{3}{11}\right)^2 \frac{\dot{r}_{dp}}{c} \right) dt \quad (3)$$

where

ΔS = increment in S-band cycle count

ΔX = increment in X-band cycle count

t = end of count time

T = length of count time interval

fo = S-band downlink carrier frequency

c = speed of light

$\dot{r}_{s/c}$ = time derivative of roundtrip path length to spacecraft

$\dot{r}_{up}$ = time derivative of apparent increase in range at received S-band frequency due to charged particles on uplink leg

$\dot{r}_{dp}$ = time derivative of apparent increase in range at received S-band frequency due to charged particles on downlink leg

Note that

$$\frac{3}{11}\,\Delta X - \Delta S\Big|_{t-T}^{t} = \frac{fo}{c}\left[1-\left(\frac{3}{11}\right)^2\right]\int_{t-T}^{t} \dot{r}_{dp}\,dt \tag{4}$$

Hence, the accumulative downlink effect during a time interval $t = nT$ is

$$SX(t) = \sum_{i=1}^{n} \frac{c}{fo}\left[1-\left(\frac{3}{11}\right)^2\right]^{-1}\left(\frac{3}{11}\right)\Delta X - \Delta S\Big|_{(i-1)T}^{iT}$$

$$= \Delta r_{dp} + K \tag{5}$$

That is, the summation of the appropriate linear combination of S- and X-band doppler counts yields a measurement of the downlink contribution to S-band range due to all charge particles to within a constant of integration.

The contribution to the total downlink range change due to the ionosphere is assumed to be known from Faraday rotation measurements. Thus, the contribution from space plasma can be computed as

$$P(t) = SX(t) - I(t) \tag{6}$$

and substituting in Eq. (1) above:

$$R(t) = SX(t) + \lambda[SX(t - \text{TPLAS}) - I(t - \text{TPLAS})$$
$$+ I(t - \text{RTLT})] \tag{7}$$

III. Error Equation for the Selected Model

We next consider the following possible sources of error in this model:

(1) The S/X doppler measurements may be in error; describe the cumulative effect of this error as $\sigma(t)$.

(2) The assumed location of the plasma may be in error; call the corresponding error in TPLAS, π.

(3) There may be a measurement or modeling error in the Faraday rotation data. The cumulative effect of this error is $\eta(t)$.

(4) There is a constant of integration K in the S/X data.

The equation for the computed range correction becomes

$$R(t) = SX(t) + \sigma(t) + \lambda[SX(t - \text{TPLAS} - \pi) - I(t - \text{TPLAS} - \pi)$$
$$+ I(t - \text{RTLT}) + \sigma(t - \text{TPLAS} - \pi) - \eta(t - \text{TPLAS} - \pi)$$
$$+ \eta(t - \text{RTLT})] \tag{8}$$

The true range correction being removed, the error in the range correction, to first order, is given by

$$ER(t) = \sigma(t) + \lambda\left[(\sigma t - \text{TPLAS}) + \Delta(SX - I)\Big|_{t-\text{TPLAS}-\pi}^{t-\text{TPLAS}}\right.$$
$$\left. + \Delta\eta\Big|_{t-\text{RTLT}}^{t-\text{TPLAS}}\right] + K \tag{9}$$

Expanding in terms of derivatives

$$ER(t) = \sigma(t) + \lambda\sigma(t - \text{TPLAS}) + \pi(\dot{SX} - \dot{I}) - \dot{\eta}\tau + K \tag{10}$$

where $\tau = \text{RTLT} - \text{TPLAS}$.

There are two principle uses for the roundtrip calibration for charged particles derived from S/X doppler data. These are:

(1) The calibration of doppler observables.

(2) The calibration of differenced range data.

For both of these phenomena, the quantity of interest is the change in the roundtrip measurement over some period of time. For doppler this period is one count time. For differenced range it is the time between the last range point taken at one station and the first range point at the next station. We are interested in the error in the range change over some period of time ΔT. Differencing Eq. (10) above yields, to first order:

$$\Delta ER = \Delta T\,[\dot{\sigma}(t) + \lambda\dot{\sigma}(t - \text{TPLAS}) + \pi(\ddot{SX} - \ddot{I}) - \tau\ddot{\eta}] \tag{11}$$

noting that the constant of integration K in the S/X data has dropped out.

IV. Results of Analysis

A. Results of the December 1, 1978 Test

Figure 2 shows the total downlink range change for the December 1, 1978 pass divided into components due to ionosphere and space plasma. Figure 3 shows the raw DRVID data points at S-band and a reconstructed S-band DRVID curve based on S/X doppler data and the model described in Section II. There is no absolute level information in the S/X doppler data, so the absolute level of the derived DRVID curve has been adjusted to minimize the RMS difference. Figure 4 shows the residuals from this adjustment. The RMS fit error of the overall fit is comparable to the raw DRVID noise level as determined by linear detrending of short segments of the raw data.

