4

Mathematics

and Scientific Inquiry

"Misha is only 3. You have to be 4 to join our club."

"I'm squooshing one raisin into every marshmallow. They're beanbag chairs for bugs."

"Add more water to the other glass to make them even."

"First I did the napkins. Then I put spoons on top. Last the plates with pop-up pebble pies."

"I won't be 5 until after José. But then it will be my turn to be the oldest."

"My new shoes cost my mom eleventy-eight-one dollars."

"I'm running faster. I bet I get to the top of the hill before you!"

"When the big hand points down at the 6, I'm going to clap my hands for clean-up."

"Big yellow rings in this box and small ones there. Same for the big and little green squares."

"You be the daddy kitty because you're the tallest. She's the baby because she's the littlest. And I'll be the mommy so I can sit in the middle."

Quite naturally, and without recognizing them as such, young children develop ideas about mathematics in the course of their day-to-day lives. These children's remarks, for example, overheard in a preschool classroom, reflect an interest in "mathematical" subjects that matter to them—age, speed, time, size, order. Summarizing the early mathematics knowledge base, Baroody (2000) notes:

> Researchers have accumulated a wealth of evidence that children between the ages of 3 and 5 years of age actively construct a variety of fundamentally important informal mathematical concepts and strategies from their everyday experiences. Indeed, this evidence indicates that they are predisposed, perhaps innately, to attend to numerical situations and problems. (61)

Thus, in the past 25 years, studies of the development of early mathematics have switched from looking at what children *cannot* do to what they *can* do. Observations of children during free play, for example, show them engaged in mathematical explorations and applications, and sometimes these are surprisingly advanced (Ginsburg, Inoue, & Seo 1999).

The typical early childhood curriculum, however, incorporates little in the way of thoughtful and sustained early mathematics experiences (Copley 2004). If mathematics is included, it tends to be limited to number, particularly counting. Yet young children also spontaneously explore topics such as patterns, shapes, and the transformations brought about by processes such as adding and subtracting (Ginsburg, Inoue, & Seo 1999), and there are foundational mathematical understandings children need to develop in these areas.

Along with changing our ideas about *what* children can understand has come rethinking of *how* to foster early mathematical development.

> Because young children's experiences fundamentally shape their attitude toward mathematics, an engaging and encouraging climate for children's early encounters with mathematics is important. It is vital for young children to develop confidence in their ability to understand and use mathematics—in other words, to see mathematics as within their reach. (NAEYC & NCTM 2002)

Researchers and practitioners have developed different systems for categorizing the mathematical areas in which young children demonstrate interest and ability (e.g., Campbell 1999; Greenes 1999). In 2000 the National Council of Teachers of Mathematics (NCTM) published *Principles and Standards in School Mathematics,* which includes standards for grades preK–2. And in 2002 NAEYC published a joint position paper with NCTM supporting its standards and offering recommendations for early mathematics education. The NCTM standards are now widely cited in the field and used by many state departments of education and local school districts to develop comprehensive early mathematics curricula in preschool programs and the primary grades.

The NCTM standards define five *content* areas: Number & Operations, Geometry, Measurement, Algebra, and Data Analysis & Probability. The standards are described in this chapter, along with their application in the preschool years. NCTM also defines five *process* standards, consistent with the strategies suggested here: Problem Solving, Reasoning and Proof, Connections, Communication, and Representation. Problem solving and reasoning, as the position statement phrases it, are the "heart" of mathematics:

> While content represents the what of early childhood mathematics education, the processes . . . make it possible for children to acquire content knowledge. These processes develop over time and when supported by well-designed opportunities to learn. Children's development and use of these processes are among the most long-lasting and important achievements of mathematics education. Experiences and intuitive ideas become truly mathematical as the children reflect on them, represent them in various ways, and connect them to other ideas. (NAEYC & NCTM 2002)

For further explanations of NCTM's process standards and examples, see the work of Clements (2004) and Copley (2004), as well as the National Council of Teachers of Mathematics Web site (www.nctm.org).

Scientific inquiry and its relationship to mathematics

Young children are doing math when they measure and graph the daily growth of bean seedlings, when they notice the changing patterns of shadows on a wall, or when they predict how many more cups of sand it will take to fill a hole and then check by counting. But they are also doing science. Gelman and Brenneman point out that "to do science is to predict, test, measure, count, record, date one's work, collaborate and communicate" (2004, 156). Because of this close connection between mathematics and science, I have chosen to include "the doing of science"—*scientific inquiry*—in this chapter.

> Science inquiry refers to the diverse ways in which scientists study the natural world and propose explanations based on the evidence derived from their work. Inquiry also refers to the activities of students in which they develop knowledge and understanding of scientific ideas, as well as an understanding of how scientists study the natural world. (NCSESA 1996, 23)

Within science, scientific inquiry is perhaps the area that has been most investigated because of young children's evident interest in observing and thinking about the world (Eshach & Fried 2005). Inquiry skills are in evidence when preschoolers are:

▸ raising questions about objects and events around them;

▸ exploring objects, materials, and events by acting upon them and noticing what happens;

▸ making careful observation of objects, organisms, and events using all of their senses;

▸ describing, comparing, sorting, classifying, and ordering in terms of observable characteristics and properties;

▸ using a variety of simple tools to extend their observations;

▸ engaging in simple investigations in which they make predictions, gather and interpret data, recognize simple patterns, and draw conclusions;

▸ recording observations, explanations, and ideas through multiple forms of representation including drawings, simple graphs, writing, and movement;

▸ working collaboratively with others; and

▸ sharing and discussing ideas and listening to new perspectives. (Worth & Grollman 2003, 18)

Doing inquiry and learning about inquiry are "critical content areas" of science. Like any skills, they need to be learned and practiced—and this is where the other content areas of science make their appearance. Teachers doing inquiry-based science choose meaningful subject matter from the life, physical, earth, and space sciences for young children to practice on. In other words,

> Inquiry and subject matter are both important and cannot be separated. Children may need direct instruction for some specific skills, such as learning to use a magnifier. And they can practice some skills on their own, such as categorization. But children develop their inquiry skills as they investigate interesting subject matter, and children build theories about interesting subject matter through the use of inquiry skills. (Worth & Grollman 2003, 156)

The connections between early mathematics and science are reflected in NAEYC's own program accreditation criteria for science (NAEYC 2005, Criterion 2.G) and technology (Criterion 2.H). The NAEYC criteria note that children should have opportunities to collect and represent data, make inferences about what they observe, and have access to technology. These opportunities and the associated learning are also explicit in the guidelines for early mathematics education (Criterion 2.F).

Young children's development in mathematics and scientific inquiry

Young children, like those quoted at the beginning of the chapter, start with only an intuitive or experiential understanding of mathematics. They don't yet have the concepts or vocabulary they need to *use* what they intuitively know or to *connect* their knowledge to school mathematics. The preschool teacher's task is "to find out what young children already understand and help them begin to understand these things mathematically From ages 3 through 6, children need many experiences that call on them to relate their knowledge to the vocabulary and conceptual frameworks of mathematics—in other words, to 'mathematize' what they intuitively grasp" (NAEYC & NCTM 2002).

The goal of early mathematics education, then, is to build "mathematical power" in young children (Baroody 2000). This power has three components: a positive disposition to learning and using mathematics; understanding and appreciating the importance of mathematics; and engaging in the process of mathematical inquiry. Turning children's early and spontaneous mathematics play (child-guided experience) into an awareness of mathematical concepts and skills is at the heart of intentional teaching in this area.

Similarly, in science we want to capitalize on children's "natural inclination to learn about their world" (Landry & Forman 1999, 133), expose them to the uses and benefits of scientific processes in everyday life, and involve them in scientific inquiry as they figure out how the world works. Here, too, children are naturally inclined to explore their surroundings. But they depend on us to give them a rich environment for inquiry and to develop their child-guided discoveries into a growing understanding of how science works.

To promote children's science inquiry, the intentional teacher takes care to design the physical setting, plan the areas of science children will focus on, and establish overall goals for learning. Once children begin exploring, however, "what actually happens emerges from a dynamic interaction among the children's interests and questions, the materials, and the teacher's goals" (Worth & Grollman 2003, 158).

In early mathematics and science, free exploration is important—but by itself it is not enough. There are concepts, principles, and vocabulary that children will not construct on their own. Even for those areas in which their investigations are key,

Materials That Promote Mathematical and Scientific Exploration

Children can manipulate, count, measure, and ask questions about almost any object or kind of material. Yet there are some things teachers should make sure to have in their classrooms to promote exploration and thinking about the components of mathematics and scientific inquiry.

