Self-consistent, unbiased rms-emittance estimates for data measured with a single current amplifier

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Motivation

In the past rms-emittances have been quoted as single numbers without any associated uncertainty estimate, because the usual analysis methods gave a wide range of answers depending on the analysis method.

This presentation will show that a self-consistent, unbiased elliptical exclusion (SCUBEEx) analysis can provide unbiased estimates of rms-emittances and their uncertainty. The uncertainty accounts only for variations and inconsistencies in the measured background and does not account for any instrumentation deficiencies.

Reliable estimates of rms-emittances and their uncertainty allow to

- compare focusability and transportability of ion beams on an absolute and reliable scale
- perform simulations with more realistic ion beam data

RMS-Emittance Definitions

For current c(x,x') measured with position- and velocity-coordinates x, x':

The terms:

$$\left\langle x^2 \right\rangle = \frac{\sum_{all} x^2 c(x, x')}{\sum_{all} c(x, x')}$$

$$\left\langle x'^2 \right\rangle = \frac{\sum_{all} x'^2 c(x, x')}{\sum_{all} c(x, x')}$$

$$\left\langle xx' \right\rangle = \frac{\sum_{all} xx' c(x, x')}{\sum_{all} c(x, x')}$$

The Twiss parameters:

$$a = -\frac{\left\langle xx' \right\rangle}{e}$$

$$b = -\frac{\left\langle x^2 \right\rangle}{e}$$

$$g = -\frac{\left\langle x'^2 \right\rangle}{e}$$

$$e = \sqrt{\left\langle x'^2 \right\rangle \left\langle x^2 \right\rangle - \left\langle xx' \right\rangle^2}$$

Current Measurement Issues

- In the absence of real current, a perfectly zeroed current amplifier produces an equal amount of positive and negative readings in a random sequence. Being weighted with the current, the contributions of these “zero” readings to the rms-emittance cancel each other. The emittance estimates remain unbiased.
- However, it is impractical to perfectly zero amplifiers, and therefore a small bias is common. A positive amplifier bias leads to overestimating the rms-emittance, a negative amplifier bias leads to underestimating the rms-emittance.
- Even a small current amplifier bias can significantly affect the rms-emittance estimates because
  - the small bias current values are multiplied with a large range of x and x’, including the highest possible values, and
  - the measured data are normally dominated by “zero” measurements.

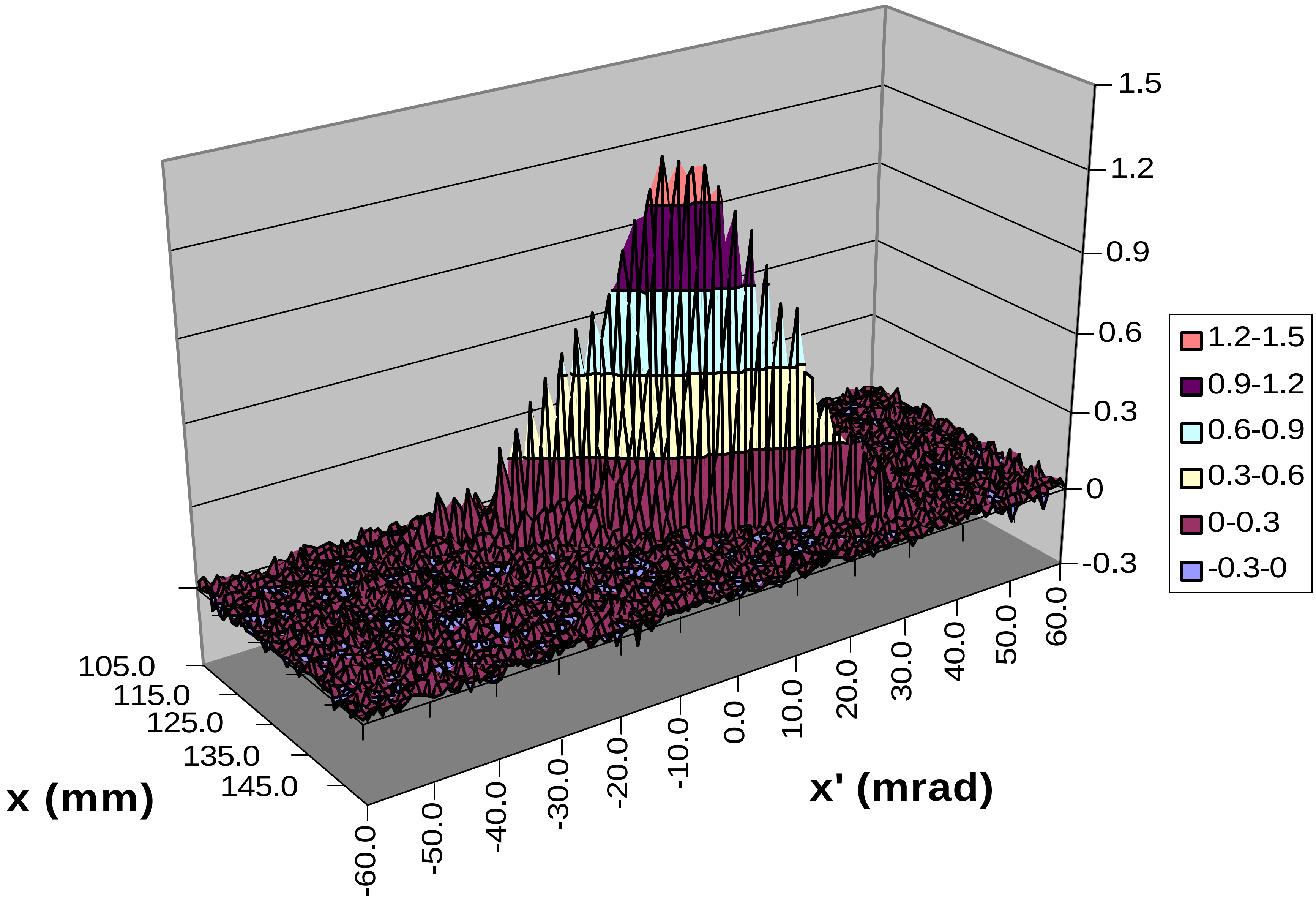
Current Measurement Precautions

- To minimize current measurement problems one has to
  - maximize the measured current signal and
  - minimize the measured noise signal and
  - perform zero current emittance measurements and
  - zero the current amplifier as good as practically possible.
- However, to obtain a good resolution, one tends to measure a very small fraction of the beam, and therefore it is likely to become noticeable that
  - current amplifiers are not perfectly stable but tend to drift, and that
  - the background with beam often differs from the background measured without beam.
- Therefore emittance estimates frequently employ appropriate data filtering methods.

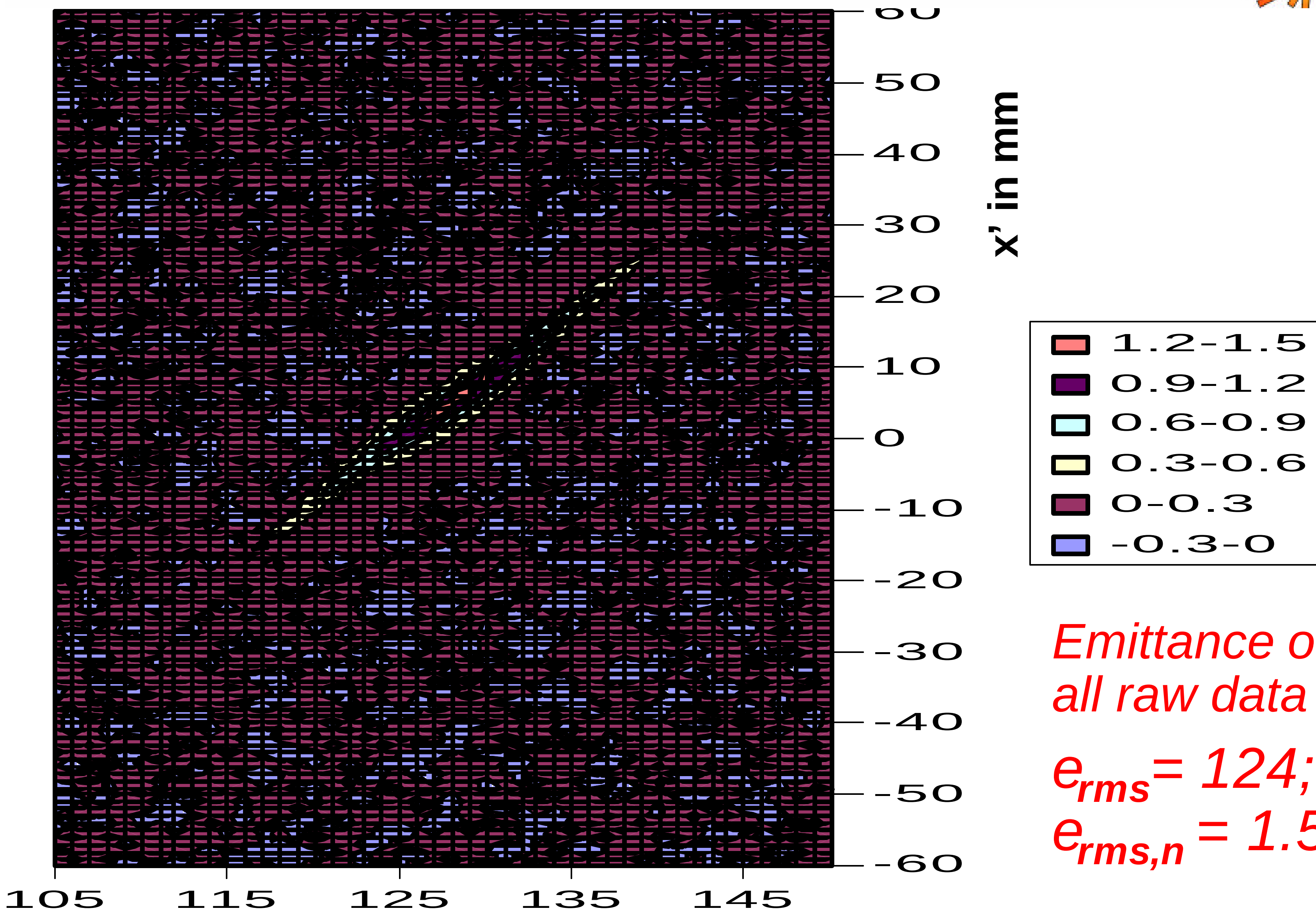
Common Data Filtering Methods

- The effect of bias and other spurious problems on rms-emittance estimates can be reduced or eliminated by
  - restricting the analysis to the core of the beam (90% emittance, Gaussian analysis [$e = f\{\ln(1/(1-f))\}$], etc.)
  - "Background" subtraction, sometimes accompanied by negative number clipping, to eliminate all "detached islands".
  - thresholding where every current value below a certain threshold is ignored or set to zero. The threshold can be established with a histogram, a threshold analysis, or experience.
  - exclusion where every current value outside a certain boundary in the x-x' plane is ignored or set to zero. Boundaries can be elliptical, trapezoidal, or custom-trimmed (cosmetic surgery).
- All methods above tend to underestimate the emittance because they aim at excluding low-intensity currents and hence are likely to exclude some low-intensity real-currents found in the beam halo.
- Unlike cosmetic criteria, statistical criteria can separate the real current from the background in a self-consistent manner.
  - Such a method, SCUBEEx, will be demonstrated with emittance data from an expanding ion beam and the LBNL beam.

Expanding beam emittance data (EBED)



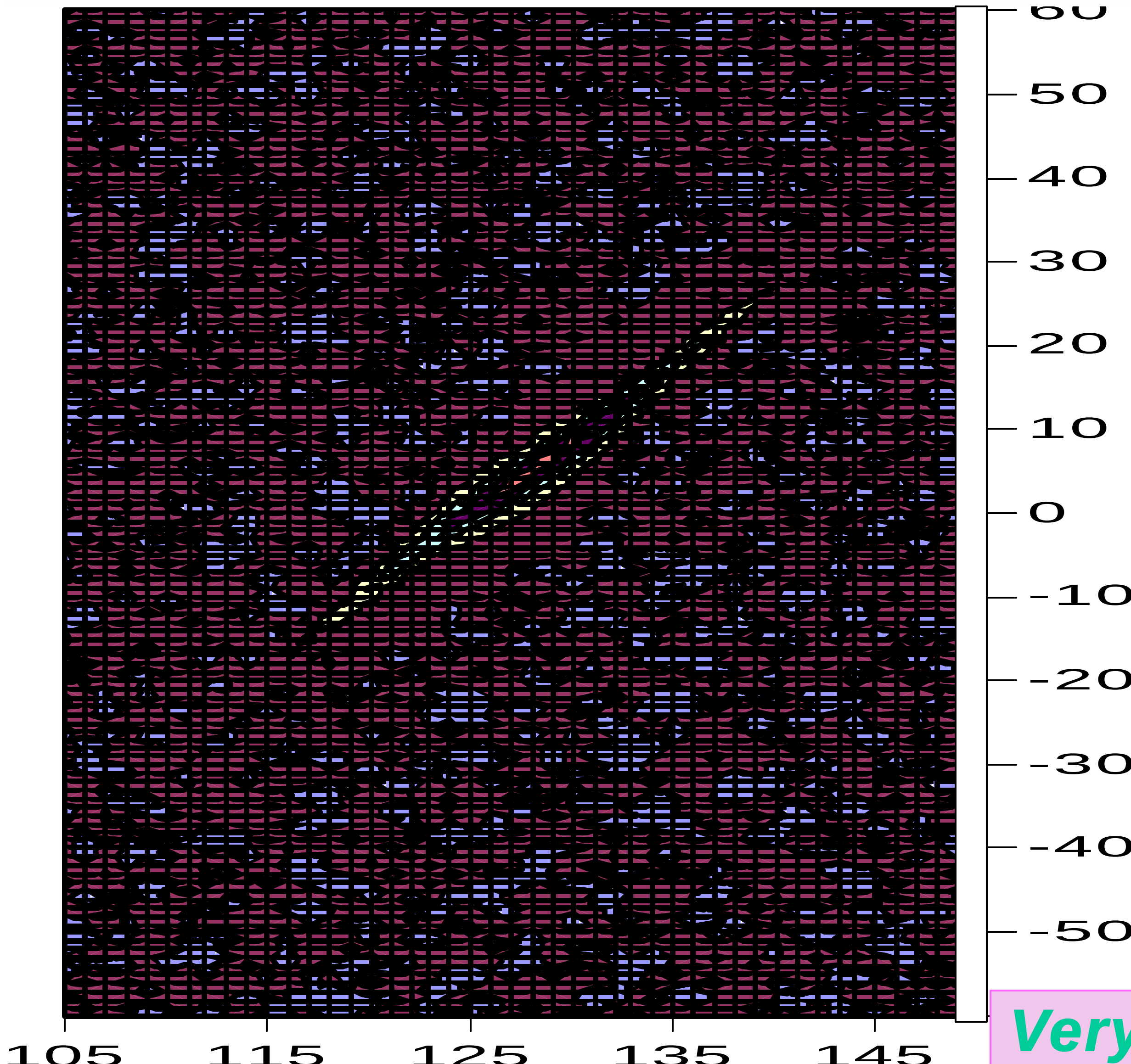
Expanding beam emittance data (EBED)