Figures 5 and 6 display the corresponding results for X-band DRVID. Again, the RMS error over the entire fit is comparable to the data noise level. The apparent discontinuity in the residuals at about 12:20 can definitely be ascribed to subreflector motion.

B. Initial Results of the April 13, 1977 Test

Figure 7 shows the total downlink range change for the April 13, 1977 pass divided into components due to ionosphere and space plasma. Figure 8 shows the raw S-band DRVID and the S/X derived DRVID for the April 13, 1977 Viking 2 pass. Figure 9 is a plot of the residuals from this fit. There is an obvious linear trend in the entire pass of residuals which a linear least squares fit gives as −3.98 RU/hr. In terms of the allowable time gap of six hours between differenced range points specified in the Voyager SIRD, this amounts to a contribution of about 1.75 meters out of a total error budget of 6.4 meters. The SIRD allocates only 0.5 meters for this error.

C. A Detective Story

What are the possible sources of such an error? Some likely candidates are:

(1) An error in setting the night time level for the ionosphere.

(2) Errors in the S/X data. Possibly frequency offset between S-band and X-band synthesizer frequencies.

(3) An error in modeling location of plasma.

In order to test item 3, a series of fits was made, assuming locations for the plasma ranging from near Earth to near spacecraft.

Figure 10 shows the RMS residuals from these fits. In none of these runs was there any substantial reduction in the apparent linear trend in the residuals.

The measurements of ionospheric electron density from which the assumed ionosphere contribution is computed are derived from data taken by a Faraday rotation polarimeter. These rotation data are supplied modulo 180 degrees of rotation. As a result, the absolute electron density level may be in error by an amount corresponding to one or more 180° rotation mods.

Re-examination of Section III, Eq. (11) shows that the contribution to range change from an error in the ionosphere model is of the order of

$$\Delta \dot{R} = \ddot{\eta}\,(\text{TPLAS} - \text{RTLT})$$

The second derivative of the ionosphere error due to setting the night time level incorrectly amounts to 0.049 m/hr^2 per mod. For this case, TPLAS − RTLT is −825 seconds, resulting in a contribution of

$$\Delta \dot{R} = \frac{0.049 \text{ m}}{\text{hr}^2} \cdot -825 \text{ sec} \cdot \frac{1 \text{ hr}}{3600 \text{ sec}} \text{per mod} \cdot \frac{7.05 \text{ RU}}{\text{m}}$$

or

$$\Delta \dot{R} = -0.079 \frac{\text{RU}}{\text{hr}} \text{ per mod}$$

Thus an error of −3.98 RU/hr would require about a +50 mod ionosphere error, which is at least an order of magnitude too large.

Finally, the minimum settable discrepancy between the S-band and X-band synthesizer frequencies is 10^{-5} Hz. The fractional error would then contribute

$$\frac{10^{-5} \text{ Hz}}{\text{VCO Hz}} \cdot \frac{48 \text{ VCO RU}}{\text{sec}} \cdot \frac{3600 \text{ sec}}{\text{hr}} \left[1 + \left(\frac{240}{221}\right)^2\right] = 3.76 \text{ RU/hr}$$

which is exactly the error we are looking for.

D. Final Result for the April 13, 1977 Test

Figures 11 through 14 show the raw DRVID data and the derived DRVID assuming a 1×10^{-5} Hz discrepancy between the S- and X-band synthesizer frequencies and the residuals of the fit between the two at both S- and X-band. The residuals are flat to the noise level of the data. Figure 15 shows the

RMS fit error as a function of the assumed position of the space plasma.

V. Conclusions

For the December 1, 1978 test, the roundtrip calibrations derived from downlink S/X data are equivalent to the measured roundtrip effect at both S- and X-band to the level of observability in the DRVID data. A glance at the computed plasma portion of the downlink range change (Fig. 2) shows that it is within one or two range units of being a straight line over the entire pass. In terms of the error equations derived in Section III, this means that the resulting roundtrip calibration on this particular occasion is quite insensitive to the assumed position along the ray path of such a plasma contribution.

For the April 13, 1977 test, the derived data from S/X doppler match the measured DRVID at both S- and X-band to what appears to be the noise level of the DRVID data. There is a well-defined minimum in the RMS fit as a function of the assumed plasma location at or near the nominal value (see Fig. 15). These results together provide a validation of the assumptions upon which the S/X calibrations are currently based.

