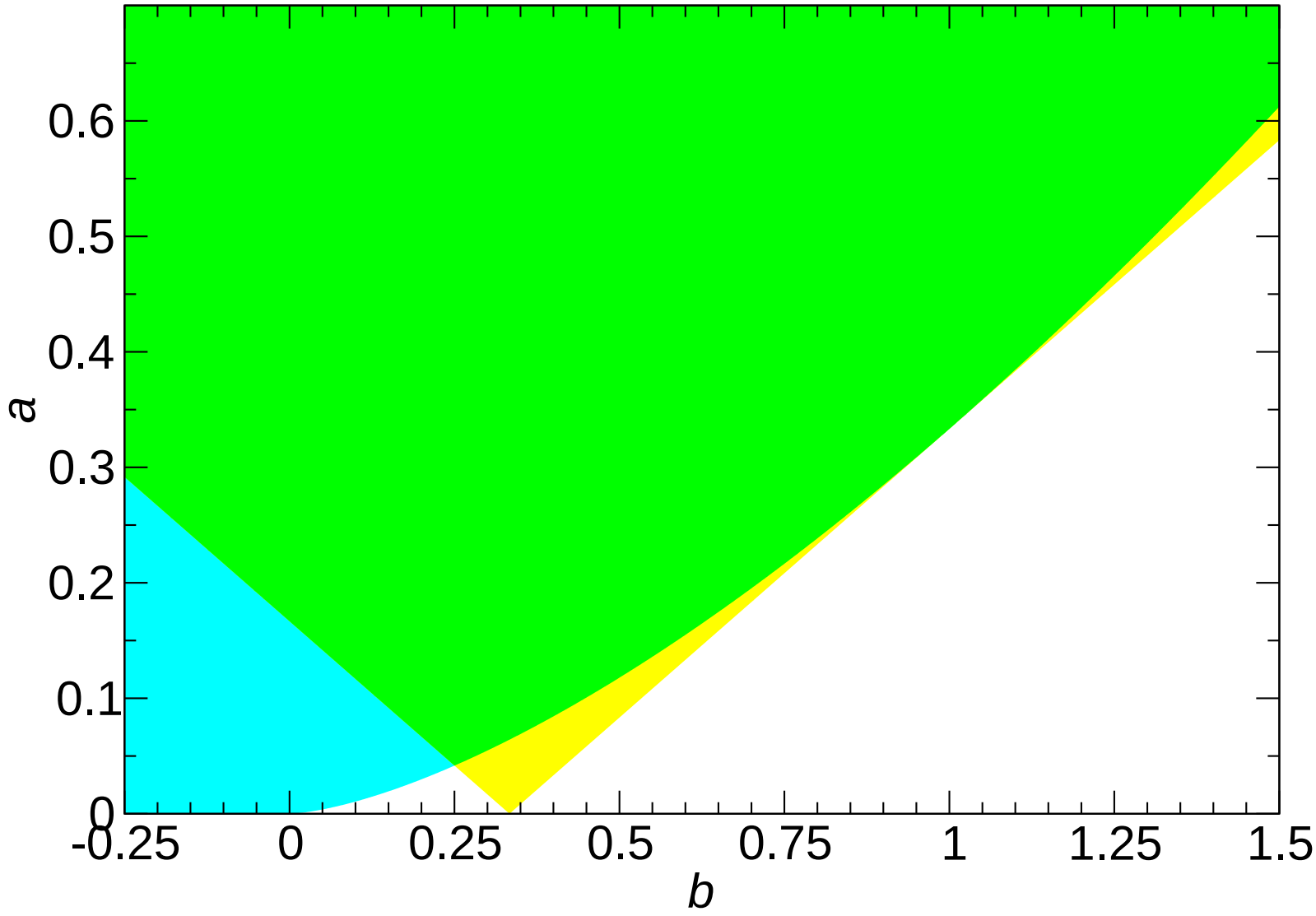
Ellipse Distortion in FFAGs

J. Scott Berg
Advanced Accelerator Group Meeting
17 March 2005

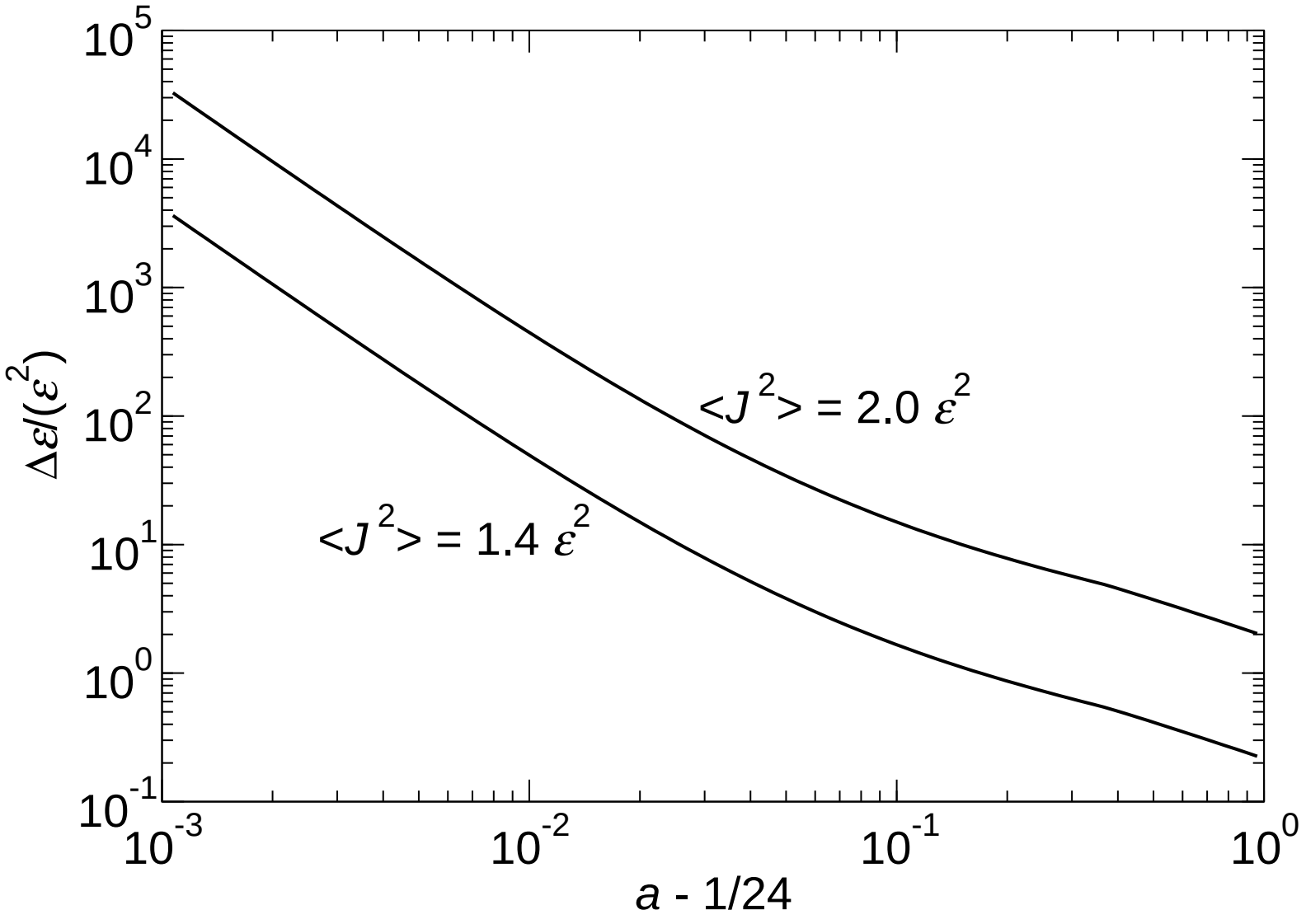
Review of Last Week

- Longitudinal dynamics in FFAG lattice is parametrized by dimensionless parameters a (scaled voltage) and b (time offset)
- There is an allowed region in that parameter space
- I have a method for computing emittance growth as a function of a and b, minimizing over ellipse orientation in phase space
- I can minimize that emittance growth over b, and find emittance growth as a function of a
- Emittance growth as computed is a funny parameter: it can be negative, for instance.
- Better to minimize “ellipse distortion”: keep an ellipse elliptical
- Follow the same process with ellipse distortion