Emittance of all raw data

e_{rms} = 124;
$e_{rms,n}$ = 1.53

edited Expanding beam emittance data (eEBED)



Eliminating most right data column out of 46 columns reduces rms-emittance of raw data by 17%

- 1.2-1.5
- 0.9-1.2
- 0.6-0.9
- 0.3-0.6
- 0-0.3
- -0.3-0

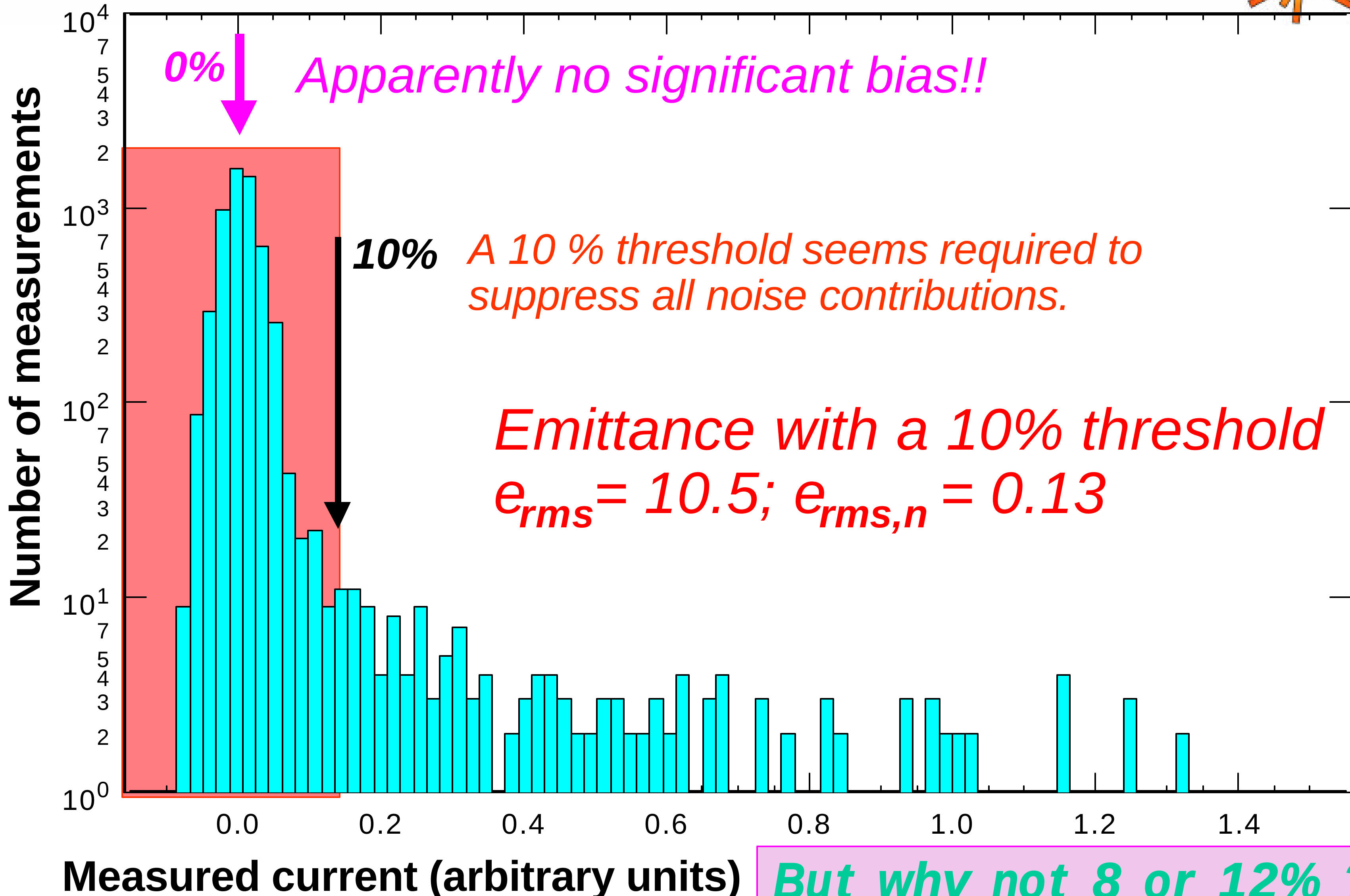
Emittance of all raw data

e_{rms}= 102;

$e_{rms,n}$ = 1.27

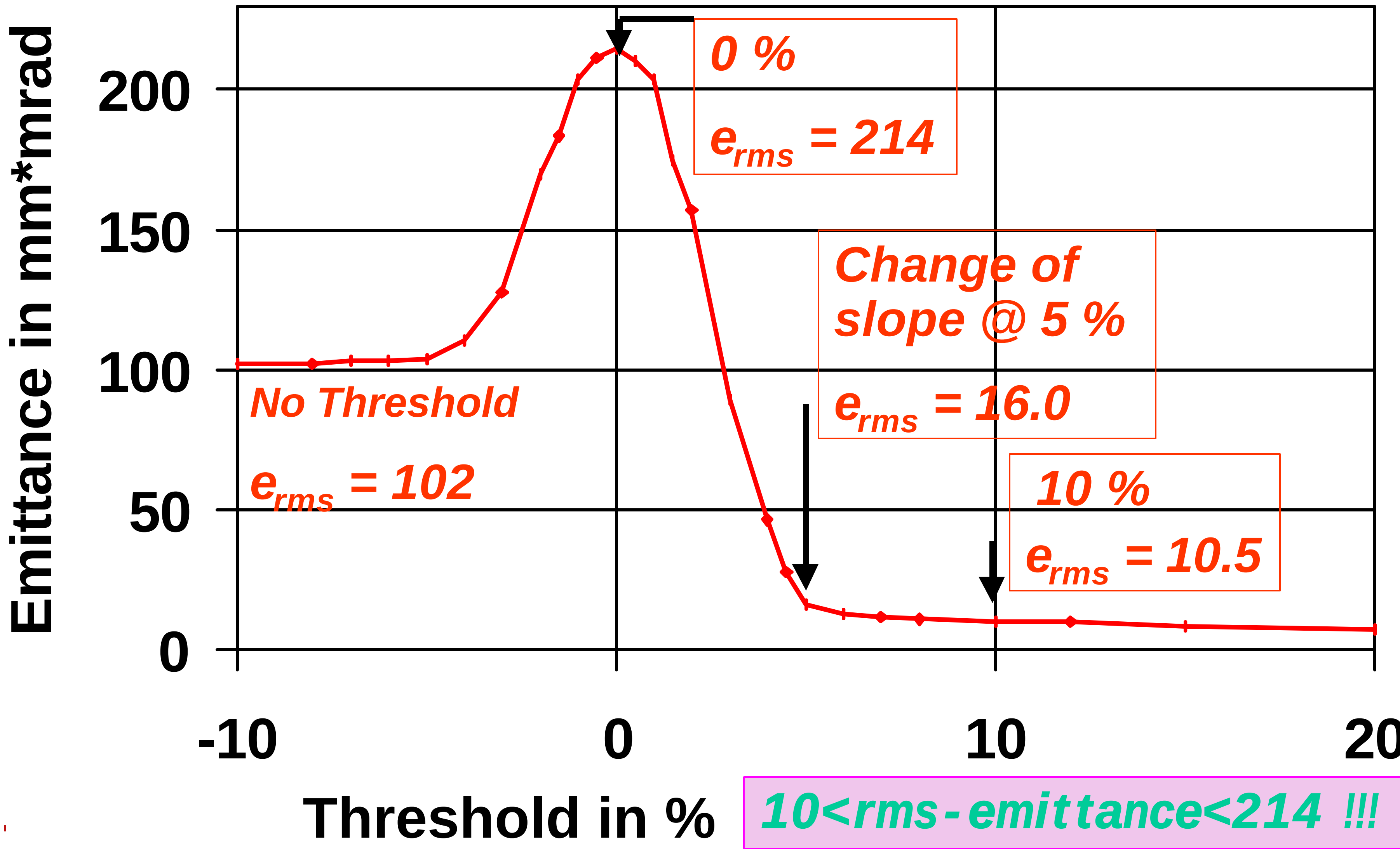
Very sensitive to noise!

Histogram Analysis of EBED



But why not 8 or 12% ??

Threshold analysis of eEBED



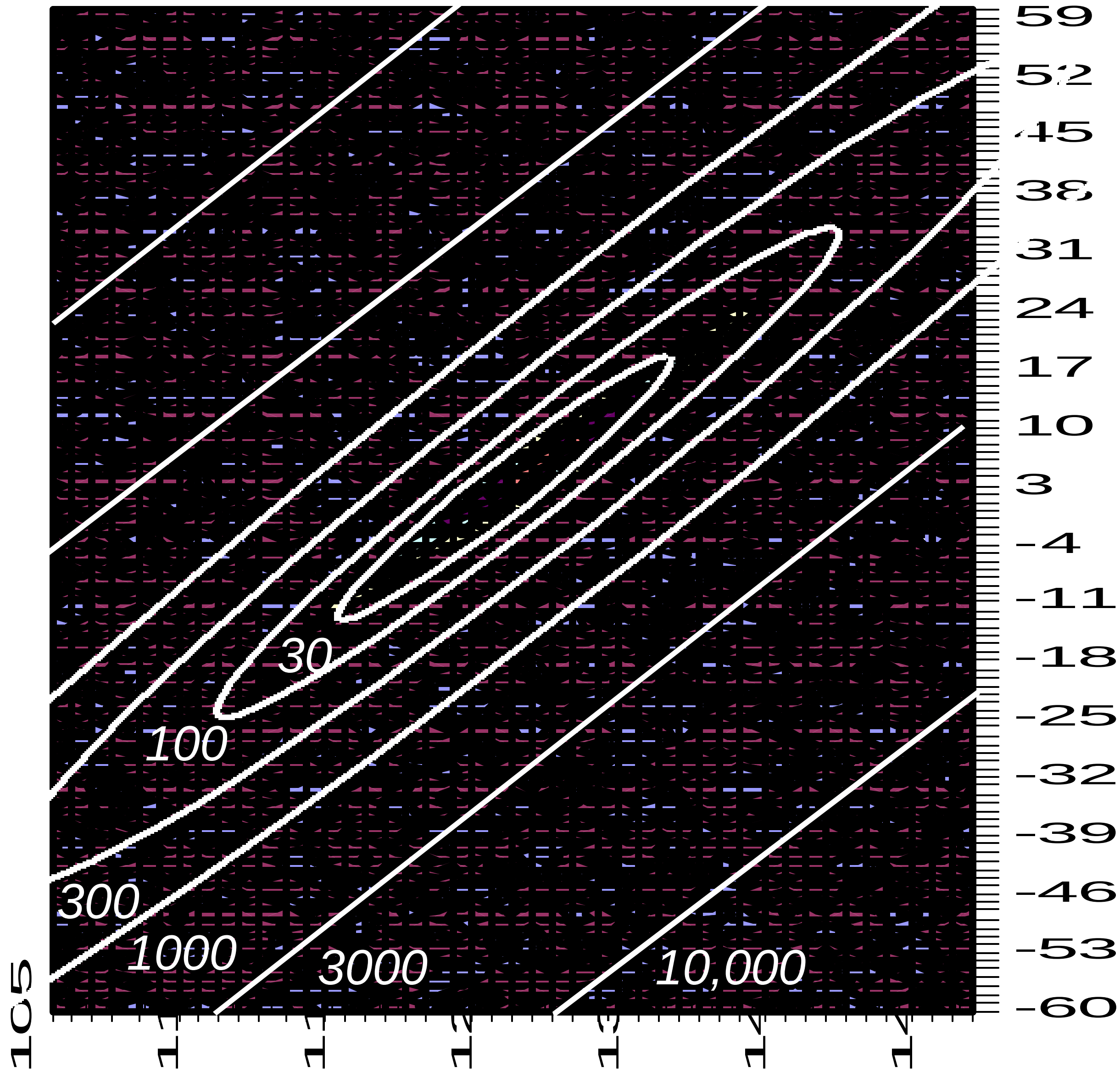
10<rms-emittance<214 !!!

Self-consistent elliptical exclusion (SCEEx) analysis

Hypothesis:

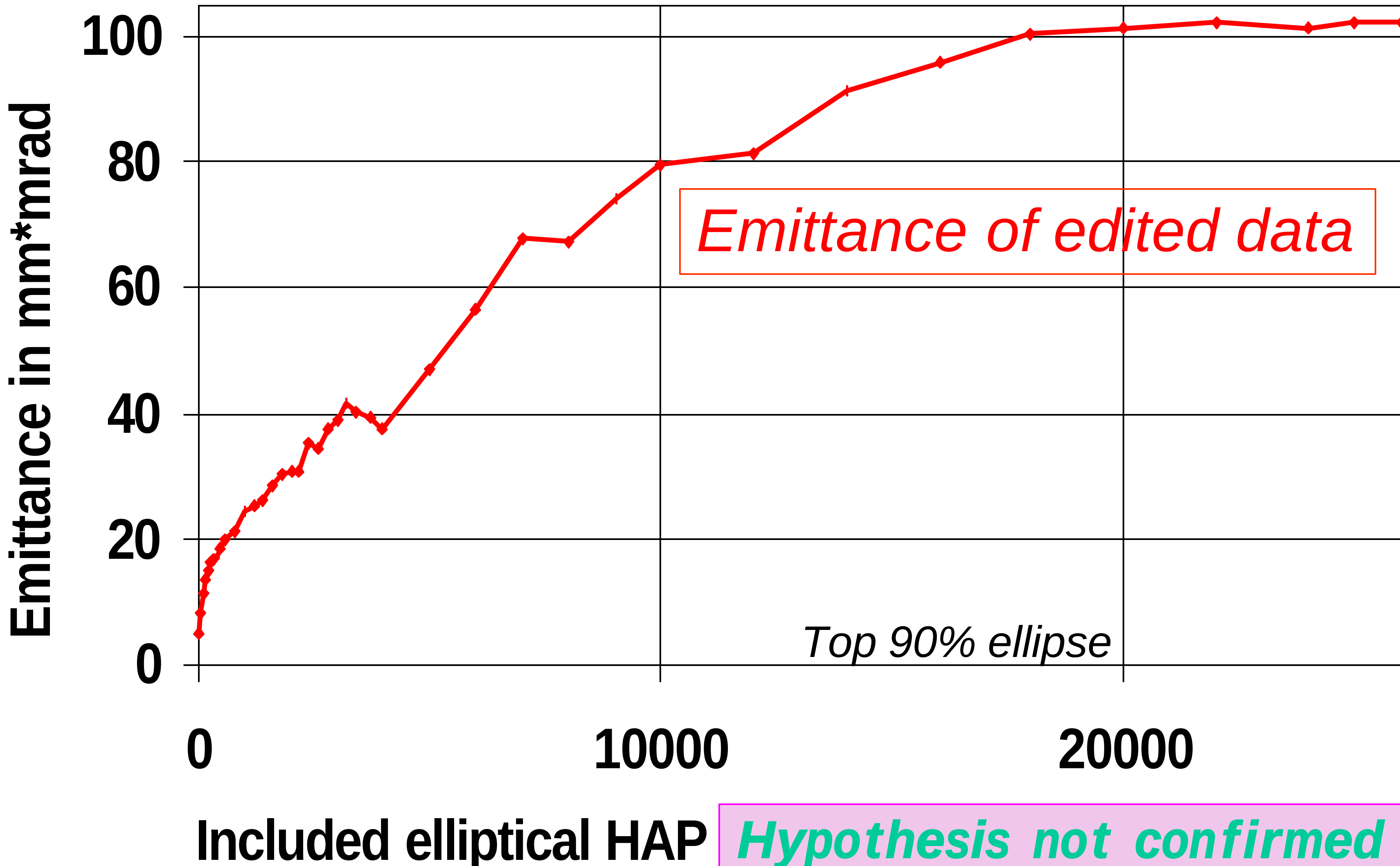
- all real current measurements are within an ellipse of a size to be determined in the analysis.
- all measurements outside the ellipse are background which should not contribute to the rms-emittance.
- the hypothesis is confirmed if the size of the ellipse can be varied over a wide range without significantly changing the resulting emittance.
- if the hypothesis is confirmed, the evaluated emittance is unbiased as all real current was included.