References

1. MacDoran, P. F., "A First-Principles Derivation of the Differenced Range Versus Integrated Doppler (DRVID) Charged-Particle Calibration Method," in *The Deep Space Network, Space Programs Summary 37-62, Vol. II*, pp. 28-33, Jet Propulsion Laboratory, Pasadena, Calif., Mar. 31, 1970.
2. Wu, S. C. and Winn, F. B., "A Technique to Determine Uplink Range Calibration Due to Charged Particles," in *The DSN Progress Report 42-41*, pp. 57-81, Jet Propulsion Laboratory, Pasadena, Calif., Oct. 15, 1977.

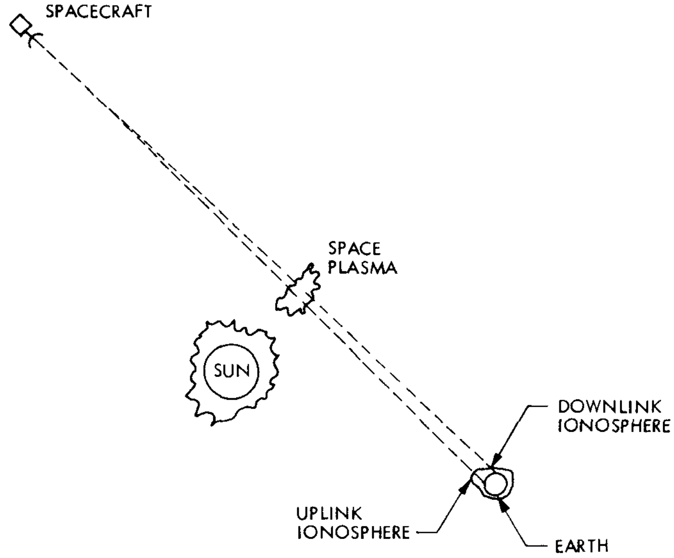

Fig. 1. Assumed charged-particle distribution for S/X calibration model

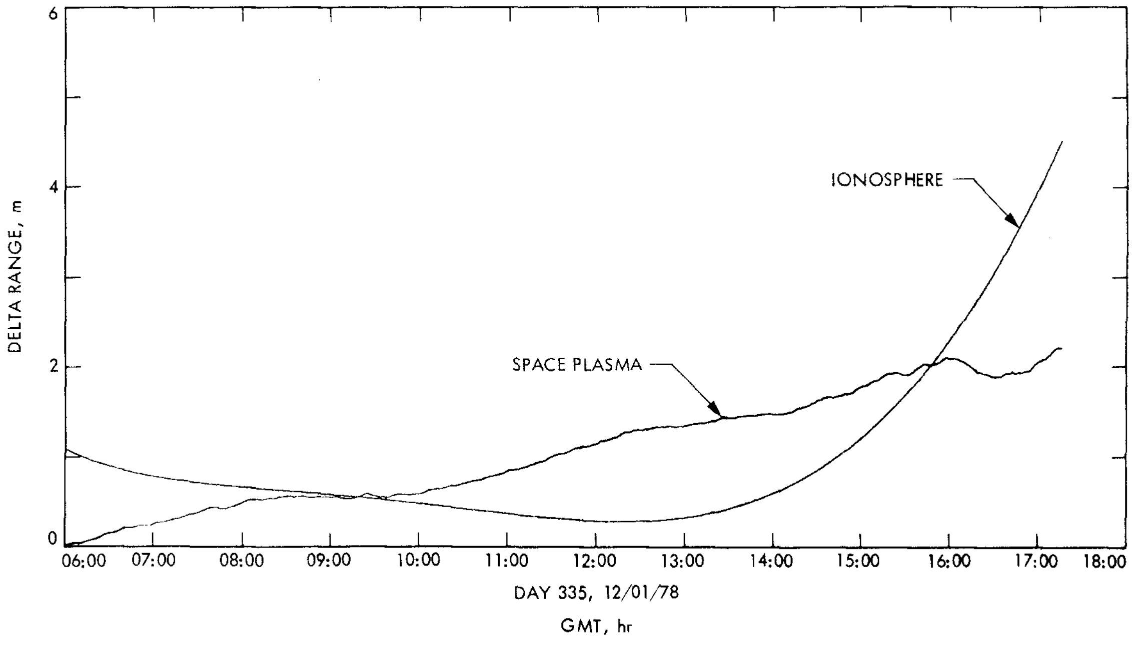

Fig. 2. Ionosphere and space plasma contributions to downlink range change on December 1, 1978

