PRECISION MEASUREMENT OF COUPLING ELLIPSES PARAMETERS IN A STORAGE RING

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Abstract

Eigen-mode coupling ellipses' tilt angles and axis ratios can be precisely measured with a Model-Independent Analysis (MIA) of the turn-by-turn Beam Position Monitor (BPM) data from resonance excitation of the betatron motion. For each BPM location one can measure 4 parameters from the two resonance excitation, which completely describe the linear coupling of the location. Results from application to PEP-II LER storage ring are presented.

INTRODUCTION

For an e+-e- collision, such as the PEP-II B-factory at the Stanford Linear Accelerator Center, a solenoid is usually imposed at the interaction point (IP). This necessarily causes beam motion coupling between horizontal and vertical planes in each of the collision storage rings. To correct these coupling effect at IP and outside the solenoid region, designed lattice includes global skew and Local skew quadrupoles inserted at the suitable locations. However, during the commissioning and operation, the linear coupling correction is not automatic; it usually requires dedicated effort of which precise measurement of the linear coupling is the key. There has been previous attempt in studying linear couplings [1]. In this paper, taking PEP-II Low-Energy Ring (LER) as an example, we show how we precisely measure the linear coupling by measuring the eigen-mode coupling ellipses' tilt angles and axis ratios to survey the linear couplings along an entire storage ring.

HIGH-RESOLUTION TURN-BY-TURN ORBITS

To overcome the synchrotron radiation damping, we make two resonance excitations, one at the horizontal tune and the other at the vertical tune, for turn-by-turn BPM buffer data acquisition of about 2000 turns. Each excitation would allow for 2 sets of data stored in two matrices, one for the horizontal and the other for the vertical beam centroid readings. Columns of the matrices represent BPMs while rows represent the turns. Excluding the closed-orbit mode, one can, through an automatical testing process for zooming, choose the right number of turns ($<$ 1800 turns for 2000 turn data) such that making Fourier transformations on columns, one can match one and only one stand-out peak Fourier mode to the resonance frequency (tune) to obtain a cosine-like and a sine-like orbits from the mode for each of the excitation [2]. Therefore the two resonance excitations can offer a complete set of 4 clean betatron-motion orbits for one turn. To obtain the next turn orbits, one simply use the same number of turns for Fourier transformation but advance one turn for the block sub-matrix to be performed with the Fourier transformation. Repeat the process until one gets about 200 turns of the high resolution orbits. Alternatively, one can perform the zoomed Fast Fourier Transformation(FFT) as described above for only once and then keeps the only one peak mode for inverse FFT to get high resolution turn-by-turn orbits. These consecutive-turn orbits from resonance excitations form two eigen-mode coupling ellipses at each double-view BPM location. Shown in Figure 1 are eigen-mode coupling ellipses at 4 double-view BPM locations, where, in each plot, the horizontal eigen-mode coupling ellipse is colored in blue while the vertical eigen-mode coupling ellipse is colored in red.

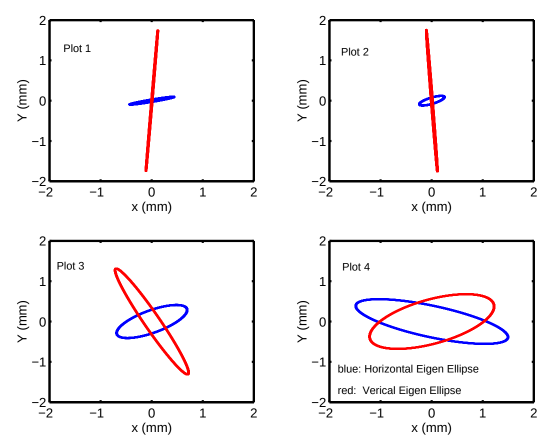

Figure 1: Eigen-mode coupling ellipses at 4 double-view BPM locations of the PEP-II LER. The top 2 are at the two BPMs beside the IP while the bottom 2 are at the tenth BPMs from IP in each side. (data acquired on September 30, 2003.)