Elliptical exclusion analysis of eEBED



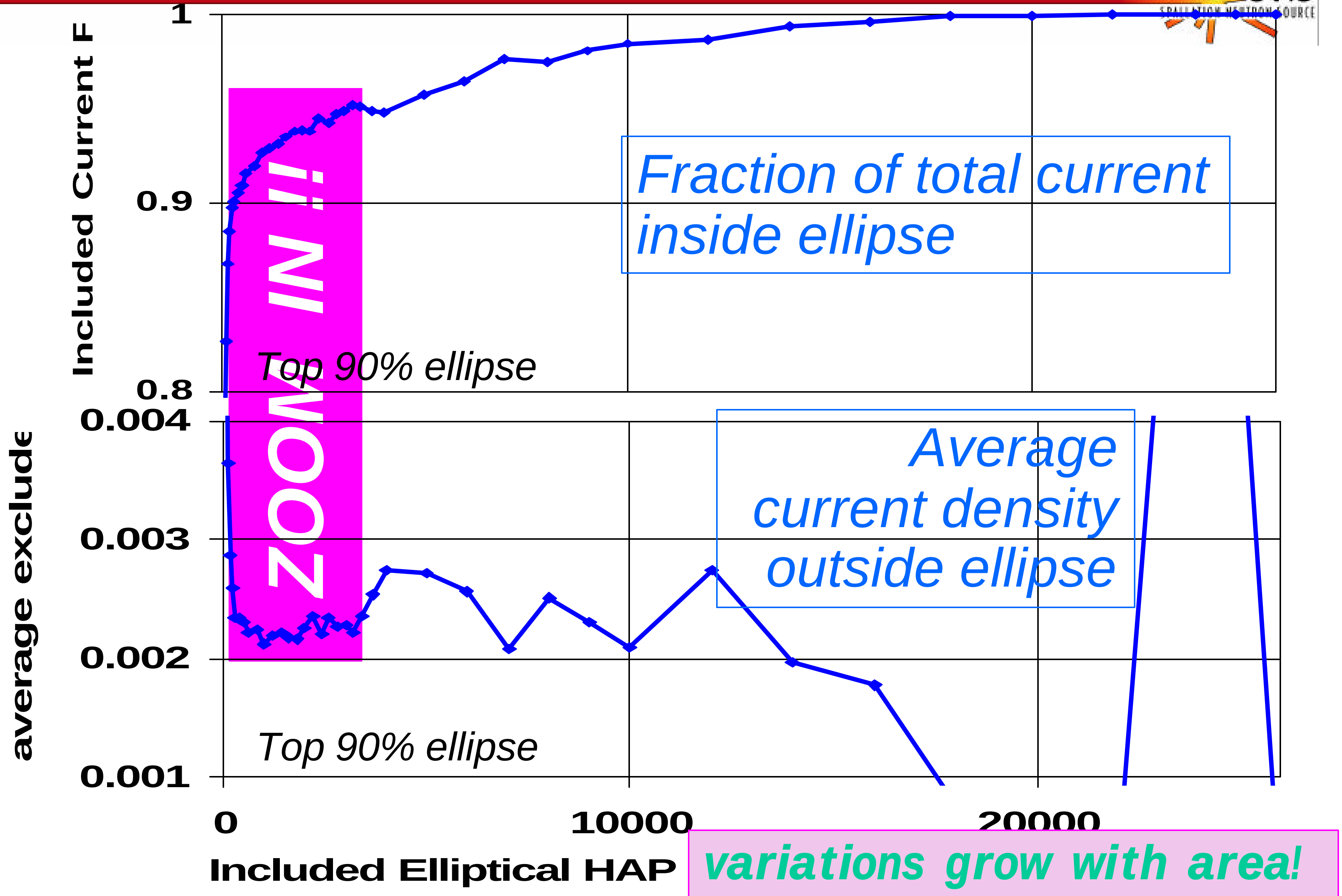
The size of the ellipses is characterized by HAP, the product of their half-axes in mm-mrad.

Elliptical exclusion analysis of eEBED

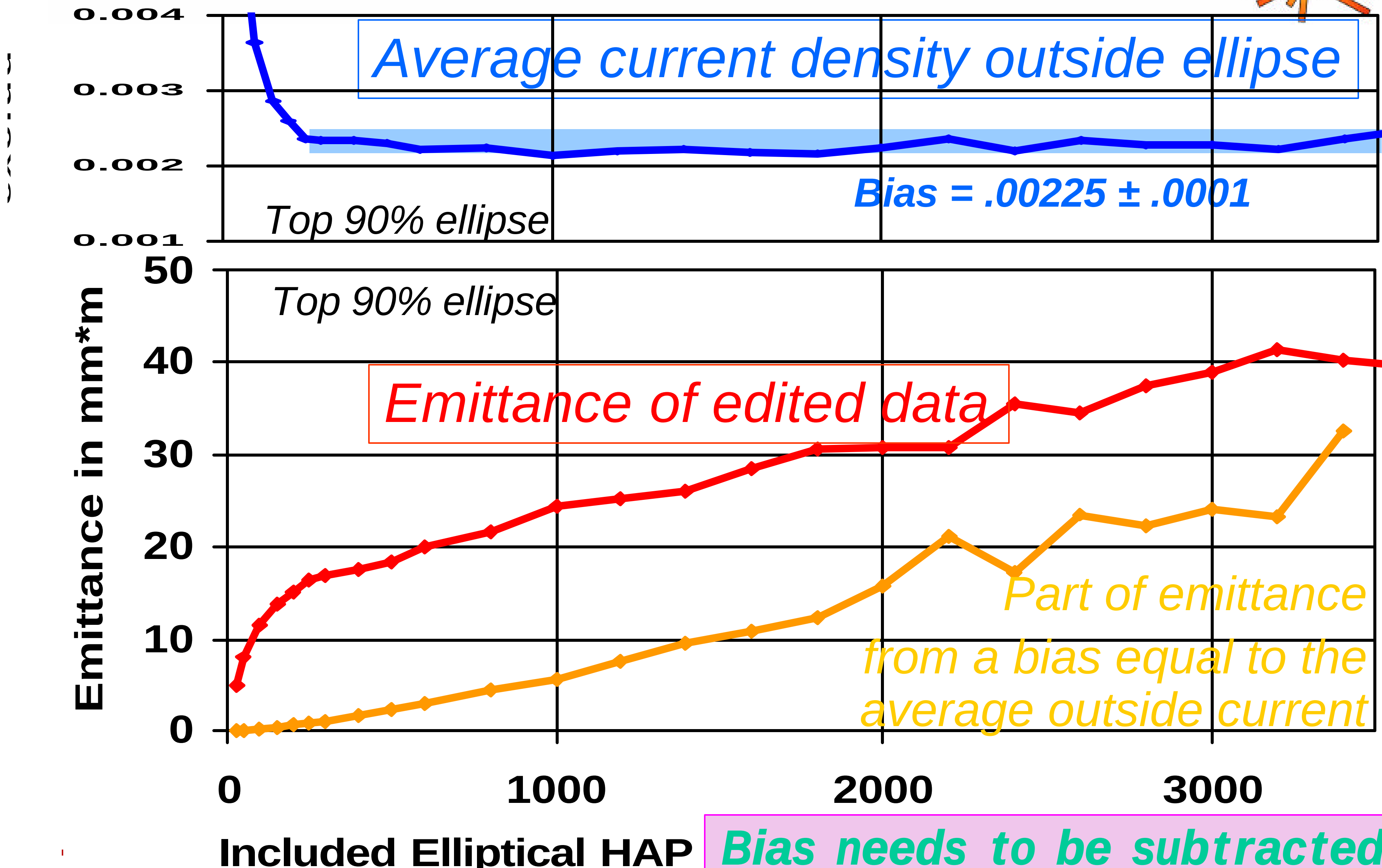


Hypothesis not confirmed !!

Elliptical exclusion analysis of eEBED

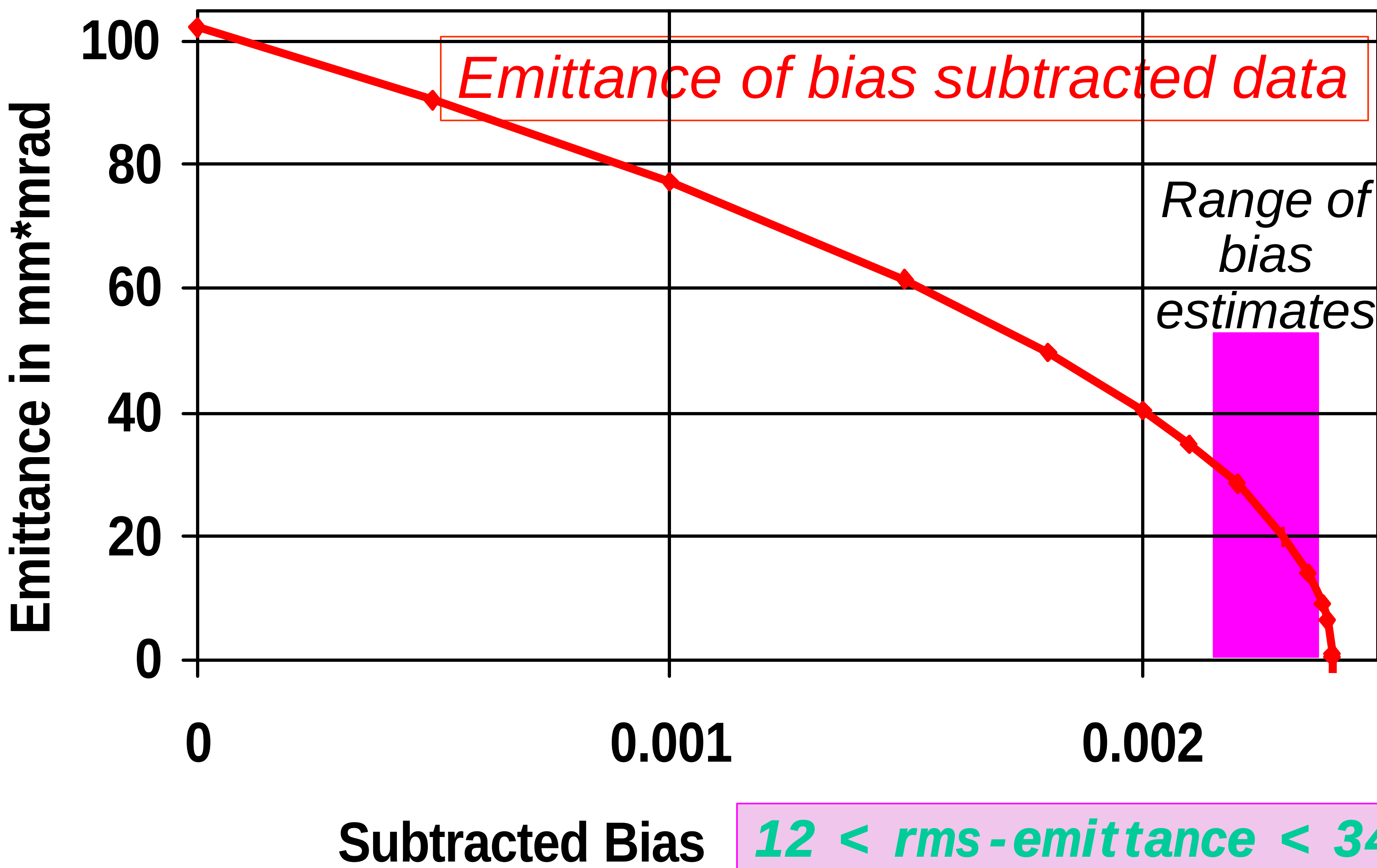


Elliptical exclusion analysis of eEBED



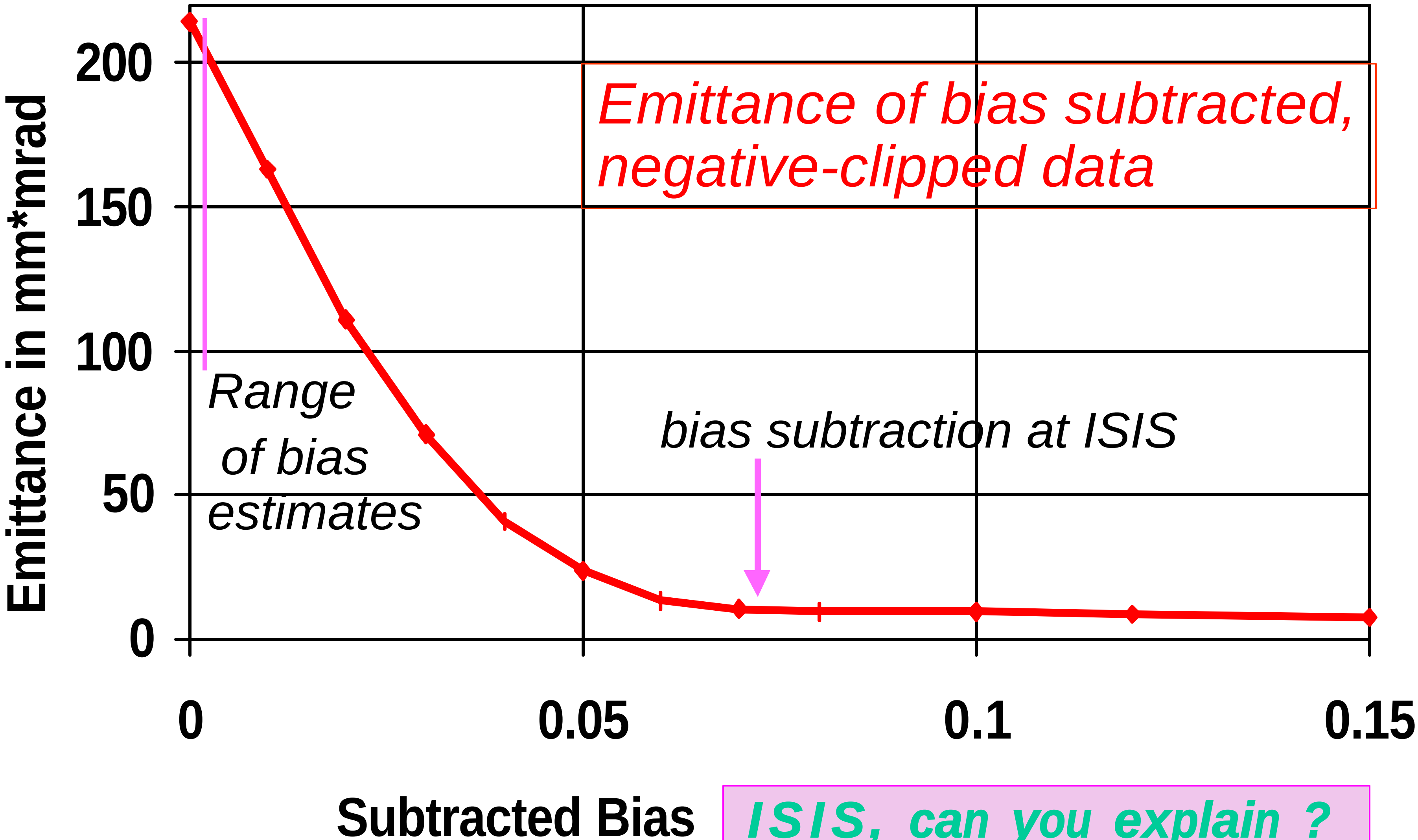
Bias needs to be subtracted !