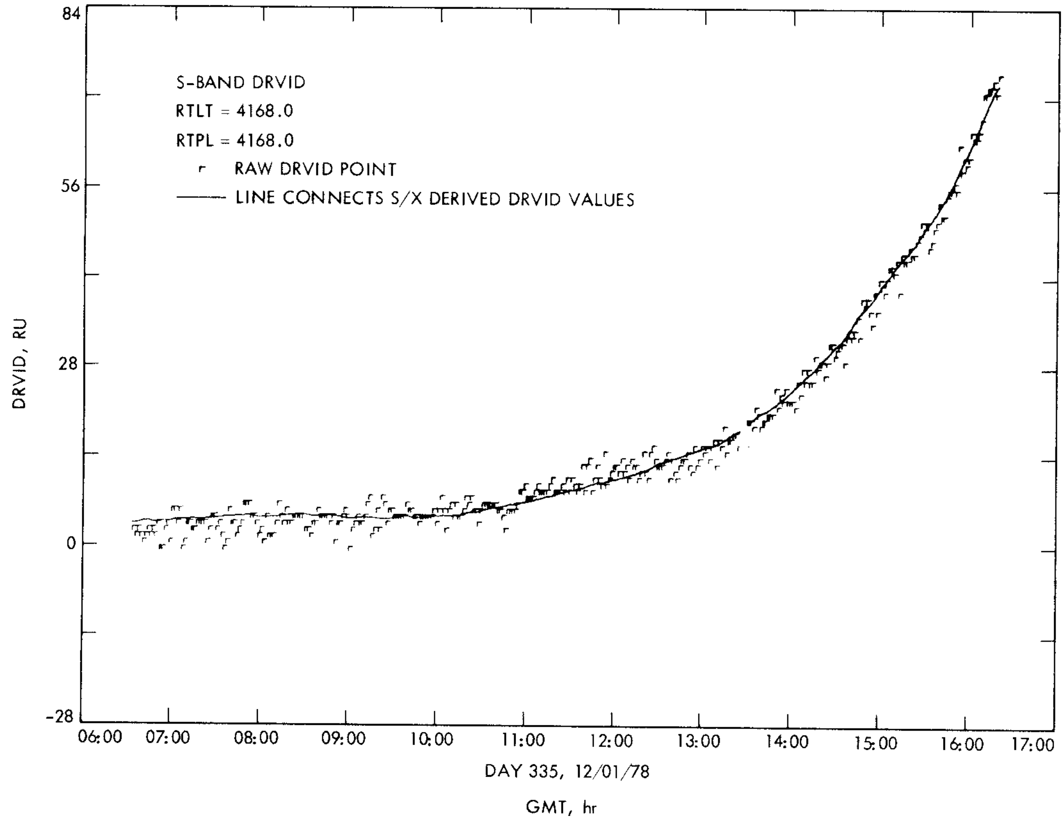

Fig. 3. Raw and derived S-band DRVID for December 1, 1978

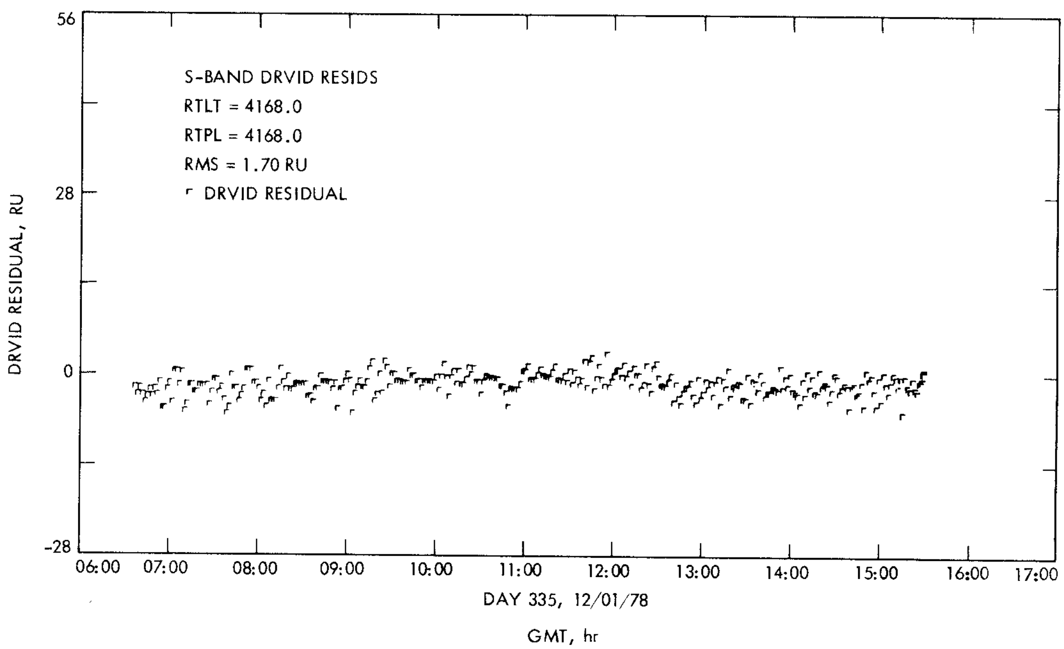

Fig. 4. S-band DRVID residuals (raw-derived) for December 1, 1978

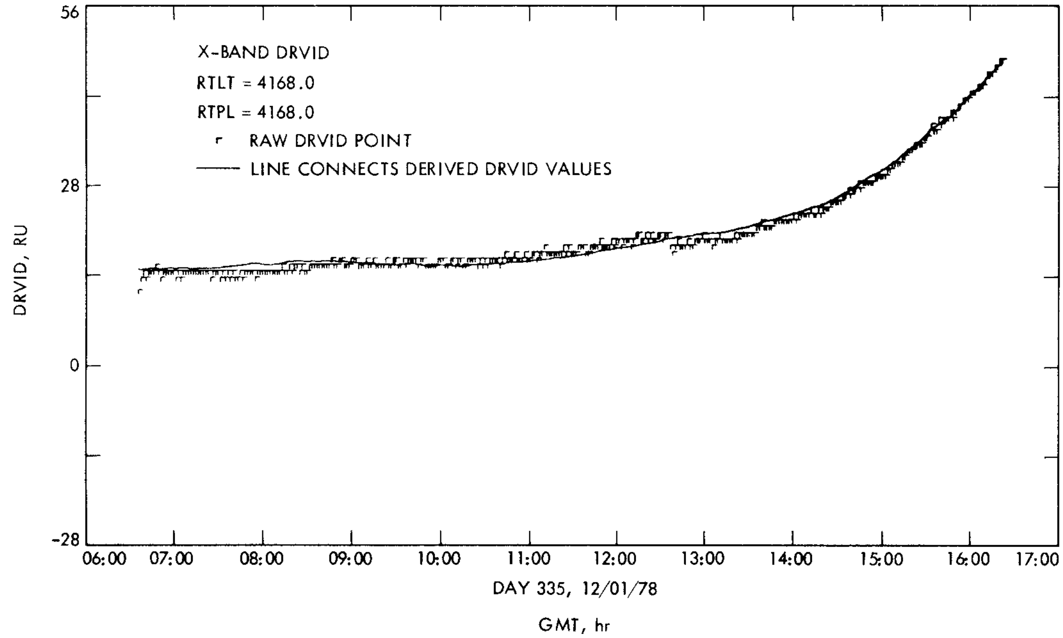