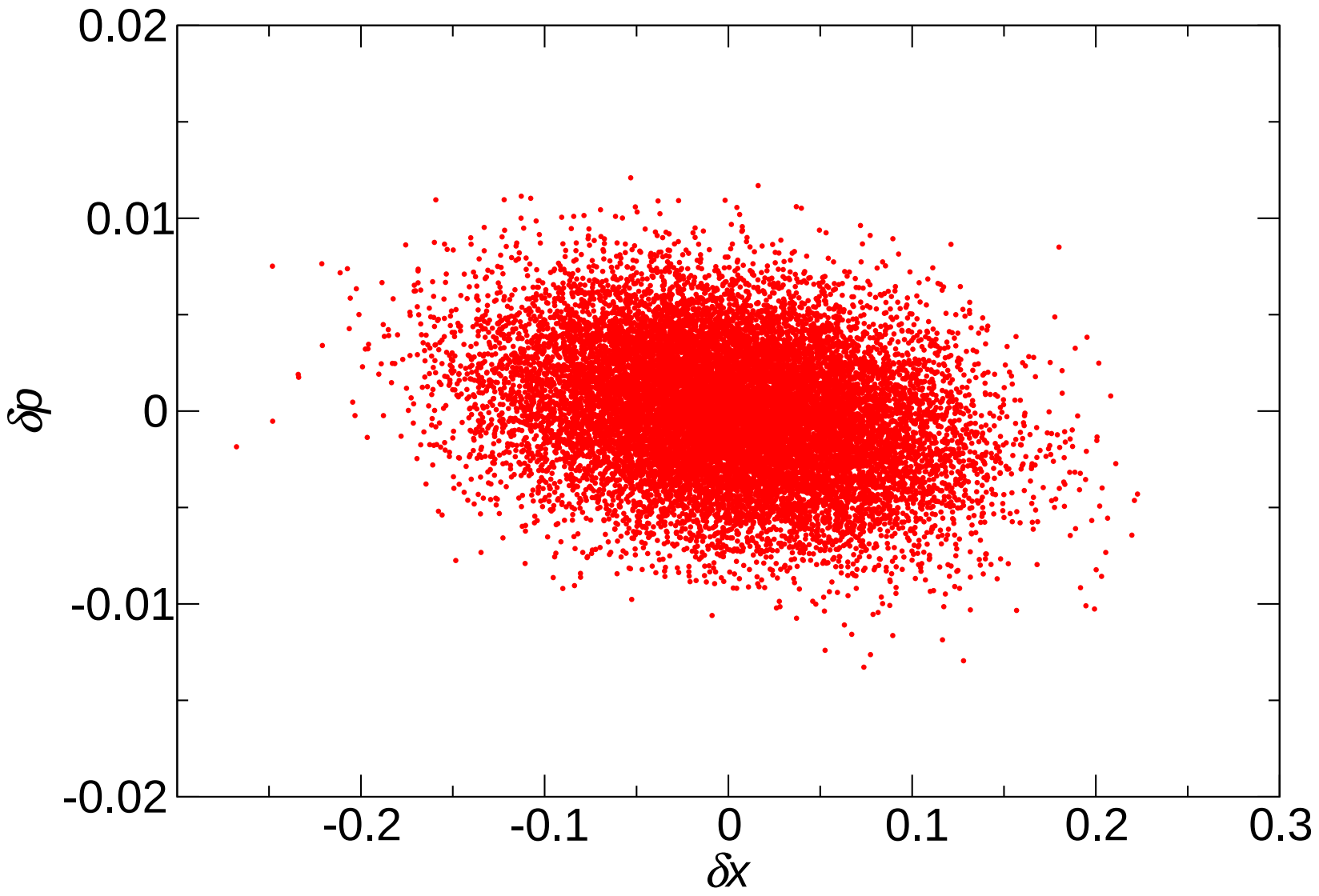
Allowed Region of Parameter Space



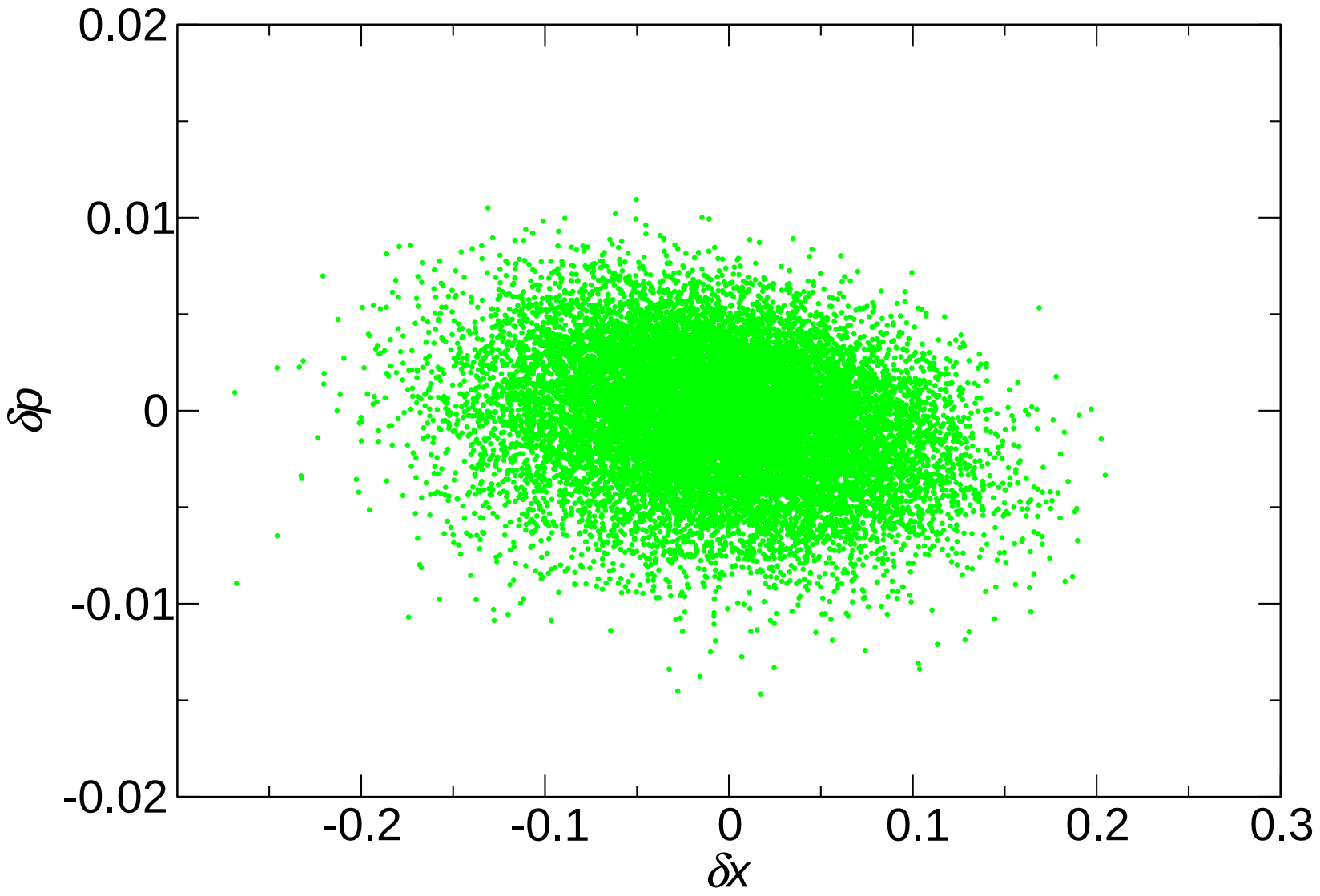
Emittance Growth vs. a



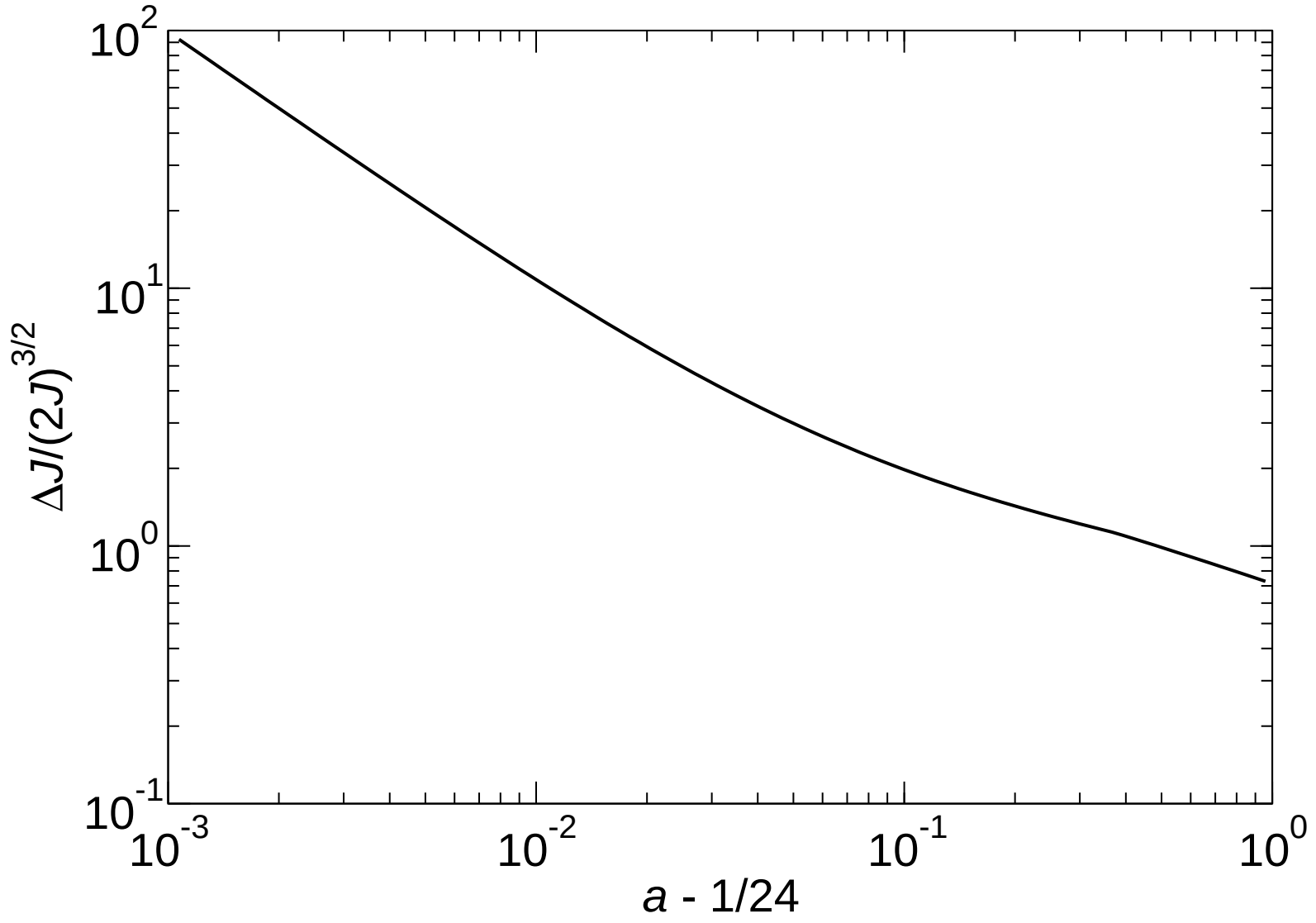
Optimal Orientation



After FFAG



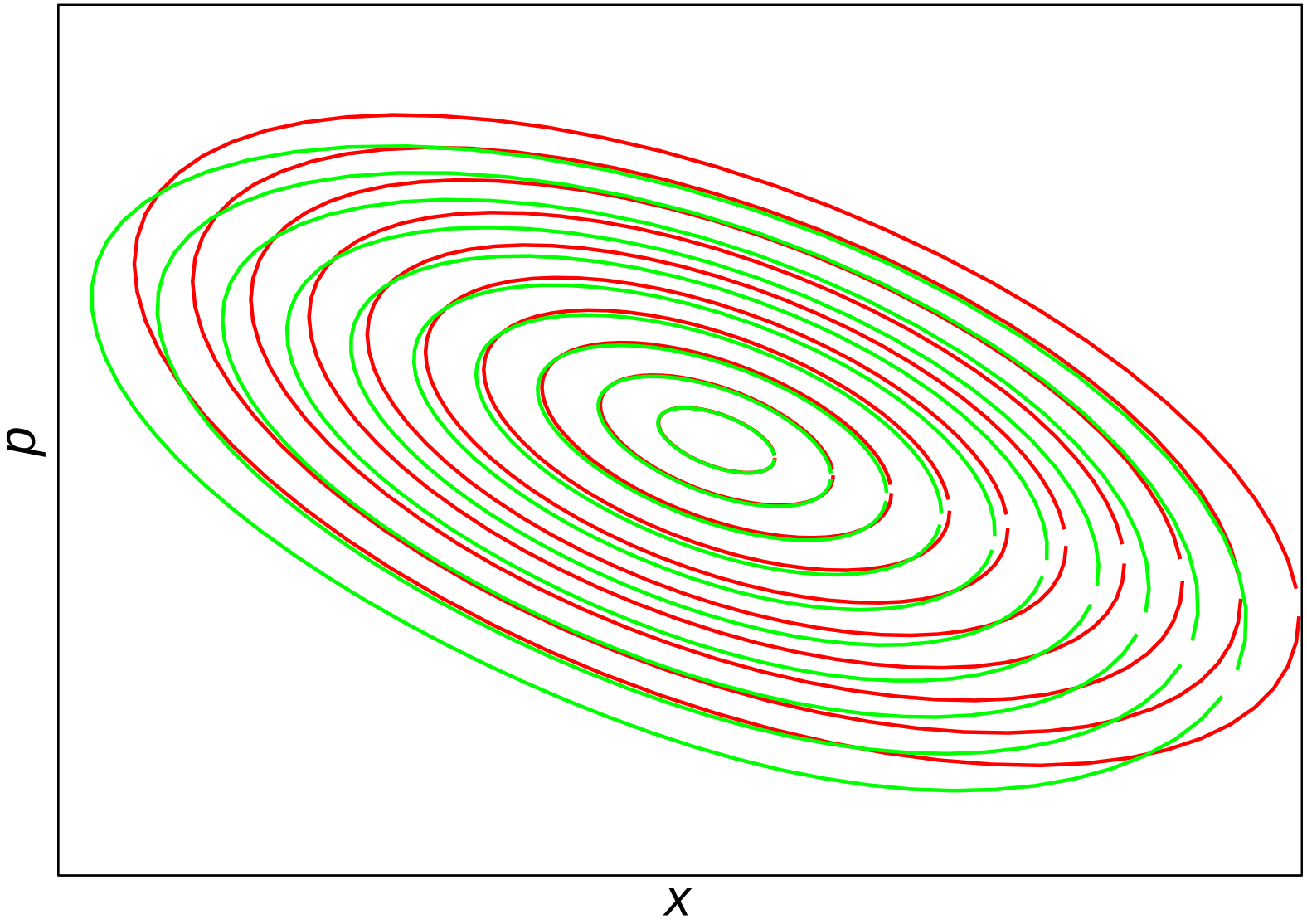
Ellipse Distortion vs. a



Analysis

- Note different qualitative behaviors
  - Emittance growth was proportional to ϵ^2; action distortion is proportional to $(2J)^{3/2}$. Equivalently, radius distortion is proportional to r^2.
  - Coefficient is proportional to $(a - 1/24)^{-1}$, whereas for emittance growth it was $(a - 1/24)^{-2}$

Improved Computation

- Leaving out two effects
  - Amplitude-dependent shift of the ellipse center
  - Amplitude-dependent distortion of the ellipse shape
  - If we include these, then we don't care where the center of the ellipse is; we only care about the outer boundary enclosing all particles
- Including these effects, action distortion will be proportional to $(2J)^{5/2}$, or radius distortion proportional to r^4
  - This gives significantly less distortion for small radii
- Still working on the computation. . .

Ellipse Distortion vs. Amplitude