TILT ANGLE MEASUREMENT

For a horizontal eigen-mode coupling ellipse in the x-y plane, we define the tilt angle, α_x as the counterclockwise angle of its long axis with respect to the x axis. If we make a counterclockwise rotation of the x-y axis for an angle that is equal to the tilt angle α_x, then the ellipse long axis matches the new x axis, and the orbits' new coordinates are given by $x_i' = x_i \cos\alpha_x + y_i \sin\alpha_x$, and $y_i' = y_i \cos\alpha_x - x_i \sin\alpha_x$. Considering a Least-Square fitting that the sum, $D(\alpha_x)$, of the distances of the ellipses' orbits to the long axis be the minimum, one would have $dD^2(\alpha_x)/d\alpha_x = 0$, where $D^2(\alpha_x) = \sum_i y_i'^2 =$

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$\sum_i y_i^2 \cos^2 \alpha_x + \sum_i x_i^2 \sin^2 \alpha_x - \sum_i x_i y_i \sin 2\alpha_x$, and so one obtains

$$\alpha_x = \frac{1}{2} tan^{-1} \frac{2\sum_i x_i y_i}{\sum_i (x_i^2 - y_i^2)}. \tag{1}$$

By the same token, For a vertical eigen-mode coupling ellipse in the x-y plane, we define the tilt angle, α_y as the counterclockwise angle of its long axis with respect to the y axis and obtain

$$\alpha_y = -\frac{1}{2} tan^{-1} \frac{2\sum_i x_i y_i}{\sum_i (y_i^2 - x_i^2)}. \tag{2}$$

The tilt angles for the horizontal eigen-mode coupling ellipses shown in Figure 1 are respectively $(12.0811^o, 21.6461^o, 25.9331^o, -15.5012^o)$ for Plots 1-4. The corresponding vertical eigen-mode coupling ellipses tilt angles are $(-3.8063^o, 3.5516^o, 28.3203^o, -69.2074^o)$.

AXIS RATIOS MEASUREMENT

The horizontal eigen-mode coupling ellipse equation is given by $\frac{x'^2}{a_x^2} + \frac{y'^2}{a_y^2} = 1$, where a_x is the long axis and a_y is short axis. Letting $X' = x'^2$, $Y' = y'^2$, $A = \frac{1}{a_x^2}$, and $B = \frac{1}{a_y^2}$, one have a linear equation of

$$AX' + BY' = 1, \tag{3}$$

which can be solved through Least-Square fitting of (X_i', Y_i')s for the ratio of the short axis length over the long axis length given by

$$(a_y/a_x) = \sqrt{A/B}. \tag{4}$$

By the same token, one can get the axis ratio for the vertical eigen-mode coupling ellipse's. Shown in figure 3 is the measurement of the eigen-mode coupling ellipse axis ratios at LER double-view BPM locations.

The axis ratios for the horizontal eigen-mode coupling ellipses shown in Figure 1 are respectively $(0.0630, 0.2867, 0.3560, 0.2494)$ for Plots 1-4. The corresponding vertical eigen-mode coupling ellipses axis ratios are $(0.0044, 0.0078, 0.1075, 0.4166)$.

TILT ANGLE AND AXIS RATIO CALCULATION

Let us first assume that the one-turn linear map, the 4-by-4 matrix M, has been decoupled and normalized as given by the factorization [3], $M = CARA^{-1}C^{-1}$, where

$$R = \begin{pmatrix} \cos\mu_x & \sin\mu_x & 0 & 0 \\ -\sin\mu_x & \cos\mu_x & 0 & 0 \\ 0 & 0 & \cos\mu_y & \sin\mu_y \\ 0 & 0 & -\sin\mu_y & \cos\mu_y \end{pmatrix},$$

$$A = \begin{pmatrix} \sqrt{\beta_x} & 0 & 0 & 0 \\ \frac{-\alpha_x}{\sqrt{\beta_x}} & \frac{1}{\sqrt{\beta_x}} & 0 & 0 \\ 0 & 0 & \sqrt{\beta_y} & 0 \\ 0 & 0 & -\frac{\alpha_y}{\sqrt{\beta_y}} & \frac{1}{\sqrt{\beta_y}} \end{pmatrix},$$

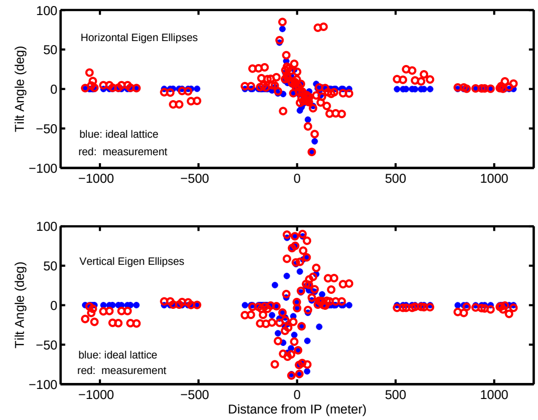

Figure 2: Comparison of the PEP-II LER eigen-mode coupling ellipses tilt angles between measurement and the ideal lattice model. (data acquired on September 30, 2003.)

