Methods for Computing Navigation Accuracy Category (NAC) for Traffic Information Service-Broadcast (TIS-B)

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Outline

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Motivation

- Aircraft Surveillance Applications (ASA)[1] require accuracy of reported position and velocity to determine if acceptable performance is met for intended use
- Horizontal position and velocity accuracies for Automatic Dependent Surveillance-Broadcast (ADS-B) and TIS-B are conveyed by single integers:
  - Navigation Accuracy Category for Position, NAC_p
  - Navigation Accuracy Category for Velocity, NAC_v

[1]Reference: Minimum Aviation System Performance Standards for Aircraft Surveillance Applications (ASA), RTCA Do-289

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Definitions -- NAC_P and NAC_V

- NAC_P[1]:
  - The Estimated Position Uncertainty (EPU) is the 95% accuracy bound on horizontal position
  - EPU is defined as the radius of a circle, centered on a reported position, such that the probability of the actual position being outside the circle is 0.05
  - Horizontal NAC_P is an index to EPU
    - Example: EPU = 1 nmi ⇔ NAC_P = 4

- NAC_V[1] is an index to the 95% accuracy of the least accurate rate component

[1]Reference: Minimum Aviation System Performance Standards for Aircraft Surveillance Applications (ASA), RTCA Do-289

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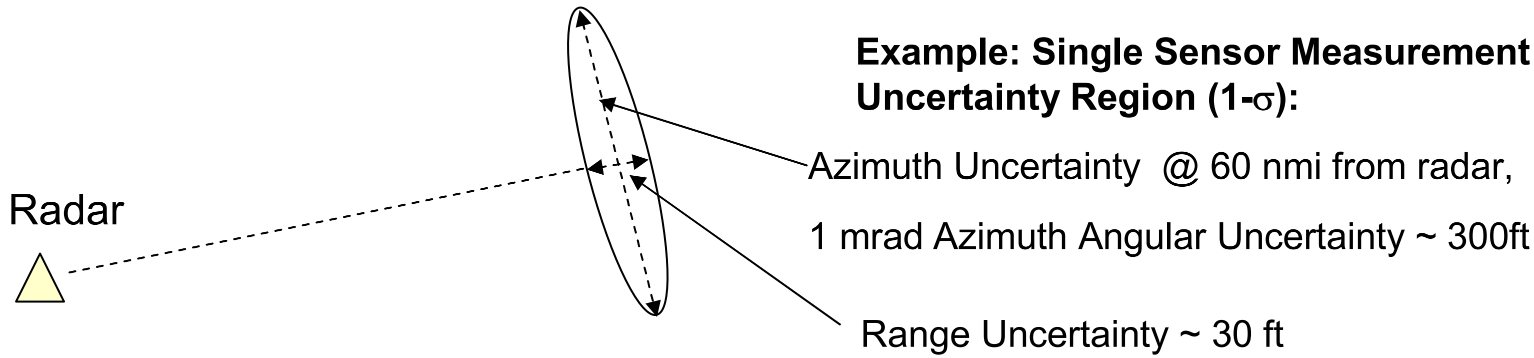
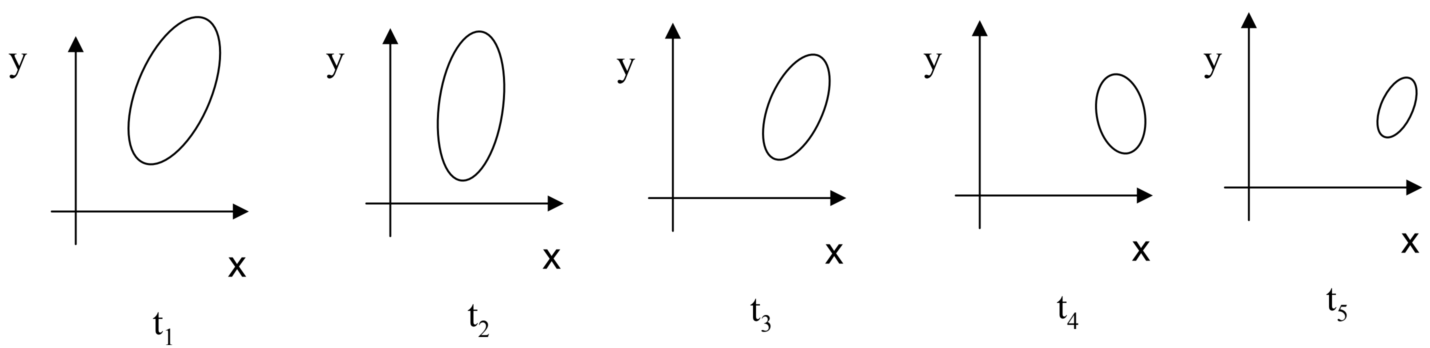
Characteristics of Uncertainty Region for TIS-B Reports