Fig. 5. Raw and derived X-band DRVID for December 1, 1978

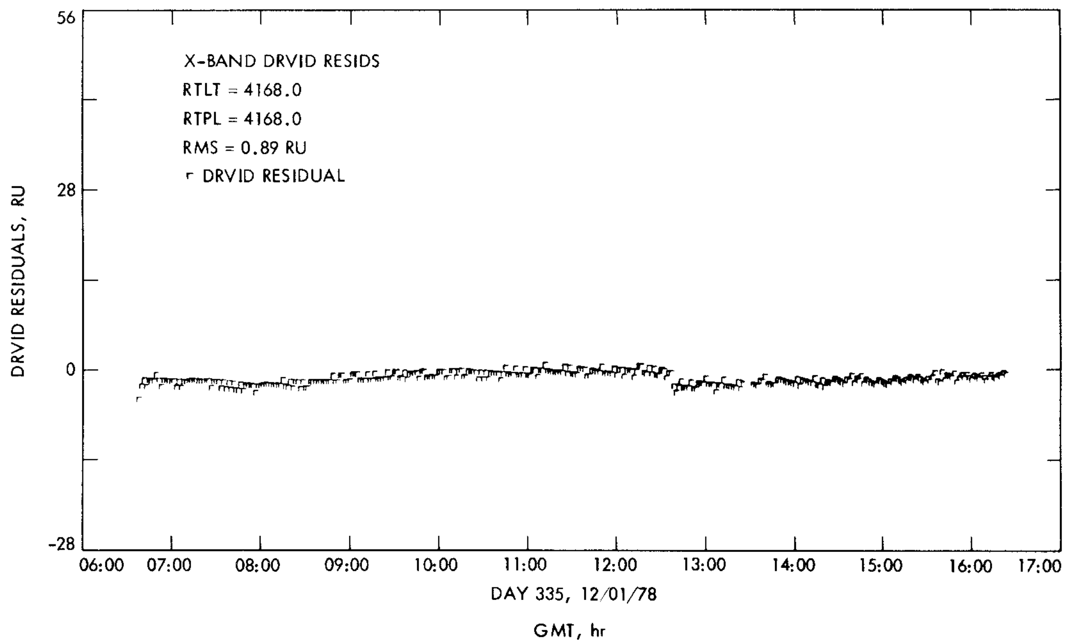

Fig. 6. X-band DRVID residuals (raw-derived) for December 1, 1978

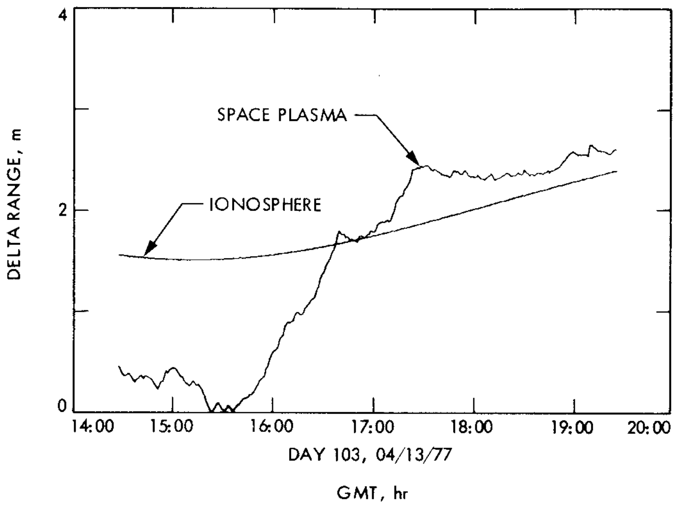

Fig. 7. Ionosphere and space plasma contributions to downlink range change on April 13, 1977