Number and operations

- Printed items containing numbers and mathematical or scientific symbols—e.g., signs, labels, brochures, advertisements with charts and graphs
- Things with numbers on them—e.g., calculators, playing cards, thermometers, simple board games with dice or spinners
- Numbers made of wood, plastic, or cardboard (make sure they are sturdy so children can hold, sort, copy, and trace them)
- Discrete items children can easily count—e.g., beads, blocks, shells, poker chips, bottle caps
- Paired items to create one-to-one correspondence—e.g., pegs and pegboards, colored markers and tops, egg cartons and plastic eggs

Geometry and spatial sense

- Materials and tools for filling and emptying—water, sand; scoops, shovels
- Everyday things to fit together and take apart—e.g., Legos, Tinkertoys, puzzles, boxes and lids, clothing with different types of fasteners
- Attribute blocks that vary in shape, size, color, thickness
- Tangram pieces
- Wooden and sturdy cardboard blocks in conventional and unconventional shapes
- Containers and covers in different shapes and sizes
- Materials to create two-dimensional shapes—e.g., string, pipe cleaners, yarn
- Moldable materials to create three-dimensional shapes—e.g., clay, dough, sand, beeswax
- Things with moving parts—e.g., kitchen utensils, musical instruments, cameras
- Books that feature shapes and locations, with illustrations from different perspectives
- Photos of classroom materials and activities from different viewpoints
- Materials that change with manipulation or time—e.g., clay, play dough, computer drawing programs, sand, water, plants, animals
- Materials to explore spatial concepts (over/under, up/down) and to view things from different heights and position—e.g., climbing equipment, empty boxes (large cartons from appliances and furniture), boards
- Maps and diagrams

children do not always construct mathematical or scientific meanings from them. Clements (2001) suggests teachers consider whether children's thinking is developing or stalled. "When it is developing, they can continue observing. When it is stalled, it is important to intervene" (Seo 2003, 31). In this way, adult-guided experience supplements child-guided exploration.

Teaching and learning in mathematics and scientific inquiry

Young children need many opportunities to represent, reinvent, quantify, generalize, and refine their experiential and intuitive understandings that might be called "premathematical" or emerging

Measurement

- Ordered sets of materials in different sizes—e.g., nesting blocks, measuring spoons, pillows, paint-brushes, drums
- Ordered labels so children can find materials and return them to their storage place—e.g., tracings of measuring spoons in four sizes on the pegboard in the house center
- Storage containers in graduated sizes
- Materials that signal stopping and starting—e.g., timers, musical instruments, tape recorders
- Materials that can be set to move at different rates of speed—e.g., metronomes, wind-up toys
- Things in nature that move or change at different rates—e.g., slow- and fast-germinating seeds, insects that creep and scurry
- Unconventional measuring tools—e.g., yarn, ribbon, blocks, cubes, timers, ice cubes, containers of all shapes and sizes
- Conventional measuring tools—e.g., tape measures, scales, clocks, grid paper, thermometers, measuring spoons, graduated cylinders

Patterns, functions, and algebra

- Materials with visual patterns—e.g., toys in bright colors and black-and-white, dress-up clothes, curtains, upholstery
- Materials to copy and create series and patterns—e.g., beads, sticks, small blocks, pegs and peg boards, writing and collage materials
- Shells and other patterned items from nature
- Original artwork and reproductions featuring patterns—e.g., weavings, baskets
- Pattern blocks
- Routines that follow patterns
- Stories, poems, and chants with repeated words and rhythms
- Songs with repetitions in melody, rhythm, and words
- Computer programs that allow children to recognize and create series and patterns

Data analysis

- Tools for recording data—e.g., clipboards, paper, pencils, crayons, markers, chalk
- Materials for diagramming or graphing data—e.g., newsprint pads and easels, graph paper with large grids, posterboard
- Small objects to represent counted quantities—e.g., buttons, acorns, pebbles
- Boxes and string for sorting and tying materials into groups
- Sticky notes and masking tape for labeling

mathematics. To do this effectively, intentional teachers design programs so that children encounter concepts in depth and in a logical sequence:

> Because curriculum depth and coherence are important, unplanned experiences with mathematics are clearly not enough. Effective programs also include intentionally organized learning experiences that build children's understanding over time. Thus, early childhood educators need to plan for children's in-depth involvement with mathematical ideas. . . . Depth is best achieved when the program focuses on a number of key content areas rather than trying to cover every topic or skill with equal weight. (NAEYC & NCTM 2002)

This need to focus on a limited number of key concepts and skills at each level is further highlighted in NCTM's amendment of its own standards, *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (NCTM 2006). Intentional teachers use a variety of approaches and strategies to achieve this focused emphasis. They integrate mathematics into daily routines and across other domains in the curriculum, but always in a coherent, planful manner. This means that the mathematics experiences they include "follow logical sequences, allow depth and focus, and help children move forward in knowledge and skills"; it does not mean "a grab bag of experiences that seem to relate to a theme or project" (NAEYC & NCTM 2002).

In addition to integrating mathematics in children's play, classroom routines, and learning experiences in otherwise nonmathematic parts of the curriculum, intentional teachers also provide carefully planned experiences that focus children's attention on a particular mathematical idea:

> Helping children name such ideas as horizontal or even and odd as they find and create many examples of these categories provides children with a means to connect and refer to their just-emerging ideas. Such concepts can be introduced and explored in large and small group activities and learning centers. Small groups are particularly well suited to focusing children's attention on an idea. Moreover, in this setting the teacher is able to observe what each child does and does not understand and engage each child in the learning experience at his own level. (NAEYC & NCTM 2002)

Research points us to the materials and activities that foster the development of mathematical and scientific concepts. Young children are concrete, hands-on learners. They need to manipulate materials to construct ideas about the physical properties of objects and their transformation. Spontaneous investigations are most common with discrete play objects such as Legos, blocks, or puzzles and continuous materials such as sand, water, or clay (Ginsburg, Inoue, & Seo 1999). Children tend to use mathematical and scientific inquiry most frequently during construction or pattern-making activities. Computers can also play a role in early mathematics and science education, if the technology is used appropriately (Hyson 2003). (See the box **Computer Technology.**)

Perhaps less expected is the finding that mathematical and scientific thinking is fostered by social interaction. When students share hypotheses and interpretations, question one another, and are challenged to justify their conclusions, they are more likely to correct their own thinking (Campbell 1999). In fact, agreements and disagreements during peer-to-peer dialogue more often prompt reflection and reconsideration than does adult-delivered instruction (Baroody 2000), perhaps because many teachers underestimate children's early grasp of mathematical principles (Kamii 2000).

For that reason, understanding how children learn in the areas of early mathematics and scientific thinking is essential to meaningful teaching in this area. Research points to the effectiveness of the following general support strategies:

▸ **Encourage exploration and manipulation—** Provide materials with diverse sensory attributes and allow children sufficient time and space to discover their properties. At the same time, "the ways children use objects are often very different from those that we intend or define" (Seo 2003, 30). Teachers might not see art materials or dramatic play props as mathematical manipulatives, but children do, as they count the Velcro strips on a smock or the rooms in a dollhouse. Indeed, there is nothing they do not count! It's the same for science: The classroom pet may be there mostly for the purpose of promoting social responsibility, but children are also observing Sniffy the guinea pig's habits and life cycle.

▸ **Observe and listen—**Attend to the questions children ask. The problems they pose for themselves or to adults offer a window into their mathematical and scientific thinking.

▸ **Model, challenge, and coach—**Demonstrate hands-on activities that children can imitate and modify. Provide experiences that stretch their thinking. Discuss what does (not) work, pose questions, and suggest alternative approaches to finding a solution.

▸ **Encourage reflection and self-correction—**When children are stuck or arrive at an incorrect mathematical solution or scientific explanation, do not jump in to solve the problem or correct their reasoning. Instead, provide hints to help children reconsider their answers and figure out solutions or alternative explanations on their own.

▸ **Provide the language for mathematic and scientific properties, processes, and relationships—** Introduce the language for children to label their observations, describe transformations, and share the reasoning behind their conclusions.

▸ **Play games with mathematical elements—** Games invented for or by children offer many opportunities, for example, to address issues of (non)equivalence, spatial and temporal relations, and measurement.

▸ **Introduce mathematical and scientific content—** Children enjoy good books about counting, especially when these invite participation. Storybooks and nonfiction texts are also a wonderful way to introduce real-life problems whose solutions depend on mathematical reasoning. (See **Resources** for a Web site that suggests books for young children related to early mathematics and science, as well as other early learning topics.) Similarly, children are fascinated by observing the natural world —things that float or sink, shifting cloud patterns, plant and animal habits. Thus "science should be considered content for mathematics and literacy experiences" (Gelman & Brenneman 2004, 156).