- **Uncertainty of Track Position and Rate derived from Radars are not *Circular* but *Elliptical* in the Horizontal Plane.**



1-σ Uncertainty Region of Tracked Position



- **Uncertainty Ellipse Orientation varies with Time and is Rotated with respect to the Reference Coordinate Frame.**

Uncertainty Characterized by Filtered State Covariance

- **Modern Trackers are based on Kalman Filter (e.g., SENSIS, STARS, MICROEARTS) and provide a measure of the error statistics through the covariance**
- **The covariance of a column vector** $\mathbf{E_k} = [\Delta x \; \Delta\dot{x} \; \Delta y \; \Delta\dot{y}]$

where *(Δx, Δy)* are the position and $(\Delta\dot{x}, \Delta\dot{y})$ **are the rate errors in the horizontal plane is given by**

$$
\begin{aligned}
\mathbf{P} &= Cov[\mathbf{E_k}] = E[\mathbf{E_k}\mathbf{E_k}^{\mathbf{T}}] \\
&= \begin{bmatrix} E[\Delta x \;\; \Delta x] & E[\Delta x \;\; \Delta\dot{x}] & E[\Delta x \;\; \Delta y] & E[\Delta x \;\; \Delta\dot{y}] \\ E[\Delta\dot{x} \;\; \Delta x] & E[\Delta\dot{x} \;\; \Delta\dot{x}] & E[\Delta\dot{x} \;\; \Delta y] & E[\Delta\dot{x} \;\; \Delta\dot{y}] \\ E[\Delta y \;\; \Delta x] & E[\Delta y \;\; \Delta\dot{x}] & E[\Delta y \;\; \Delta y] & E[\Delta y \;\; \Delta\dot{y}] \\ E[\Delta\dot{y} \;\; \Delta x] & E[\Delta\dot{y} \;\; \Delta\dot{x}] & E[\Delta\dot{y} \;\; \Delta y] & E[\Delta\dot{y} \;\; \Delta\dot{y}] \end{bmatrix} \\
&= \begin{bmatrix} \sigma_x^2 & \sigma_{x\dot{x}}^2 & \sigma_{xy}^2 & \sigma_{x\dot{y}}^2 \\ \sigma_{x\dot{x}}^2 & \sigma_{\dot{x}}^2 & \sigma_{\dot{x}y}^2 & \sigma_{\dot{x}\dot{y}}^2 \\ \sigma_{xy}^2 & \sigma_{\dot{x}y}^2 & \sigma_y^2 & \sigma_{y\dot{y}}^2 \\ \sigma_{x\dot{y}}^2 & \sigma_{\dot{x}\dot{y}}^2 & \sigma_{y\dot{y}}^2 & \sigma_{\dot{y}}^2 \end{bmatrix}
\end{aligned}
\tag{1}
$$

- **Diagonal elements are the variances, off-diagonal encode correlations**

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Decoupled Position and Velocity Covariance

- Covariance Matrix can be partitioned[1] into position and rate terms as a property of the multivariate Gaussian distribution of $\mathbf{E_k}$.