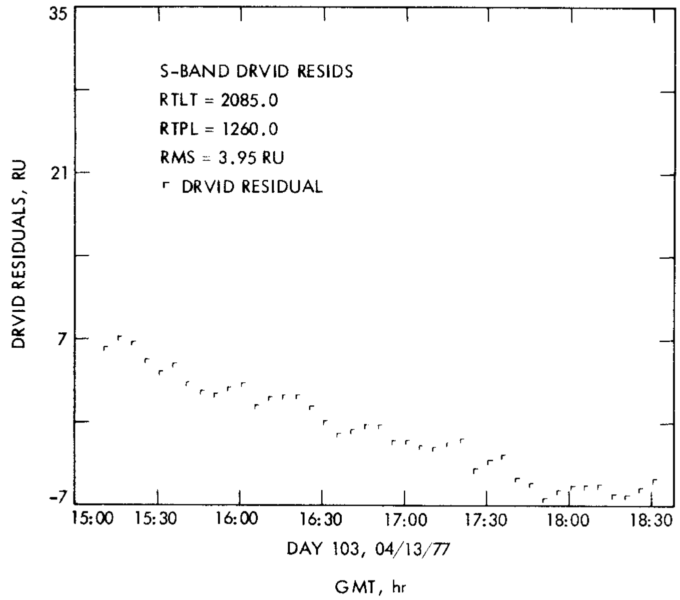

Fig. 9. Initial S-band DRVID residuals (raw-derived) for April 13, 1977

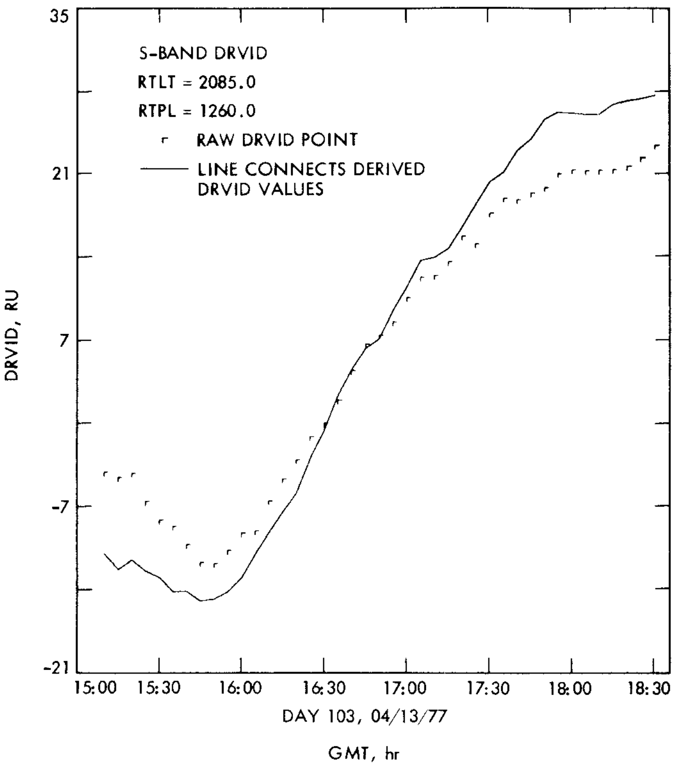

Fig. 8. Initial raw and derived S-band DRVID for April 13, 1977

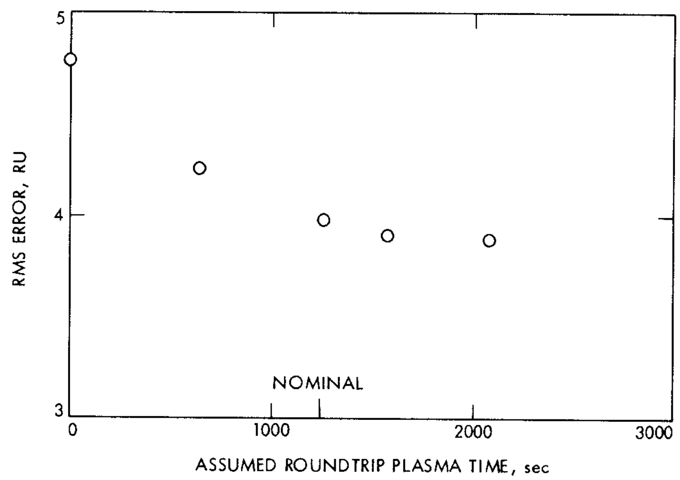