$$C = \begin{pmatrix} I\cos\phi & \bar{W}\sin\phi \\ -W\sin\phi & I\cos\phi \end{pmatrix},$$

where $\cos\phi$, $\sin\phi$, and $\tan\phi$ to appear later, may be replaced with $\sinh\phi$, $\cosh\phi$, and $\tanh\phi$ respectively for certain cases,

$$W = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

and its conjugate $\bar{W} = -SW^T S$, where

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

For horizontal eigen motion we have, in the eigen-mode plane at the n^{th}-turn,

$$\begin{pmatrix} x \\ p_x \end{pmatrix}_n = \begin{pmatrix} \sqrt{\beta_x} & 0 \\ -\frac{\alpha_x}{\sqrt{\beta_x}} & \frac{1}{\sqrt{\beta_x}} \end{pmatrix} \begin{pmatrix} \cos n\mu_x & \sin n\mu_x \\ -\sin n\mu_x & cosn\mu_x \end{pmatrix} \begin{pmatrix} \cos\mu_{ox} & \sin\mu_{ox} \\ -\sin\mu_{ox} & \cos\mu_{ox} \end{pmatrix} \begin{pmatrix} A_{ox} \\ 0 \end{pmatrix}$$
$$= A_{ox}\sqrt{\beta_x} \begin{pmatrix} \cos\theta_x \\ -\frac{1}{\beta_x}(\alpha_x\cos\theta_x + \sin\theta_x) \end{pmatrix}$$

where A_{ox} is the resonance excitation normalized amplitude in the eigen space, and $\theta_x = n\mu_x + \mu_{ox}$. Transferring to the measurement (coupled) frame, at the n^{th}-turn, we have $x = A_{ox}\sqrt{\beta_x}\cos\phi\cos\theta_x = A_x\cos\theta_x$, which would yields $\cos\theta_x = x/A_x$ and $\sin\theta_x = \sqrt{A_x^2 - x^2}/A_x$, where $A_x = A_{ox}\sqrt{\beta_x}\cos\phi$. We would also have $y = -A_{ox}\sqrt{\beta_x}\sin\phi(a\cos\theta_x - \frac{b}{\beta_x}(\alpha_x\cos\theta_x + \sin\theta_x)) = -A_x tan\phi(a\frac{x}{A_x} - \frac{b}{\beta_x}(\alpha_x\frac{x}{A_x} + \frac{\sqrt{A - x^2 - x^2}}{A_x})$. We therefore get the ellipse equation as $(D_x^2 tan^2\phi)x^2 + 2[(a - \frac{b\alpha_x}{\beta_x})tan\phi]xy + y^2 = A_x^2\frac{b^2}{\beta_x^2}tan^2\phi$, where $D_x = \sqrt{(a - \frac{b\alpha_x}{\beta_x})^2 + \frac{b^2}{\beta_x}}$. Counterclockwise rotating axis angle

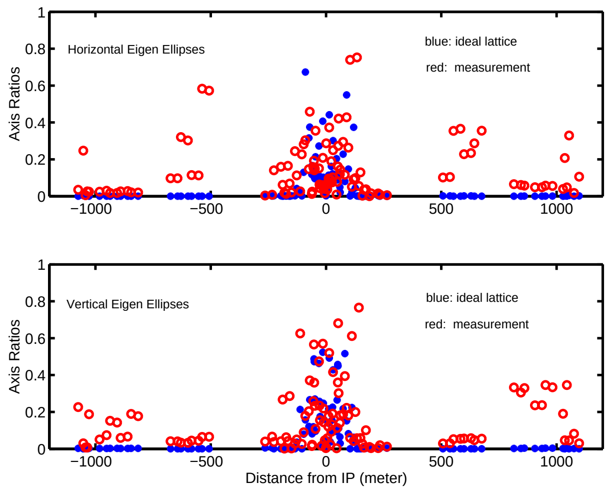

Figure 3: Comparison of the PEP-II LER eigen-mode coupling ellipses (short vs. long) axis ratios between measurement and the ideal lattice model. (data acquired on September 30, 2003.)

