# Automatic In-Plane Rotation for Doubly Oblique Cardiac Imaging 

P. Kellman ${ }^{1}$, J. A. Derbyshire ${ }^{1}$, E. R. McVeigh ${ }^{1}$<br>${ }^{1}$ Laboratory of Cardiac Energetics, NHLBI, National Institutes of Health, DHHS, Bethesda, MD, United States

## Synopsis

A method for automatically calculating the in-plane rotation for doubly oblique slice geometry is proposed. Doubly oblique slice orientation is commonly used in applications such cardiac imaging, and results in an in-plane rotation of the body cross-section. In-plane rotation minimizes wrap artifacts for a given FOV. It is often desirable to adjust the in-plane orientation to ensure a minimum alias free field-of-view (FOV) in the phase-encode direction. This is particularly important for reduced FOV imaging such as UNFOLD or parallel MR methods, SENSE or SMASH.

## Theory \& Methods

A doubly oblique imaging plane is generally specified by its normal vector $\mathbf{n}=[\mathrm{nx}, \mathrm{ny}, \mathrm{nz}]^{\mathrm{T}}$ (superscript ${ }^{\mathrm{T}}$ denotes transpose) or rotation matrix ( $\mathbf{M}$ ), and is typically prescribed graphically. Consider the body to be a cylindrical ellipsoid. The intersection of an oblique plane, defined by the equation $\mathbf{r}^{\mathrm{T}} \cdot \mathbf{n}=0\left(\right.$ with $\left.\mathbf{r}=[\mathrm{x}, \mathrm{y}, \mathrm{z}]^{\mathrm{T}}\right)$, and a cylindrical ellipsoid with short and long axes defined by SAX and LAX, respectively, defined by equation (1): $\alpha x^{2}+\beta y^{2}=\frac{x^{2}}{(L A X / 2)^{2}}+\frac{y^{2}}{(S A X / 2)^{2}}=1 \quad$, is an ellipse in the oblique plane. The derivation of the in-plane rotation angle $\vartheta$ follows. If the oblique plane coordinates are transformed from the body coordinates ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) into $\left(x_{1}, y_{1}, z_{1}\right)$ such that $z_{1}=0$ is in the plane, then the ellipse parameters in $\left(x_{1}, y_{1}\right)$ may be derived as follows. Let the coordinate transformation from $\mathbf{r}=[x, y, z]^{T}$ into $\mathbf{r}_{1}=\left[x_{1}\right.$, $\left.\mathrm{y}_{1}, \mathrm{z}_{1}\right]{ }^{\mathrm{T}}$ be defined by the matrix rotation $\mathbf{r}=\mathbf{M} \bullet \mathbf{r}_{1}$. Then the ellipse in ( $\mathrm{x}_{1}$, $\mathrm{y}_{1}$ ) may be found by substituting into (1) as: $\alpha x^{2}+\beta y^{2}=\alpha\left(m_{11} x_{1}+m_{12} y_{1}\right)^{2}+\beta\left(m_{21} x_{1}+m_{22} y_{1}\right)^{2}=A x_{1}^{2}+B x_{1} y_{1}+C y_{1}^{2}=1, \quad$ with $\quad \mathrm{z}_{1}=0 \quad$ and $\quad$ with the coefficients, $\quad \mathrm{A}, \quad \mathrm{B}, \quad$ and $\quad \mathrm{C}, \quad$ derived $\quad$ as: $A=\alpha m_{11}^{2}+\beta m_{21}^{2}, \quad B=2\left(\alpha m_{11} m_{12}+\beta m_{21} m_{22}\right), C=\alpha m_{12}^{2}+\beta m_{22}^{2}$. The ellipse in ( $\mathrm{x}_{1}, \mathrm{y}_{1}$ ) is rotated by the angle $\vartheta$ and has long and short axes defined as LAX ${ }^{\prime}$ and $\mathrm{SAX}^{\prime}$. Define the rotated coordinates $\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)$ as: $\left[\mathrm{x}_{1} \mathrm{y}_{1}\right]^{\mathrm{T}}=[\cos (\vartheta)-\sin (\vartheta) ; \sin (\vartheta) \cos (\vartheta)] \bullet\left[\mathrm{x}_{2} \mathrm{y}_{2}\right]^{\mathrm{T}}$. Then substituting the equation for the ellipse in ( $\left.\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ yields the ellipse in $\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right): a x_{2}^{2}+b x_{2} y_{2}+c y_{2}^{2}=1$, where the coefficients may be derived as:

$$
\begin{aligned}
a & =A \cos ^{2}(\vartheta)+B \cos (\vartheta) \sin (\vartheta)+C \sin ^{2}(\vartheta) \\
b & =B \cos ^{2}(\vartheta)-2(A-C) \cos (\vartheta) \sin (\vartheta)-B \sin ^{2}(\vartheta) \\
c & =A \sin ^{2}(\vartheta)-B \cos (\vartheta) \sin (\vartheta)+C \cos ^{2}(\vartheta)
\end{aligned}
$$

with ellipse parameters calculated as:

$$
\begin{aligned}
& \cot (2 \vartheta)=\frac{A-C}{B} \\
& L A X^{\prime}=2 / \sqrt{a} \\
& S A X^{\prime}=2 / \sqrt{c}
\end{aligned}
$$

Thus the in-plane rotation angle $\vartheta$ is calculated in terms of the body SAX and LAX dimensions. It is further shown by substitution that the angle $\vartheta$ is determined solely by the ellipticity (ratio) $\mathrm{e}=$ LAX/SAX and rotation matrix $\mathbf{M}$.
Figure 1 illustrates an example of a body ellipse with $\mathrm{e}=2$ intersected by a doubly oblique plane with normal $\left[\begin{array}{lll}1 & 1 & 1\end{array}\right]$ resulting in the ellipse in bold which has an inplane rotation of $\vartheta=23^{\circ}$ in the plane defined by the box labeled A. After in-plane rotation the box labeled B with same dimensions is aligned with the body ellipse and will not have wrap for phase encode along short axis direction. For $\mathrm{e}>2$, the rotation angle shown in Fig. 2 for $\mathbf{n}=\left[\begin{array}{lll}1 & 1 & 1\end{array}\right]$ is very insensitive to e for a wide range of normal vector directions. Thus an in-plane rotation angle may be calculated automatically from the normal vector using a default value for $\mathrm{e}=2.0$ which will be within several degrees for a wide range of interest.
Cardiac imaging was performed on a GE $1.5 \mathrm{~T} \mathrm{CV} / i$ scanner using a gated-segmented FGRE sequence with $320 \times 240 \mathrm{~mm}^{2}$ FOV.

## Results

Fig. 3 shows example images of a single SAX doubly oblique cardiac image (a) before and (b) after in-plane rotation has been applied. In this example, the body ellipticity was approximately $\mathrm{e}=2.0$, and the normal vector was $[-0.48 \quad 0.69-0.55]$, and the rotation angle was calculated to be $34^{\circ}$.

## Discussion

In-plane rotation is desirable for doubly oblique imaging, e.g., cardiac applications, particularly for reduced FOV accelerated imaging such as SENSE. The proposed method provides an approx. solution automatically eliminating the requirement for additional measurements. Another benefit is the elimination of alias artifacts including wrap of arms and shoulders due to the fact that these features may be placed in the frequency readout direction.

Figure 1. Illustration of doubly oblique imaging plane intersecting an ellipsoidal cylinder resulting in an ellipse (bold) with in-plane rotation (plane A) which may be rotated for minimum alias free FOV (plane B) with phase encode direction along SAX dimension.


Figure 2. In-plane rotation angle versus ellipticity for doubly oblique plane with normal with normal [llll $\left.\begin{array}{lll}1 & 1 & 1\end{array}\right]$ illustrating relative insensitivity of angle for e>2.


Figure 3. Example doubly oblique SAX cardiac images (a) before and (b) after automatic in-plane rotation.