Fig. 10. Initial RMS fit error vs. assumed plasma location on April 13, 1977

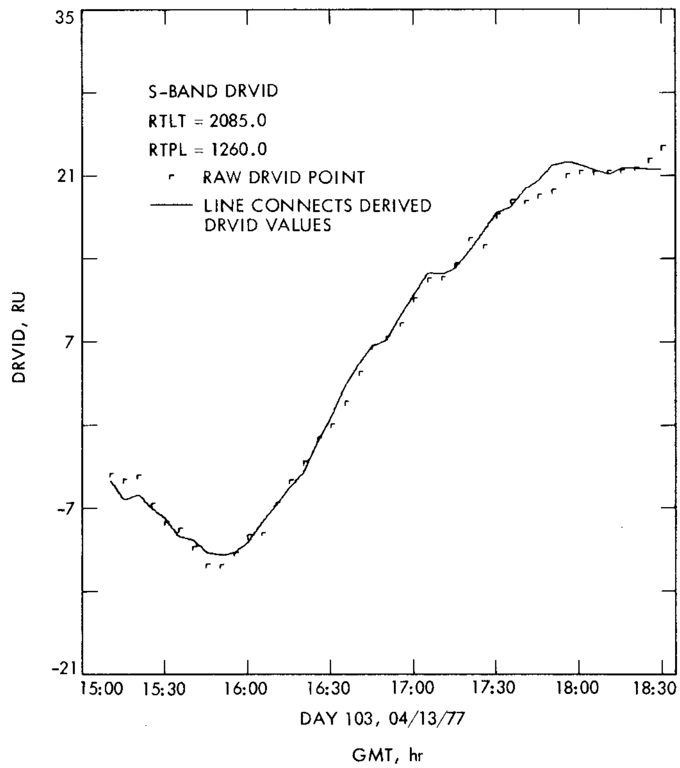

Fig. 11. Final raw and derived S-band DRVID for April 13, 1977

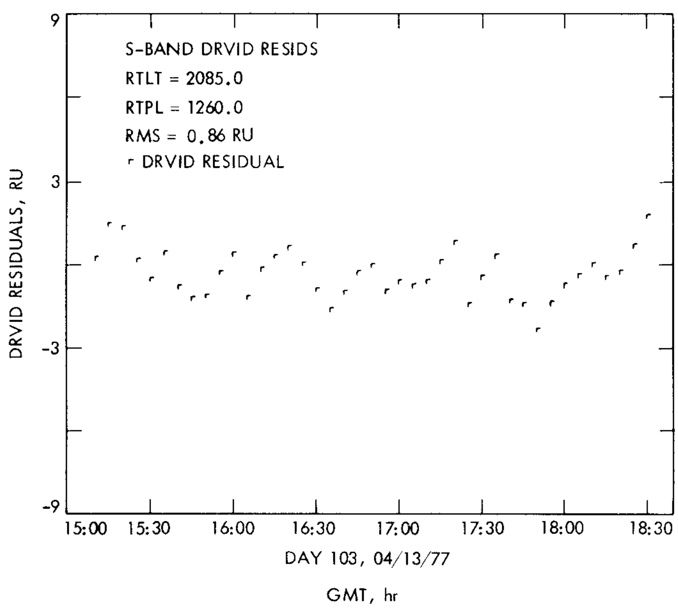

Fig. 12. Final S-band DRVID residuals (raw-derived) for April 13, 1977

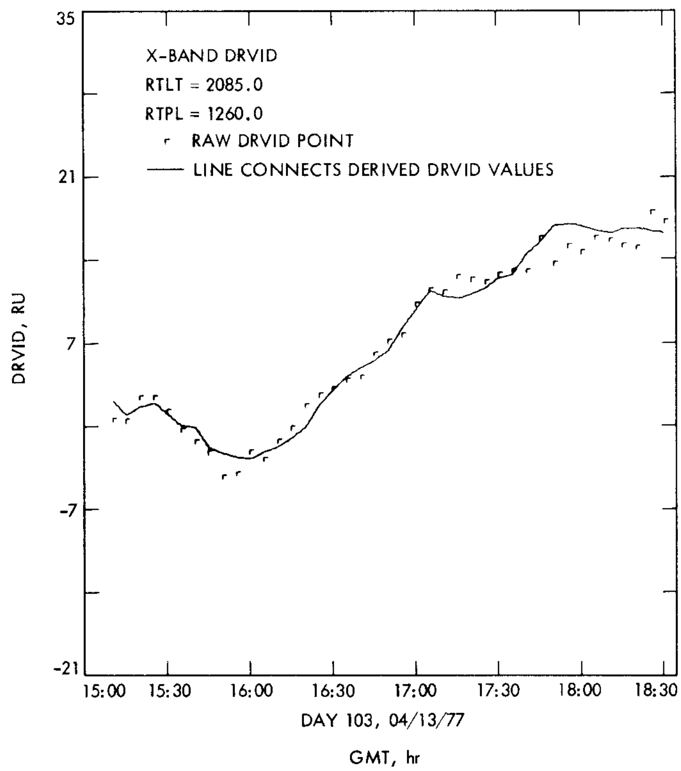