Bias subtraction analysis of eEBED



12 < rms-emittance < 34

Bias subtraction with negative clipping of eEBED



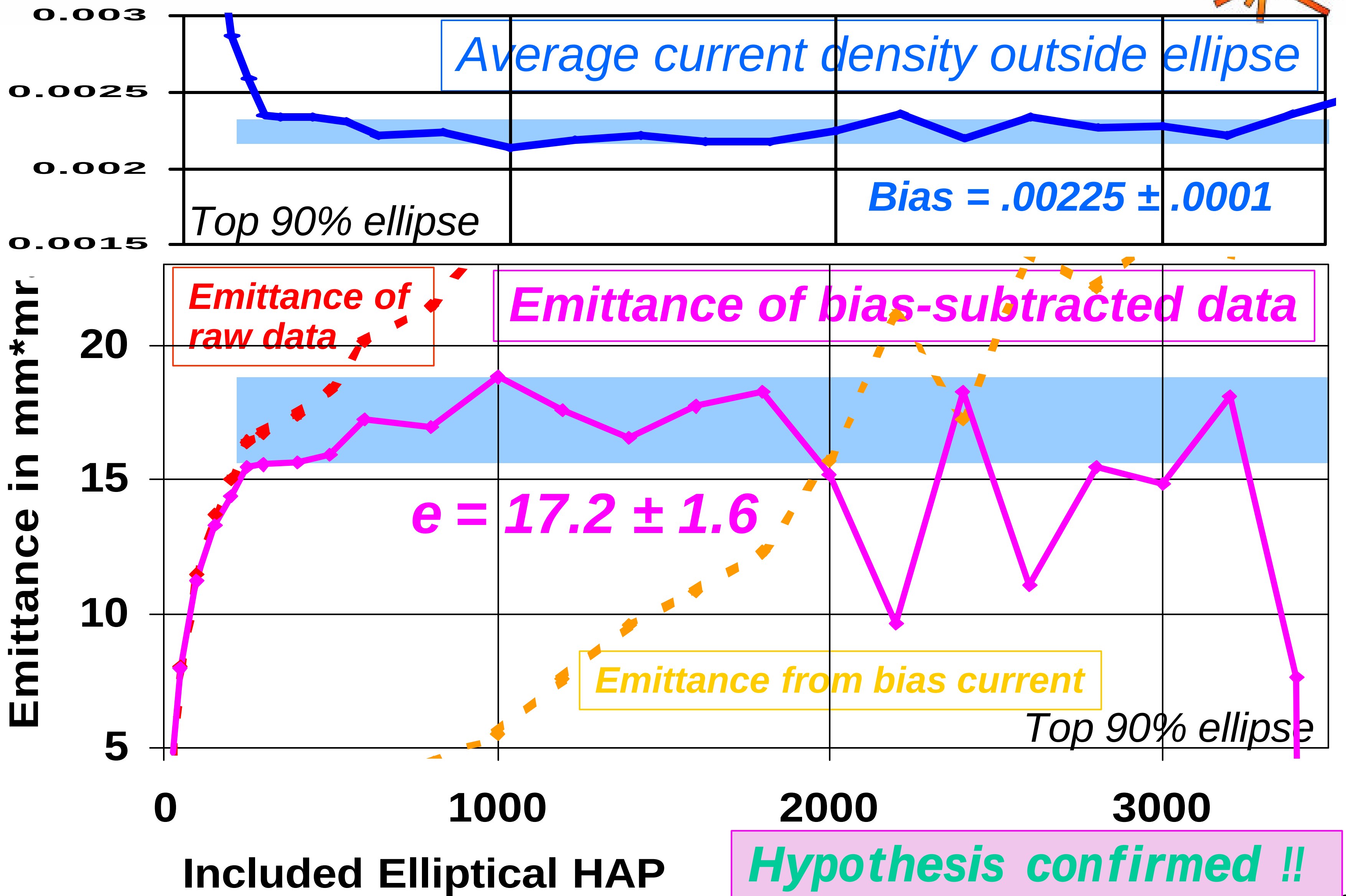
ISIS, can you explain ?

Unbiased elliptical exclusion analysis (SCUBEEx)

Refined Hypothesis:

- all real current measurements are within an ellipse of a size to be determined in the analysis.
- all current measurements outside the ellipse are noise plus bias, with the bias to be subtracted from all data.
- the hypothesis is confirmed if the size of the ellipses can be varied over a wide range without significantly changing the resulting emittance.
- If the hypothesis is confirmed, the evaluated emittance is unbiased as the bias has been subtracted while all real current was included.
- the statistical uncertainty of the evaluated emittance can be estimated from the variation of the emittance when varying the size of the ellipse.

SCUBEEx Analysis of eEBED



Hypothesis confirmed !!

Unbiased elliptical exclusion analysis (SCUBEEx)

Detailed procedure:

- threshold data to eliminate most of the background but retain most of the real data.
- eliminate first moments of remaining data.
- determine twiss parameters of the remaining data.

- use determined alpha and beta and select an elliptical area.
- determine the average background outside the elliptical area
- subtract determined background from all data
- eliminate first moments of data inside the selected elliptical area
- evaluate emittance of data inside the selected elliptical area.

  - repeat the five steps above with different elliptical areas.
  - plot the evaluated emittances as a function of elliptical area.

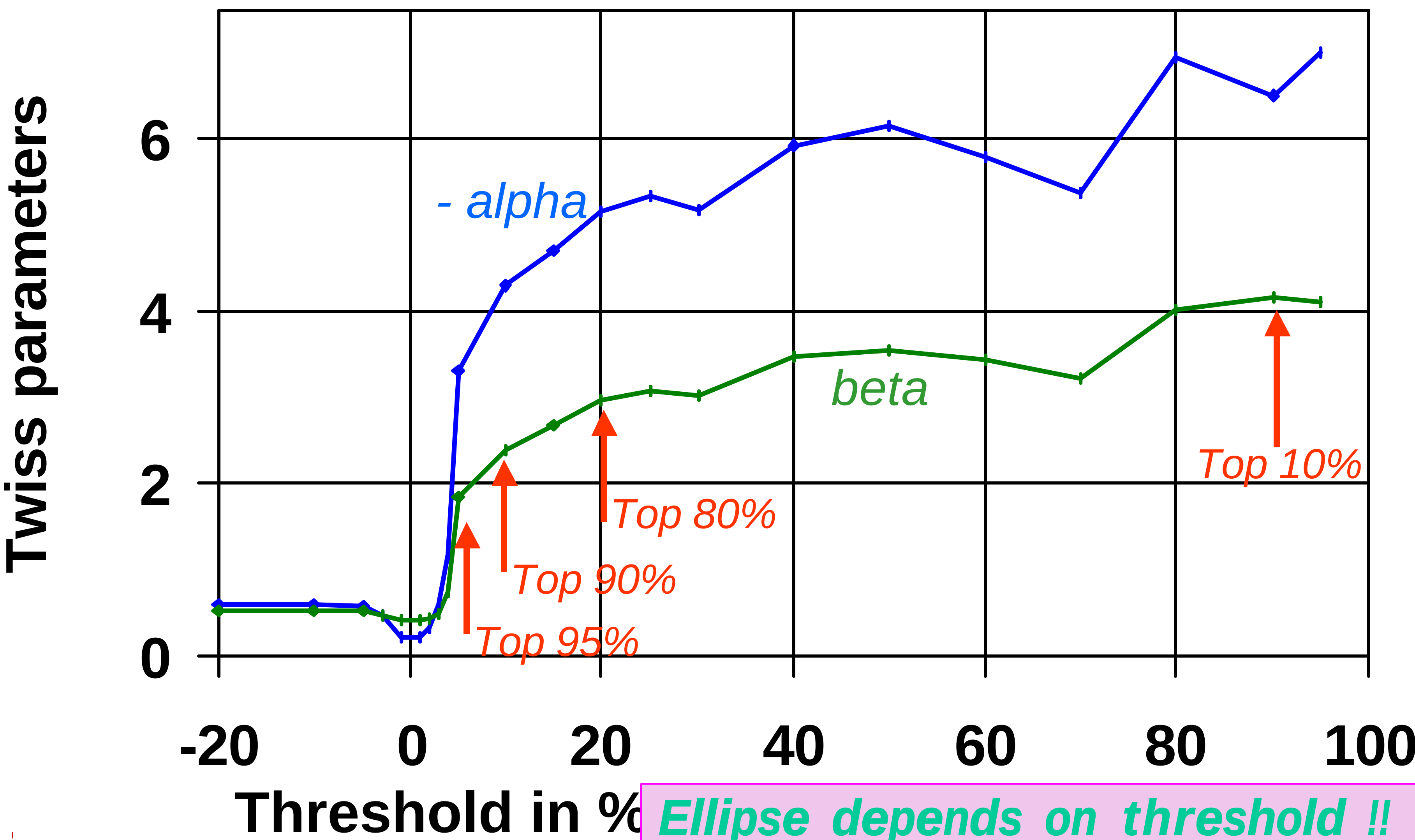
- use plot to estimate emittance and its uncertainty.

Robustness tests of SCUBEEx analysis

To verify robustness of the unbiased estimates we need to vary all parameters which remained constant when varying the elliptical area:

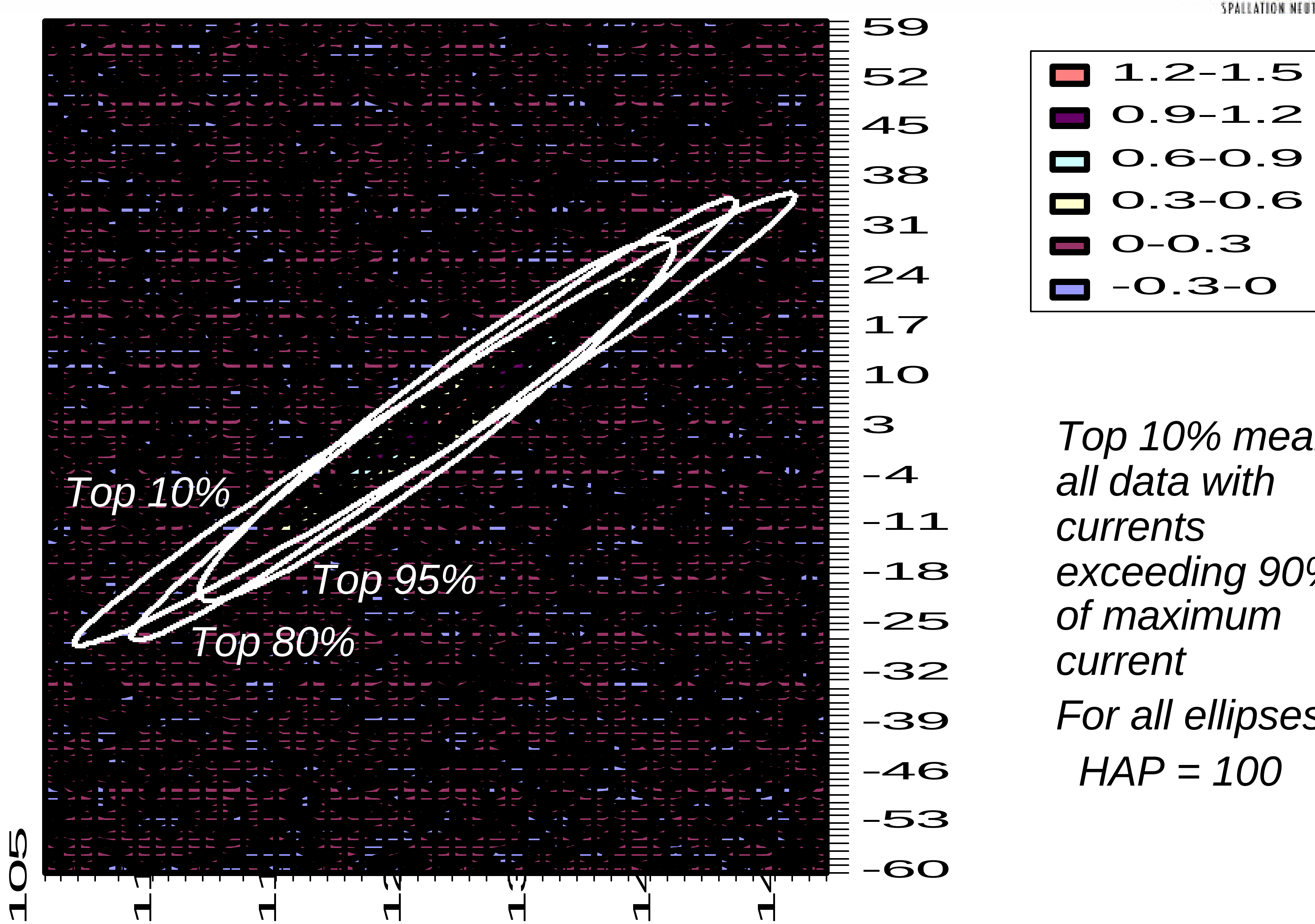
- Vary the initially selected threshold parameter which will change alpha and beta, or the orientation and aspect ratio of the selected ellipse.
- Adjust estimate and uncertainty if needed

Threshold analysis of eEBED



Ellipse depends on threshold !!