$$\mathbf{P} = Cov[\mathbf{E_k}] = E[\mathbf{E_k}\mathbf{E_k}^{\mathbf{T}}]$$

$$= \begin{bmatrix} \sigma_x^2 & \sigma_{x\dot{x}}^2 & \sigma_{xy}^2 & \sigma_{x\dot{y}}^2 \\ \sigma_{x\dot{x}}^2 & \sigma_{\dot{x}}^2 & \sigma_{\dot{x}y}^2 & \sigma_{\dot{x}\dot{y}}^2 \\ \sigma_{xy}^2 & \sigma_{\dot{x}y}^2 & \sigma_y^2 & \sigma_{y\dot{y}}^2 \\ \sigma_{x\dot{y}}^2 & \sigma_{\dot{x}\dot{y}}^2 & \sigma_{y\dot{y}}^2 & \sigma_{\dot{y}}^2 \end{bmatrix} \qquad (2)$$

$$\mathbf{P}_{pos} = Cov[\mathbf{E}_{pos}] = E[\mathbf{E}_{pos}\mathbf{E}^{\mathbf{T}}_{pos}]$$

$$= \begin{bmatrix} \sigma_x^2 & \sigma_{xy}^2 \\ \sigma_{xy}^2 & \sigma_y^2 \end{bmatrix} \qquad (3)$$

$$\mathbf{P}_{rate} = Cov[\mathbf{E}_{rate}] = E[\mathbf{E}_{rate}\mathbf{E}^{\mathbf{T}}_{rate}]$$

$$= \begin{bmatrix} \sigma_{\dot{x}}^2 & \sigma_{\dot{x}\dot{y}}^2 \\ \sigma_{\dot{x}\dot{y}}^2 & \sigma_{\dot{y}}^2 \end{bmatrix} \qquad (4)$$

- Calculation of $\mathrm{NAC_p}$ and $\mathrm{NAC_v}$ are decoupled.
- Remainder of talk is focused on algorithmic development of $\mathrm{NAC_p}$.

[1]Reference: B. Nobel, J.W. Daniel, Applied Linear Algebra, Prentice Hall, Nov. 1987

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Derivation of 1-σ Uncertainty Ellipse from Covariance

- **Bivariate Gaussian density of $\mathbf{E}_{pos}$ is:**

$$f_{\mathbf{E}_{pos}}(\mathbf{e}_{pos}) = \frac{\left(\det \mathbf{P}_{pos}\right)^{-1}}{2\pi} \exp\left(-1/2\left(\mathbf{e}_{pos}{}^{T}\mathbf{P}_{pos}{}^{-1}\mathbf{e}_{pos}\right)\right) \qquad (5)$$

- **Contours of $f_{\mathrm{E}}(\mathrm{e}_{pos})$ *described by:***

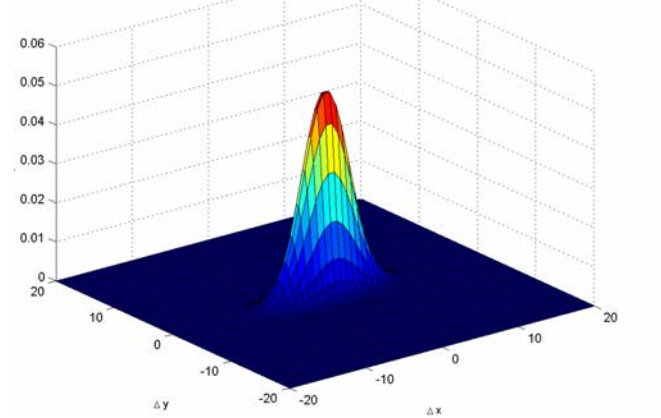
$$\begin{aligned} C &= \mathbf{e}_{pos}^{T}\mathbf{P}_{pos}^{-1}\mathbf{e}_{pos} \\ &= \begin{bmatrix}\Delta x & \Delta y\end{bmatrix}\begin{bmatrix}\sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2\end{bmatrix}^{-1}\begin{bmatrix}\Delta x \\ \Delta y\end{bmatrix} \\ &= \frac{1}{1-\rho^2}\left(\frac{\Delta x^2}{\sigma_x^2} - 2\rho\frac{\Delta x \Delta y}{\sigma_x \sigma_y} + \frac{\Delta y^2}{\sigma_y^2}\right) \quad \textit{where} \quad \rho = \frac{\sigma_{xy}^2}{\sigma_x \sigma_y} \end{aligned} \qquad (6)$$