Fig. 13. Final raw and derived X-band DRVID for April 13, 1977

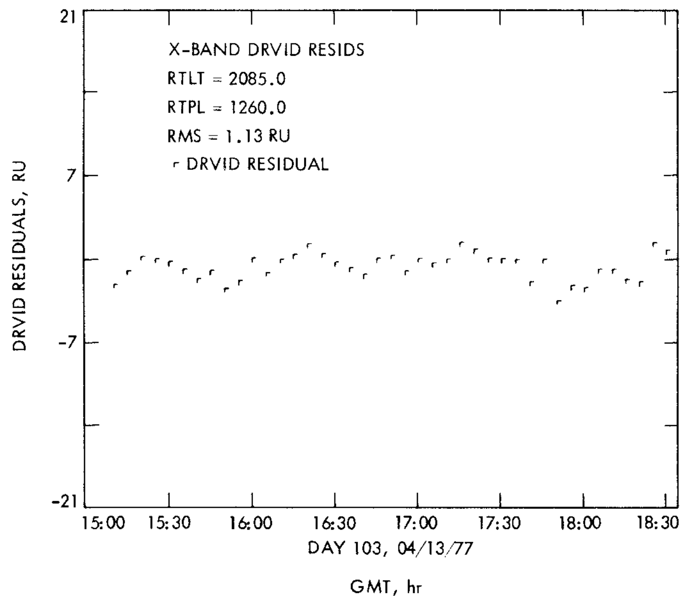

Fig. 14. Final X-band DRVID residuals (raw-derived) for April 13, 1977

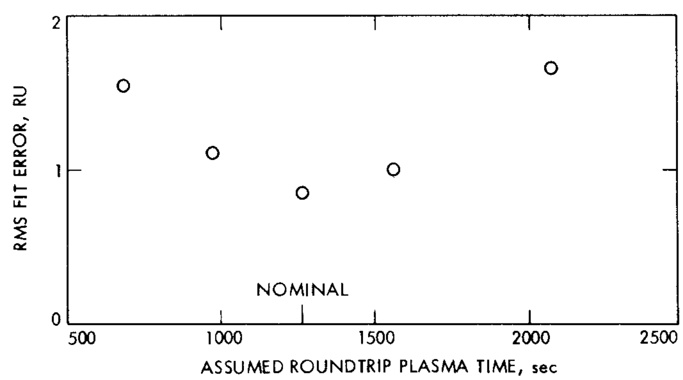

Fig. 15. Final RMS fit error vs. assumed plasma location on April 13, 1977