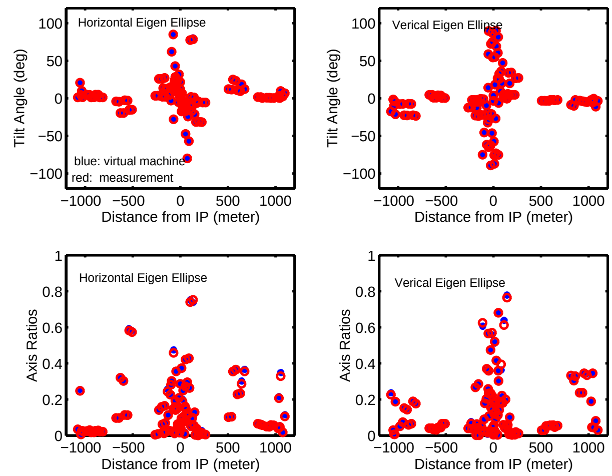

Figure 4: Comparison of the PEP-II LER eigen-mode coupling ellipses' tilt angles and (short vs. long) axis ratios between measurement and the virtual machine obtained through Green's functions fitting [4]. (Sept. 30, 2003.)

by ψ_x such that in the new coordinate frame, the x'y' cross term disappear would yield

$$\psi_x = \frac{1}{2}tan^{-1}(\frac{-2(a-\frac{b\alpha_x}{\beta_x})tan\phi}{1-D_x^2tan^2\phi}),$$

and the right ellipse equation would be given by Eq. 3 while the axis ratio given by Eq. 4, where A and B are given by

$$A = \frac{\cos^2\psi_x D_x^2 tan^2\phi + \sin^2\psi_x + (a-\frac{b\alpha_x}{\beta_x})\sin 2\psi_x tan\phi}{A_x^2\frac{b^2}{\beta_x^2}tan^2\phi},$$

$$B = \frac{\sin^2\psi_x D_x^2 tan^2\phi + \cos^2\psi_x - (a-\frac{b\alpha_x}{\beta_x})\sin 2\psi_x tan\phi}{A_x^2\frac{b^2}{\beta_x^2}tan^2\phi}.$$

By the same token one can calculate the tilt angle for the vertical eigen motion as

$$\psi_y = -\frac{1}{2}tan^{-1}(\frac{2(d+\frac{b\alpha_y}{\beta_y})tan\phi}{1-D_y^2tan^2\phi}),$$

where $D_y = \sqrt{(d+\frac{b\alpha_y}{\beta_y})^2+\frac{b^2}{\beta_y^2}}$. The vertical eigen-mode coupling ellipse axis ratio (short axis over long axis) would be given by

$$(a'_x/a'_y) = \sqrt{\frac{\cos^2\psi_y D_y^2 tan^2\phi + \sin^2\psi_y + (d+\frac{b\alpha_y}{\beta_y})\sin 2\psi_y tan\phi}{\sin^2\psi_y D_y^2 tan^2\phi + \cos^2\psi_y - (d+\frac{b\alpha_y}{\beta_y})\sin 2\psi_y tan\phi}}.$$

Comparison of the LER eigen-mode oupling ellipses tilt angles between measurement and the ideal lattice model are shown in Figure 2. Figure 3 shows the corresponding comparison of the axis ratios. These linear coupling ellipses' tilt angles and axis ratios can also be used to check if a virtual machine [4] is reliable by comparing these coupling parameters to see if they match each other automatically as shown in Figure 5.

SUMMARY

From resonance excitations of the two betatron motions, one can obtain two eigen-mode coupling ellipses in the transverse coordinates (x,y) space by tracing turn-by-turn high-resolution orbits that are extracted with MIA from the BPM buffer data. One can then measure the tilt angle and the axis ratio for each of the coupling ellipse. Therefore, at each double-view BPM location, one can obtain 4 coupling parameters (two tilt angles and two axis ratios) that would describe the linear coupling completely. On the other hand, one can also calculate the corresponding parameters from the one-turn linear maps of the lattice model and so compare them with the measurement results to check the machine optics. Examples from PEP-II LER measurement on September 30, 2003 show that the PEP-II LER linear coupling pattern follows the lattice design as expected. Also, this high-resolution eigen-mode coupling ellipse measurement technique can be combined with the IP optics measurement [5] to improve the results.

Work supported by Department of Energy contract DE-AC02-76SF00515.

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