- **This is an ellipse that is rotated by an angle θ.**

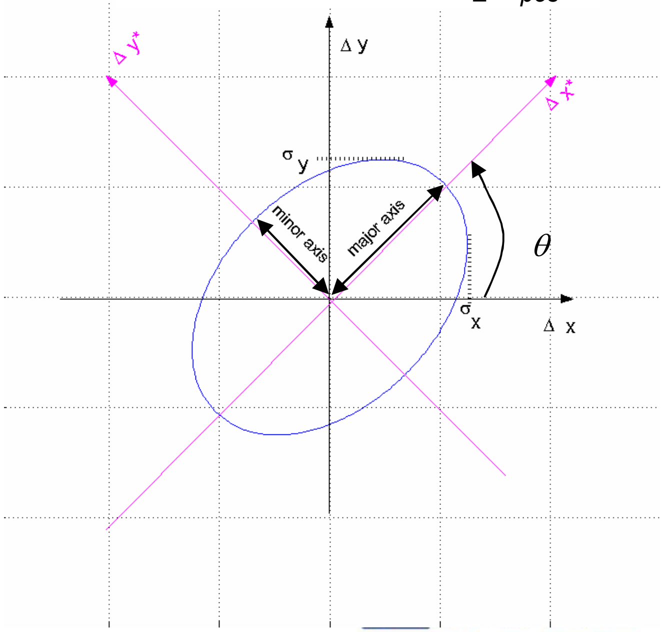
$$\theta = \frac{1}{2}\arctan\left(\frac{2\rho\sigma_x\sigma_y}{\sigma_x^2 - \sigma_y^2}\right) \qquad (7)$$

- **In rotated coordinate frame components of uncertainty are independent.**

Gaussian Density Function



1-σ Contour Plot of $f_{\mathbf{E}}(\mathbf{e}_{pos})$



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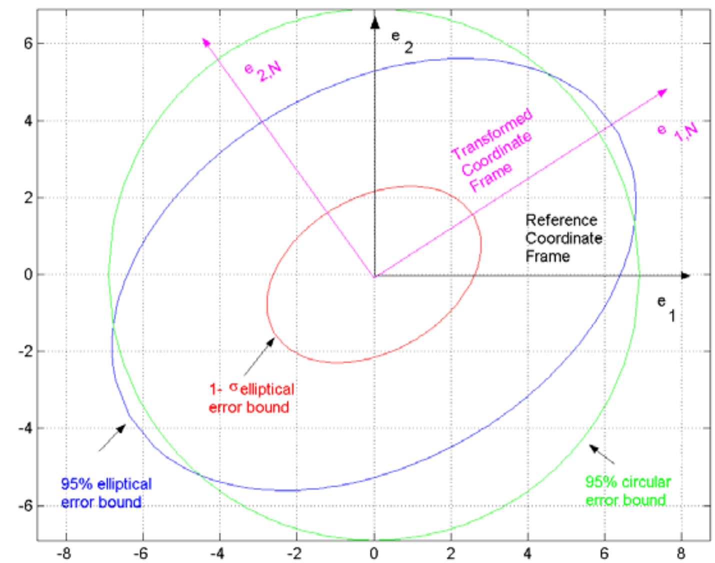
Derivation of 95% Circular Uncertainty (NAC) from 1-σ Uncertainty Ellipse