▸ **Encourage peer interaction—**As noted above, children can sometimes explain mathematical and scientific ideas to their peers more effectively than adults can. Sharing ideas, particularly conflicting ones, prompts children to articulate and, where necessary, modify their understanding.

In general, an investigative approach works better than a purely didactic one. Begin with a "worthwhile task, one that is interesting, often complex, and creates a real need to learn or practice. Experiencing mathematics in context is not only more interesting to children but more meaningful" (Baroody 2000, 64). It also makes learning in both mathematics and science more likely and more lasting. As Ginsburg and his colleagues note,

> [Rote instruction that does not emphasize understanding] does little to inculcate the spirit of mathematics—learning to reason, detect patterns, make conjectures, and perceive the beauty in irregularities—and may

> instead result in teaching children to dislike mathematics at an earlier age than usual. Clearly the early childhood education community should not implement at the preschool and kindergarten levels the kinds of activities that the National Council of Teachers of Mathematics is trying to eliminate in elementary school! (Ginsburg, Inoue, & Seo 1999, 88)

Fitting the learning experience to the learning objective

The rest of this chapter describes what preschoolers learn as they begin to acquire mathematical literacy across NCTM's five content areas: Number & Operations, Geometry, Measurement, Algebra, and Data Analysis & Probability. Some of NCTM's standards, however, have labels that seem too sophisticated for what happens in preschool mathematics. That is, are young children doing what older children and adults know as "geometry," "algebra," or "probability" in preschool? In writing its standards document, NCTM opted to use one label for each area across the entire age range, from preK to grade 12, for a purpose. It wanted to emphasize that for each content standard, children at *every* age are learning aspects of math that relate to that standard. In this chapter, we have modified the labels slightly to be more descriptive of the preschool learnings: hence, "Geometry and spatial sense" and "Patterns, functions, and algebra" and simply "Data analysis."

Of those five areas, **number and operations, geometry and spatial sense,** and **measurement** are areas particularly important for 3- to 6-year olds, because they help build young children's "foundation" for mathematics learning:

> For this reason, researchers recommend that algebraic thinking and data analysis/probability receive somewhat less emphasis in the early years. The beginnings of ideas in these two areas, however, should be woven into the curriculum where they fit most naturally and seem most likely to promote understanding of the other topic areas. Within this second tier of content areas, patterning (a component of algebra) merits special mention because it is accessible and interesting to young children, grows to undergird all algebraic thinking, and supports the development of number, spatial sense, and other conceptual areas. (NAEYC & NCTM 2002)

This chapter describes concepts and skills in the three key areas, as well as in **patterns, functions, and algebra** and **data analysis.** Each section is divided into those concepts and skills that seem most likely to be learned, or best learned, through children's own explorations and discoveries (child-guided experience) versus those concepts and skills in which adult-guided experience seems to be important in going beyond, as well as contributing to, what children learn through their independent efforts. As with every other domain, of course, this division is not rigid.

Number and operations

This is the first of three areas that NCTM (2000; 2006) has identified as being particularly important for preschoolers. In the preschool years, number and operations focuses on six elements, or goals for early learning. (For further explanations and examples, see Clements 2004 and Copley 2004.) *Counting* involves learning the sequence of number words, identifying the quantity of items in a collection (knowing that the last counting word tells "how many"), and recognizing counting patterns (such as 21-22, 31-32, 41-42 . . .). *Comparing and ordering* is determining which of two groups has more or less of some attribute (comparing them according to which has the greater or lesser quantity, size, age, or sweetness, for example), and seriating, or ordering, objects according to some attribute (length, color intensity, loudness). *Composing and decomposing* are complementary: Composing is mentally or physically putting small groups of objects together (e.g., two plus three blocks makes five blocks), while decomposing is breaking a group into two or more parts (e.g., five spoons is two spoons plus two spoons plus one spoon).

Adding to and taking away is knowing that adding to a collection makes it larger and subtracting makes it smaller. When this understanding is combined with counting and (de)composing, children can solve simple problems with increasing efficiency. *Grouping and place value* are related: Creating sets of objects so each set has the same quantity creates "groups." Grouping in sets of 10 is the basis for understanding place value later (i.e., making groups of 10 and then counting the "leftovers"). *Equal partitioning* is dividing a collection into equal parts, a prerequisite to children's understanding of division and fractions.

To develop the mathematical understanding and skills encompassed in these six areas, preschoolers need an optimal blend of child-guided and adult-guided experiences. Because early mathematical development depends so much on manipulating objects, it is important that young children have ample opportunities to work with materials that lend themselves to ordering, grouping and regrouping, and so on. Children "intuit" certain properties and processes from their spontaneous explorations, while adults help them explore these ideas and provide the mathematical vocabulary to describe the numerical properties and transformations they observe. Adults also challenge children to try additional transformations and to reflect on the results. These experiences and the role of the intentional teacher are described below.

Of the key knowledge and skills in the area of number and operations, intuiting number and its properties, as well as performing informal arithmetic seem to develop best with child-guided experience. On the other hand, adult-guided experience seems to prove helpful for counting and numeration, as well as for performing simple arithmetic.

C Child-guided experience is especially important for learnings such as:

C1 Intuiting number and its properties

Even before they learn how to count, young children come to an informal understanding of quantity and equivalence. For example, they can identify small quantities (up to four or five) by eyeballing them. They use one-to-one correspondence to establish equivalence (e.g., matching each blue bead with a yellow one to see that there are equal quantities of each). And they can make equal sets (i.e., make groups) by putting one in each pile, then another in each pile, and so on (e.g., to distribute an equal number of pretzels to each person at the table). Although lacking a formal knowledge of "sets" in a strict mathematical sense (defined as a collection of distinct elements, such as a set of squares versus triangles), young children can create groups and recognize when items share all or some attributes with other items in a group.

Mathematicians, researchers, and practitioners agree that a central objective of early mathematics education is developing children's "number sense" —an intuition about numbers and their magnitude, their relationship to real quantities, and the kinds of operations that can be performed on them. Early number sense includes this eyeballing ability, called "subitizing" (recognizing quantities by sight alone, usually for quantities of four or fewer) and establishing one-to-one correspondence, which is the foundation of counting (i.e., linking a single number name with one, and *only one,* object).

An ability to identify equivalence is also fundamental to understanding number. Most 3-year-olds can recognize equivalence between collections of one to four objects (e.g., two hearts and two squares) without actually counting items. They can also recognize equal collections in different arrangements as being the same (e.g., three squares on the top and two on the bottom has the same number as one square on top and four on the bottom). Most 4-year-olds can make auditory-visual matches, such as equating the sound of three dings with the sight of three dots. These findings suggest that by age 3, children have already developed a nonverbal representation of number, although it's unclear what this mental representation is like or how accurate it is. Regardless, they can clearly represent and compare objects even before they can count them (Baroody 2000).

The part-part-whole concept is the understanding that a whole number (e.g., 7) can be represented as being made up of parts (e.g., 4 and 3, or 5 and 2, or 6 and 1). Part-part-whole representation is a precursor to number operations, helping children understand addition and subtraction; most 3- and 4-year-olds can describe the parts of whole numbers up to 5, with understanding of larger numbers developing around age 6 (Copley 2000, 58–59).

Teaching strategies. Intuition develops with experience. Teachers help young children develop their number sense by surrounding them with a number-rich environment offering many opportunities to work with materials and processes that rely on numbers and their operations, as shown in the following examples:

- Display materials around the classroom printed with numerals and mathematical or scientific symbols. Make sure the numerals are large enough for children to see and are placed at their eye level. Include manipulatives in the shape of numerals made of wood or cardboard, as well as toys and other items with numerals on them.
- Offer materials and games that convey the concept of number, such as dominoes and dice. Encourage children to explore them and to find matches; for example, "Can you find another domino with the same number of dots?"
- Label and describe number phenomena that occur naturally in the children's play; for example, "There are four wheels on Katie's truck and two more, or six, on Donald's" and "You found the second mitten for your other hand."
- Provide materials that allow children to explore one-to-one correspondence, such as nuts/bolts and cups/saucers. Children will also make one-to-one correspondences with any sets of materials they are playing with; for example, giving each bear a plate or ball.
- Include materials that can be broken down and divided into smaller parts, such as a lump of clay that can be divided into smaller balls or a piece of fruit that can be sliced or separated into sections. Unit blocks, Legos, and other toys with equal-size parts that children can build up and then break down into components also work well.
- Offer materials that are the same in some ways but different in others; for example, blocks of the same shape in different colors. When children use these materials, make observations that highlight their attributes; for example, "All the blocks in your tower are square, but only some blocks are red."

