Ellipse Distortion in FFAGs

Number of Stages in FFAGs

FFAG Electron Model Lattices

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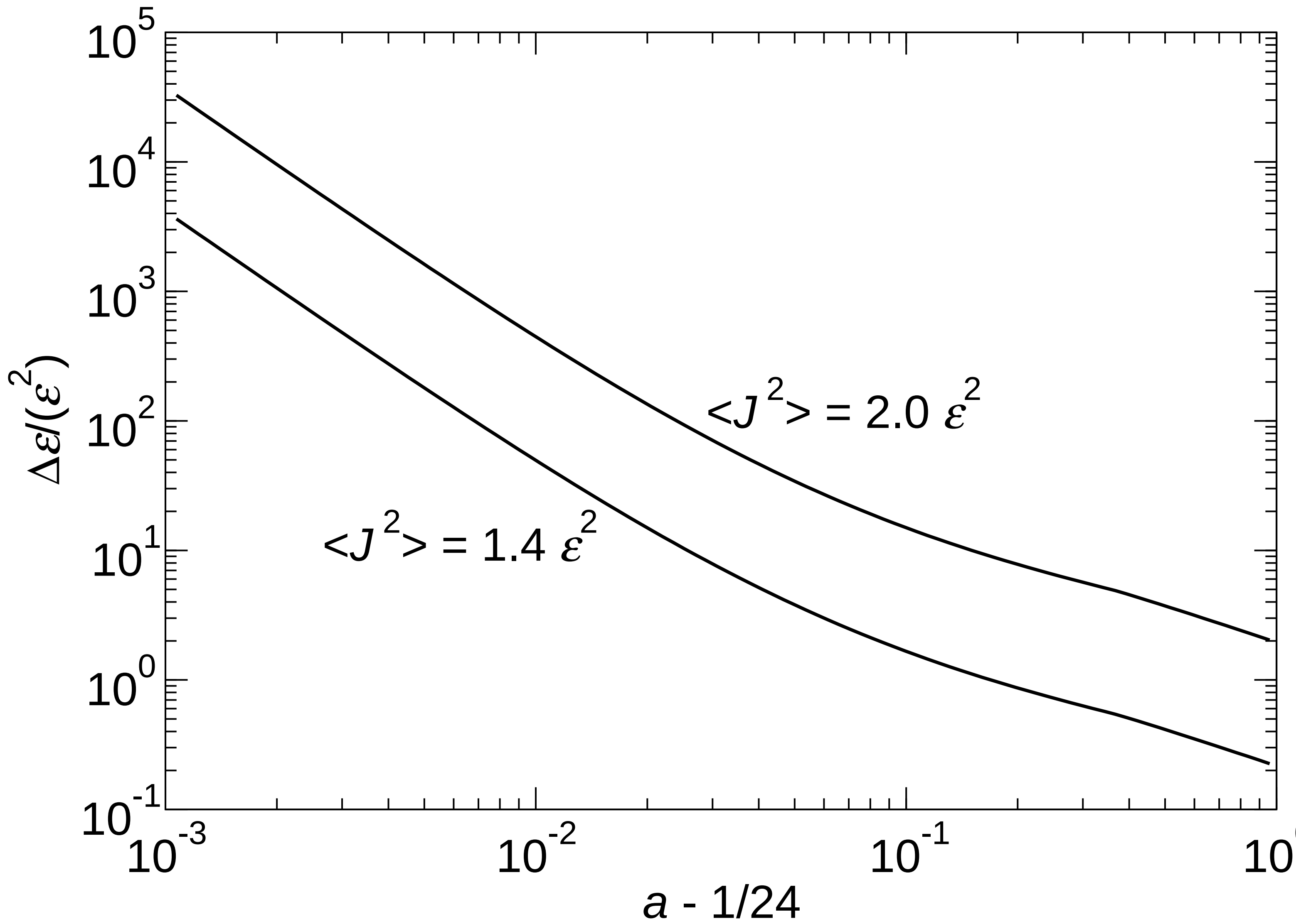
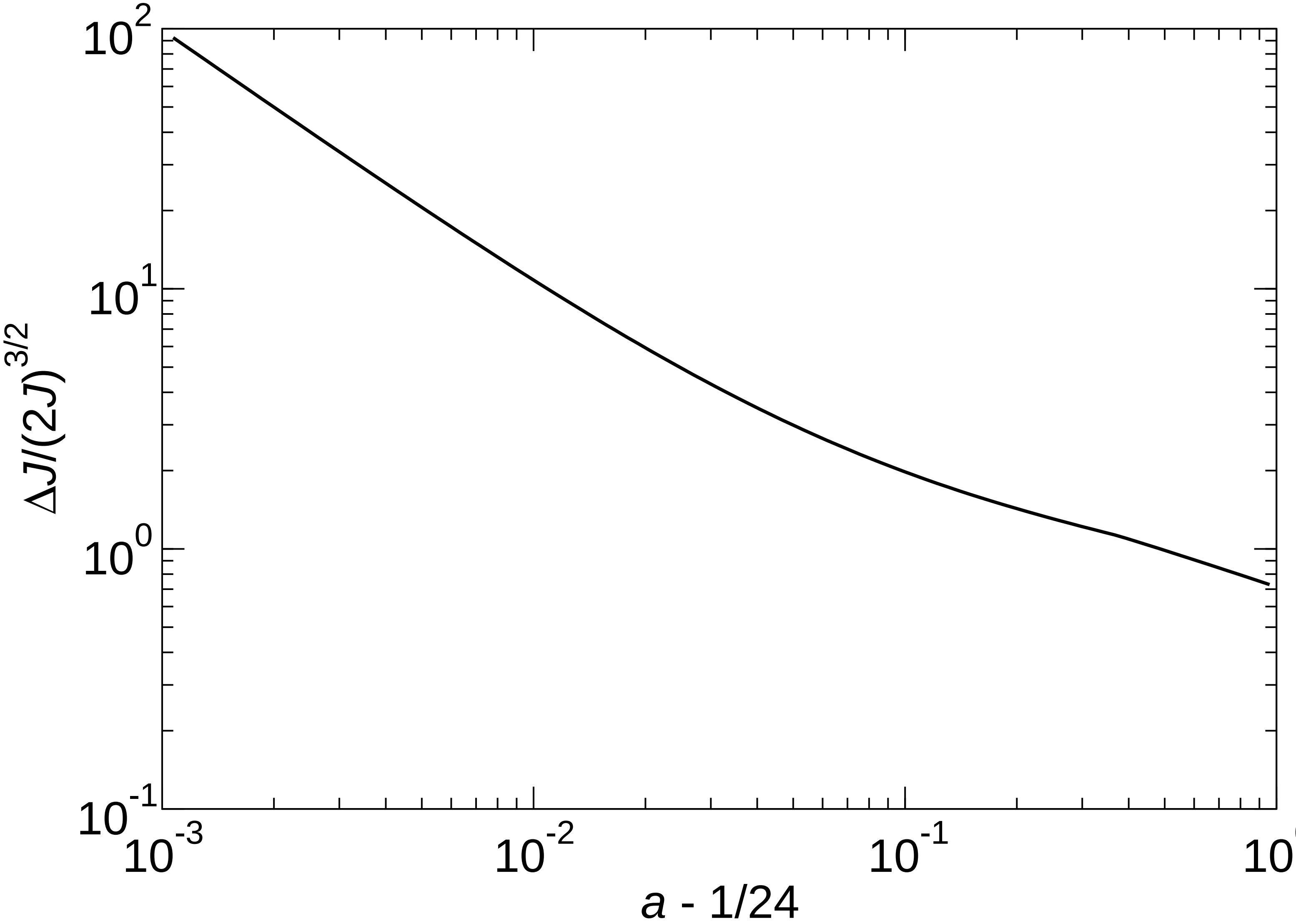
Ellipse Distortion in FFAGs: Review