- **Covariance provides 1-σ Uncertainty**
  - **Desired to find simple scaling of 1-σ major axis to find NAC**
- **95% Elliptical Error Bound**
  - **Requires simple 2.4477 scaling of 1-σ ellipse**
  - **Scaling is independent of eccentricity of ellipse**
- **95% *Circular* Uncertainty found by two-dimensional integration of $f_E(e_{pos})$ over a circular domain**
  - **No closed form solution**
  - **Real-time numerical integration not practical**
  - **No single scaling of 1-σ major axis works for all eccentricities**
  - **Two methods explored:**
    - **Exact**
      - **Offline Numerical Integration to calculate scalar factor of 1-σ major axis parameterized by eccentricity**
      - **Calculate eccentricity via eigenvalues of filtered covariance.**
    - **Bounded**
      - **Fixed scale factor to derive NAC from 1-σ uncertainty**

Description of the Three Error Bounds



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Exact Method

- Calculate the scaling factor as a function of the ratio of major-to-minor axes
- The axes of the ellipse are computed from the eigenvalues of the partitioned Covariance matrix. The eigenvalues, λ_1 and λ_2, of a symmetric 2x2 matrix

$$P = \begin{bmatrix} \sigma_x^2 & \sigma_{xy}^2 \\ \sigma_{xy}^2 & \sigma_y^2 \end{bmatrix}$$

are given by the equation

$$\lambda_{1,2} = \frac{\left(\sigma_x^2 + \sigma_y^2\right) \pm \sqrt{\left(\sigma_x^2 - \sigma_y^2\right)^2 + 4\sigma_{xy}^4}}{2} \qquad (8)$$

$$\text{"1-}\sigma\text{" axis}_{\text{major}} = \max(+\sqrt{\lambda_1}, +\sqrt{\lambda_2}) \qquad (9)$$

$$\text{"1-}\sigma\text{" axis}_{\text{minor}} = \min(+\sqrt{\lambda_1}, +\sqrt{\lambda_2}) \qquad (10)$$

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Exact Method (cont'd)

- **Equation of 95% Uncertainty Region (NAC):**

$$0.95 = \iint_{\{x,y:x^2+y^2<radius\}} \frac{1}{2\pi\sigma_x\sigma_y} e^{-\frac{1}{2(1-\rho^2)}\left(\frac{x^2}{2\sigma_x^2} - 2\rho\frac{xy}{\sigma_x\sigma_y} - \frac{y^2}{2\sigma_y^2}\right)} dxdy \qquad (11)$$

- **Applying change of basis (rotation of covariance) and change of variables:**

$$0.95 = \int_0^{2\pi} \int_{r<\frac{k\sqrt{\lambda_{major}/\lambda_{minor}}}{\sqrt{\cos^2\theta + \sqrt{\lambda_{major}/\lambda_{minor}}\sin^2\theta}}} \frac{1}{2\pi} e^{-\left(\frac{r^2}{2}\right)} r dr d\theta \qquad (12)$$

- **Containment radius = $k^*(\lambda_{major})^{1/2}$**
- **Build solution table offline**
  - **Solve for k**
  - **Parameterized by $\sqrt{\lambda_{major}/\lambda_{minor}}$**

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Exact Method - Performance

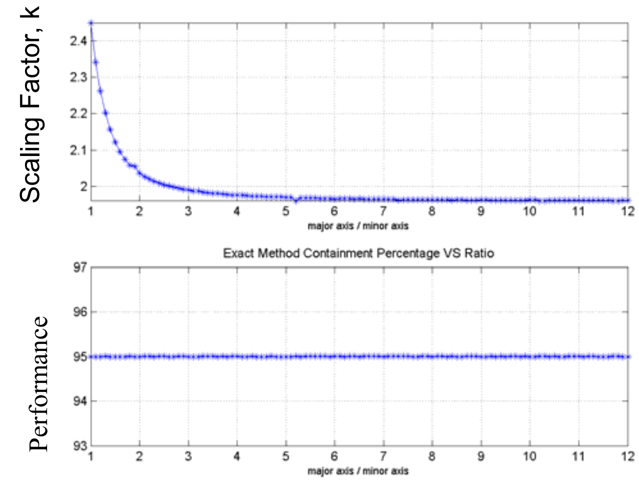
- **Variable scaling factor, *k*, computed via numerical integration for a constant 95% containment region.**
  - **Range of values for *k*: 2.4477 to 1.9625**