C2 Performing informal arithmetic

"Informal" arithmetic is something similar to adding and subtracting nonquantitatively—that is, without using numbers or other written symbols. Even before receiving formal instruction, preschoolers often are able to solve simple nonverbal addition and subtraction problems (e.g., two children are drawing at the table when a third child sits down; one child fetches "another" piece of paper for her). Children begin by acting out problems with objects (setting out two blocks, and then "adding on" one more). Later they can substitute representations (e.g., tally marks on paper) for the physical objects and form mental representations (visualizing two blocks, then adding one more). Teachers can be helpful in encouraging representation (NCTM 2000).

Forming mental representations is significant: "They understand the most basic concept of addition—it is a transformation that makes a collection larger. Similarly, they understand the most basic concept of subtraction—it is a transformation that makes a collection smaller" (Baroody 2000, 63). Preschoolers can attain this basic understanding of operations on their own, especially when adults support its development. The understanding is fundamental to later success in school mathematics.

In kindergarten, children sometimes solve simple multiplication (grouping) and division (partitioning) problems by direct modeling with objects. For example, in the problem, "Jane has 10 pennies and wants to give 2 to each friend. How many friends can she give pennies to?" the child would make piles with 2 pennies each and count the number of piles to arrive at "five friends." If the problem were stated as "Jane has 10 pennies and wants to give the same number to each of five friends. How many pennies will each friend get?" the child would put a penny in each of five piles and repeat the process until the pennies ran out, then count the number in each pile to arrive at "2 pennies per friend." With adult guidance, these informal strategies are replaced by formal number knowledge and counting strategies (Campbell 1999).

Teaching strategies. Because preschoolers tend to think concretely, handling objects and working with visual representations help them carry out and understand operations. Below are examples of strategies teachers can use to promote this learning:

- Provide many small items that children can group and regroup, adding and subtracting units.
- Pose simple addition and subtraction problems in the course of everyday experiences. For example, after a child sets the table, say "Remember that Thomas is out sick today" or "Mrs. King is going to join us for snack" and see whether they subtract or add a place setting. Or during block time, say "Jane wants to make her wall one row higher. How many more blocks will she need?"
- Pose simple multiplication or division problems that children can solve using concrete objects. For example, give a child a collection of objects at small group and say "Give the same number to everyone." Or say, "There are five children and everyone wants two scarves to wave in the wind. How many scarves will we need to bring outside?"

A Adult-guided experience is especially important for learnings such as:

A1 Counting and numeration

For young children, counting and numeration (reading, writing, and naming numbers) involves understanding numbers, which is knowing the number names and the position of each one in the sequence, ordinal numbers (e.g., first, second, . . .), and cardinal numbers (one, two, . . .); notation, which is reading and writing numerals and recognizing the simple mathematical symbols +, –, and = ; counting, which is determining quantity and equivalence; and sets, which involves creating and labeling collections and understanding "all" and "some." These are each elaborated below.

As with learning letter names and shapes, children cannot acquire knowledge of number names and numerals unless adults give them this information. At times, children will ask, "How do you write *three*?" or "What comes after 10?" but the intentional teacher also is proactive in introducing the vocabulary and symbols children need to understand and represent mathematical ideas (Campbell 1999). With adult guidance, children can then apply this knowledge to solve problems, including those of measurement and data analysis.

Early counting is finding out "how many," which is a powerful problem-solver and essential to comparing quantities. Research (Gelman & Gallistel 1978) has identified five principles of counting: (1) stable order (2 always follows 1); (2) one-to-one correspondence (each object is assigned a unique counting name); (3) cardinality (the last counting name identifies "how many"); (4) order irrelevance (objects can be counted in any order without changing the quantity); and (5) abstraction (any set of objects can be counted). Adult-guided experience helps preschoolers develop these understandings.

Older preschoolers use counting to determine that two sets of objects are equivalent. Between the ages of 3 and 4, as they acquire verbal counting skills, children gain a tool more powerful than their earlier subitizing for representing and comparing numbers, including collections larger than four items. They recognize the "same number name" principle (two collections are equal if they share the same number name, despite any differences in physical appearance). Children generalize this principle to any size collection they can count. Similarly, by counting and comparing two unequal collections, preschoolers can discover the "larger number" principle (the later a number word appears in the sequence, the larger it is). By age 4, many preschool children can name and count numbers up to 10 and compare numbers up to 5. When they have ample opportunities to learn the counting sequence, children often learn to name and count to 20 by age 5 (Clements 2004). They are also fascinated by large numbers, such as 100 or "a gazillion," even if they only know them as number names without a true sense of their value.

Equal partitioning builds on and is related to the concept of equivalence. Equal partitioning is the process of dividing something (e.g., a plate of eight cookies) into equal-size parts (e.g., to serve four children). Children as young as 4 or 5 begin to solve such problems concretely, using strategies such as dividing the objects into the requisite number of piles (four) and then counting how many are in each pile (Baroody 2000).

Teaching strategies. Psychologist Howard Gardner says, "Preschoolers see the world as an arena for counting. Children want to count everything" (1991, 75). Being creative, teachers can invent or take advantage of many situations to count objects and events in children's daily lives. For example:

- Notice things children typically compare (the number of blocks in a tower, their ages), and provide materials and experiences based on these observations. Think of fun and unusual things to count; for example, the number of mosquito bites on an ankle.
- Make numerals prominent. Place numerals of different materials, sizes, and colors throughout the classroom. Provide cards with dots and numerals for children to explore, sort, and arrange in order. Use numerals on sign-up sheets so children can indicate not only the order but also how many turns they want or for how long (two minutes, three flips of the sand-timer). Children can indicate their preferences with numerals or other marks (stars, checks, hash marks).
- Use written numerals and encourage children to write them. For example, when they play store, encourage them to write size and price labels, orders, and the amount of the bill.
- Use everyday activities for number learning and practice. For example, as children gather or distribute countable materials, engage them in counting at clean-up (counting items as they're collected and put away), small group (handing out one glue bottle per child), and choice time (distributing playing cards). At snack or mealtimes, ask the table setter to count children to determine how many place settings are needed. Pose simple number problems such as "Our group has six, but Celia is sick today. How many napkins will we need?" or "How many cups of sand will it take to fill the hole?"
- Use games as a natural yet structured way to develop counting skills. Examples include board games with dice (moving a piece the corresponding number of places) or physical movement challenges (counting the number of times the tossed beanbag lands in the bowl).
- Use children's own questions as the springboard for teachable moments. For example, Baroody (2000) imagines an incident when Diane says to her teacher,

> "My birthday is next week, how old will I be? Will I be older than Barbara?" The teacher could simply answer, "You'll be 4, but Barbara is 5 so she's still older." Or, the teacher can respond by saying, "Class, Diane has some interesting questions with which she needs help. If she is 3 years old now, how can she figure out how old she'll be on her next birthday?" The teacher could follow up by posing a problem involving both *number-after* and *number-comparison* skills: "If Barbara is 5 years old and Diane is 4 years old, how could we figure out who is older?" (65)

- Use children's literature. Not only are there many appealing counting books, but there are storybooks in which mathematics is used to solve a problem. For example, read books where the story is about sharing a quantity of something fairly. Before the problem is solved in the book, ask children to suggest solutions by trying them out with materials or working through simple ideas in their heads. Children can work alone or in pairs. After reading the book, encourage them to comment on the solution(s) in the text. As a follow-up, they might role-play the same or similar situations using props you supply or they make themselves.

A2 Performing simple arithmetic

Younger preschoolers perform simple arithmetic qualitatively. Older preschoolers, however, begin to add and subtract whole numbers quantitatively—that is, using numerals to abstractly represent numbers of objects, rather than physically manipulating or visualizing the objects. They are able to do this because they can hold a representation of quantities in their minds. For example, they may say out loud, "Two and one more is three" or "If Kenny isn't here today, I only need four napkins." Although they can do this most readily with numerals up to 5, some preschoolers can handle numbers up to 10.

Research shows they may also be capable of adding and subtracting very simple fractions. For example, when researchers hid part of a circle behind a screen and then hid another fraction, children could visually identify what the total amount was. They understood that two halves made a whole, a half plus a quarter circle resulted in a three-quarters circle, and so on (Mix, Levine, & Huttenlocher 1999). Such research suggests that children can grasp the basic idea behind simple fractions if adults pose interesting challenges.