- Longitudinal dynamics in FFAG lattice is parametrized by dimensionless parameters a (scaled voltage) and b (time offset)
- There is an allowed region in that parameter space
- I have a method for computing emittance growth and ellipse distortion as a function of a and b, minimizing over ellipse orientation in phase space
- I can minimize these quantities over b, and find emittance growth as a function of a
- Emittance growth as computed is a funny parameter: it can be negative, for instance.
- Better to minimize “ellipse distortion”: keep an ellipse elliptical

Emittance Growth vs. a



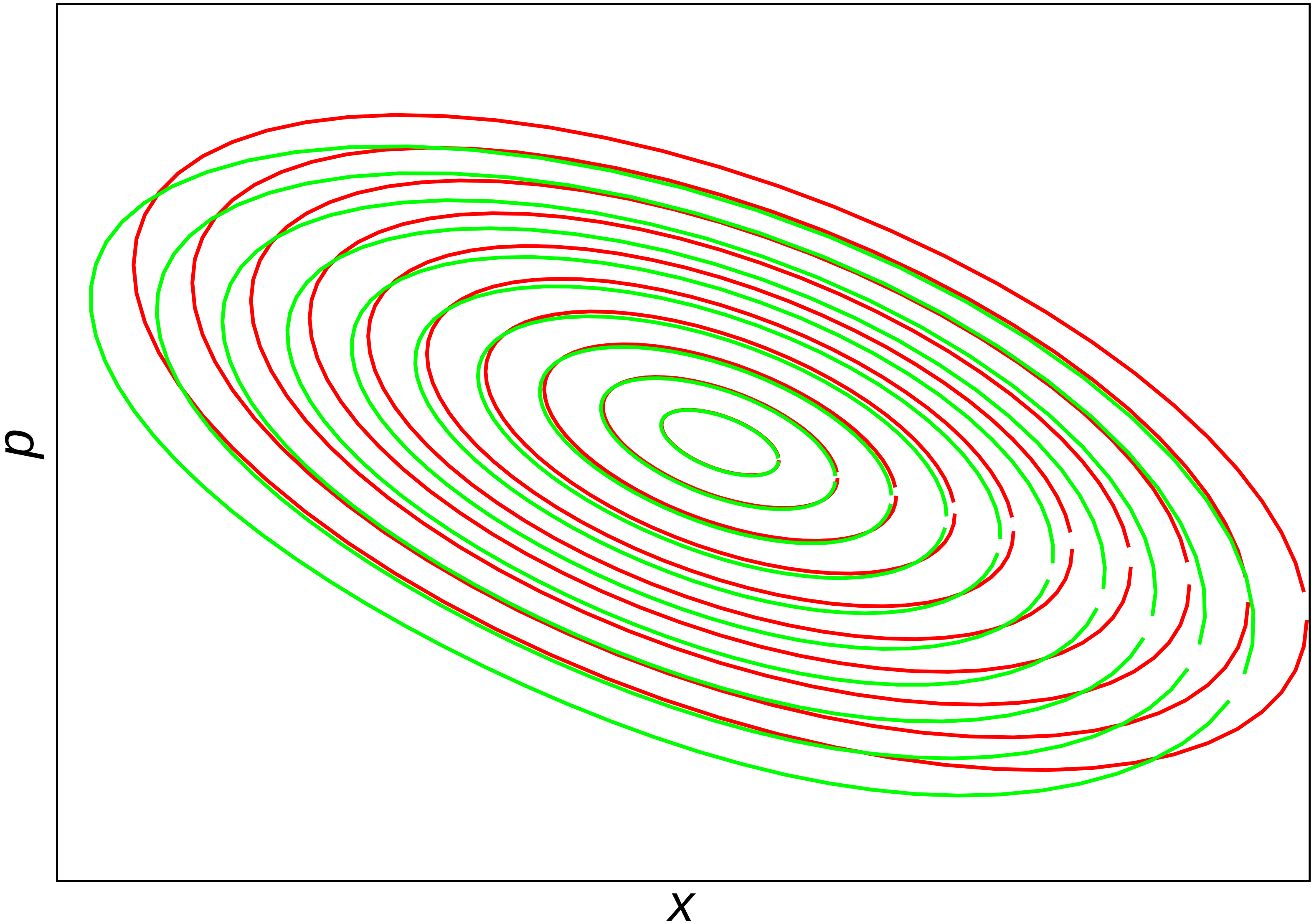
Ellipse Distortion vs. a