- **The scaling factors can be:**
  - **Interpolated between exact solutions in a look-up table.**
  - **Approximated by the following expression**

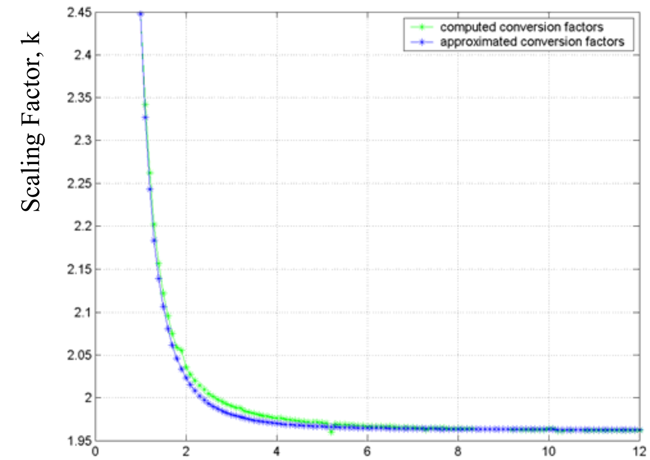
$$k = \frac{(2.4477 - 1.9625)}{ratio^3} + 1.9625 = \frac{.4852}{ratio^3} + 1.9625$$

$$\text{where } ratio = \frac{\text{"}1-\sigma\text{"}\, axis_{\text{major}}}{\text{"}1-\sigma\text{"}\, axis_{\text{minor}}} = \frac{\max(+\sqrt{\lambda_1}, +\sqrt{\lambda_2})}{\min(+\sqrt{\lambda_1}, +\sqrt{\lambda_2})} \qquad (13)$$

Scaling Factor and Performance of Exact Method



Approximation of Scaling Factor



Ratio of Major-to-Minor Axis

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Bounded Method: Root-Sum-Squared (RSS)

- **The RSS method:**
  - **Containment radius = RSS_k defined by:**

$$RSS_k = \sqrt{(k * \sigma_x)^2 + (k * \sigma_y)^2} = k\sqrt{\sigma_x^2 + \sigma_y^2} \qquad (14)$$

  - **Uses the components from the covariance matrix. Equivalent to root-sum-squared of the square roots of the eigenvalues**

$$RSS_k = \sqrt{\left(k * \sqrt{\lambda_1}\right)^2 + \left(k * \sqrt{\lambda_2}\right)^2} = k\sqrt{\lambda_1 + \lambda_2} \qquad (15)$$

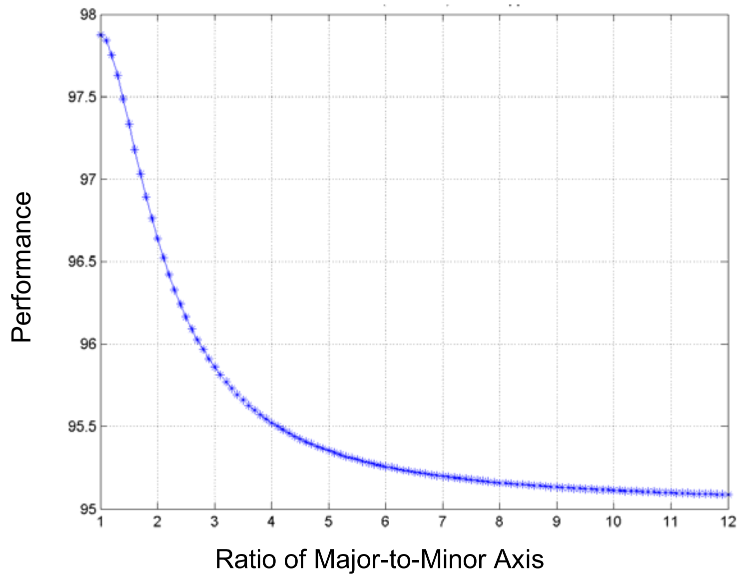
  - **Converges to exact solution for highly elongated ellipse, $\lambda_1 >> \lambda_2$**