Teaching strategies. Arithmetic follows fixed rules or conventions. Like combining letters into words, performing operations on numbers depends on knowing these rules. With support from their teachers, preschoolers are capable of solving simple arithmetic problems that come up in play and exploration. They are also motivated to use arithmetic "like grown-ups." Teachers can therefore readily implement strategies such as the following to enhance young children's early understanding and use of arithmetic:

■ Use real objects when helping children work through arithmetic problems. For example, if a child is building a tower of three blocks, count them with the child, and ask how many blocks there would be if the child added two more to make it taller. Wonder aloud how many blocks would be left if the child made it three blocks shorter. The child can add or subtract the actual blocks and count the result to determine the answer.

■ Pose challenges that build on children's interests. For example, if a child has drawn a picture of a dog, wonder aloud whether the child can draw a dog "twice as big" or "half as big."

■ Encourage children to use arithmetic to answer their own questions. For example, if a child says, "My daddy wants to know how many cupcakes to bring for my birthday tomorrow," you could reply "Well, there are 16 children and two teachers. Plus your daddy, and your brother will be here, too. How can we figure out how many cupcakes you'll need to bring?"

■ Encourage children to reflect on their arithmetic solutions rather than telling them if they're right or wrong. When children are stumped (though not yet frustrated) or arrive at erroneous answers, resist the temptation to give the answer or correct them. Instead, offer comments or pose questions that encourage them to rethink their solutions. Baroody (2000) gives this example:

> Kamie concluded that 5 and two more must be 6. Instead of telling the girl she was wrong and that the correct sum was 7, her teacher asked, "How much do you think 5 and one more is?" After Kamie concluded it was 6, she set about recalculating 5 and two more. Apparently, she realized that both 5 and one more and 5 and two more could not have the same answer. The teacher's question prompted her to reconsider her first answer. (66)

■ Start with one fraction at a time. For example, children are fascinated by the concept of "one half." If they learn—really learn—through repeated experiences that half means two parts are the same and together they make up a whole, then they can generalize this concept later to thirds, quarters, and so on.

Geometry and spatial sense

This is the second of three areas NCTM (2000; 2006) has identified as being particularly important for preschoolers. In the preschool years, learning about geometry and spatial sense focuses on four elements: *Shape* refers to the outline or contour (form) of objects and comprises identifying two- and three-dimensional shapes. *Locations, directions, and coordinates* refers to understanding the relationship of objects in the environment. *Transformation and symmetry* is the process of moving (sliding, rotating, flipping) shapes to determine whether they are the same. It also involves building larger shapes from smaller shapes, a common construction activity in preschool. *Visualization and spatial reasoning* is creating mental images of geometric objects, examining them, and transforming them. At first children's mental representations are static; that is, children cannot manipulate them. Later children can move and transform images mentally; for example, deciding whether a chair will fit in a given space or imaging a puzzle piece rotated.

Spatial concepts and language are closely related; for example, where someone stands determines whether he is "in front" of or "behind" another object. "Thus, it is important that young children be given numerous opportunities to develop their spatial and language abilities in tandem" (Greenes 1999, 42). Because society has specific conventions for labeling various shapes, transformations, and especially concepts of position, location, and so on, teachers especially need to enhance children's descriptive vocabulary in this domain.

Of the key knowledge and skills in the area of geometry and spatial sense, child-guided experience seems most helpful for creating familiarity with two- and three-dimensional shapes and their attributes, as well as for orienting self and objects in space. To create, name, and transform shapes, on the other hand, as well as to articulate position, location, direction, and distance, adult-guided experience seems necessary.

C Child-guided experience is especially important for learnings such as:

C1 Familiarity with two- and three-dimensional shapes and their attributes

For young children, shape knowledge is a combination of visual and tactile exploration, which begins in infancy. During the preprimary years, NCTM expects children to recognize, name, build, draw, compare, and sort two- and three-dimensional shapes. Although most adults support children's recognition of two-dimensional shapes, they often overlook the need to give children experiences with three-dimensional shapes, which focus their attention on geometrical features. For example, exploring the rolling of cylinders and other shapes helps children to understand the properties of the circle versus the ellipse. These skills involve perceiving (differentiating) such attributes as lines and cubes; circles, cylinders, and globes; sides and edges; corners, angles, and so on. Preschoolers are also engaged in investigating transformations with shapes (composing and decomposing), and they demonstrate an intuitive understanding of symmetry. (Note: Children need adult-guided experiences to learn to accurately label and describe transformations and symmetry.)

Teaching strategies. Communication skills are important in all areas of mathematics, but especially so in geometry. Spatial concepts and language are closely related—words facilitate an understanding of such concepts as on top of, next to, behind, and inside. For example:

- Introduce both two- and three-dimensional shapes, giving children opportunities to explore them. Include both regular and irregular shapes. Engage children in drawing and tracing the shapes. Provide models (drawings, molds, maquettes) and tools children can use to trace or copy them. Visual and physical shapes help young children grasp the essential attributes of each.
- Encourage children to sort shapes and provide reasons for their groupings. Encourage them to describe why objects are *not* alike.
- Encourage children to combine (compose) and take apart (decompose) shapes to create new shapes; for example, combining two triangles to make a square or rectangle (composing), and vice versa (decomposing). Engage them in discussions about these transformations.
- Provide materials that have vertical (i.e., left/right halves are identical) or horizontal (i.e., top/bottom halves are identical) symmetry; for example, doll clothes, a teeter-totter, and a toy airplane. For contrast, provide similar but asymmetric materials; for example, a glove, slide, and toy crane. Engage children in discussing how the two sides (or top and bottom) of objects are the same (symmetrical) or different (asymmetrical).

C2 Orienting self and objects in space

Spatial relations—how objects are oriented in space and in relation to one another—are the foundation of geometry, which involves understanding and working with the relationships of points, lines, angles, surfaces, and solids. Compared with toddlers, preschoolers navigate their bodies and move objects with greater skill and confidence. Younger children still tend to see and describe space from their own perspective (egocentrism), but older preschoolers can begin to represent and describe things from another person's point of view (perspective taking).

Teaching strategies. Because mathematics is the search for relationships, early instruction should focus on physical experiences through which children construct understandings about space. Teachers do this primarily by providing materials and allowing children ample time to explore them:

- Create different types of space in the classroom and outdoor area—small spaces for children to maneuver into and around; large open areas where children can move about freely; spaces to crawl over and under, in and out, up and down, and around and through. Ask and talk with children about their relationships with objects and with one another.
- Provide materials, time, and ample space to build with construction toys. For example, notice all the relative dimension and position concepts Trey and his friends used when they made a "bus" with large wooden blocks and invited their classmates and the teacher to get onboard:

> The group quickly decided the bus was too *small*, so they made it *bigger* by adding many more seats. The children worked hard *fitting* the big wooden blocks *end-to-end* to make the bus *longer.* They made a "driver's seat" *up front* and made a "steering wheel" to fit *on top* of the "dashboard." They also decided to build a "refrigerator" in the *back* of the pretend bus. Trey said it needed to be "on the back wall, but in the *middle* of the aisle." (Tompkins 1996b, 221)

- Provide other materials to move and rearrange; for example, doll house furniture or pedestals to display artwork. Provide materials children can use to organize and construct collages.

A Adult-guided experience is especially important for learnings such as:

A1 Creating, naming, and transforming shapes

The ability to accurately name, describe, and compare shape, size (scale), and volume is important for children to acquire during the preschool years. With appropriate experiences and input they learn to transform shapes to achieve a desired result and describe the transformation ("I'm making this bridge longer by adding more blocks at the end and holding it up in the middle"). They can also create and label symmetry in their two- and three-dimensional creations. Language is critical in all these activities. Therefore, as vocabulary expands, so does geometric understanding.

Teaching strategies. Building on preschoolers' explorations of shapes, teachers should explicitly focus the children's attention on features and what the shapes will do (e.g., "Which of these shapes can roll?") and provide words for these characteristics. Children should be given opportunities to identify shapes in various transformations, including reflections and rotations and (de)compositions. For example:

- Comment and ask children about differences in the size and scale of things that interest them; for example, their own bodies, food portions, piles of blocks. Encourage them to alter two- and three-dimensional materials and comment on the transformations, including whether their manipulations resulted in regular or irregular forms.
- Identify and label shapes and their characteristics throughout the children's environment (classroom, school, community). Go on a shape hunt in the classroom (e.g., a "triangle search"). Use increasingly sophisticated vocabulary words; for example, say "On our walk, let's look for all the square signs" or "You used cubes and rectangular blocks to build your dollhouse." Remember to supply names of three-dimensional as well as two-dimensional shapes.