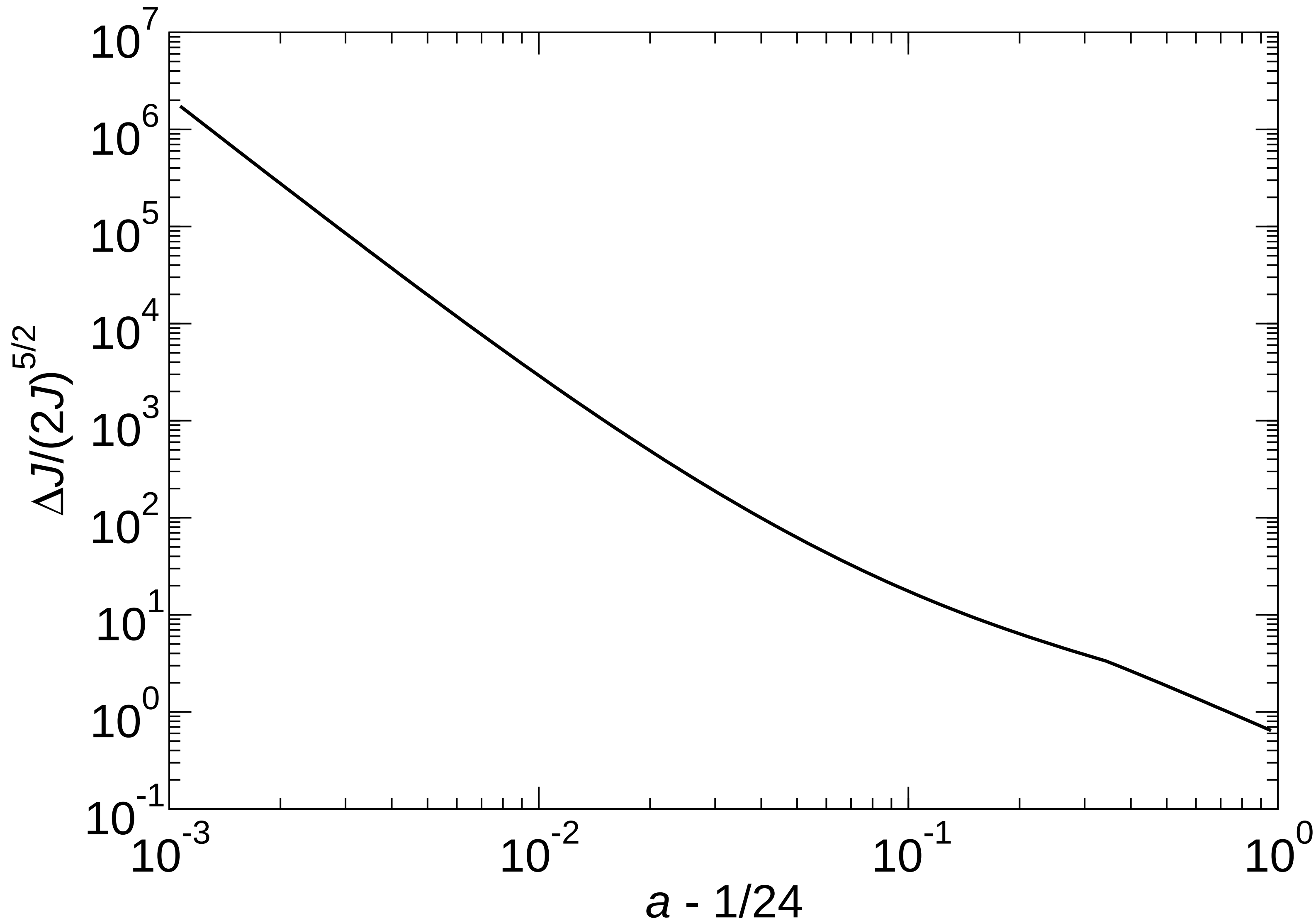
Include Center Shift, Ellipse Shape

- At higher amplitudes, ellipses were still elliptical, but shifted and changed shape
- If we only care about what happens at large amplitude (neutrino factory), then don't consider these shifts and shape changes to be distortions
- Different characteristic behavior
  - $\Delta J \propto (2J)^{5/2}$, compared to $\Delta J \propto (2J)^{3/2}$ without shift removed, or $\delta\epsilon \propto \epsilon^2$ for emittance growth
  - $\Delta J \propto (a - 1/24)^{-3}$, compared to $\Delta J \propto (a - 1/24)^{-1}$ without shift removed, or $\Delta\epsilon \propto (a - 1/24)^{-2}$ for emittance growth