Robustness of SCUBEEx Analysis of eEBED

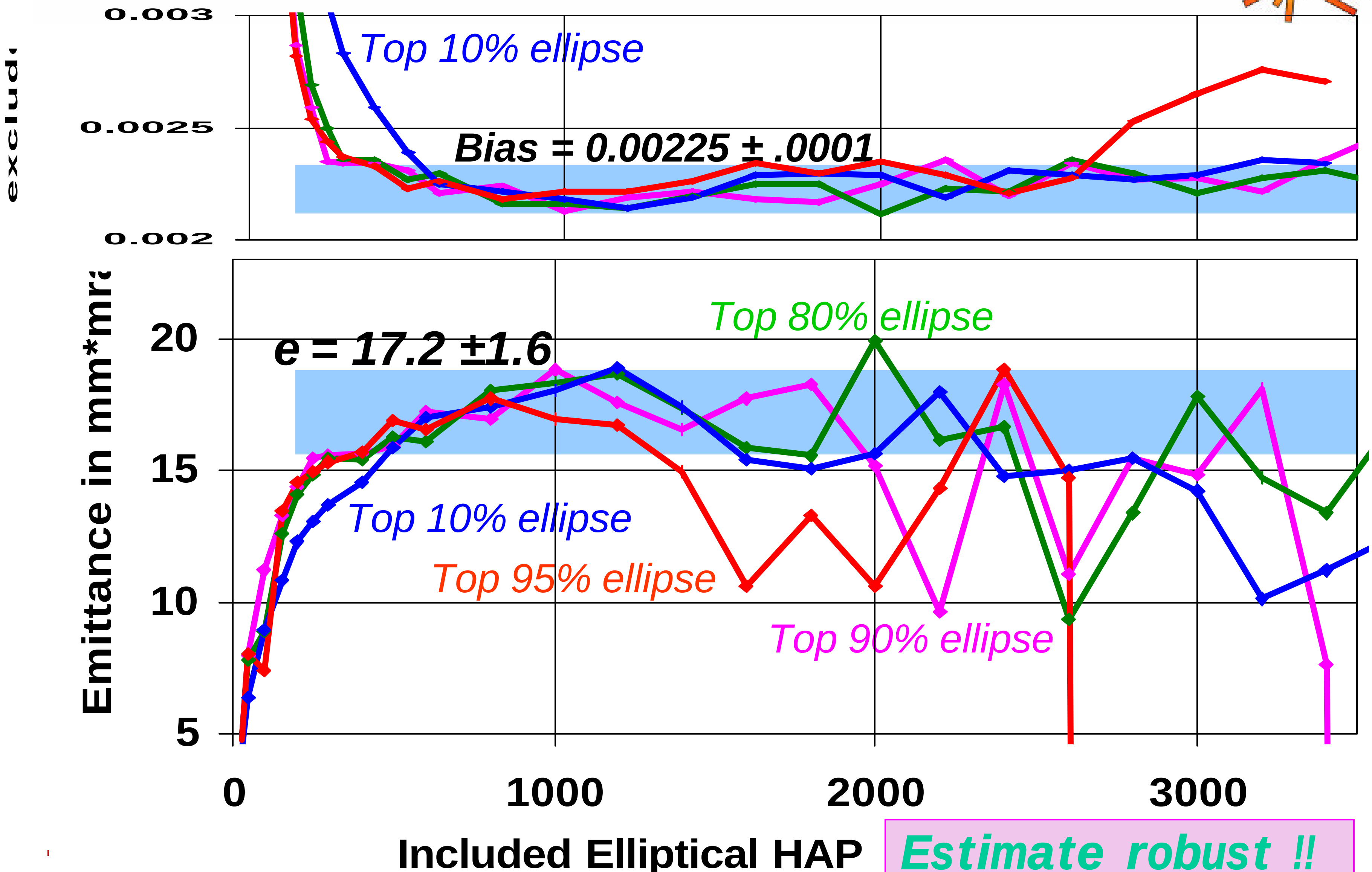


Top 10% means: all data with currents exceeding 90% of maximum current

For all ellipses:

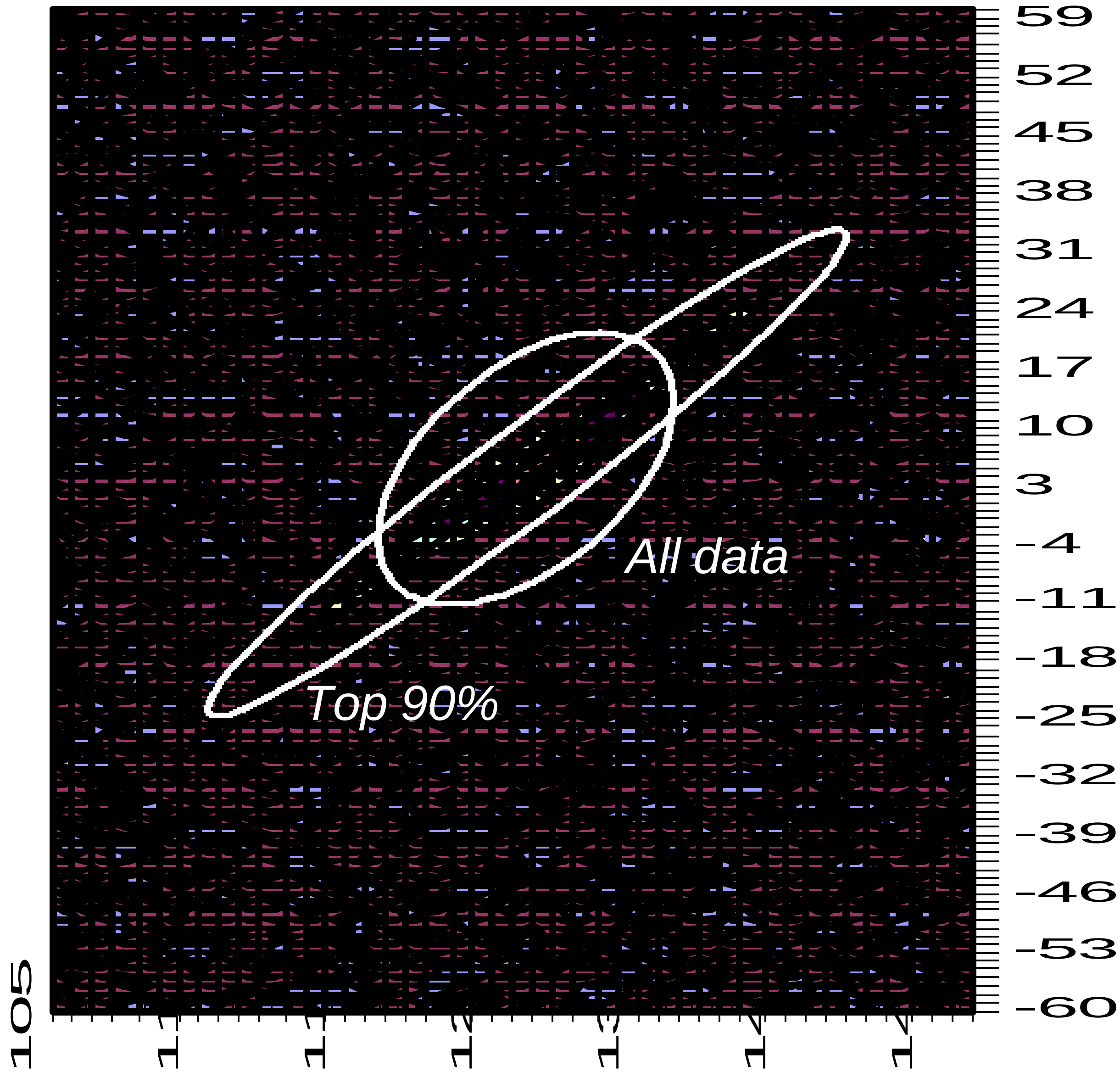
HAP = 100

Robustness of SCUBEEx Analysis of eEBED



Estimate robust !!

Robustness of SCUBEEx Analysis of eEBED

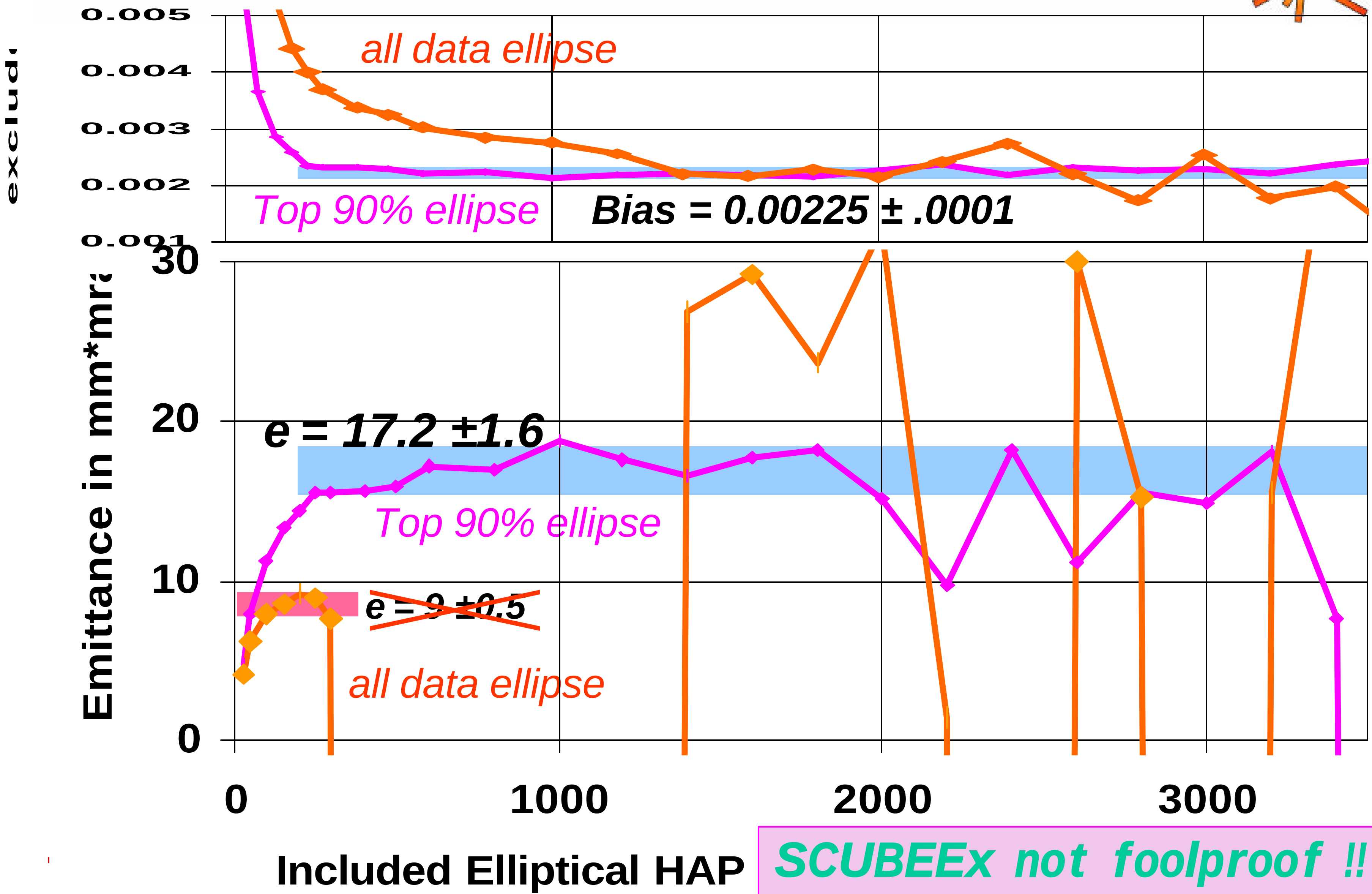


Top 90% means: all data with currents exceeding 10% of maximum current

For all ellipses:

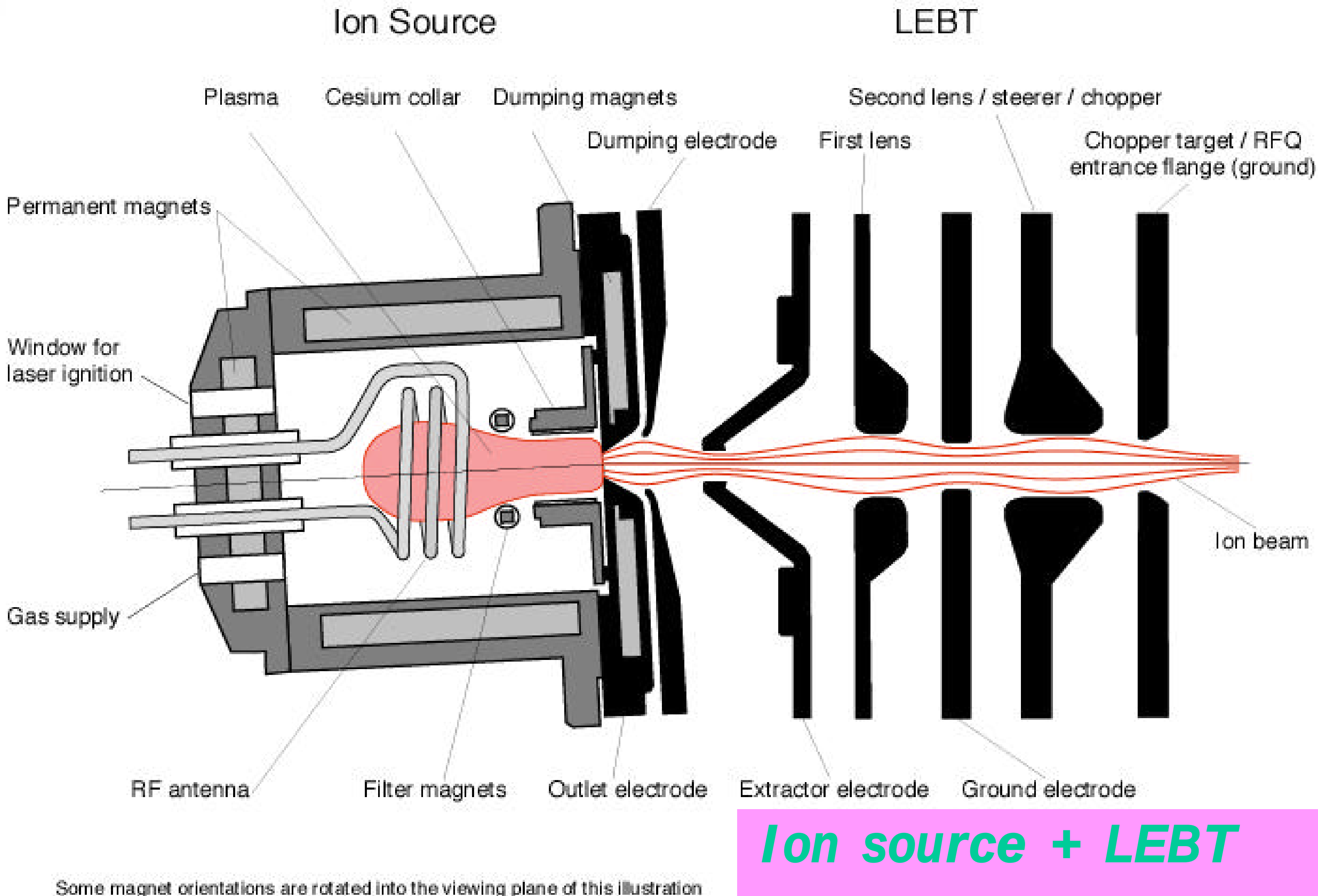
HAP = 100

Robustness of SCUBEEx Analysis of eEBED



SCUBEEx not foolproof !!