- Encourage the exploration of shapes beyond conventional ones such as circles, squares, and triangles. Young children enjoy hearing and learning names such as *cylinder* and *trapezoid.* Even if they do not fully grasp the meaning and characteristics, they become attuned to the variety of spatial phenomena in the world. Also important is giving children diverse examples of triangles and other shapes, not just the equilateral triangle that is the only example offered in many classrooms.
- Use printed materials to focus on shape. Cut out photographs from magazines that feature shape pictures and encourage children to sort them. Create a shape scrapbook for the book area. Encourage children to build structures like those in story and information books. Refer to the books and talk with children about their choice of materials, how they match the attributes in the illustrations, and how they are recreating or modifying the structures or both.
- Challenge children to imagine what their structures would look like with one or more elements transformed, for example, in location or orientation. Encourage them to represent and verify their predictions. For example, in *Building Structures with Young Children*, Chalufour and Worth share this note from a preschool teacher:

> I brought a whiteboard and markers over to the block area because Abigail was having a hard time imagining what her tower would look like if it were built with the blocks placed vertically instead of horizontally, as she had done. Not only did it help her to see a drawing of a tower built with verticals, but Adam came up to the drawing and pointed to one of the blocks near the top of the drawing, declaring that he didn't think it would balance on top of the one under it. So he and Abigail proceeded to use the drawing to build a tower and, lo and behold, Adam was right! Tomorrow I'm going to invite him to tell the group about the event. We can ask Adam how he knew that the vertical wouldn't balance. (2004, 45)

A2 Articulating position, location, direction, and distance

Expectations in this area involve concepts of position and relative position, direction and distance, and location (NCTM 2000). With appropriate adult guidance, preschoolers can use position and direction words and follow orientation directions. They also are able to begin moving beyond their egocentric perceptions to predict another's perspective. For example, with experience they can describe how someone else would see something from his perspective, and can give appropriate directions or instructions to another person.

Teaching strategies. Teachers need to supply vocabulary, of course. But preschoolers still master such ideas through a combination of concrete experience and mental imagery, so teachers need to provide many opportunities for them to represent these concepts in two- and three-dimensional ways:

- Make comments and ask questions that focus on location and direction; for example, "You attached the sides by putting a long piece of string between the two shorter ones" or "Where will your road turn when it reaches the wall?" Comment on naturally occurring position situations, such as "Larry is climbing the steps to the slide, Cory's next, and last is Jessica."
- Use various types of visual representations to focus on these concepts. Engage children with making and interpreting maps—for finding a hidden object, for example. Children can draw diagrams of the classroom, their rooms at home, and other familiar places. Ask them about the placement of the objects. Comment on the location of things in their drawings using position words; for example, "You have a big poster over your bed" or "What's poking out behind the curtain?" Ask children to draw, paint, build, or use their bodies to represent favorite books featuring characters or objects in relation to one another. For example, ask them to draw the three bears sitting around the table or to lie down next to one another like the three bears in their beds. Because society has specific conventions for labeling various concepts of position, location, and so on, teachers are especially needed to enhance children's descriptive vocabulary in this domain.
- Create occasions for children to give directions; for example, when helping one another or leading during large group. This requires them to use position and direction words such as "Hold the top and

push down hard into the dough" or "Stretch your arms over your head and then bend down to touch your toes." Encourage children to volunteer as the leader.

■ Use movement to focus on spatial concepts. Provide objects that can be thrown safely, such as beanbags and foam balls, and interact with children about distance. Use simple movement directions for games and dances at large group such as Hokey Pokey. Invent variations to games and dances by frequently modeling the adding of a new twist. Get the children engaged in making up variations of their own.

■ Talk about trips children take with their families or about walks and field trips with the class. For example, "Does your grandma live close to you or far away?" or "We took a long ride to the zoo on the bus, but after we parked, it was just a short walk to the monkey cage."

Measurement

This is the third of three areas NCTM (2000; 2006) has identified as being particularly important for preschoolers. In the preschool years, learning about measurement focuses on two elements: *Attributes, units, and processes* refers to developing concepts about size and quantity, arranging objects to compare them, estimating differences (e.g., by eyeballing, lifting), and quantifying differences with nonstandard (e.g., footsteps) and standard (e.g., tape measure) tools. *Techniques and tools* comprises learning measuring rules such as starting at zero, aligning or equalizing beginning points, and not allowing gaps. It also includes becoming familiar with standard measuring tools such as rulers, scales, stopwatches, and thermometers. As with spatial concepts, measurement benefits from language, especially comparison words.

Of the key knowledge and skills in the area of measurement, comparing (seriating) or estimating without counting or measuring seems to develop best with child-guided experience, while adult-guided experience seems integral for counting or measuring to quantify differences.

C Child-guided experience is especially important for learnings such as:

C1 Comparing (seriating) or estimating without counting or measuring

Young children are able to grasp the basic concept of one thing being bigger, longer, heavier, and the like, relative to another. Making comparisons is the beginning of measurement. According to NCTM's standards (2000), preschoolers should be engaged in comparing length, capacity, weight, area, volume, time, and temperature.

At first, children make qualitative comparisons by matching or ordering things ("Stacy is the short one, and Bonnie is tall" or "My cup holds more water than yours") rather than quantitative comparisons that use counting or measuring. To estimate, they use their various senses, such as eyeballing (visual), lifting (kinesthetic), or listening (auditory). They may compare length by aligning blocks on the bottom and seeing how much they stick out on top, or listen to instruments to compare their loudness.

Teaching strategies. Teachers can draw on children's interest in comparing to focus their attention on quantitative and qualitative differences. Examples abound in mathematical and scientific applications, including those suggested here:

■ Make comments and ask questions using comparison words ("Which of these is longer?" or "Does everyone have the same number of cookies now?") Ask children whether they think something is wider (softer, heavier, louder, colder) than something else.

■ Provide ordered sets of materials in different sizes, such as nesting blocks, measuring spoons, pillows, paintbrushes, and drums. Affix ordered labels that children can use to find materials and return them to their storage place; for example, four sizes of measuring spoons traced on the peg board in the house center. Provide storage containers in graduated sizes.

■ Encourage children to move at different rates throughout the day and comment on relative speed. Make transitions fun by asking children to proceed to another area or activity "as slow as a snail" or "as fast as a rocket." Acknowledge their observations about speed and what affects it. A preschool teacher shared this anecdote:

> At outside time, James was pushing two children around the playground on the toy taxi. When the adult asked if she could have a ride, he said "Sure." After going around two more times, James stopped the taxi and said, "Get off. You're too fat and I can't go fast." Acknowledging the validity (if not the kindness) of his observation, the teacher got off the taxi so he could move at a faster clip. (Graves 1996, 208).

- Call children's attention to graduated changes in nature. Comment on seasonal fluctuations in temperature (e.g., "It feels colder now than it did when we went to the pumpkin patch. We're wearing heavier jackets."). Plant a garden and ask children how long they think it will take before the seeds germinate, the vegetables are ready to eat, and so on.

A Adult-guided experience is especially important for learnings such as:

A1 Counting or measuring to quantify differences

Many older preschoolers and kindergarten children are able to understand the idea of standard units, and with well-conceived learning experiences, they can begin to determine differences in quantity by systematic measurement. They use their knowledge of number to make comparisons. At first, they use nonstandard units such as how many steps it takes to cross the schoolyard in each direction or the number of song verses to clean up different areas of the room. With teachers' assistance, they acquire the understanding that it is useful to employ conventional units and measuring devices, such as inches on a ruler or minutes on a clock.

Teaching strategies. There are many opportunities throughout the day for children to engage in measurement; for example, when they are building something or resolving a dispute. However, it usually does not occur to preschoolers to measure or quantify things to solve these problems. Adults can actively encourage children to use measurement in these situations. For example:

- Provide conventional and unconventional measuring devices, and encourage children to use them to answer questions or solve problems. Conventional devices include rulers, tape measures, clocks, metronomes, kitchen timers, and spring and balance scales. Unconventional ones include string or paper towel tubes for length, sand timers for duration, grocery bags for volume (three bags of blocks were needed to make a tall tower, only one bag to make a short one), unmarked bags of clay or sand

Gathering Data to Resolve a Social Conflict

Undertaking an investigation with adult help is one way to resolve disputes (for more, see **Chapter 5**). Some conflict situations lend themselves to collecting information that can be quantified and interpreted to reach a fair solution. In this example, a group of older preschoolers resolve a dispute by measuring heights and are surprised by the result!

John, Liza, and Devon argued about the order of turn-taking to drive the big truck around the playground. They decided the biggest person should go first, then the next biggest, then the smallest. They would make a list and check off each name as that person finished a turn. John chalked *J, D, L* on the blacktop and moved to get on the truck. But the conflict was not yet over:

Devon: Hey! I'm bigger than you. I get to go first!