Ellipse Distortion vs. Amplitude



Ellipse Distortion, Shift Removed



Stages in FFAGs

- Last week looked at 3 vs. 4 stages to get from 2.5 to 20 GeV
- Found 3 stages slightly better than 4
- Look at 2 stages also
  - Result: significantly worse
  - More cells, larger apertures, fewer turns
  - But fields and magnet lengths lower
- Cost per GeV at low energy stays pretty flat
  - For 2.5 GeV to something ring: 2.1 GeV cost me 30.1 PB/GeV
  - for something to 20 GeV ring: 2.9 GeV cost me 18.3 PB/GeV
  - Almost certainly better to given low energy a SMALLER range

Stages in FFAGs: Table

Minimum total energy (GeV)	2.5	4.2	7.1	11.9	2.5	5.0	10.0	2.5	7.1
Maximum total energy (GeV)	4.2	7.1	11.9	20.0	5.0	10.0	20.0	7.1	20.0
a	0.20	0.16	0.12	0.09	0.17	0.12	0.08	0.12	0.07
Number of cells	34	38	46	57	50	63	82	101	152
D length (cm)	77	90	108	122	63	78	97	47	65
D radius (cm)	13.2	10.7	8.7	7.0	13.4	10.0	7.4	13.8	8.7
D pole tip field (T)	4.6	5.8	6.6	7.9	4.5	5.8	7.1	4.4	6.1
F length (cm)	98	117	137	164	96	115	141	88	114
F radius (cm)	21.4	18.6	15.7	13.2	21.2	16.6	13.1	20.8	13.6
F pole tip field (T)	2.7	3.3	3.8	4.3	2.7	3.5	4.3	2.7	3.9
Number of cavities	26	30	35	38	42	48	56	88	97
RF voltage (MV)	331	382	434	477	534	606	704	1114	1230
Turns	5.2	7.6	11.4	17.7	4.7	8.5	15.0	4.2	11.3
Circumference (m)	144	174	228	306	204	279	400	389	653
Decay (%)	3.6	3.8	4.4	5.4	4.2	5.1	6.5	5.8	9.1
Machine cost (PB)	53.0	56.7	61.5	68.1	74.8	78.9	88.9	138.1	142.0
...per GeV (PB/GeV)	31.1	19.8	12.8	8.4	29.9	15.8	8.9	30.2	11.0
Marginal decay cost (PB)	18.0	18.9	21.9	27.1	21.1	25.6	32.3	28.9	45.5
Total machine cost (PB)	239.3				242.7			280.1	
Total decay cost (PB)	85.9				78.9			74.5	

FFAG Electron Model Lattices

- 1.3 GHz RF frequency
  - 3 GHz requires longer rings
- 20 cm RF drift, 5 cm drift between magnets
- Doublet Lattices
- Multiple of 6 cells (Carol)
- Low energy tunes: 0.39 (H), 0.24 (V), based on muon optimized doublet lattices
- Probably limited by pole tip: look at various values
  - Air core magnets allow adjustability of fields
  - But may be limited to 0.1 T pole tip field
- $a = 1/12$, 3 mm normalized acceptance

Cell Parameters

Cells	42	48	54	36	42	48	36	42	48
Pole Tip Field (T)	0.1	0.1	0.1	0.2	0.2	0.2	0.3	0.3	0.3
$\Delta E/V_{\text{cell}}$	389	528	683	374	524	692	427	519	774
D Quad Length (mm)	139	122	109	78	68	61	51	45	41
D Quad Radius (mm)	18	16	15	15	14	13	14	13	13
F Quad Length (mm)	119	110	103	66	60	56	44	40	38
F Quad Radius (mm)	30	27	25	26	24	22	24	22	20
Cavity Voltage (kV)	26	19	15	27	19	14	23	17	13
Circumference (m)	21.3	23.1	24.9	14.2	15.9	17.6	12.4	14.1	15.8

Analysis

- $\Delta E/V_{\text{cell}}$ is approximately the number of cell-turns
  - This is, I think, the merit factor for testing muon acceleration
    - ⋆ Existing muon designs require this to be 500–1500
    - ⋆ Other problems want small time-of-flight variation also, I think
  - Proportional to n^2/L_{cell}
  - Gain quickly with increasing n
    - ⋆ L_{cell} reduces with increasing n due to lower dipole
- Increasing pole tip field helps $\Delta E/V_{\text{cell}}$ slightly
  - Cost: aspect ratio of magnets is worse: more end contributions
- I like 42 cells, 0.2 T
  - Get at least 500 cell-turns
  - Higher pole tips not worth the bad aspect ratio
  - 0.1 T very long, but may be limited to 0.1 T for air core