The LBNL RF-driven volume H^- source for SNS



Ion source + LEBT

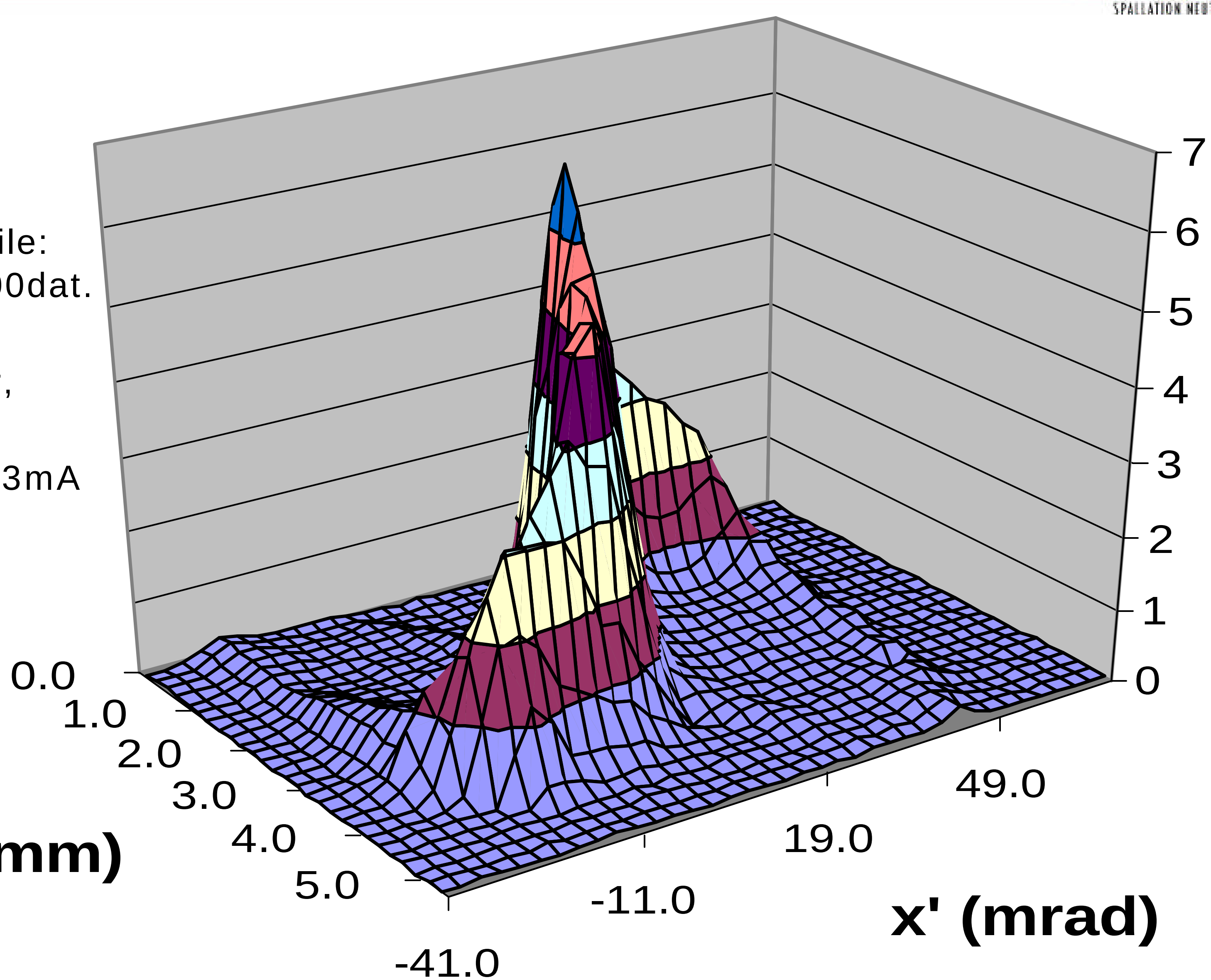
Aberrations expected !

LBNL H⁻ source y-emittance data

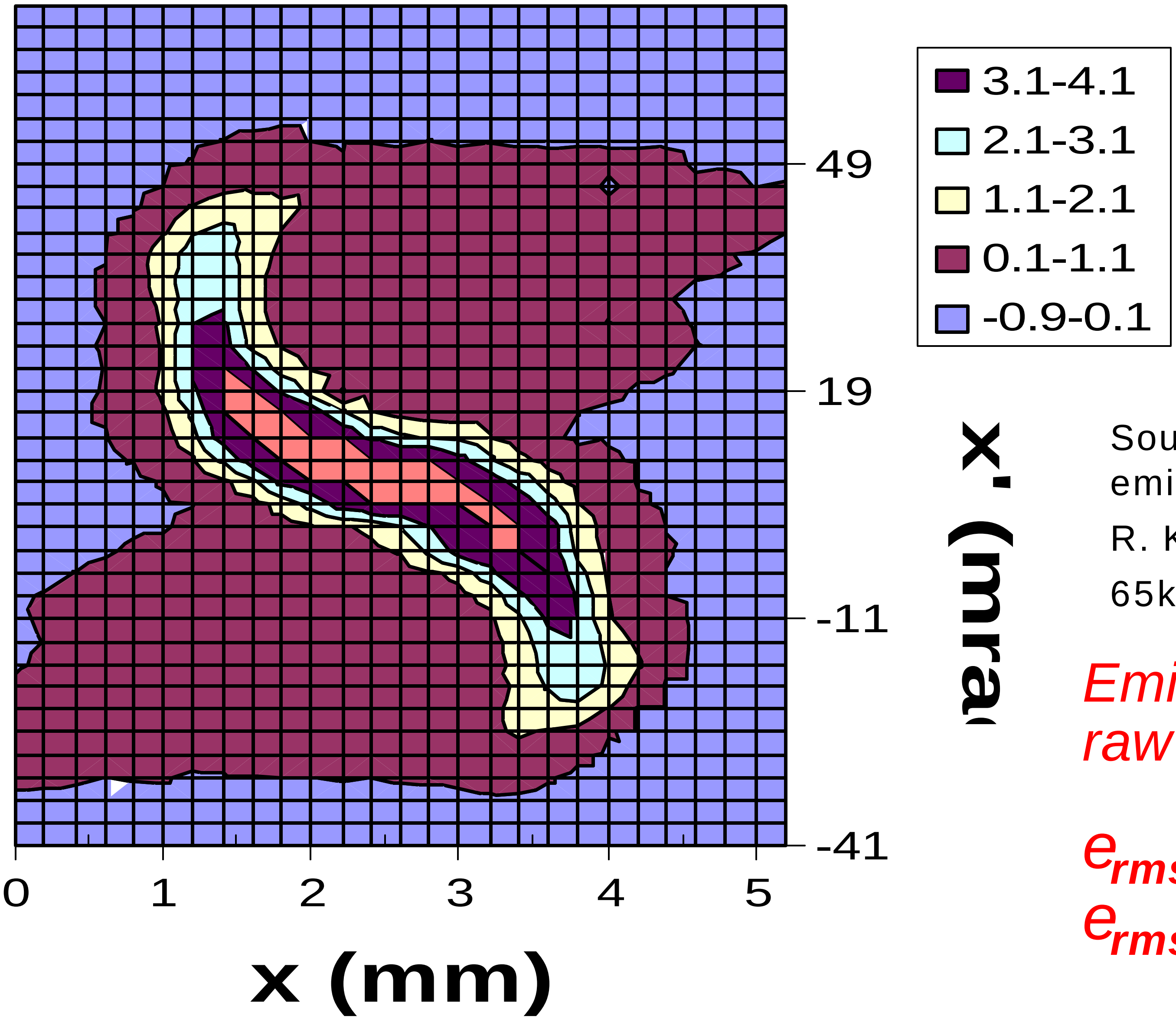
Source file: emi5Y200dat.xls;

R. Keller, LBNL;

65kV - 33mA



LBNL H⁻ source y-emittance data



Source file: emi5Y200dat.xls;

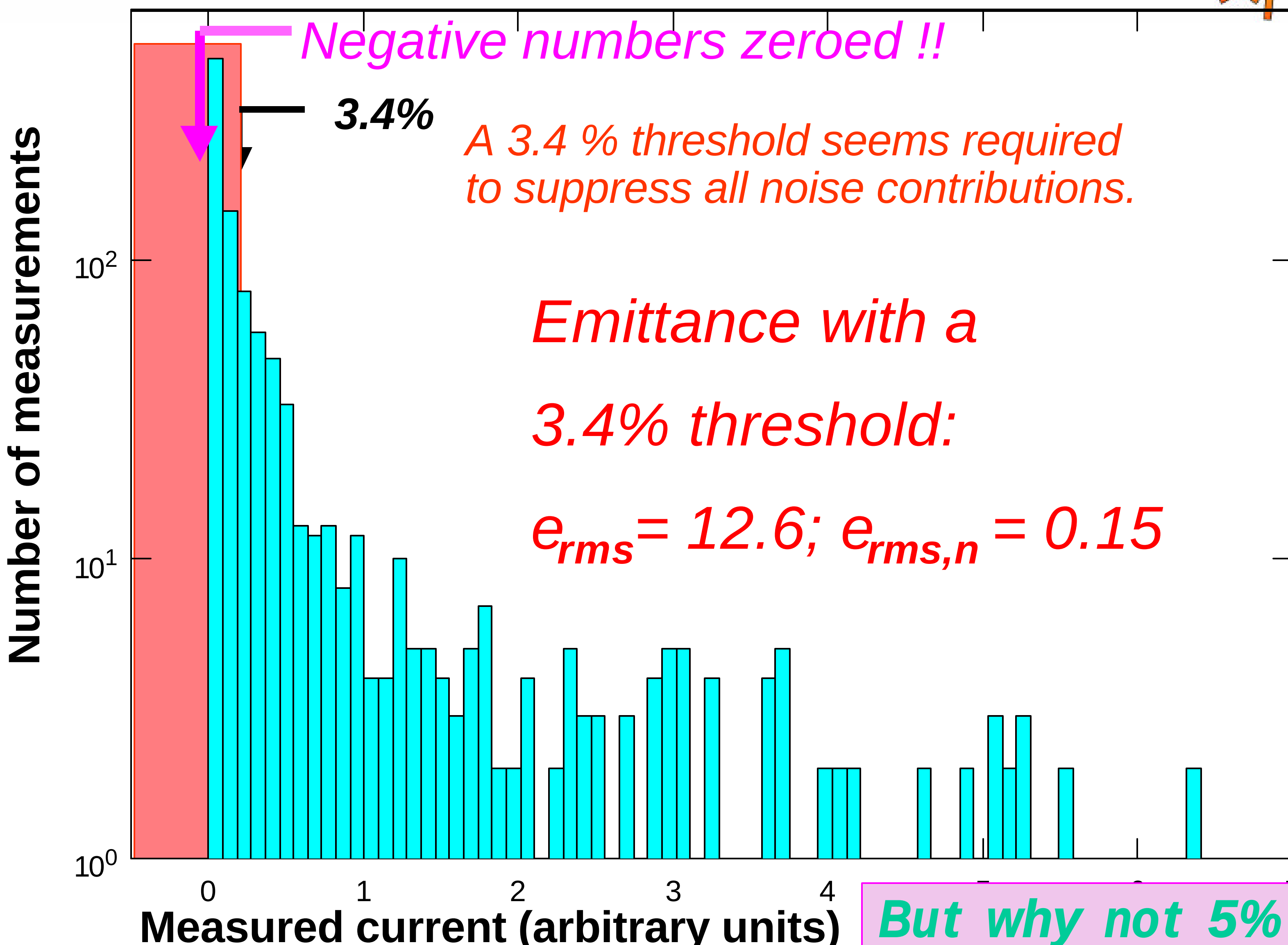
R. Keller, LBNL;