- **The containment region derived from the RSS radius will vary depending on the eccentricity of the error ellipse because it uses a fixed scaling factor**

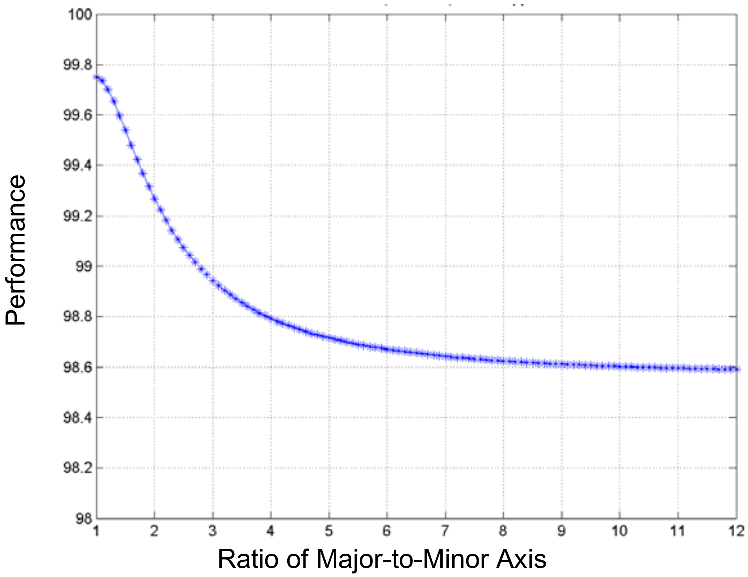
Performance of RSS Method

- **Scaling extrema from Exact Method: Circular = 2.4477 and Elongated Ellipse = 1.9625**

Performance of RSS using Scaling Factor = 1.9625



Performance of RSS using Scaling Factor = 2.4477



  - **For scaling factor = 1.9625 converges to 95% as ratio increases, but exceeds the 95% threshold by 3% at ratio = 1**
  - **For scaling factor = 2.4477 the containment region only approaches 98.6% and exceeds the threshold by 5% when ratio = 1.**

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Algorithm Summary for Computing NACp using Exact Method – with an Example

1. Partition the covariance matrix into position-only and rate-only submatrices.

$$P = \begin{bmatrix} 4.84e+5 & d & -3.176e+5 & d \\ d & 7.657e+2 & d & -1.452e+2 \\ -3.176e+5 & d & 6.577e+5 & d \\ d & -1.452e+2 & d & 8.877e+2 \end{bmatrix} \Rightarrow \mathbf{P}_{pos} = \begin{bmatrix} 4.84e+5 & -3.176e+5 \\ -3.176e+5 & 6.577e+5 \end{bmatrix}$$

$$\mathbf{P}_{rate} = \begin{bmatrix} 7.657e+2 & -1.452e+2 \\ -1.452e+2 & 8.877e+2 \end{bmatrix}$$

(d = don't care)

2. Find the eigenvalues of each submatrix (Equation 8). Here shown for position-only.

$$\lambda_{1,2} = \frac{(4.84e+5+6.577e+5) \pm \sqrt{(4.84e+5-6.577e+5)^2 + 4(-3.176e+5)^2}}{2} \qquad \lambda_1 = 9.0011e+5; \quad \lambda_2 = 2.4159e+5$$

$$\text{major axis} = \sqrt{\lambda_1} = 958.7417 \text{ feet; minor axis} = \sqrt{\lambda_2} = 491.5172 \text{ feet}$$