John: No you're not. I'm the biggest one.

Liza: Let's measure and find out.

The children stood against the wall and asked their teacher to make a chalk mark where each of their heads touched. Then they got the tape measure and asked their teacher to write down how many inches tall each child was. She wrote 41 next to the J, 42 next to the D, and 44 next to the L.

Liza: It's me! I'm the biggest!

Devon: Yeah, and I'm next. John is last.

John: But I can stay on the longest because there's no one after me!

for weight. Children can also develop their own devices. When children ask measurement-related questions ("Which is heavier?") or have disputes ("I am too taller by a whole, big lot!"), ask them which of these tools might help them arrive at an answer or solution.

■ Pose measurement challenges that children will be motivated to solve; for example, "I wonder how many cups of sand it will take to fill all 12 muffin tins?" Ask "How many more . . . ?" questions, such as "How many more pieces of train track will you need to close the circle?" Here's an amusing challenge shared by a curriculum developer and writer who works with preschoolers:

> I was stretched out on the floor against a wall. I said, "I wonder how many me's long the wall is?" The children thought this was very funny, but they were intrigued to figure out the answer. Some estimated by simply "envisioning" the response. Others wanted to use the direct route, having me move and stretch out while they counted the number of repetitions. When I said I was too comfortable and didn't want to move, the children had to come up with another solution. They decided that two of them equaled one of me, so they stretched in a line along the wall and counted how many of them it took. With my help, they then divided the number of children in half. (Stuart Murphy, 2004, pers. comm.)

■ When resolving social conflicts with children, ask how they could measure to guarantee a fair solution; for example, to make sure everyone gets to play with a toy the same amount of time.

■ Use visual models to help children understand and quantify differences. For example, make a daily routine chart where the length of each part in inches is proportional to its duration in minutes. Give time checks ("Five minutes to clean-up") with visual and auditory cues.

■ Create opportunities for group construction projects, such as laying out a garden, making a bed for each doll within a defined space, or recreating a supermarket after a class field trip. These often lead to situations where children have different opinions and need to "measure" to find out who is right or what solution will work. Sometimes you will need to suggest this method of resolving the difference of opinion.

■ Include units of measurement when sharing information with children; for example, "I went grocery shopping for an hour last night" or "My puppy gained 5 pounds since the last time I took him to the vet."

Patterns, functions, and algebra

In the preschool years, learning about patterns, functions, and algebra focuses on two elements: *Identifying patterns* involves recognizing and copying patterns and determining the core unit of a repeating pattern. It includes visual, auditory, and movement patterns. Deciphering patterns requires inductive reasoning, which is also a precursor to understanding probability. *Describing change* is using language to describe the state or status of something before and after a transformation. For example, "When I was a baby, I couldn't drink out of a cup" or "When we raised the ramp a little higher, my car went all the way to the book shelf."

Of the key knowledge and skills in the area of patterns, functions, and algebra, child-guided experience seems to help children recognize, copy, and create simple patterns and also recognize naturally occurring change. For children to identify and extend complex patterns and to control change, on the other hand, children seem to benefit most from adult-guided experience.

C Child-guided experience is especially important for learnings such as:

C1 Recognizing, copying, and creating simple patterns

For young children, this area encompasses an awareness of patterns in the environment (visual, auditory, temporal, movement). Preschoolers can acquire the ability to copy or create simple patterns with two elements, such as A-B-A-B or AA-BB. Even before they know the word *pattern,* children notice recurring designs or routines in their lives, whether it be on their clothing, the stripes on a kitten's back, or the order of each day's activities.

Preschoolers generally need at least three repetitions of a pattern before they can recognize or repeat it.

Teaching strategies. Patterns and series of objects or events are plentiful in the world. Teachers can actively help children become aware of common patterns and series. Simple observations and questions can lead them to notice and create regularity and repetition. For example:

■ Ask children to do or make things that involve series and patterns. For example, at small group, give children drawing or sculpting materials and invite them to represent their families—from the smallest to the biggest members. Other materials that lend themselves to pattern making include string and beads in different colors and shapes (e.g., to make a necklace), multi-colored blocks in graduated sizes (e.g., to make a train), and pegs and pegboards (e.g., to make a design).

■ Acknowledge the patterns children spontaneously create in art and construction projects. When they are busy building, acknowledge their work with a smile and a descriptive statement such as, "I see a pattern in your tower. First you used two rectangles, then you used a cylinder, and then you added two more rectangles and a cylinder" or "This reminds me of the Eiffel Tower. It's wide at the bottom and becomes narrow at the top" (Chalufour & Worth 2004, 38). Music also provides many opportunities for calling attention to patterns; for example, "You sounded two loud, one soft, two loud, and one soft beat with the rhythm sticks." Movement provides another source for constructing patterns; for example, a series of two or three steps repeated in sequence (side, side, hop, side, side, hop).

■ At large group, encourage children to move their bodies into graduated positions such as lying, sitting, and standing. Move through transitions at slow, medium, and fast paces.

■ Read and act out stories in which size, voice, or other graduated qualities play a role, such as The Three Bears or The Three Billy Goats Gruff. At small group, ask children to make beds for the three bears with play dough. At large group, have them choose which instrument the papa, mama, or baby bear would play, depending on variations in pitch or loudness.

C2 Recognizing naturally occurring change

Noticing and describing changes includes identifying what variable or variables are causal. This is a mathematics concept, but it is also prominent in science in children's developing awareness of changes in the world around them and possible reasons for these. For example, children see changes in their own bodies (e.g., getting taller) or the growth of a flower. Although they are often unable to identify the causal factor accurately, young children do make tentative guesses, both right and wrong, about the changes they see. For example, "I'm 5 today. That means I'm taller" or "The flower grew up because the wind blew on it from the bottom."

Teaching strategies. The most important strategy teachers can follow in this area is to notice and acknowledge children's awareness of changes in their environment and initiate situations in which change can be created, observed, and investigated; for example, by discussing the growth of vegetables in the school garden or experimenting with color mixing at the easel. Repeating and extending children's comments about the changes they observe is a signal you are listening to them. Calling their attention to change and showing you are interested in their reaction, and in their explanations, is also a form of acknowledgment. For example:

■ Repeat children's comments to acknowledge their spontaneous seriation. When LaToya said, "These giants are hungrier because they have bigger teeth," her teacher agreed: "Those bigger teeth will help the monsters eat lots more food."

■ Extend children's comments. Josh was washing his hands at the sink when his teacher turned on the water in the next sink full blast. Josh said "Mine is running slow." She turned down her water and said "I made mine slow*er* like yours."

■ Call children's attention to cycles in nature with concrete examples. Point out the seasonal variations in schoolyard plants or the changing thickness of children's jackets from fall (lightweight) to winter (heavy) to spring (back to light). Document changes with photographs.

A Adult-guided experience is especially important for learnings such as:

A1 Identifying and extending complex patterns

Simple patterning is something young children do spontaneously. With experience and adult input, they learn to do more. For example, older preschoolers and kindergartners are able to analyze, replicate, and extend the core unit of a complex repeating pattern with three or more elements (A-B-C-A-B-C; 1-22-3-22-1), provided they see or hear it several times (Clements 2004). They can also begin to recognize what are called "growing" patterns—that is, patterns where successive elements differ (rather than repeat) but still proceed according to an underlying principle, such as counting by ones or twos (2-4-6). The same principles apply to patterns in nature. Younger children may notice past and present seasons; older preschoolers are ready to grasp the cycling of four seasons in a year.

Teaching strategies. Young children recognize simple patterns on their own. Complex patterns are more dependent on someone pointing them out, particularly if the viewer is not looking for them in the first place. Therefore, teachers can play an especially active role in helping young children identify and create multi-part repeating and growing patterns and sequences. For example:

■ Create complex patterns, then give children art and construction materials to copy them. Encourage them to create patterns and series on their own with three or more elements.

■ Comment on the patterns children create, identifying repeating elements. For example, Leah showed a painting of "two rainbows" to her teacher. It was actually two sequences or patterns of color that were exactly the same. "Look," her teacher commented, "this rainbow has green, red, purple, yellow, and so does this one—green, red, purple, yellow" (Hohmann & Weikart 2002, 469).

■ Introduce children to the books and catalogs with complex patterns used by ceramic tilers, landscape designers (brick and paver patterns), and fiber artists (weaving, quilting, needlepoint, basketry). Decorating stores often give away books of discontinued wallpaper and rug samples. With these, engage children in describing the patterns and finding corresponding examples that contain one or more comparable repeated elements in their own environment; for example, the walkway to the school or a knitted woolen hat.