65kV - 33mA

Emittance of all raw data

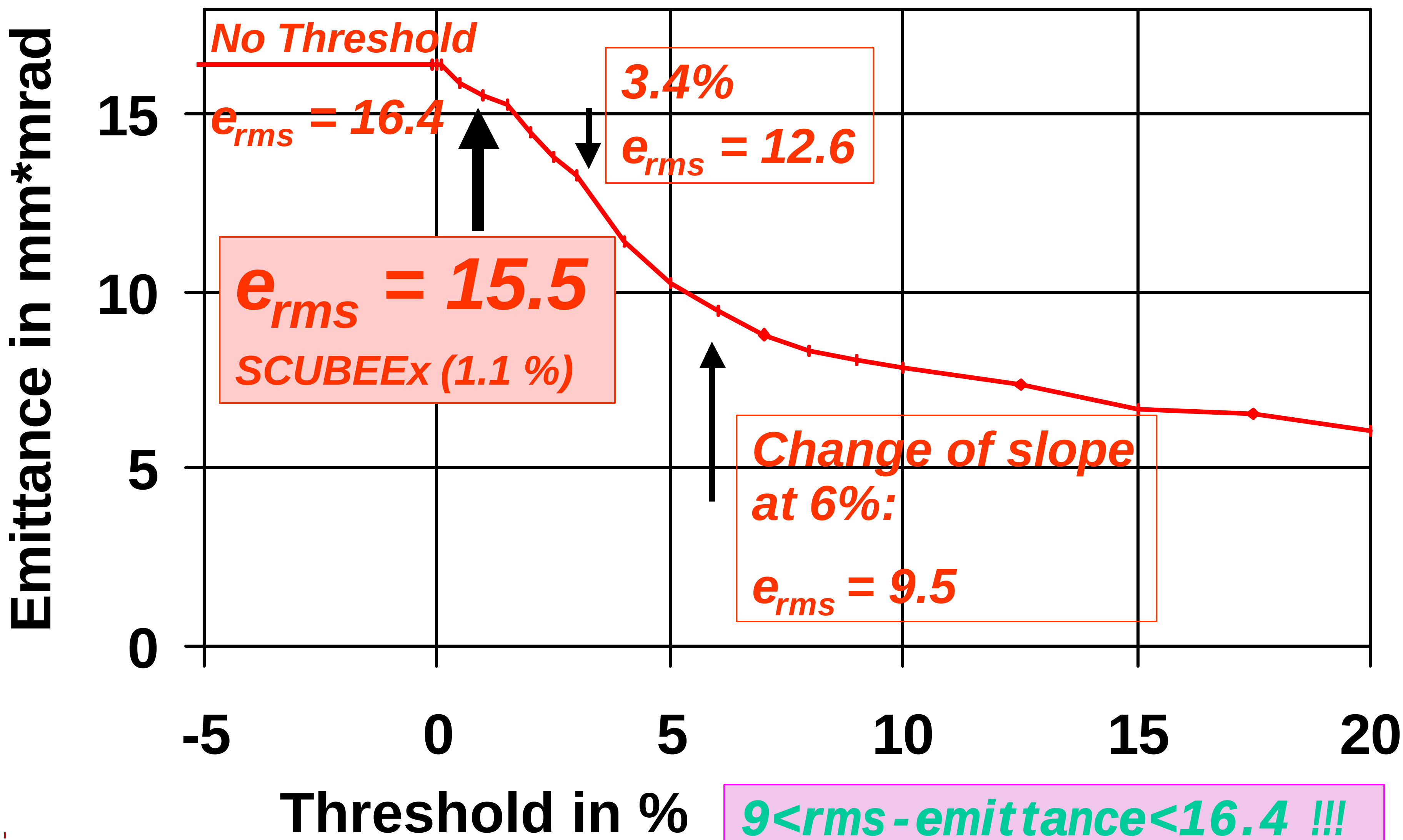
e_{rms}= 16.41;
$e_{rms,n}$ = 0.20

Histogram Analysis of the LBNL y-emittance data



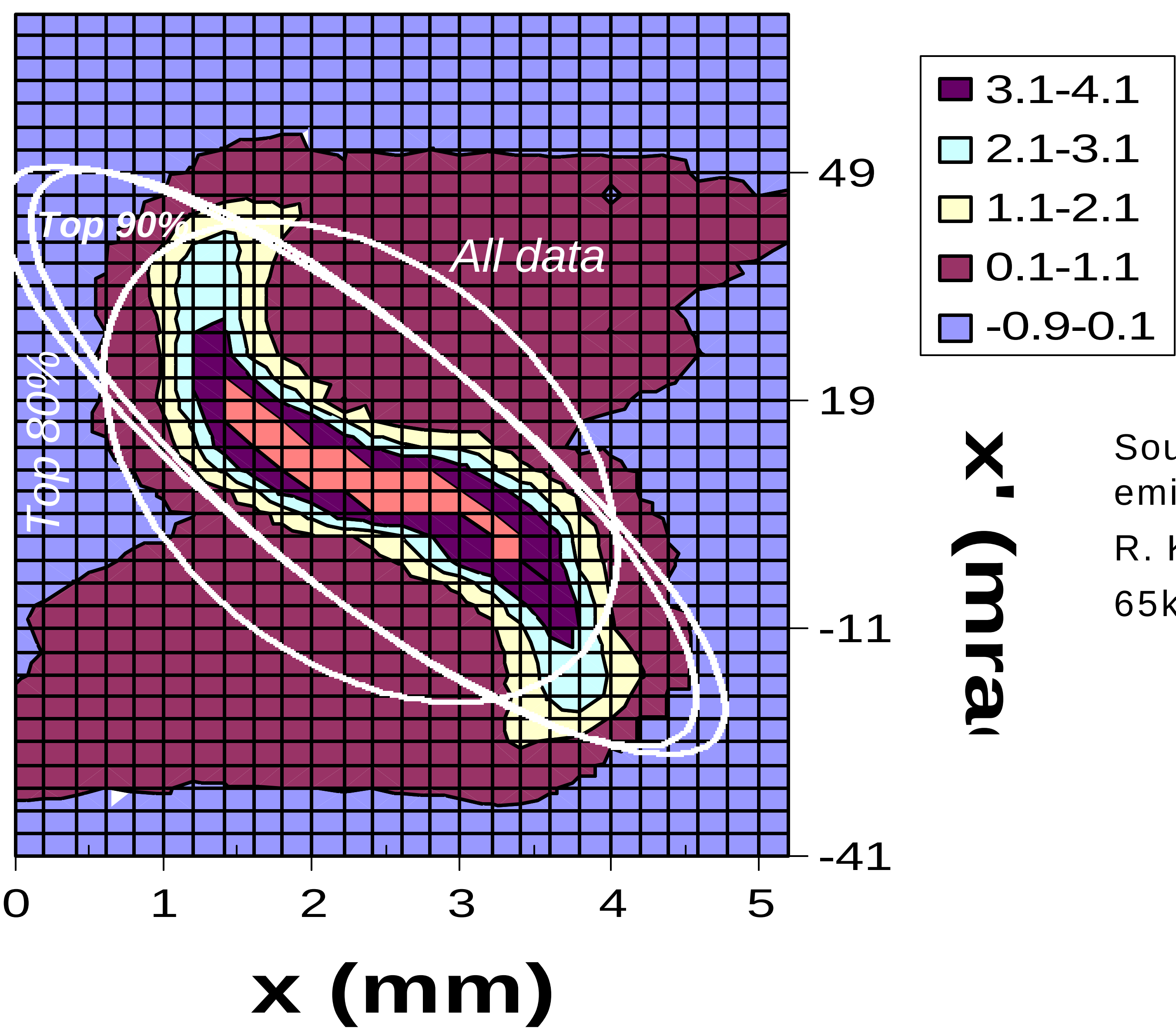
But why not 5% ??

Threshold analysis of LBNL y-emittance data



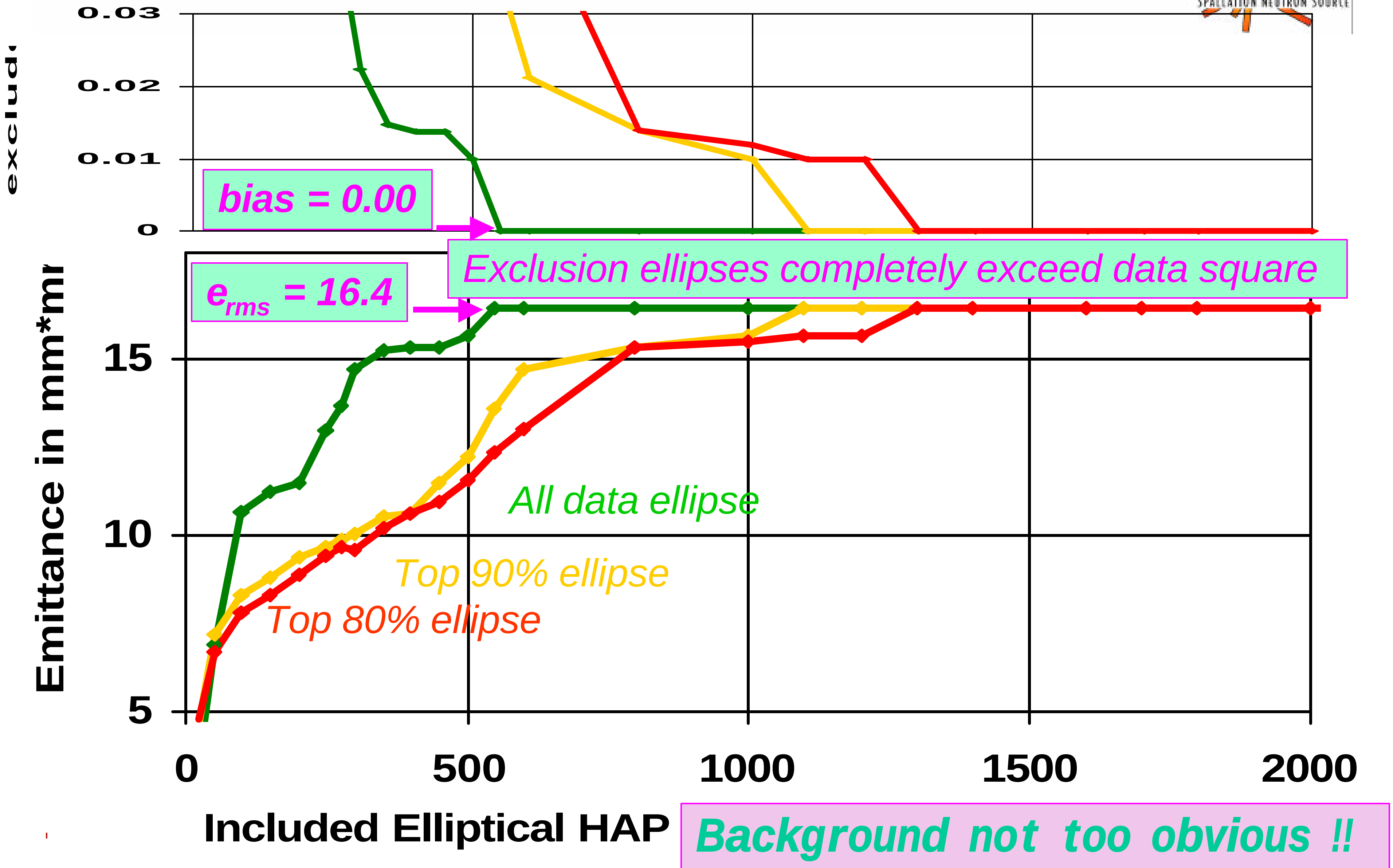
9<rms-emittance<16.4 !!!