3. Compute the real time ratio to approximate the scaling factor from Equation (11).

ratio = 9.58.7417 / 491.5172 = 1.95

$$k = \frac{0.4852}{1.95^3} + 1.9625 = 2.0279$$

4. Compute Radius of 95% Containment:

radius = k major axis = 2.0458*958.7417 [feet] = 1.96e+3 [feet] = 0.323 nmi.*

5. From Table 3-6 in ASA MASPS[1] this corresponds to $NAC_P = 6$.

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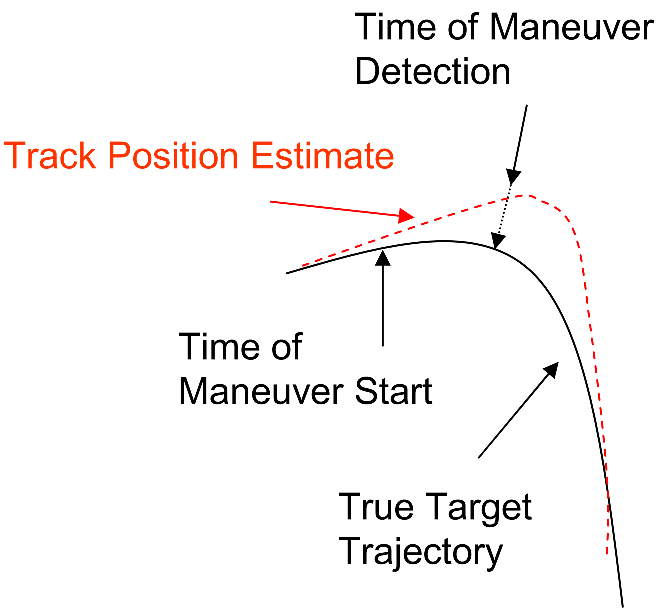
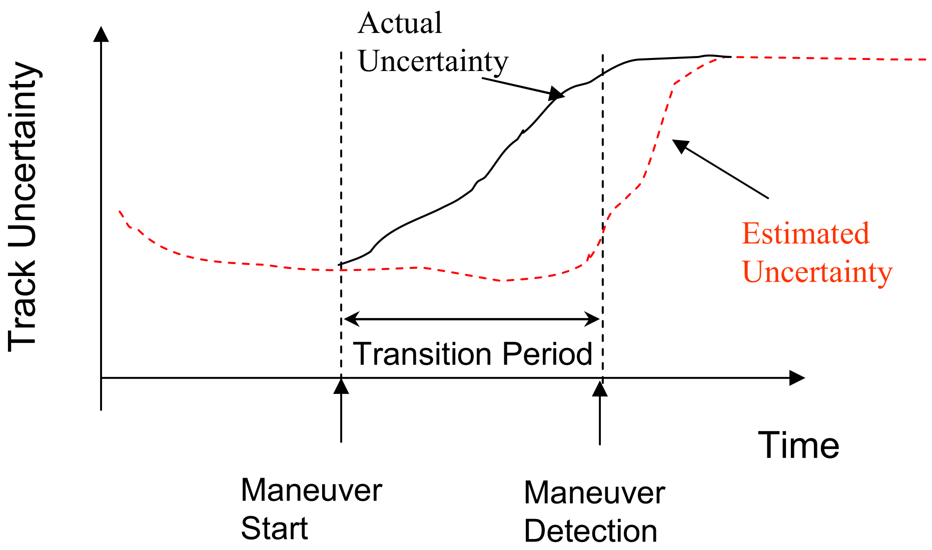
Conclusion

- **The Exact Method:**
  - **Outperforms the Bounded Method**
  - **Maintains a stable 95% containment region during straight line phase of trajectory**
  - **Computationally feasible**

Future Work





- **The estimated covariance matrix underestimates the true uncertainty during the transition period from straight line to maneuver trajectory.**
- **Characterize the uncertainty during the transition phase**
  - **Depends on:**
    - **Tracker Type (e.g., using simple Maneuver Detection or Interactive Multiple Model (IMM) to account for maneuvers)**
    - **Sensor type (i.e., measurement accuracy, update rate)**
    - **Target-to-Sensor Geometry**
    - **Number of Sensors**
    - **Type of maneuver (e.g., maximum acceleration)**