■ Call children's attention to complex patterns and sequences in their environment; for example, seasonal cycles, markings on plants and animals, art and crafts in their community. Encourage children to duplicate and extend the patterns they see. For example, collect things with complex patterns on a nature walk and have children copy and extend the patterns (or create their own comparable one) at small group or art time.

■ Provide computer programs that allow children to recognize and create series and patterns.

■ Use music to call attention to patterns. Play instrumental music with pitch, tempo, or loudness patterns and encourage children to identify them. (This works best if the children are already familiar with the music.) Sing songs with repeating patterns (where verses and chorus alternate) or growing patterns (count-down songs such as I Know an Old Lady Who Swallowed a Fly). Comment on the patterns and encourage children to identify them.

■ Use movement to focus on pattern. Older preschoolers can sequence three movements. If children can master these, encourage them to be leaders and suggest three-step sequences.

A2 Controlling change

Younger children spontaneously notice changes in themselves and their environment. Older preschoolers not only observe but can also begin to articulate the reasons for such changes. Moreover, they can deliberately manipulate variable(s) to produce a desired effect. For example, they may alter the choice of materials and their arrangement to better represent something in a collage, or alter the length and angle of a ramp to affect the speed of a toy car.

Teaching strategies. Teachers can promote awareness of and curiosity about change by fostering a spirit of inquiry in the classroom. An adult's investigative attitude is transmitted to the children. They will begin to pose the kinds of questions that scientists use when they want to know about the properties of materials and how they operate, then predict and estimate or measure the results to satisfy their curiosity. Children are eager to try different things ("manipulate variables") and see the outcomes. Here are some strategies:

- Make "I wonder what would happen if . . ." statements; for example, "I wonder what would happen if you made this end of the ramp higher."
- Ask "Suppose you wanted to . . ." questions; for example, "Suppose you wanted to make the car go slower. How do you think you could do that?"
- During social problem-solving situations (see **Chapter 5**), encourage older preschoolers to anticipate the consequences of their proposed solutions. If they foresee difficulties, have them consider how to change all or part of the solution to avoid them.

Data analysis

In the preschool years, learning about data analysis focuses on three elements: *Classifying or organizing* involves collecting and categorizing data (e.g., the favorite foods of children in the class). *Representing* is diagramming, graphing, or otherwise recording and displaying the data (e.g., a list of different foods, with check marks for every child who likes them). *Using information* involves asking questions, deciding what data is needed, and then interpreting the data gathered to answer the questions (e.g., what to have for snack).

Of the key knowledge and skills in the area of data analysis, children seem most capable of making collections and sorting/classifying by attributes when they learn through child-guided experience and seem most capable of representing gathered information when they learn through adult-guided experience.

C Child-guided experience is especially important for learnings such as:

C1 Making collections, sorting/classifying by attributes

Children love to collect and sort things. (Adults do too; science used to be primarily about collecting specimens and developing taxonomies to describe each group's characteristics.) Sorting involves noticing, describing, and comparing the attributes of things (animals, people, objects) and events. Young children can classify according to one attribute (e.g., color), and children slightly older can classify by two attributes (e.g., color and size). Examples of other attributes by which young children typically classify include shape, texture, temperature, loudness, type, and function.

Teaching strategies. Because children are natural collectors, they will eagerly initiate and respond to suggestions in this area of mathematical and scientific inquiry. By showing interest in their collecting and arranging and by asking skillful questions, teachers can extend child-guided explorations. For example:

- Encourage children to make collections of items in the classroom, natural objects gathered on field trips, and various objects they bring from home. Provide containers (bowls, boxes, baskets) for them to sort the items. Encourage them to explain and describe their collections.
- Encourage children to explain why things do *not* fit into the categories they have created. For example, pick up an object and say "Would this one fit here?"
- Provide opportunities to experiment with materials whose attributes involve all the senses, such as shape, texture, size, color, pitch, loudness, taste, and aroma.

- Acknowledge and repeat children's attribute labels, including invented ones ("This fruit feels squishy on my tongue" or "The pebbles are bumply"). Use common words to build children's vocabulary ("You used lots of blue in your painting") and introduce new language to expand their descriptive language ("This cloth feels silky" or "You used all the rectangular blocks in your tower").

A Adult-guided experience is especially important for learnings such as:

A1 Representing gathered information

Representing information for purposes of data analysis means documenting categories and quantities with numbers, diagrams, charts, graphs, counters (e.g., one button for each occurrence), and other symbols. These activities involve knowledge of both mathematics and scientific inquiry.

Teaching strategies. Children are naturally curious about their environment, but their investigations tend to be limited in scope and haphazard in procedure. Adult intervention can make children's explorations and conclusions more systematic and meaningful. Strategies such as the following help them use the "scientific method" to answer questions of interest to them:

- Provide materials children can use to record and represent data, such as clipboards and graph paper.
- Pose questions whose answering requires gathering and analyzing data; for example, "How many bags of gerbil food do we need to feed Pinky for one month?" Focus on things of particular interest to children, such as their bodies (height, age, hair color), animals and nature (types of pets), the dimensions of things they build, and what they and their friends like and dislike (foods, favorite story characters). For example, chart the ingredients children like best in trail mix, and use the data to make snacks in proportion to their tastes.
- Put a "question box" in the classroom, and help children write out and submit questions. For questions that involve data collection, ask children to suggest ways to answer them.
- Be alert to situations that lend themselves to documentation, such as construction projects that involve multiples of materials. For example, if children build a train, help them chart the number of cars or units in the track. If the cars are of different sizes, create rows or columns and encourage children to record the number of each. If train building is a recurring activity, investigate whether trains made on different days are longer or shorter, and by how many cars.

A2 Interpreting and applying information

This component of data analysis refers to making and testing predictions, drawing conclusions, and using the results of an investigation to establish or clarify facts, make plans, or solve problems.

Teaching strategies. Without adult intervention, children's scientific inquiries often end with just collecting information. They may need help to analyze the data to draw one or more conclusions. Further, children's learning is less likely to end there if teachers encourage them to apply their learning to related topics and to solving problems. Try strategies such as these:

- When disputes arise, encourage children to test out their hypotheses to resolve differences.
- Make simple summaries of the data the children have collected or displayed. For example, "So in our class we have two children who are 5 years old, eight who are 4, and six who are 3."
- Ask children for their ideas about what to do with the information they gather. For example, "Everyone likes pretzels, half of you like raisins, but there are only two check marks next to sesame sticks. So what does this mean we should put in the trail mix for our walk tomorrow?"
- Encourage children to predict the outcome, record their predictions, and then compare them with the results. For example, have each child guess the length of a wall and record their estimates. Measure the wall and then discuss whose guess was too long, too short, or just right.

* * *

In her book *The Young Child and Mathematics,* Copley (2000) asks rhetorically,

> Should we immediately correct young children's misconceptions about mathematics? Can we expect all children to solve problems in identical ways? Should we expect all the young children in a group to "get it" at the same time? . . . [T]he answer is *No!* As teachers, we need to remember that young children construct mathematical understanding in different ways, at different times, and with different materials. Our job is to provide an environment in which all children can learn mathematics. (8–9)

This chapter demonstrates that young children are eager to enter the worlds of mathematics and science. If adults create an atmosphere that encourages investigation and engages children in reflection, they will experience the small and large pleasures of these areas in their daily lives. In addition,

> Positive experiences with using mathematics to solve problems help children to develop dispositions such as curiosity, imagination, flexibility, inventiveness, and persistence that contribute to their future success in and out of school. (NAEYC & NCTM 2002)

Questions for Further Thought

1. Why do some early educators underestimate young children's mathematical and scientific abilities? What does this underestimation say about how practitioners define these subject areas and their self-perceived knowledge and skills? Are these areas considered "too hard" for children because early educators fear they are too complex for themselves?

2. How can early childhood educators change the public perception of early mathematics to encompass more than numbers and counting; for example, to include all five areas identified by mathematics educators (NCTM 2000)?

3. How can the early childhood field change the public perception of early science to include more than studying nature; for example, to include the scientific method of investigation?

4. What scientific knowledge should we expect preschoolers to master (comparable to the early standards NCTM has developed for mathematics)? What areas of substantive knowledge are appropriate or necessary for young children to have?

5. Do gender differences in mathematics and science (favoring the involvement of boys) emerge in the preschool years? If no, what lessons can we learn from early childhood practice to sustain interest in these subjects and prevent the emergence of a gender gap in later years? If yes, how can we alter our practices to instill and sustain ("fortify") lasting interest in girls?

6. How can (and should) we take advantage of emerging technologies to enhance early learning in mathematics and scientific inquiry? Is there such a thing as "bad" technology or do the (dis)advantages lie only in its application?