LBNL H⁻ source y-emittance data

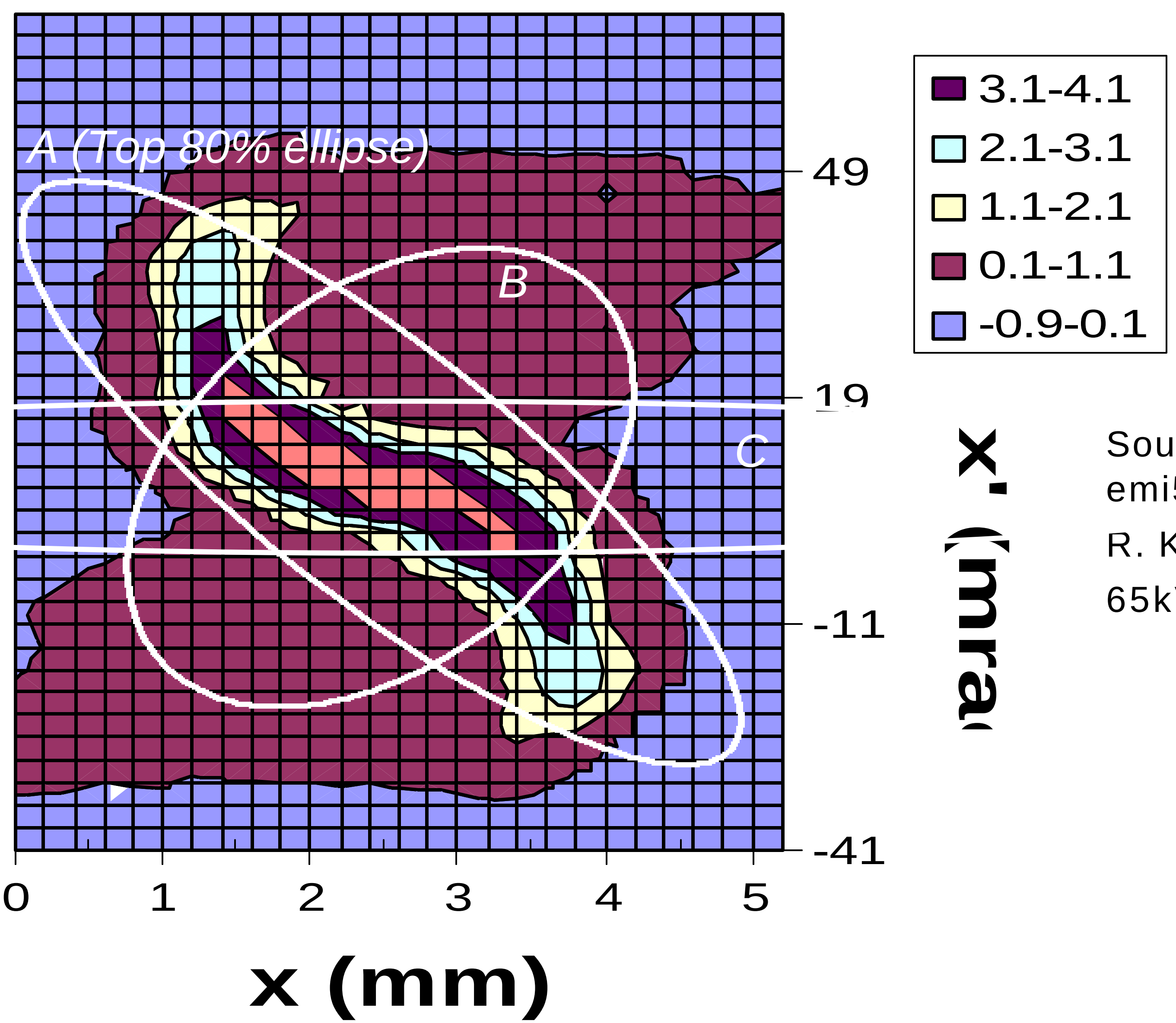


Source file:
emi5Y200dat.xls;
R. Keller, LBNL;
65kV - 33mA

SCUBEEx Analysis of LBNL y-emittance data

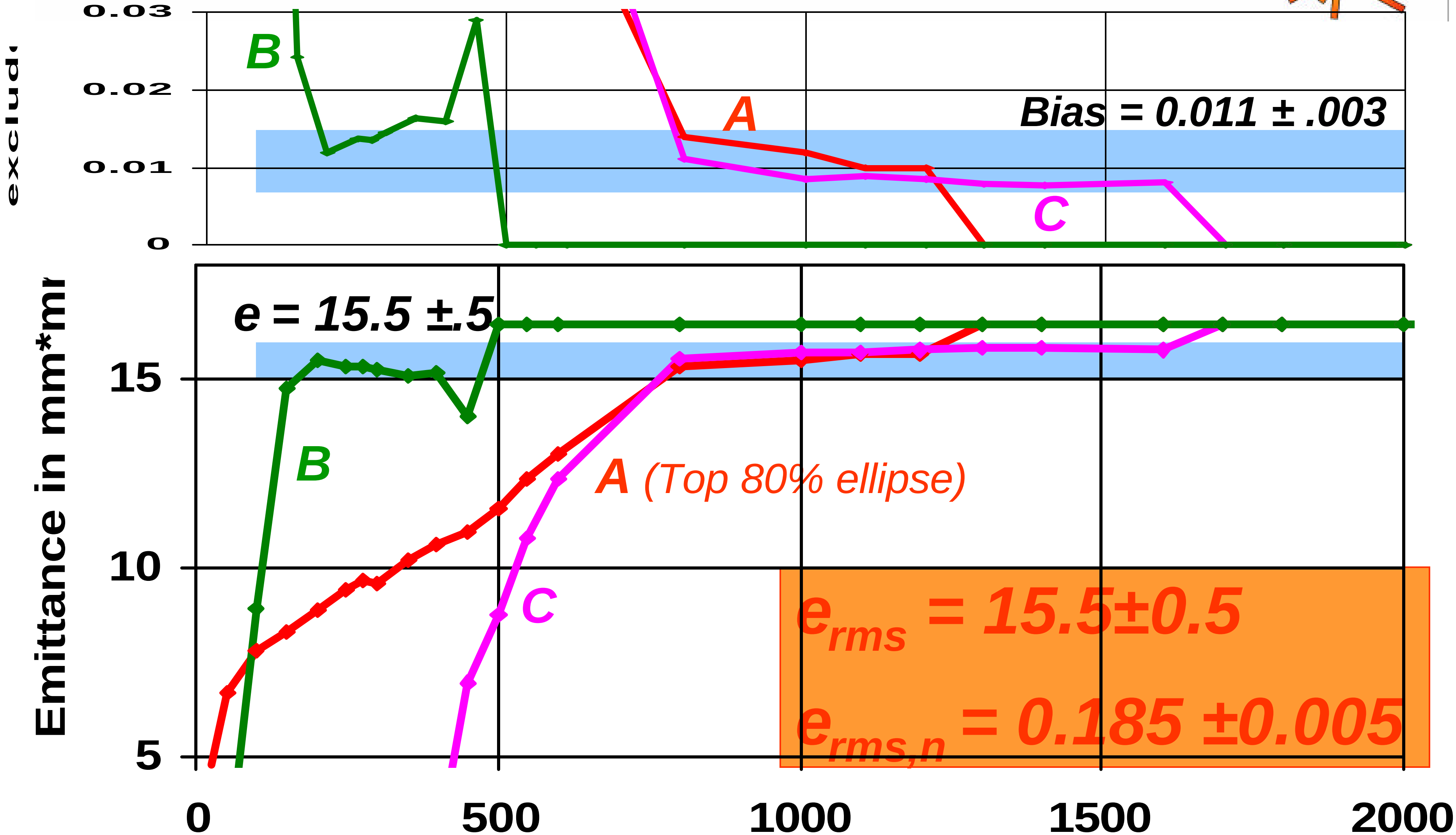


LBNL H¯ source y-emittance data



Source file:
emi5Y200dat.xls;
R. Keller, LBNL;
65kV - 33mA

SCUBEEx analysis of LBNL y-emittance data

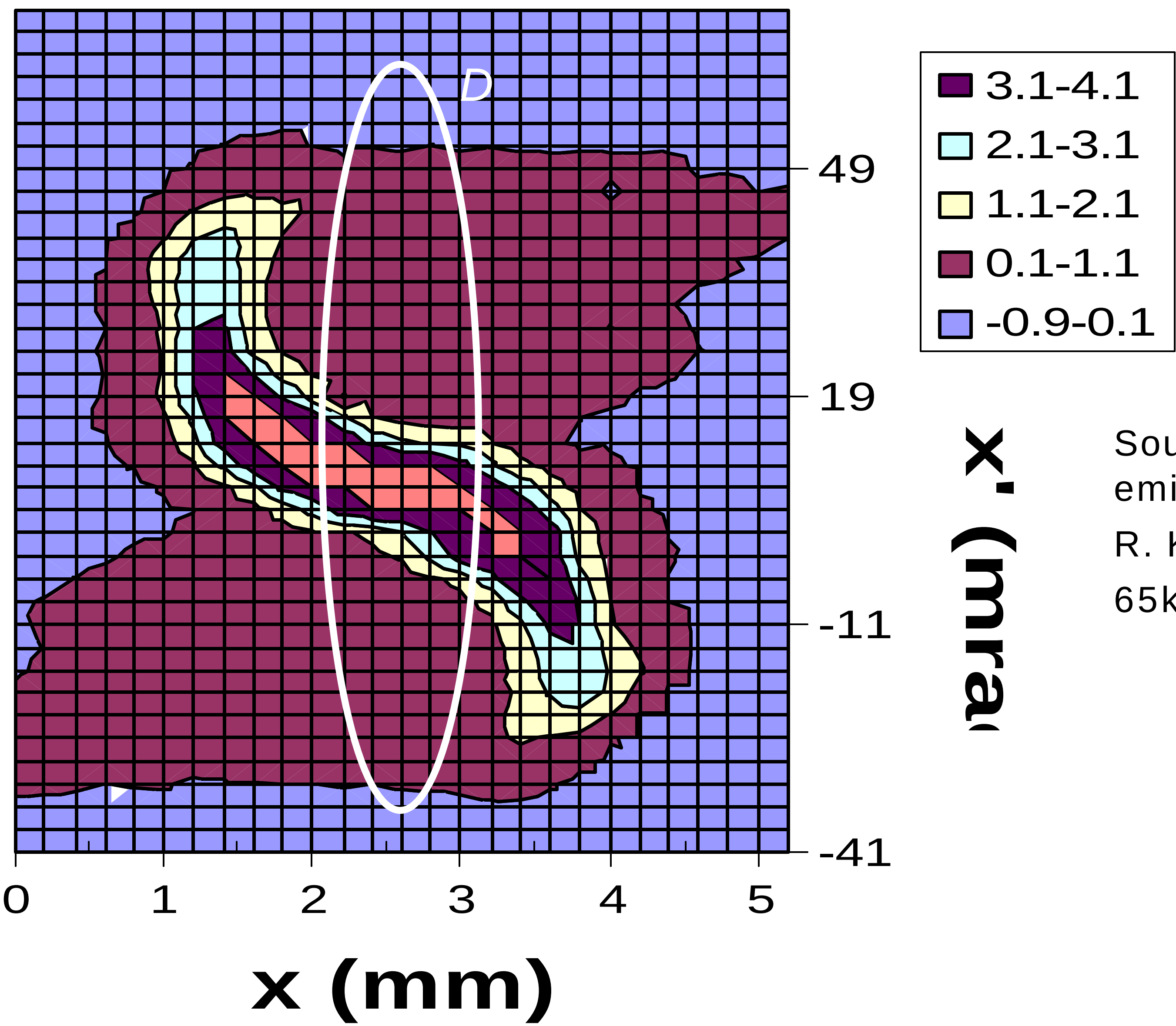


e_{rms} = 15.5±0.5

$e_{rms,n}$ = 0.185 ±0.005

Estimate robust !!

LBNL H⁻ source y-emittance data



Source file:
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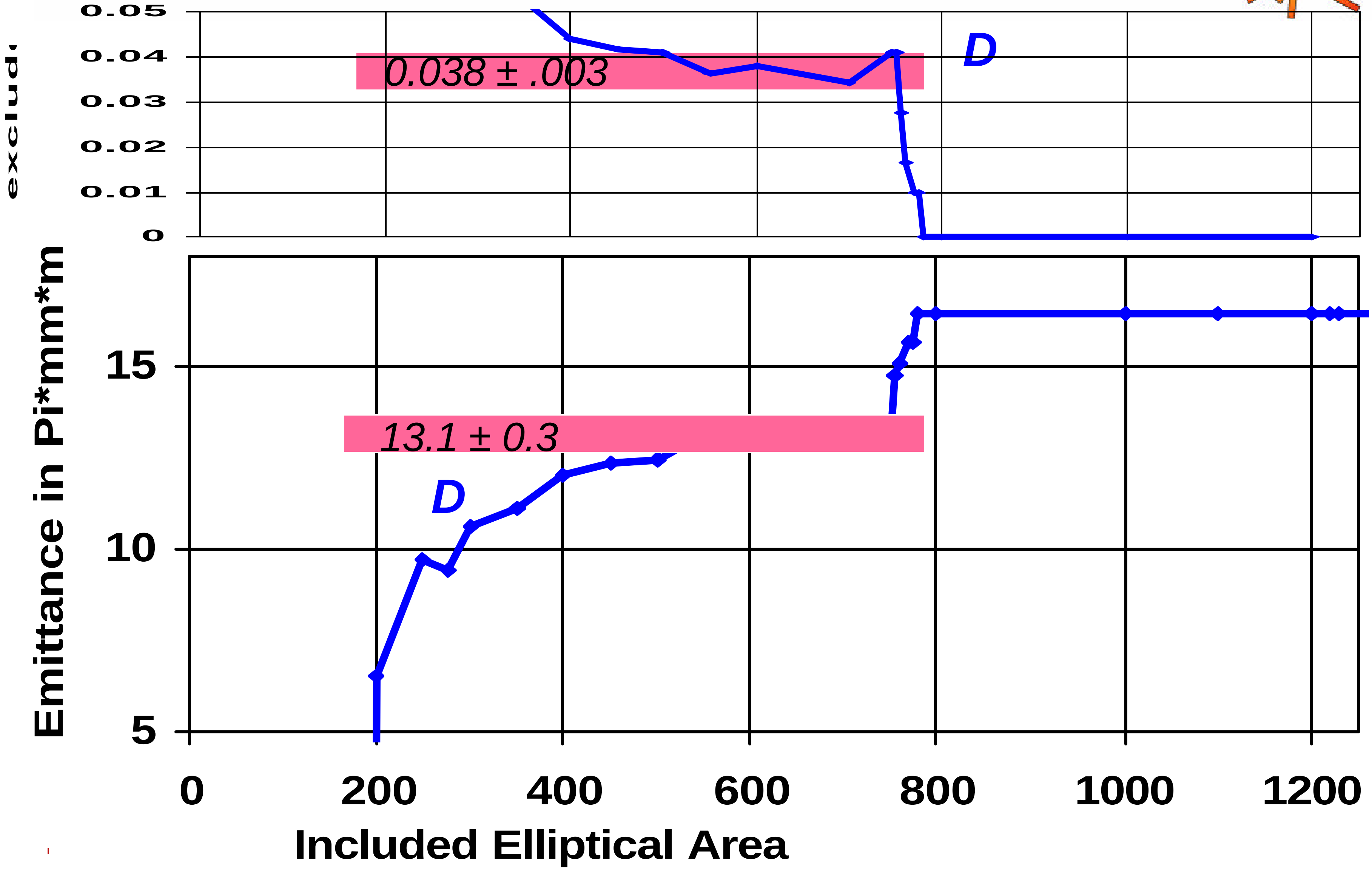
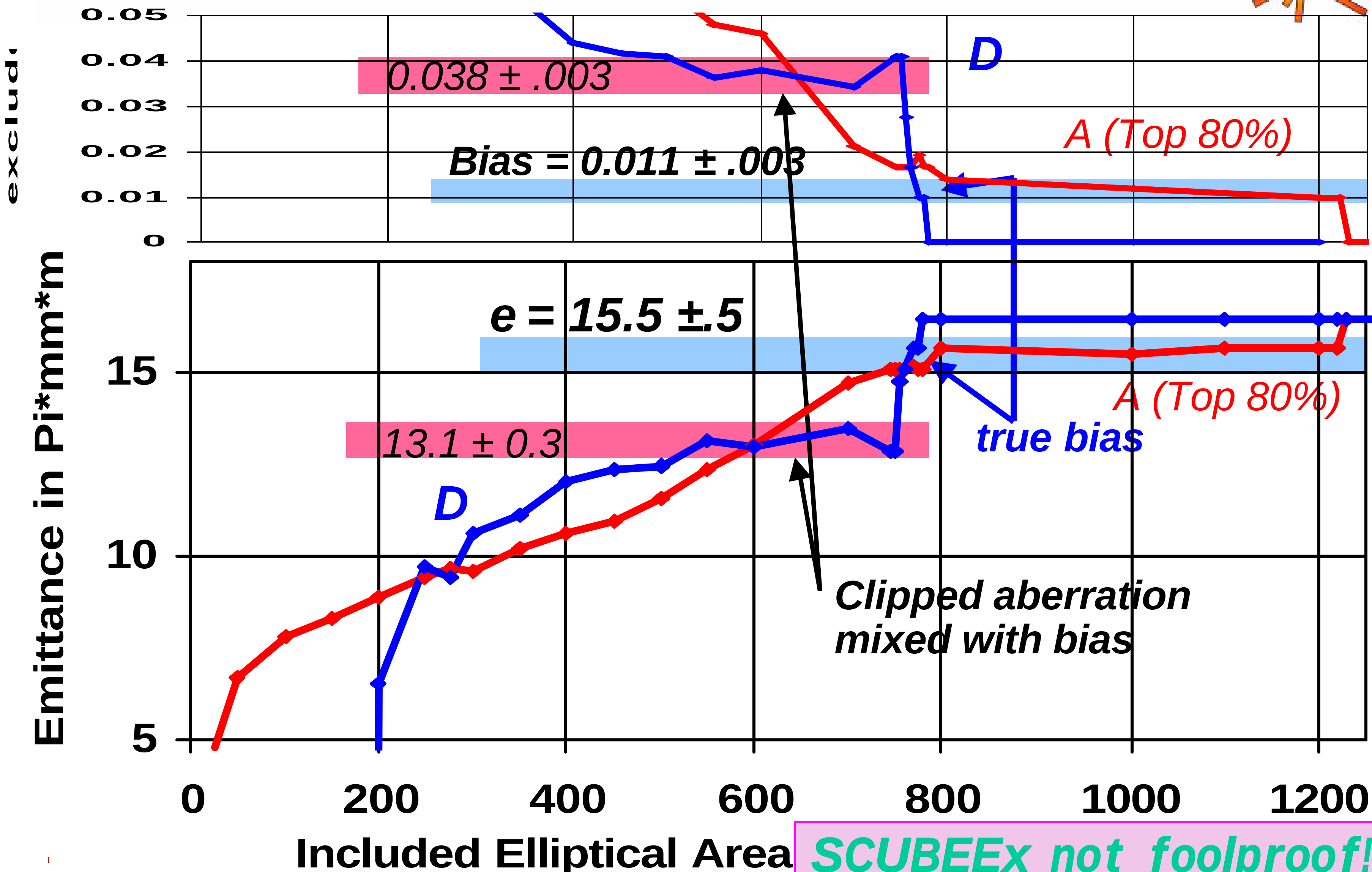
R. Keller, LBNL;
65kV - 33mA

SCUBEEx analysis of LBNL y-emittance data



SCUBEEx analysis of LBNL y-emittance data



SCUBEEx not foolproof!!

Requirements for reliable rms-emittance estimates:

- Data that significantly differ from other neighboring data should be eliminated.
- Include all negative numbers in the analysis.
- Use an exclusion boundary shape which tightly surrounds all real current measurements clearly above the noise.
- Determine the bias from data clearly free of real current measurements.
- Subtract bias from all data before calculating the rms-emittance from the data only within the exclusion boundary.
- Vary exclusion boundary shape and size to check for consistency of the bias subtraction.

To do list:

- Develop better diagnostics for background problem areas.
- Develop data display with superimposed exclusion boundary.
- Improve fitting of tightest exclusion boundary.
- Automate rms-emittance versus exclusion boundary plateau finding process.
- Expand to multi-amplifier systems.
- Develop negative-free, unbiased data matrices desired as input for simulators. A highly-local current redistribution process can be used to eliminate negative numbers from the bias-subtracted data matrix inside an all-including exclusion boundary without significantly altering distribution and rms-emittance value.

The *"Evolution versus Creation"* Act of 1999

As stipulated by the "Balanced Teaching Act" of 1999, despite overwhelming evidence supporting SCUBEEx estimates, we are required to inform you about other methods, which sometimes yield different estimates:

- Threshold analysis: threshold normally based on experience and common sense; estimates strongly dependent on threshold, data set, and thresholder.
- Exclusion analysis: exclusion boundary normally based on experience and common sense; estimates strongly dependent on exclusion boundary, data set, and excluder.
- Smallest measured range method: equivalent to exclusion; relatively good estimates, but risk of clipped tail.
- U20+ method: based on 20+ years experience, bias subtraction, and proper treatment of negative numbers. Yields 40% lower value.

A Lesson learned:

A lesson learned from the crooked E case:

ENRON

Zeroing negative numbers is unethical, illegal, punishable by law, and rarely helpful !

Conclusions:

- The expanding emittance data have a bias of approximately 0.16%, much smaller than the typical noise amplitudes of 2% (~ 1 sigma).
- The LBNL emittance data have a bias of approximately 0.16%, roughly the size of the typical noise amplitude due to zeroing negative measurements. This artifact makes new measurements desirable and such measurements are planned to be made at ORNL.
- Even small, barely noticeable bias currents and/or dc-offsets can cause significant errors in the rms-emittance when calculated from the raw data.
- Threshold analysis and exclusion analysis can rarely provide reliable estimates as these methods lack scientific criteria for choosing the cutoff parameters.
- SCUBEEx, the self-consistent, unbiased elliptical exclusion analysis can give unbiased, consistent and reliable estimates for the rms-emittances and can also estimate the uncertainty caused by background-variations and -inconsistencies.
- The LBNL developed H^- source meets the SNS emittance requirement.

Thank you for your attention !!